Dispersive HVP Updates from DHMZ

Comparisons among the $e^+e^- \to \pi^+\pi^-$ measurements

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The BMW lattice QCD collaboration

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Content of the talk

Hadronic Vacuum Polarization and Muon $(g\text{-}2)_\mu$

Dominant uncertainty for the theoretical prediction: from lowest-order HVP piece

Cannot be calculated from pQCD (low E-scale), but one can use experimental data on $e^+e^- \rightarrow$ hadrons cross section

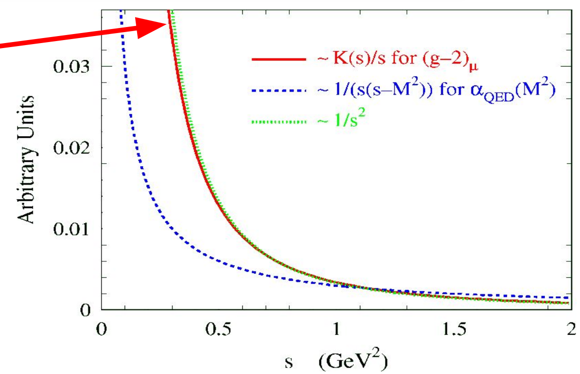
Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

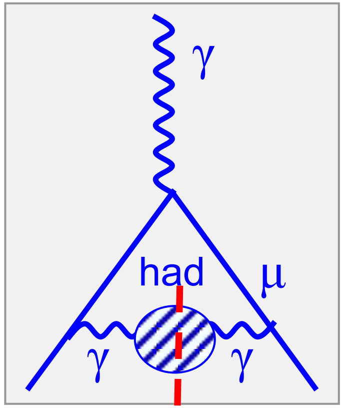
$$12\pi \; \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0 \; [e^+e^- \rightarrow \text{hadrons} \; (\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} R(s)$$

Dispersion relation

Bouchiat and Michel, 1961



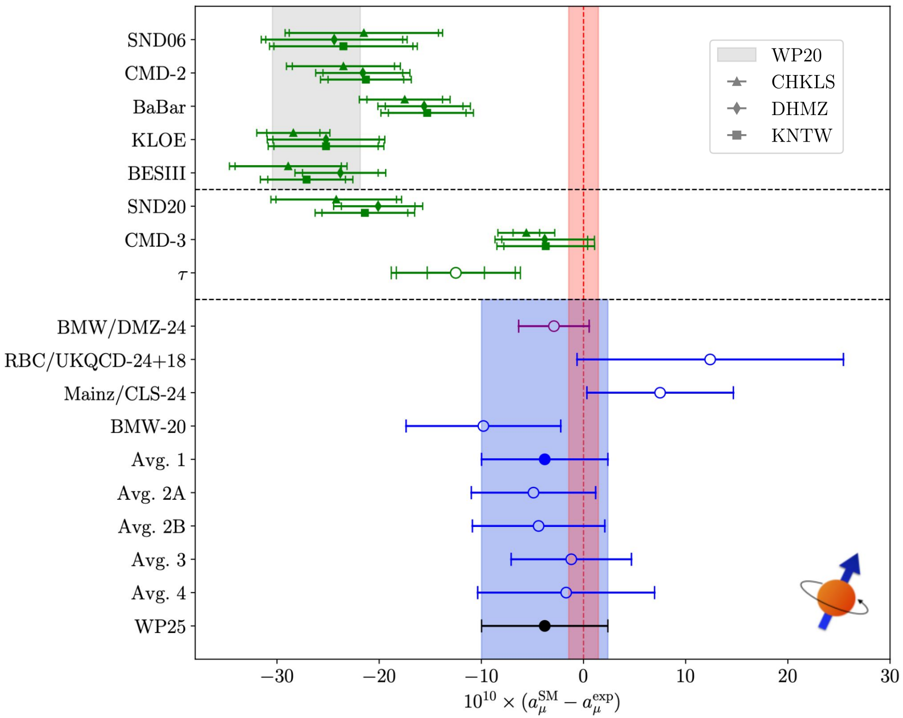


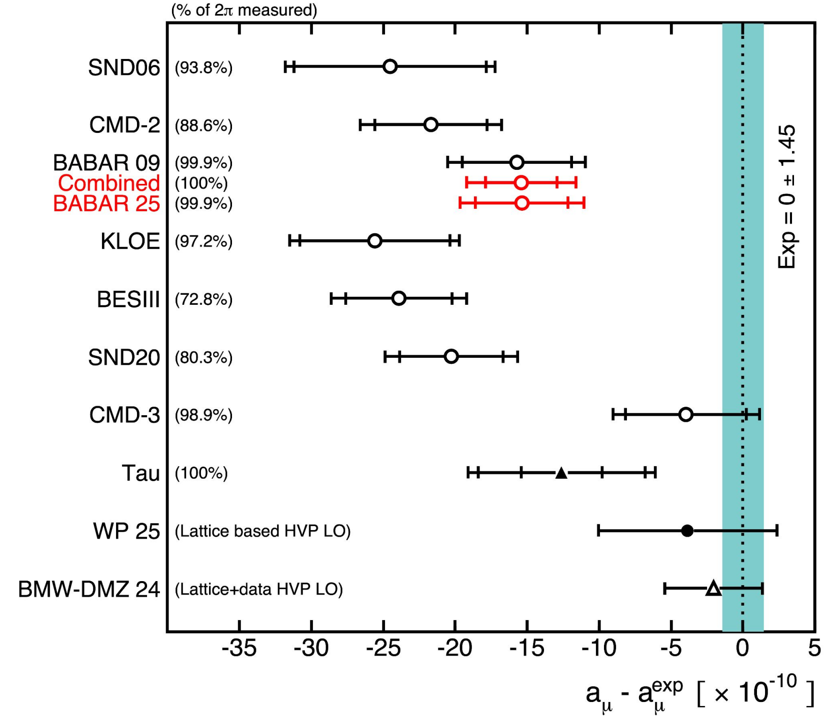
→ Precise $\sigma(e^+e^- \rightarrow \text{hadrons})$ measurements at low energy are very important

$\pi\pi$ channel: 73% (70%) of HVP contribution to a_μ (uncertainty ^{2})

→ Alternatively, one can use hadronic τ decays data + Isospin Breaking corrections

Status of a_μ





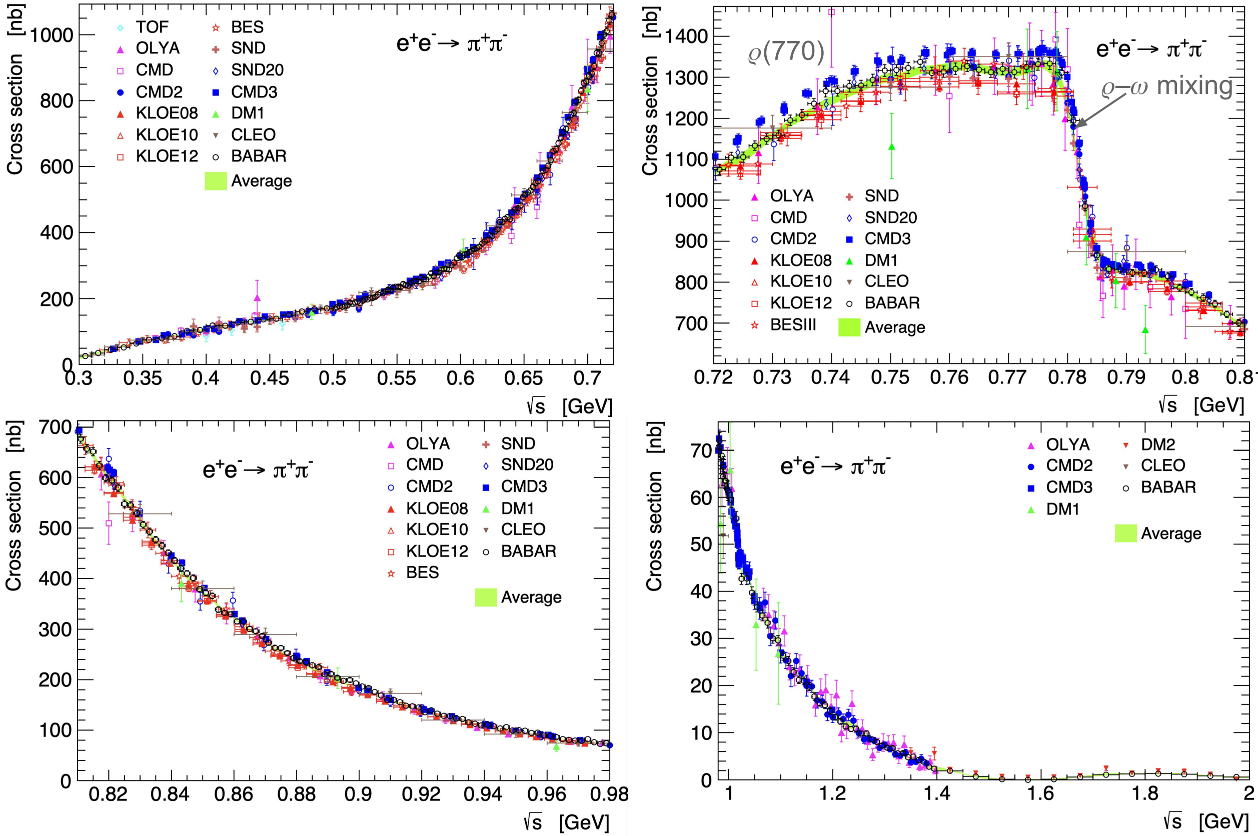
We have an interesting, long standing, multifaceted problem to solve…

→ Comparisons of dispersive results for various experimental inputs depends on the method (fit/combination) used to cover missing energy ranges; misleading "penalization" for measurements covering low- and/or high-mass range, when comparing uncertainties → Aim for more direct comparisons

→ Preliminary BABAR 25 result not yet included in the comparisons presented later in this talk

