The BabaYaga@NLO event generator

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in collaboration with
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Eighth Plenary Workshop of the Muon $g-2$ Theory Initiative 2025
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Introduction

- ⋆ In the last ~25 years BabaYaga/BabaYaga@NLO has been developed for precision luminometry at flavour factories
- ⋆ It simulates QED processes

  ↪ $e^+e^- \to e^+e^- \quad (+n\gamma)$
  ↪ $e^+e^- \to \mu^+\mu^- \quad (+n\gamma)$
  ↪ $e^+e^- \to \gamma\gamma \quad (+n\gamma)$

  with multiple-photon emission in a QED Parton Shower framework, matched with exact NLO matrix elements (NLOPS accuracy)

  $\Longrightarrow$ generation of one additional (virtual ad real) photon with exact matrix element and leading logarithmic resummation of other photons with exclusive kinematics generation, avoiding double counting

- ⋆ A theoretical precision at the $0.5 - 1 \times 10^{-3}$ level (at least for Bhabha),

  systematic comparison to independent calculations/codes and assessing the size of missing higher-order corrections (part of the $\mathcal{O}(\alpha^2 L)$ contributions, $L \equiv \log(Q^2/m^2)$)

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- ⋆ **recent improvement**
  - ⋆ **addition of di-pion final state ($e^+e^- \to \pi^+\pi^-(n\gamma)$)**

E. Budassi et al., arXiv:2409.03469

Pion pair production in e^+e^- annihilation at next-to-leading order matched to Parton Shower

Ettore Budassi,[a,b] Carlo M. Carloni Calame,[b] Marco Ghilardi,[a] Andrea Gurgone,[a,b] Guido Montagna,[a,b] Mauro Moretti,[c,d] Oreste Nicrosini,[b] Fulvio Piccinini,[b] and Francesco P. Ucci[a,b]

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- NLOPS in sQED

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- NLOPS in sQED
- pion form factor in three approaches

Extension to di-pion final state

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- NLOPS in sQED
- pion form factor in three approaches
  - F×sQED

    H. Czyz, A. Grzelinska, J.H. Kuhn, G. Rodrigo, hep-ph/0308312
  - GVMD

    F. Ignatov and R.N. Lee, arXiv:2204.12235
  - FsQED

    G. Colangelo, M. Hoferichter, J. Monnard, J.R. de Elvira, arXiv:2207.03495

Pion Form Factor approaches

- F×sQED: FF out of the loop diagrams, with a scale chosen to guarantee the cancellation of IR singularities with real radiation

- GVMD: FF as a sum over additional propagators with Breit-Wigner form

$$F_\pi^{\rm BW} \equiv \frac{1}{c_t} \sum_{v=1}^{n_r} c_v \frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$

- FsQED: FF written through its dispersive representation

$$F_\pi(q^2) = 1 + \frac{q^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{q^2}{\pi} \frac{ds'}{s'} \frac{{\rm Im} F_\pi(s')}{s' - q^2 - i\epsilon}$$

di-pion final state, angular distribution

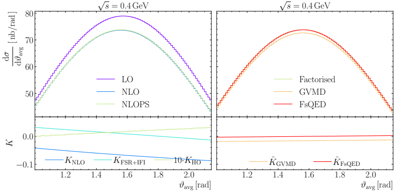
$$\vartheta_{\rm avg} \equiv \frac{1}{2}(\pi - \vartheta^+ + \vartheta^-)$$

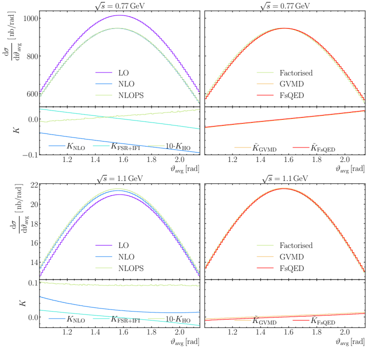
$$K_{\rm NLO} = \frac{\sigma_{\rm NLO} - \sigma_{\rm LO}}{\sigma_{\rm LO}}$$

$$K_{\rm FSR+IFI} = \frac{\sigma_{\rm NLO} - \sigma_{\rm ISR}}{\sigma_{\rm LO}}$$

$$K_{\rm HO} = \frac{\sigma_{\rm NLOPS} - \sigma_{\rm NLO}}{\sigma_{\rm LO}}$$

$$\tilde{K}_{\rm FF} = \left(\frac{d\sigma_{\rm FF}}{d\vartheta_{\rm avg}}\right)\left(\frac{d\sigma_{\rm Factorized}}{d\vartheta_{\rm avg}}\right)^{-1} - 1$$