Global combination for data comparisons

Experimental data combination (Example: $e^+e^- \rightarrow \pi^+\pi^-$ channel)



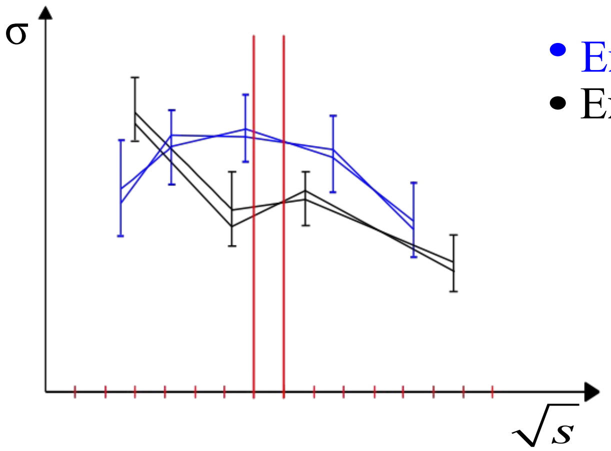
2312.02053
(DHLMZ)

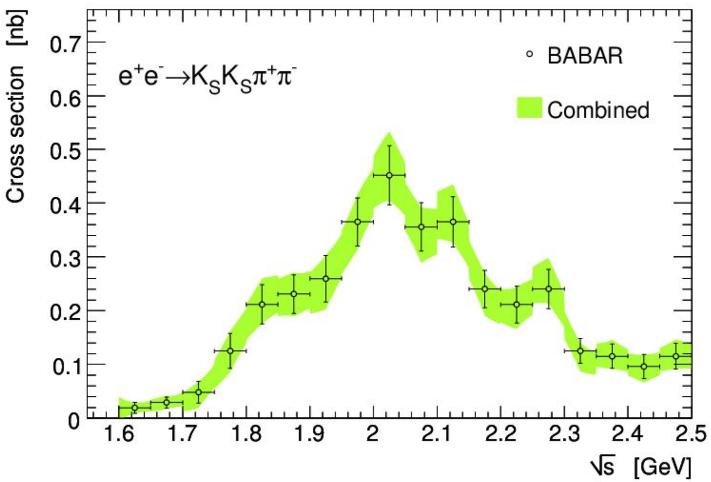
→ *Combining energy-scan and ISR "(N)LO" measurements*: In this talk used as reference for comparisons and for quantifying tensions among measurements

→ *New since g-2 Theory Initiative White Paper 1*: Large tensions, especially among KLOE & CMD3, which provide the smallest / largest cross-sections in the ρ region

Procedure and software (*HVPTools - Since 2009*) for combining cross section data with arbitrary point spacing/binning → Validated through closure test. Featuring full & realistic (i.e. not too optimistic) treatment of uncertainties and correlations (between measurements (data points/bins) of a given experiment, b. experiments, b. different channels), fully accounting for systematic tensions between experiments.

Combination procedure implemented in HVPTools software





→ Define a (fine) final binning (to be filled and used for integrals etc.)

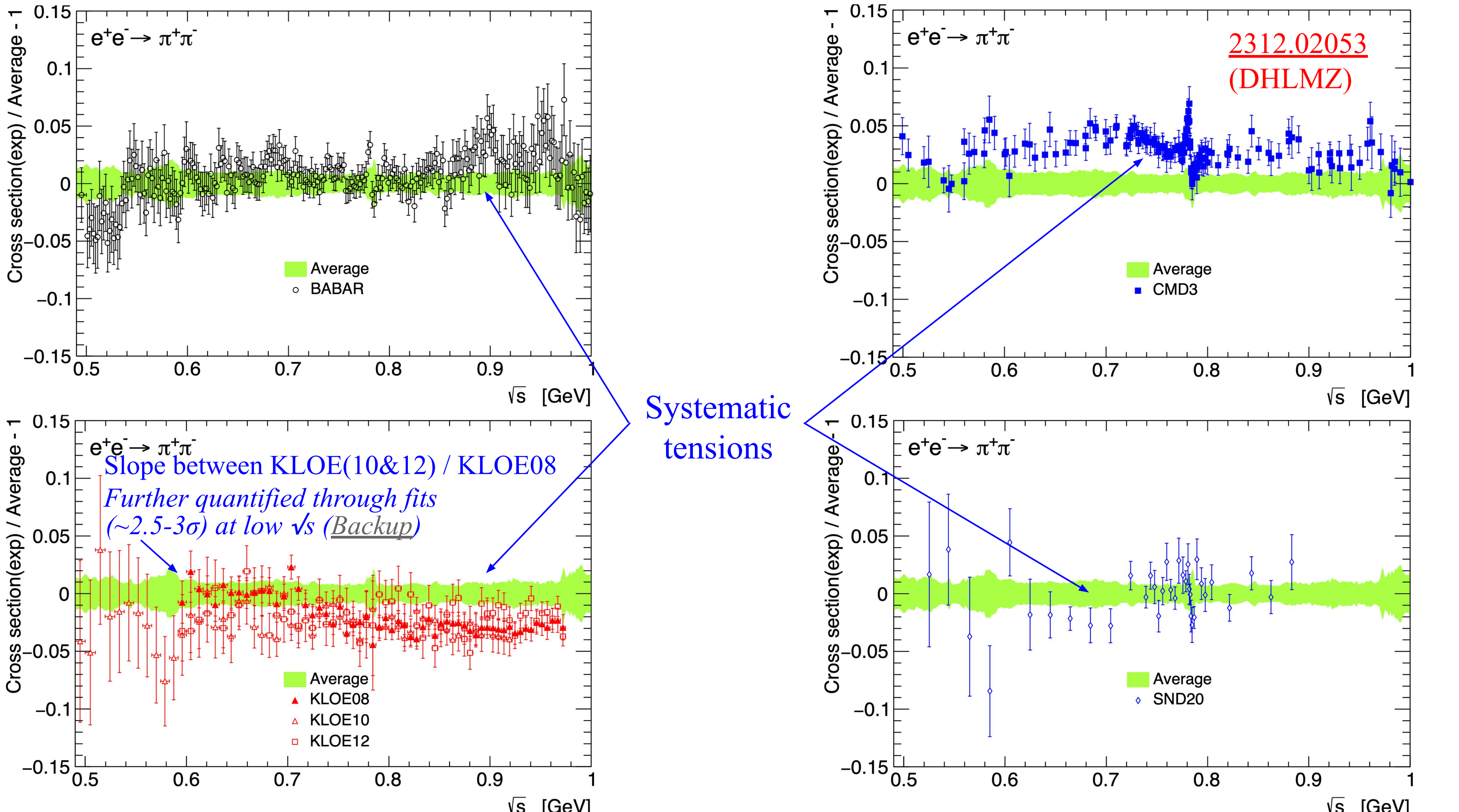
→ Linear/quadratic splines to interpolate between the points/bins of each experiment
- for binned measurements: preserve integral inside each bin
- closure test: replace nominal values of data points by Gounaris-Sakurai model and re-do the combination
  → *(non-)negligible bias for (linear)quadratic interpolation*

→ Fluctuate data points taking into account correlations & re-do the splines for each (pseudo-)experiment
- each uncertainty fluctuated coherently for all the points/bins that it impacts
- eigenvector decomposition for (statistical) covariance matrices

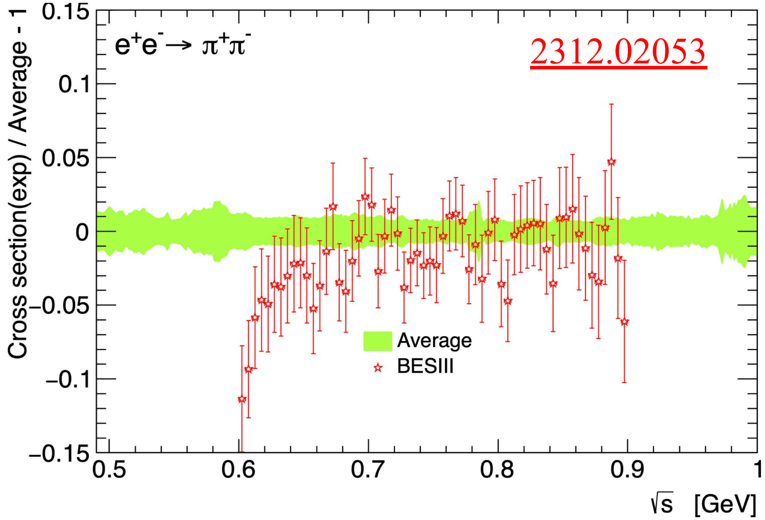
→ In each final bin, compute: average value for each measurement & its uncertainty;
correlation matrix between experiments

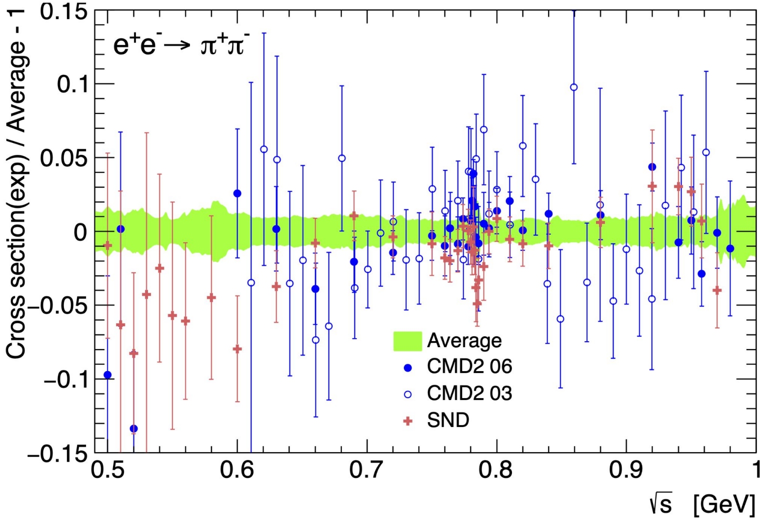
Backup

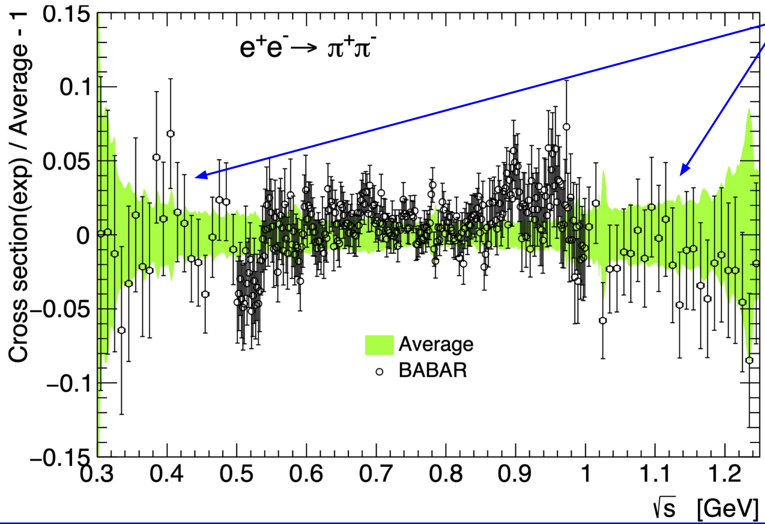
Combining the $e^+e^- \rightarrow \pi^+\pi^-$ data: relative differences



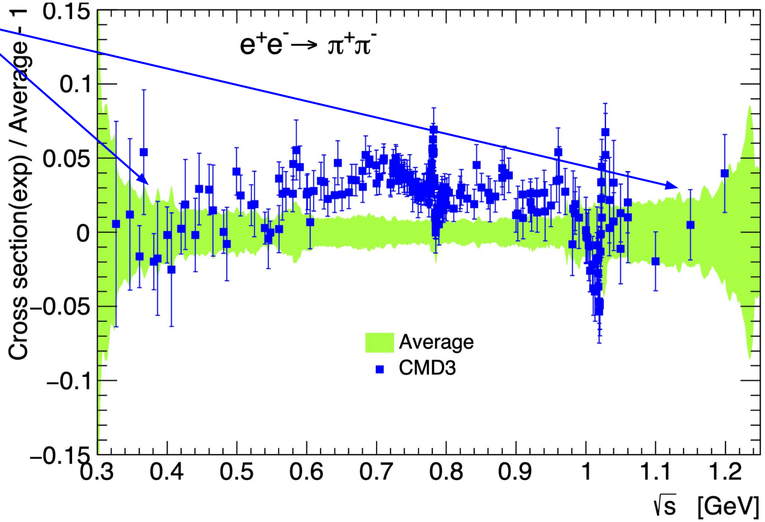
Combining the $e^+e^- \to \pi^+\pi^-$ data: relative differences







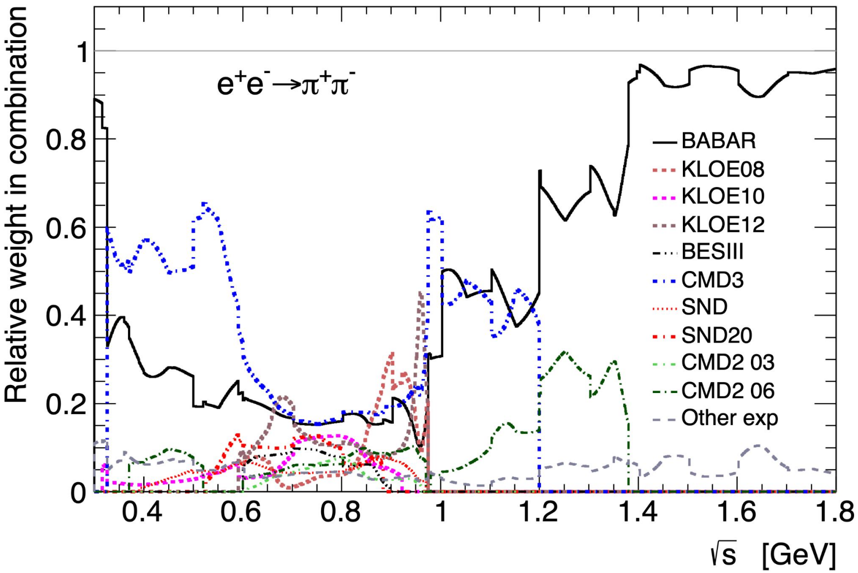
Reasonable BABAR/CMD3 agreement at low & high E

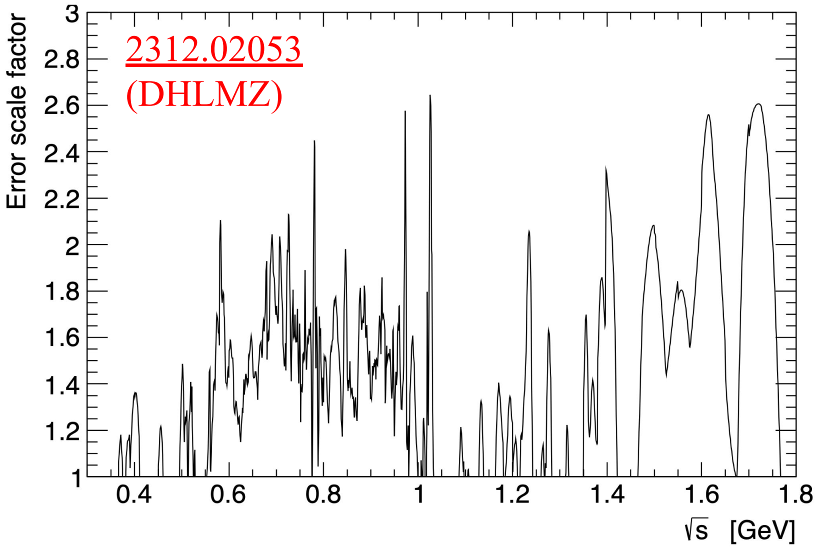


Combination procedure: weights and tension

→ For each narrow final bin minimize χ^2 to get average coefficients test locally the level of agreement

→ Average weights account for bin sizes/point-spacing of measurements: compare precisions on same footing





→ Average dominated by BaBar, CMD3, KLOE, SND20; BaBar covers full energy range

→ Enhanced tensions, especially between KLOE & CMD3, which provide the smallest / largest cross-sections in the ρ region: *clear indication of uncertainties on uncertainties (and on correlations)* - *Backup*

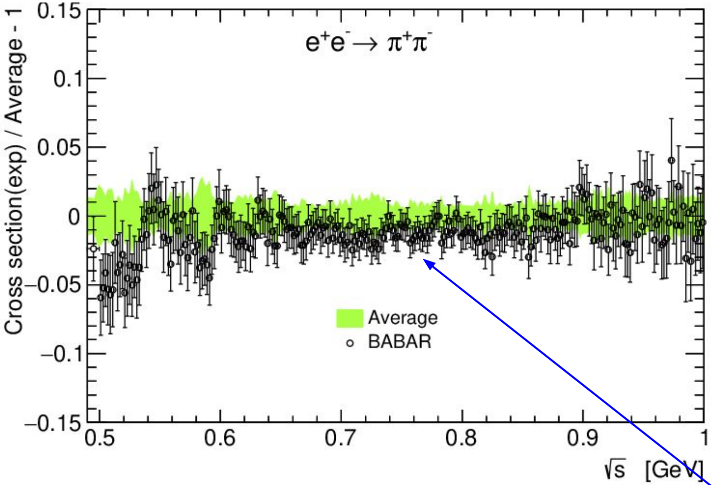
→ *Calls for conservative uncertainty treatment* in combination fit (fits / evaluation of weights)

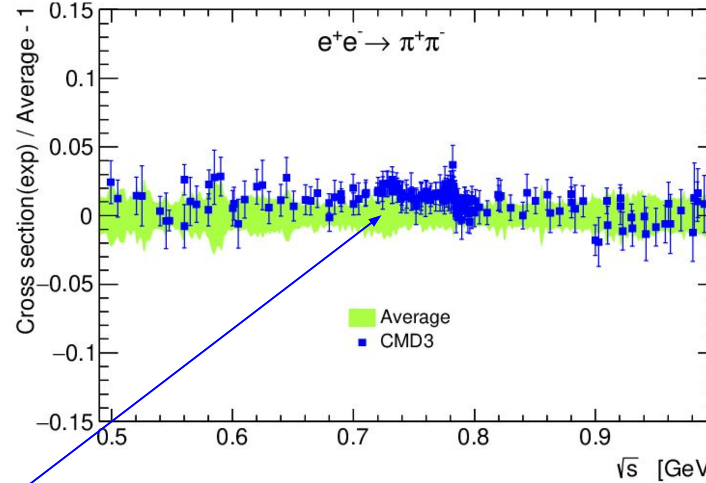
→ Systematic effects beyond the local χ^2 /ndof rescaling

Backup

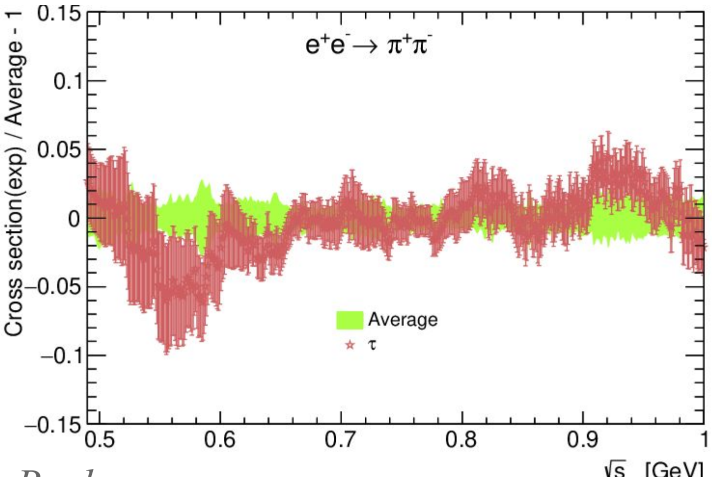
Partial combinations for data comparisons