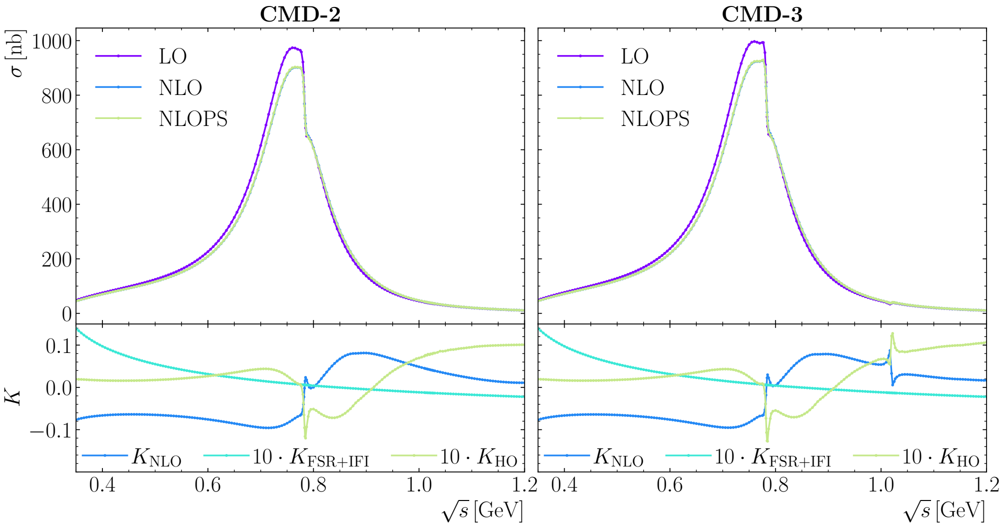


- NLO corrections of the order of 1 - 10%
- HO corrections in the range 0.1 - 1%

- pion FF in the loops w.r.t. factorized gives differences on the % scale for angular distributions

arXiv:2409.03469

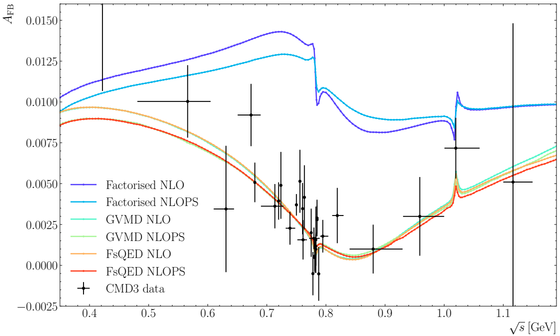
di-pion final state, resonance scan



arXiv:2409.03469

di-pion final state: charge asymmetry

$$A_{\mathrm{FB}}\left(\sqrt{s}\right) = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



arXiv:2409.03469

- FsQED and GVMD approaches give very similar predictions
- Relevant differences with the FxsQED approach (for exclusive angular observables)

WiP: Radiative events

- Target accuracy: NLOPS
  - $e^+e^- \to \mu^+\mu^-\gamma$
  - $e^+e^- \to \pi^+\pi^-\gamma$ (with F$\times$sQED, to start with)
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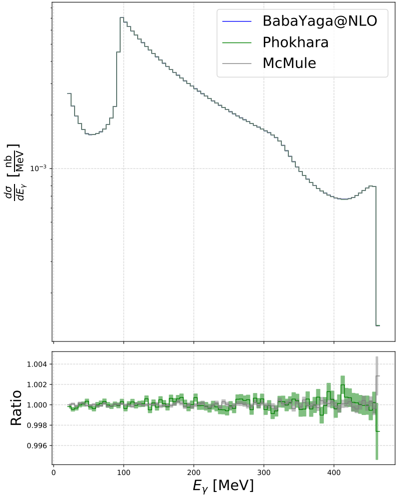
  - exact tree-level simulation of $2 \to 3$ processes $e^+e^- \to X\gamma$
  - exact QED NLO corrections to $2 \to 3$
  - consistently matched to the resummation of additional photons generated exclusively with the parton shower technique

- $e^+e^- \to \mu^+\mu^-\gamma$@NLO (KLOE-LA)