Comparisons with BaBar & CMD3 & Tau(+IB) combination



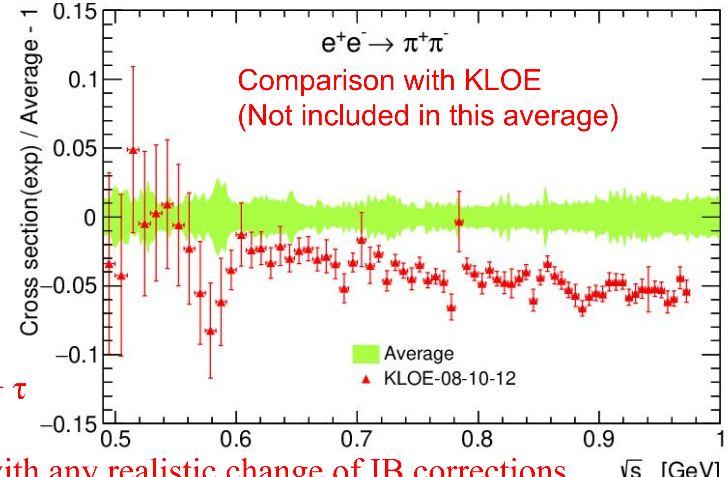


Some (reduced) systematic tensions



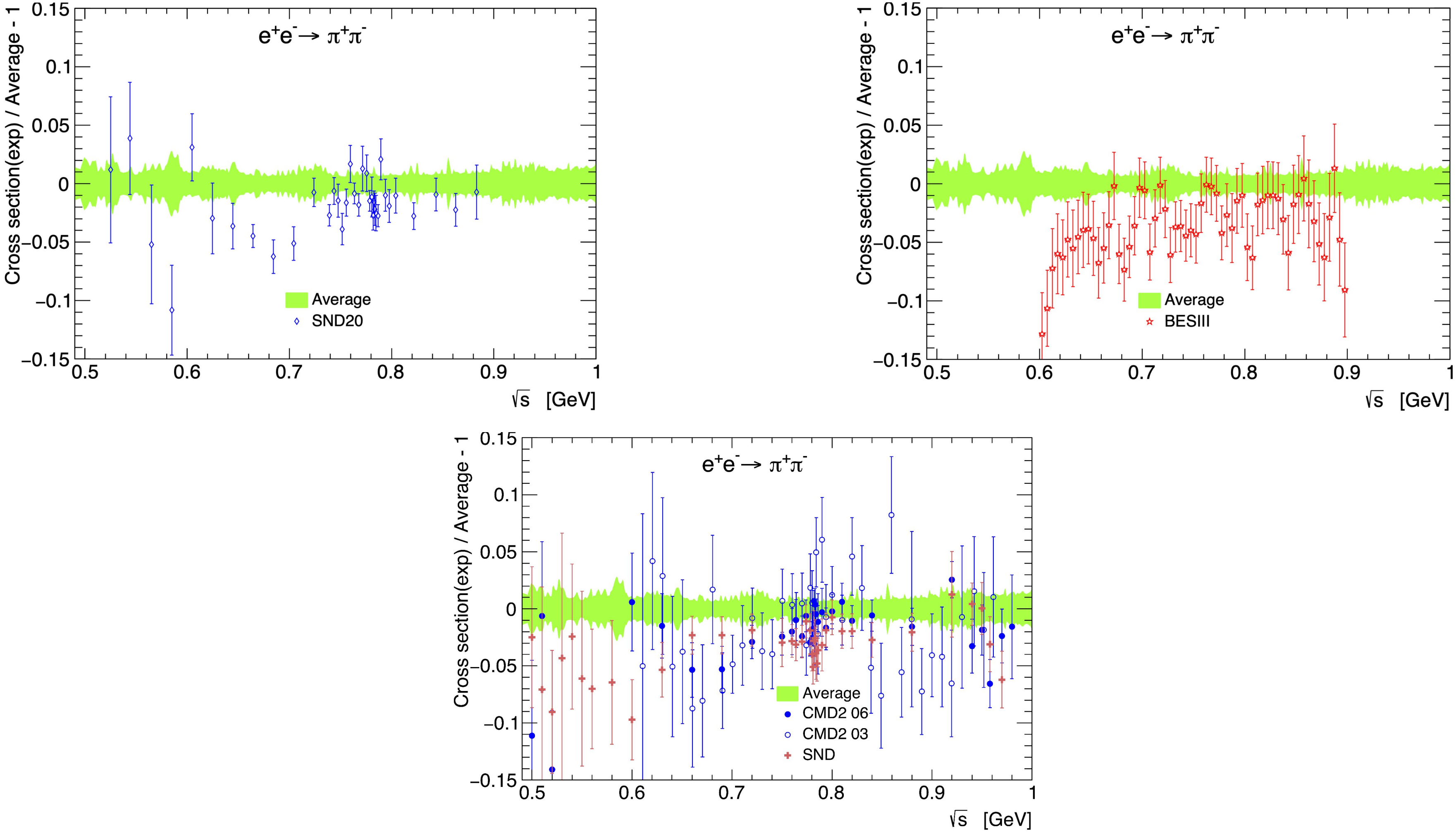
Tension (slope and shift) for KLOE vs. BABAR + CMD-3 + τ combination

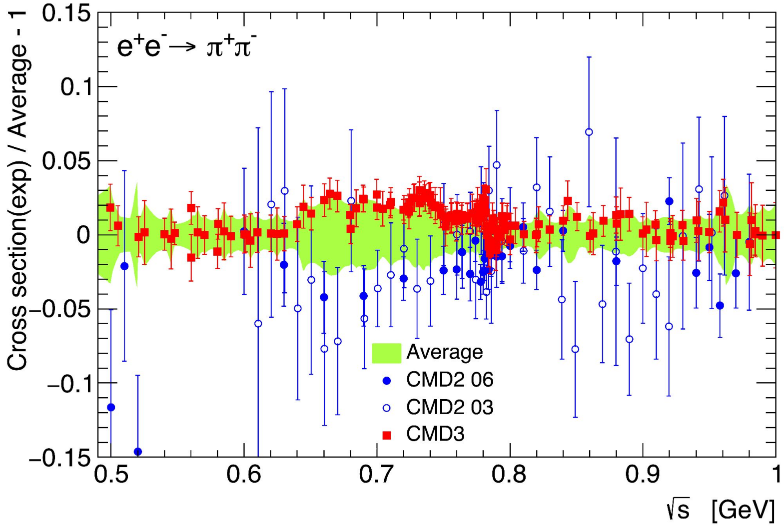
→ Not compatible with any realistic change of IB corrections

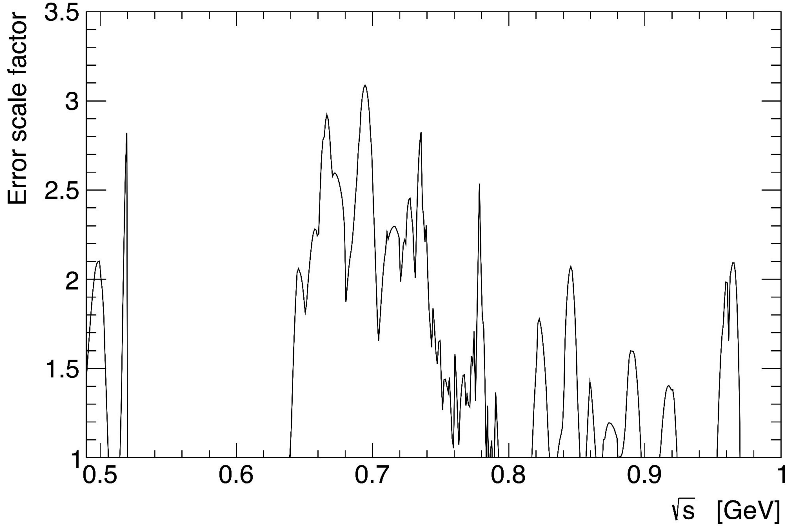


Backup

Comparisons with BaBar & CMD3 & Tau(+IB) combination







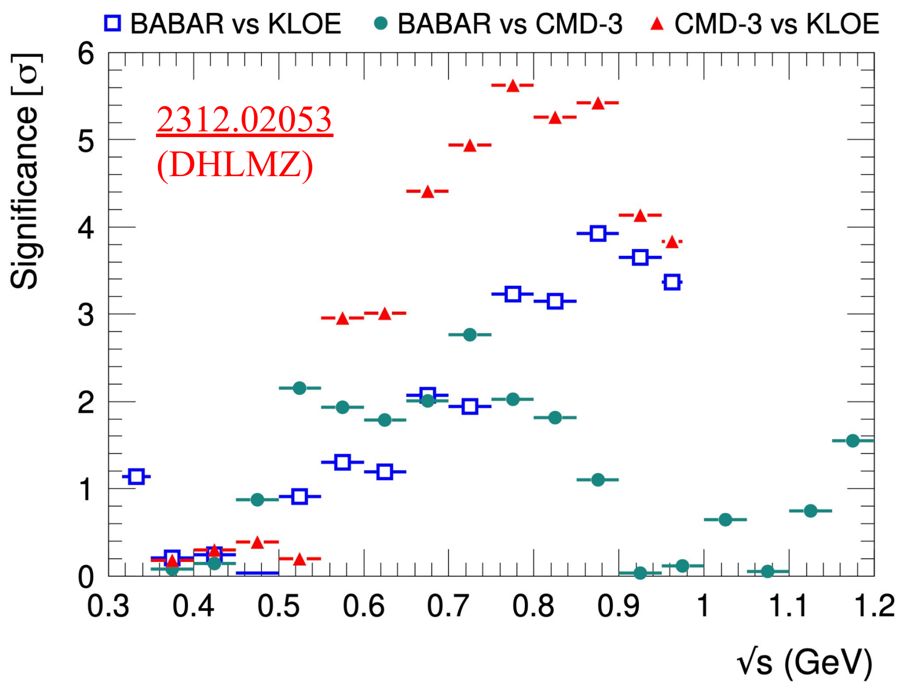
→ Employing an average between CMD2 and CMD3 for comparing the two (sets of) measurements