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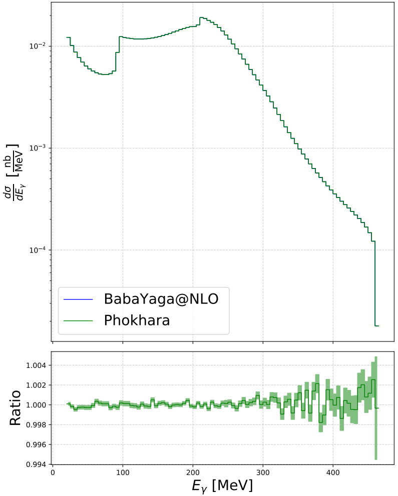

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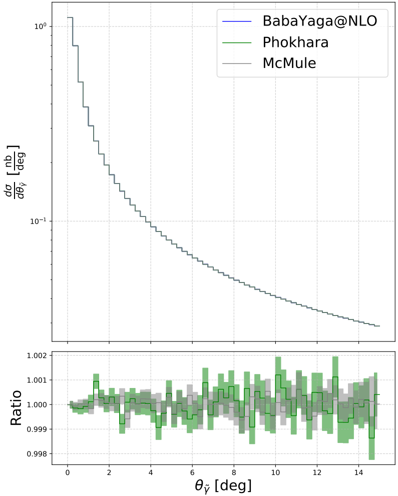

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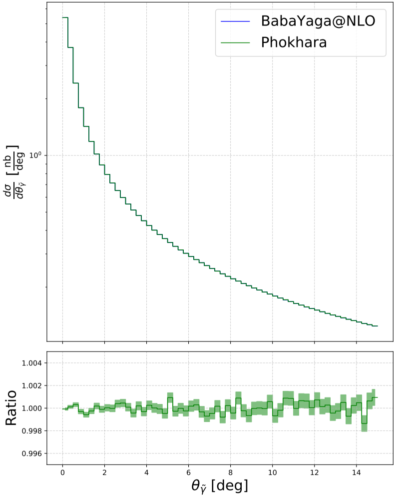
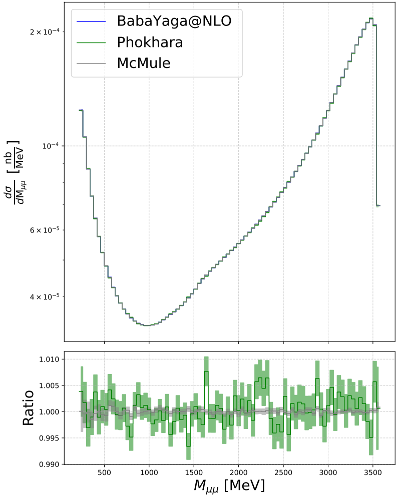


Validation of the NLO implementation (cfr. with R. Aliberti et al. arXiv:2410.22882)

- $e^+e^- \to \mu^+\mu^-\gamma$@NLO (BES)



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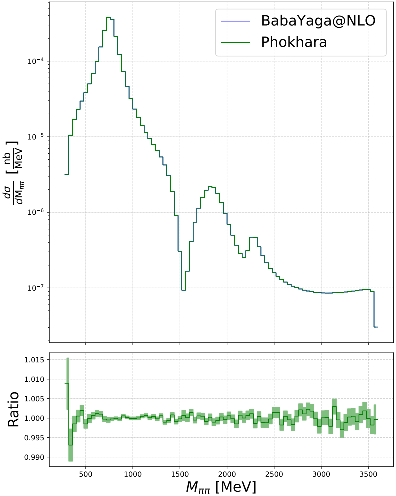
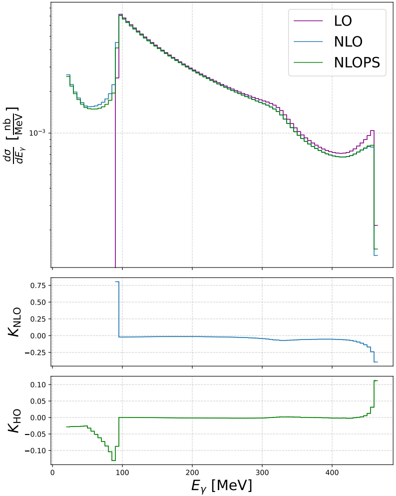


Estimate of QED higher orders (PRELIMINARY)

- $e^+e^- \to \mu^+\mu^-\gamma$@NLO (KLOE-LA)