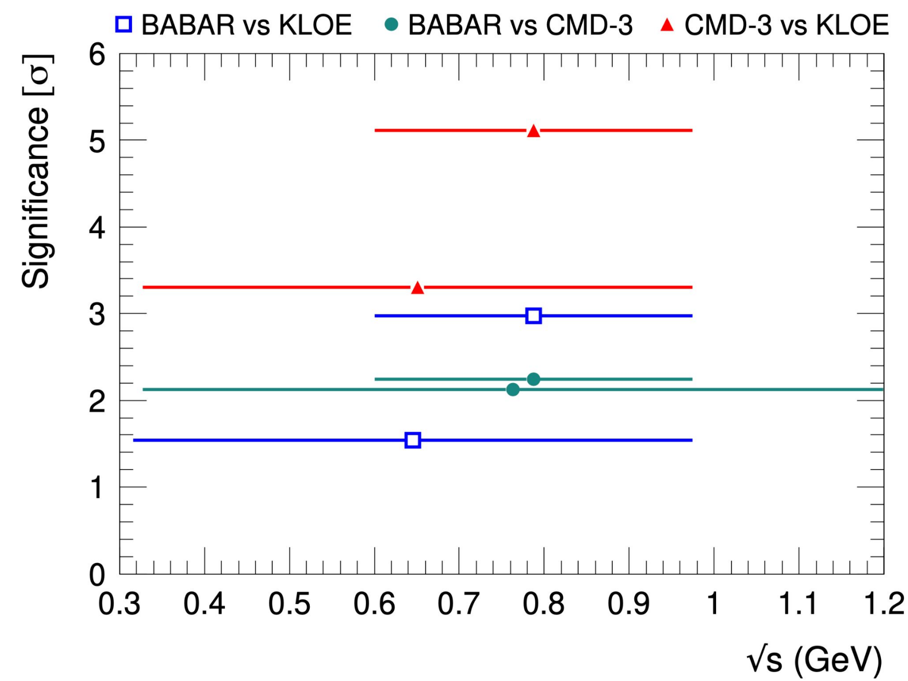
→ Note: while here the CMD3 uncertainties were treated as independent of the CMD2 ones, it would be useful to discuss about possible *estimates* of the correlations among them.

Quantitative direct comparisons for pairs of measurements

Quantitative comparisons for a_μ^{HVP}

→ Comparison of integrals computed in various restricted energy ranges, for individual e^+e^- experiments: significance of the pairwise differences between experiments, taking into account correlations

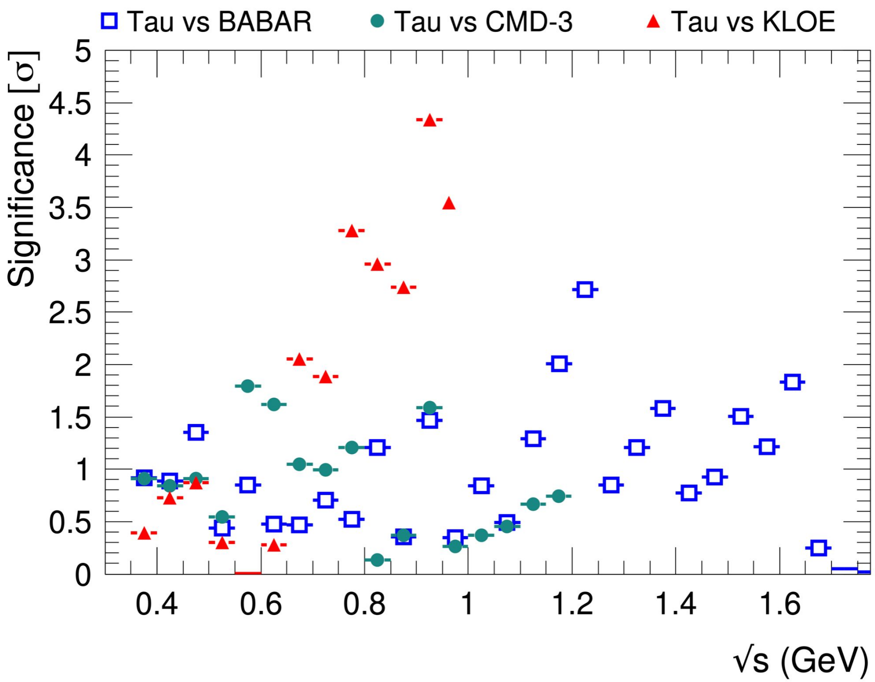


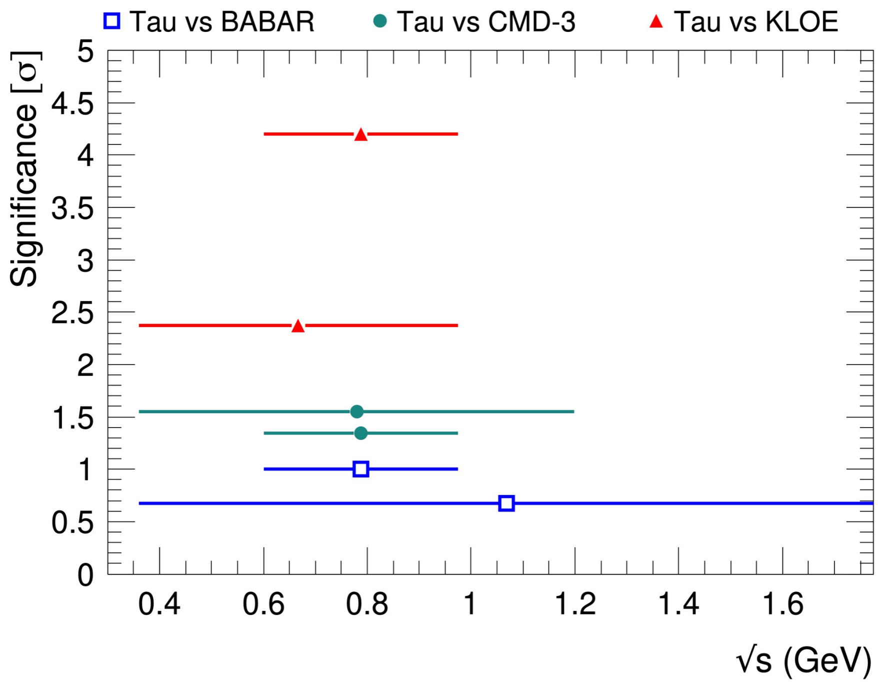


→ Largest tensions between CMD3 and KLOE

Quantitative comparisons for a_μ^{HVP}

→ Comparison of integrals computed in various restricted energy ranges, for τ / individual e^+e^- experiments: significance of the pairwise differences between experiments, taking into account correlations

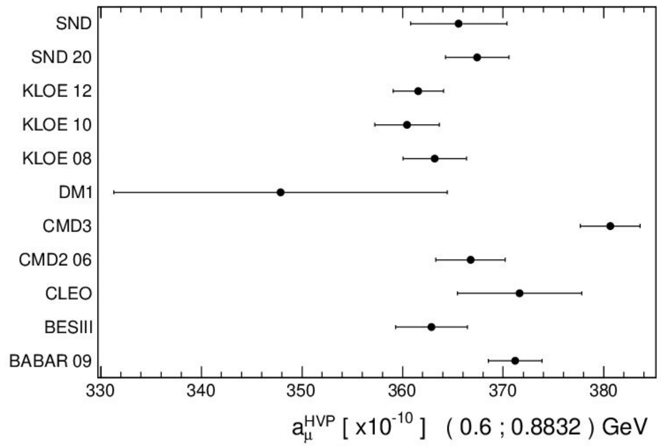


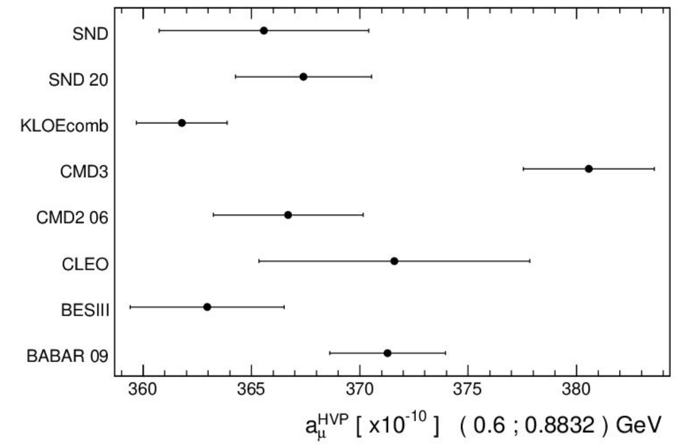


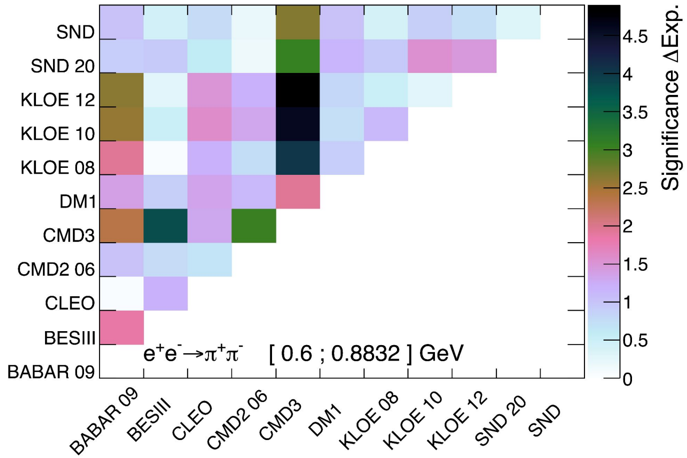
→ Largest tensions between Tau and KLOE

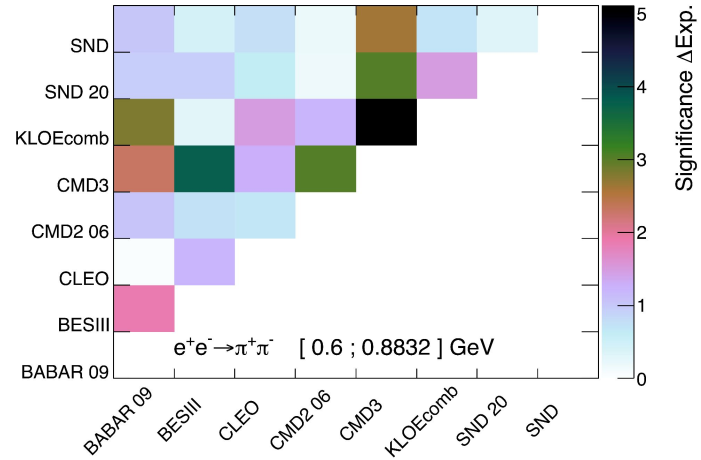
→ Good agreement among the Tau measurements (Backup)

→ Considering all the e^+e^- experiments fully covering a given energy range: numerous for ~narrow ranges

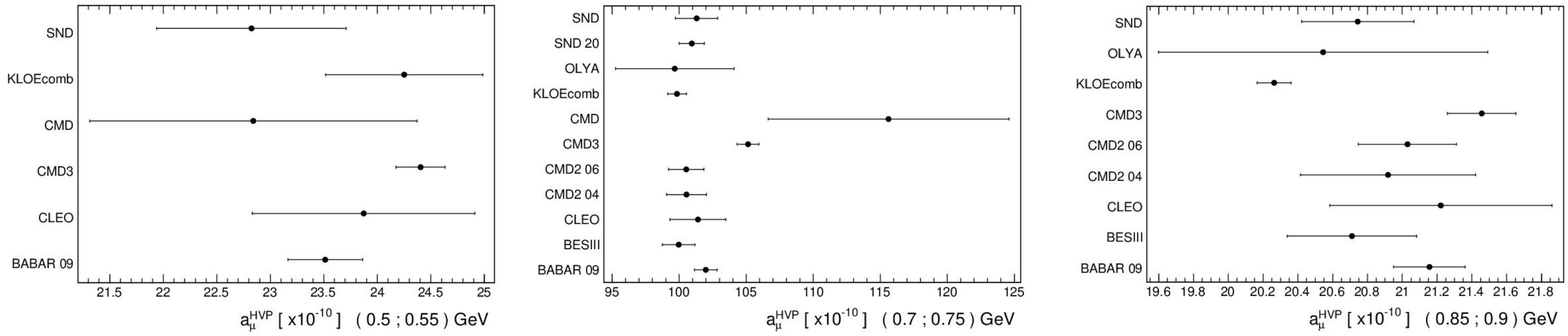


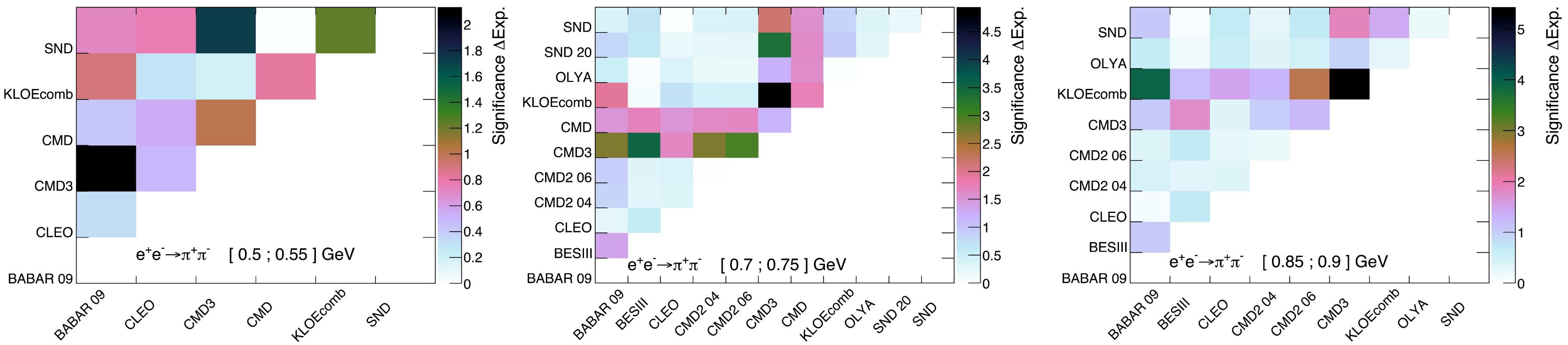






Quantitative comparisons: towards a more global picture for e^+e^- data





Remarks and conclusions

→ Detailed quantitative comparisons among measurements provide insight into the current situation for the data inputs

→ Need *rigorous* and *realistic* treatment of uncertainties and correlations at all levels

→ Caution about significance:
for numerous input measurements, resulting a_μ^{HVP} uncertainty limited by non-Gaussian systematic effects

Backup

Combine cross section data: goal and requirements

→ Goal: combine experimental spectra with arbitrary point spacing / binning

→ Requirements:

- Properly propagate uncertainties and correlations

  - *Between measurements (data points/bins) of a given experiment*
    (covariance matrices and/or detailed split of uncertainties in sub-components)
  - *Between experiments* (common systematic uncertainties, e.g. VP)
    based on detailed information provided in publications
  - *Between different channels* – motivated by understanding of the meaning of systematic uncertainties and identifying the common ones

    BABAR luminosity (ISR or BhaBha), efficiencies (photon, Ks, Kl, modeling);
    BABAR radiative corrections; $4\pi 2\pi^0 - \eta\omega$
    CMD2 $\eta\gamma - \pi^0\gamma$; CMD2/3 luminosity; SND luminosity;
    FSR; hadronic VP (old experiments)

- Minimize biases

- Optimize g-2 integral uncertainty
  (*without overestimating the precision with which the uncertainties of the measurements are known*)

Back

Combination procedure implemented in HVPTools software

For each final bin:

→ Compute an average value for each measurement and its uncertainty

→ Compute correlation matrix between experiments

→ Minimize χ^2 and get average coefficients (weights)

→ Compute average between experiments and its uncertainty

Evaluation of integrals and propagation of uncertainties:

→ Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys

→ The pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections → Integral evaluation

→ Uncertainties also propagated through $\pm 1\sigma$ shifts of each uncertainty:
- allows to account for correlations between different channels (for integrals and spectra)

→ *Checked consistency between the different approaches*

Combination procedure: weights of various measurements

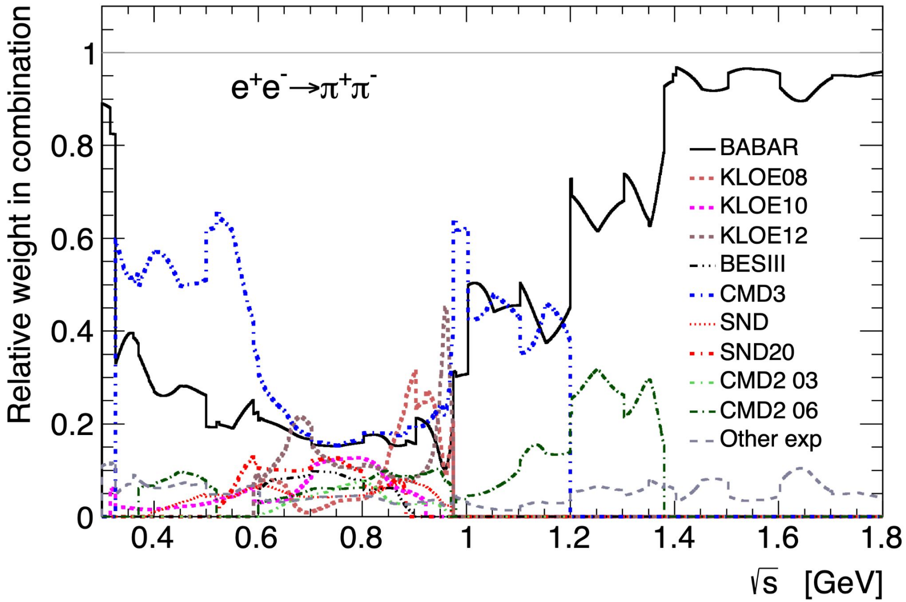
For each final bin:

→ Minimize χ^2 and get average coefficients

Note: average weights must account for bin sizes / point spacing of measurements
(do not over-estimate the weight of experiments with large bins)

→ Weights in fine bins evaluated using a common (large) binning for measurements + interpolation

→ Compare the precisions on the same footing



→ Bins used by KLOE larger than the ones by BABAR in ρ-ω interference region (factor ~3)