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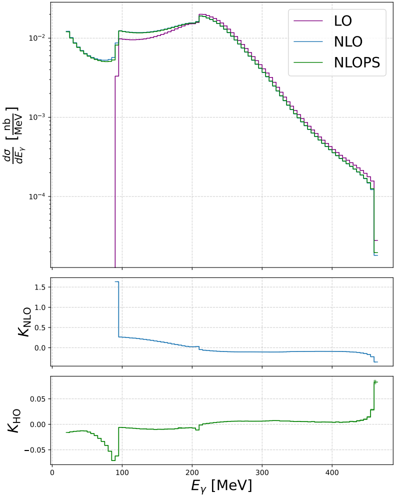

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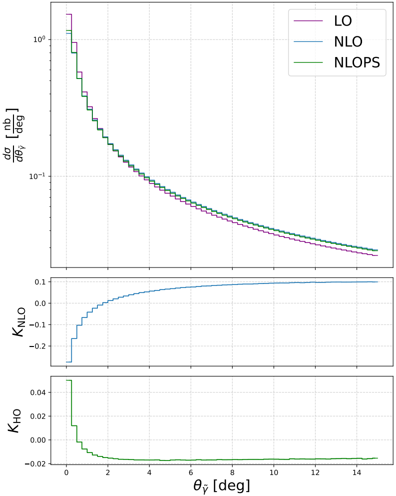

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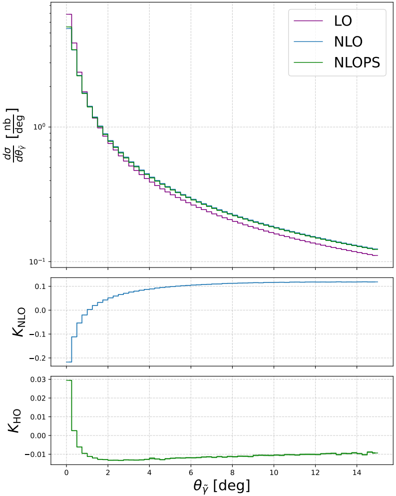

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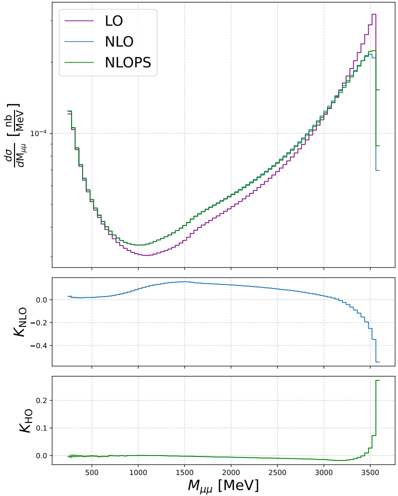

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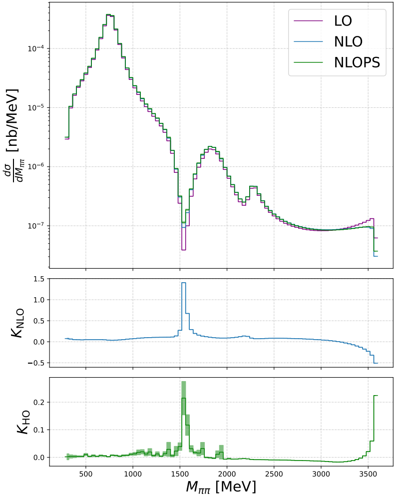
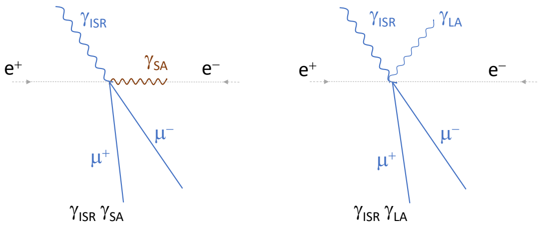


A preliminary exercise on more exclusive variables for $e^+e^- \to \mu^+\mu^-\gamma(\gamma)$

- Event selection:
  - at least a detected photon: $E^*_\gamma > 4$ GeV, $0.35 < \theta_\gamma < 2.4$ rad (LA)
  - the photon with the highest energy is tagged as $\gamma_{\rm ISR}$
  - charged tracks: $p_\perp > 0.1$ GeV, $0.4 < \theta < 2.45$ rad
  - $m_{\mu^+\mu^-} < 1.4$ GeV
- Events categorized as NLO events