→ Average dominated by BaBar, CMD3, KLOE, SND20
BaBar covering full range

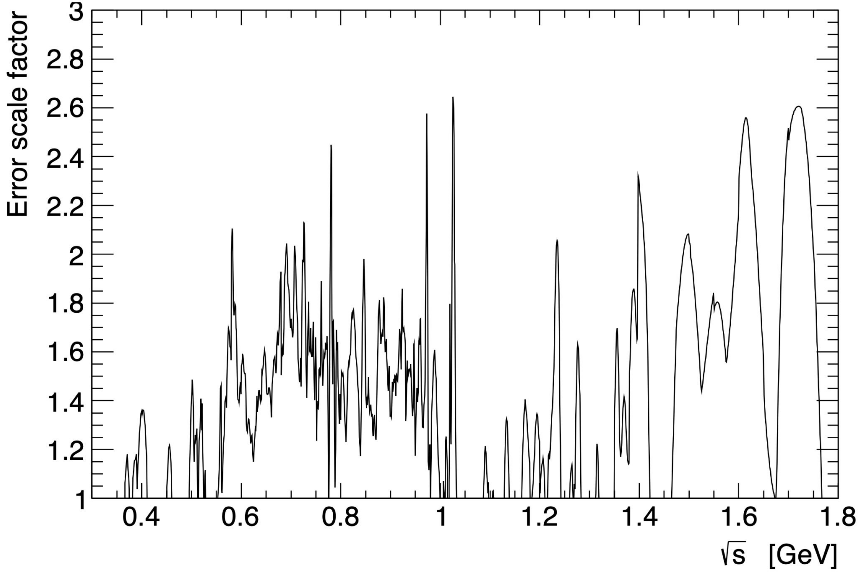
Back

Combination procedure: compatibility between measurements

For each final bin:

→ χ^2 /ndof: test locally the level of agreement between input measurements, *taking into account correlations*

→ Scale uncertainties in bins with χ^2 /ndof > 1 (PDG): *locally* conservative; Adopted by KNT since '17



→ Tension between measurements, especially between KLOE & CMD3, which provide the smallest / largest cross-sections in the ρ region:

Indication of underestimated uncertainties

Motivates conservative uncertainty treatment

in combination fit (evaluation of weights / fits based on analyticity & unitarity to constrain uncertainties)

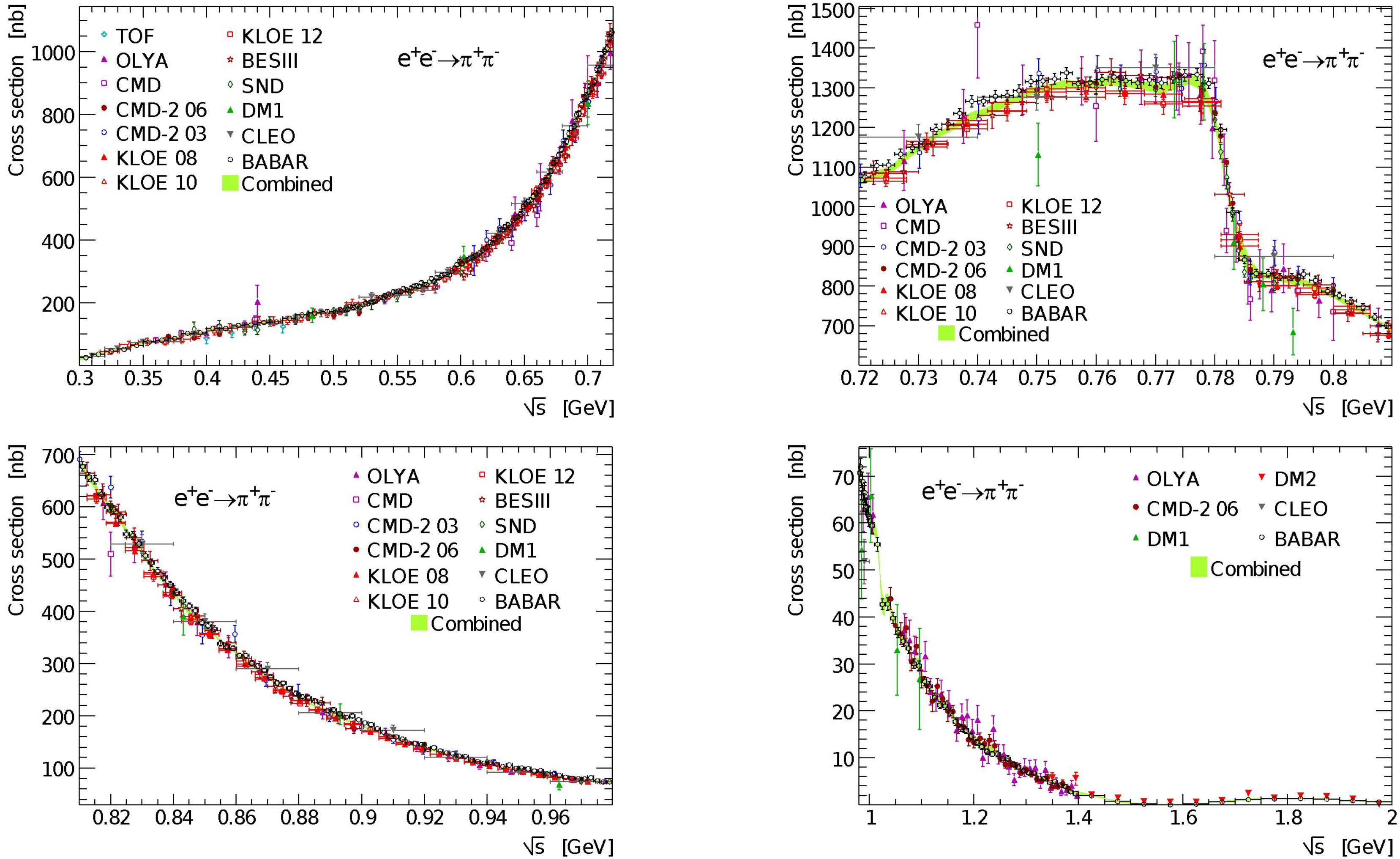
→ Observed (systematic) tension between measurements, beyond the local χ^2 /ndof rescaling

→ (*Since 2019*) Included extra (dominant) uncertainty: 1/2 difference between integrals w/o either BABAR or KLOE

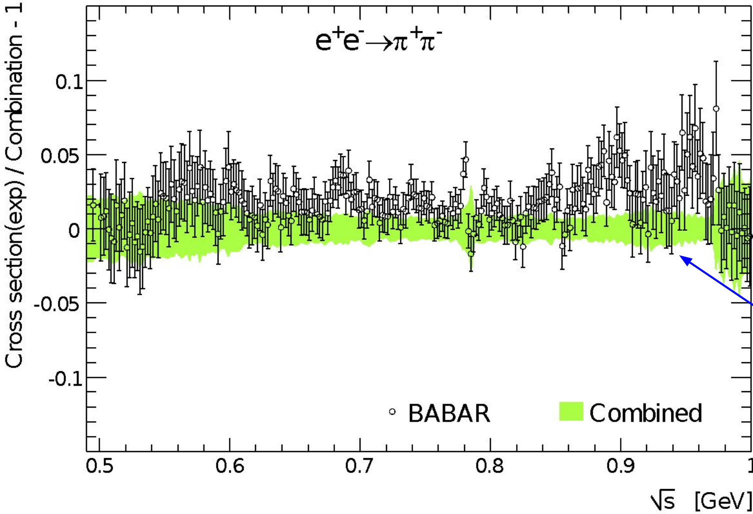
However, *tensions are larger now and we need to understand their source!*

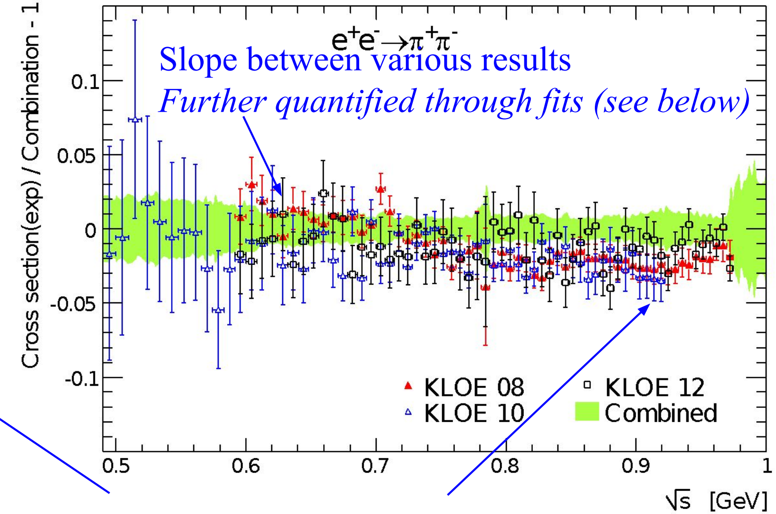
CMD2 / CMD3 tension still open question.

Combination for the $e^+e^- \to \pi^+\pi^-$ channel (DHMZ '19)



More on the combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel (DHMZ ’19)



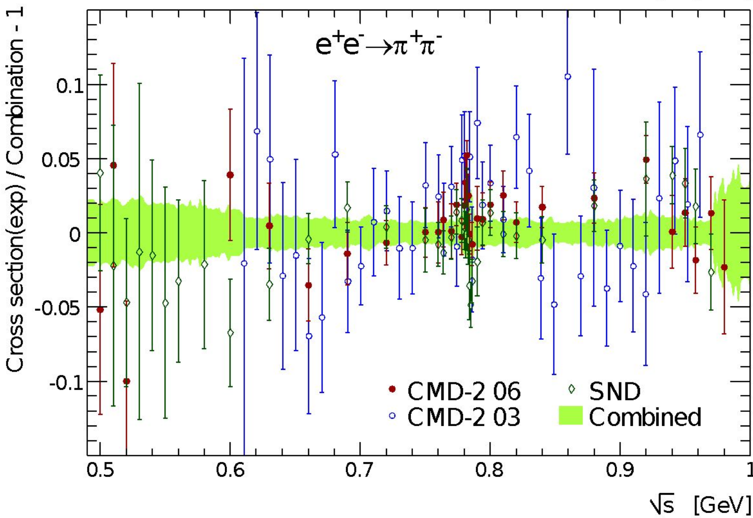


Slope between various results

Further quantified through fits (see below)