J.P. Lees et al. (BaBaR Coll.), arXiv:2308.05233

NLO

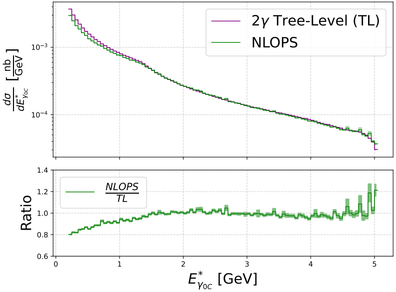


- p_{0C} = missing momentum, with $E^*_{0c} > 200$ MeV

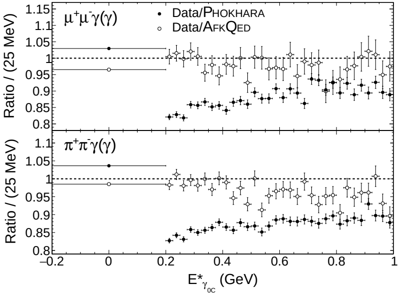
- at least an additional photon with $E_\gamma > 200$ MeV with LA evt. selection

Preliminary results of the simulation (I)

- $e^+e^- \to \mu^+\mu^-\gamma$

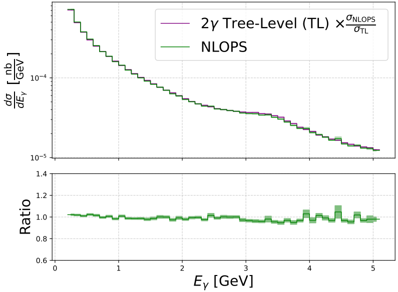


- (BaBaR Coll.), arXiv:2308.05233



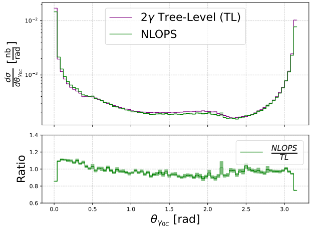
Preliminary results of the simulation (II)

- $e^+e^- \to \mu^+\mu^-\gamma$ (LA evt. sel.)

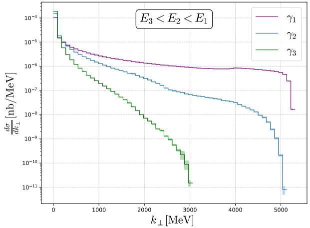


- the same pattern is also found in data/Phokhara

- γ_{0C} angular distribution



- $k_\perp$ of the three most energetic photons



Summary and outlook

- BabaYaga@NLO is a generator for QED processes with NLOPS accuracy
- recent extension to $e^+e^- \to \pi^+\pi^-(\gamma)$
  - various pion FF parameterizations available in the code
  - three different approaches for the inclusion of the pion FF in the calculation
- in progress: simulation of radiative final states $\mu^+\mu^-\gamma$, $\pi^+\pi^-\gamma$, $e^+e^-\gamma$ with NLOPS accuracy
  - factorized pion FF for $\pi^+\pi^-\gamma$
- perspectives:
  - provide a tool available for simulation and useful for the estimate of the theoretical uncertainties associated to th predictions
  - other pion FF approaches also for radiative processes
  - extension of the code to other interesting processes

THANK YOU!

Relevant references

⋆ **BabaYaga** core references:

- E. Budassi et al., arXiv:2409.03469 — BabaYaga@NLO for $e^+e^- \to \pi^+\pi^-$
- C.M. Carloni Calame et al., Phys. Lett. B **798** (2019) 134976 — BabaYaga@NLO EW corrections at high energies for $e^+e^- \to \gamma\gamma$
- Barzè et al., Eur. Phys. J. C **71** (2011) 1680 — BabaYaga with dark photon
- Balossini et al., Phys. Lett. **663** (2008) 209 — BabaYaga@NLO for $e^+e^- \to \gamma\gamma$
- Balossini et al., Nucl. Phys. **B758** (2006) 227 — BabaYaga@NLO for Bhabha
- C.M. Carloni Calame et al., Nucl. Phys. Proc. Suppl. **131** (2004) 48 — BabaYaga@NLO
- C.M. Carloni Calame, Phys. Lett. B **520** (2001) 16 — improved PS BabaYaga
- C.M. Carloni Calame et al., Nucl. Phys. B **584** (2000) 459 — BabaYaga

⋆ Related work:

- R. Aliberti et al., "Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions", Scipost Phys. Comm. Rep. 9 (2025)
- S. Actis et al.
  "Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data", Eur. Phys. J. C **66** (2010) 585
  Report of the Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies
- C.M. Carloni Calame et al., JHEP **1107** (2011) 126
  NNLO massive pair corrections