Local tension & systematic trends

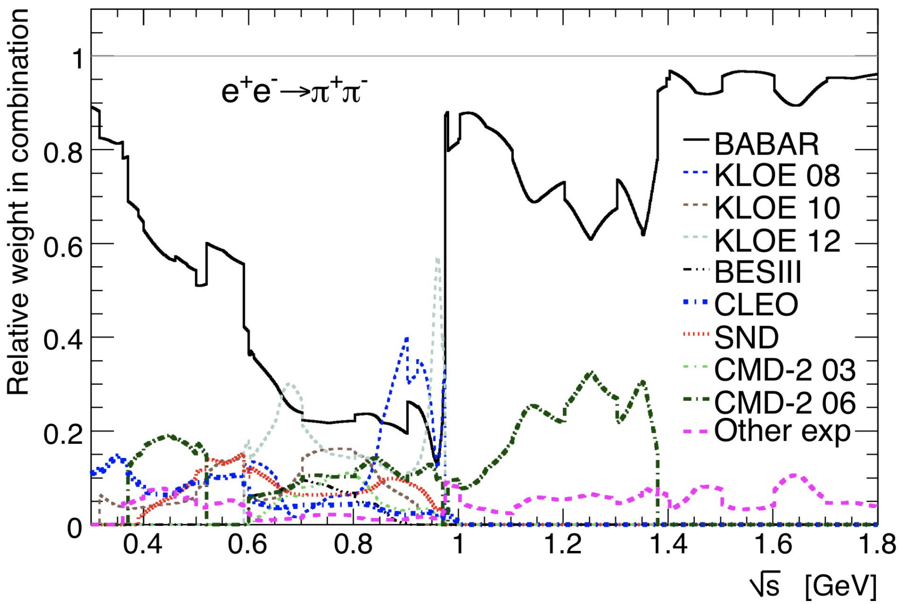
Indication of “uncertainties on uncertainties” (i.e. unaccounted biases)

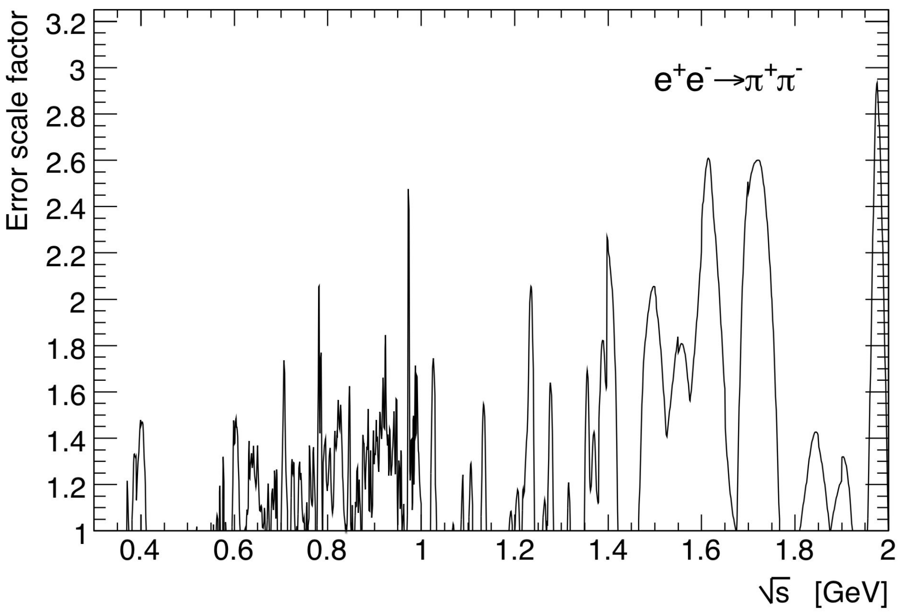


Other experiments not yet precise enough to discriminate

(see however update from SND: ~significant tension with KLOE above 720 MeV)

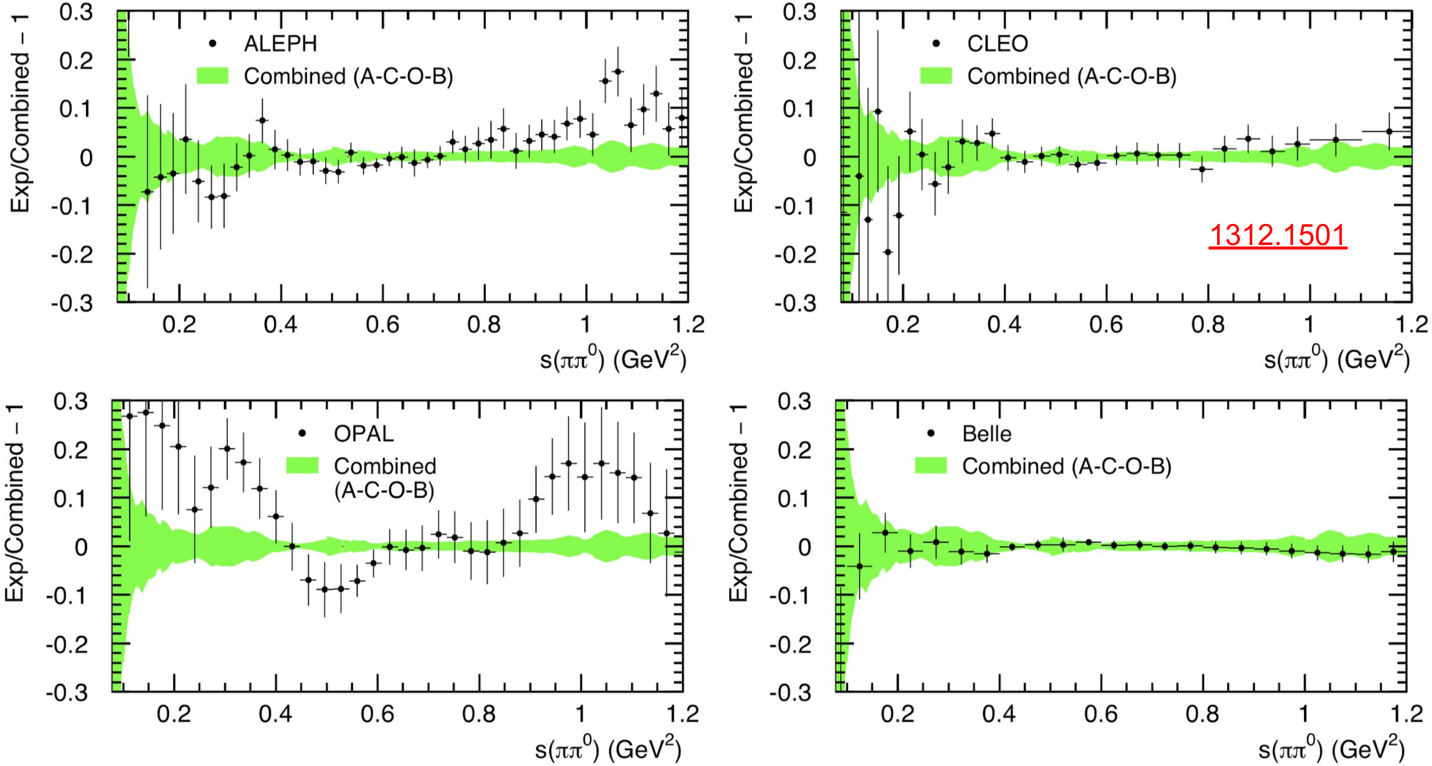
Combining the $e^+e^- \to \pi^+\pi^-$ data: weights and tension (DHMZ '19)





Combining the τ data in the ππ channel

→ τ hadronic spectral functions ($\pi\pi^0$ channel) from various experiments



→ Normalisation: branching fractions best determined by ALEPH (large boost, high granularity)

→ Shape best determined by Belle (high statistics); improvements @ Belle II

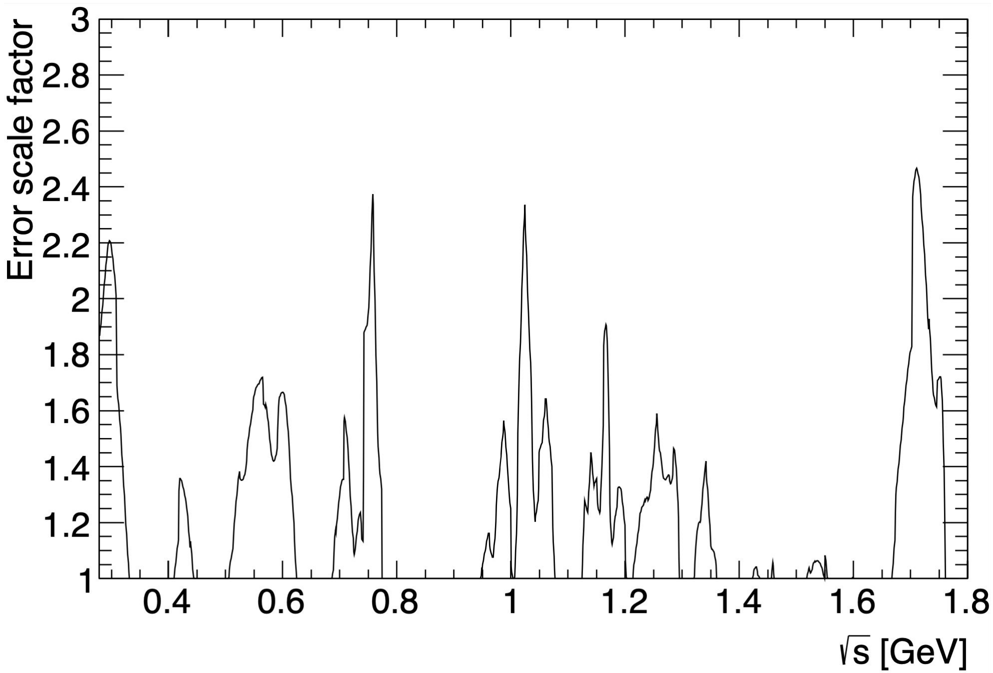
Back

Combination: compatibility between measurements

For each final bin:

→ χ^2 /ndof: test locally the level of agreement between input measurements, *taking into account correlations*

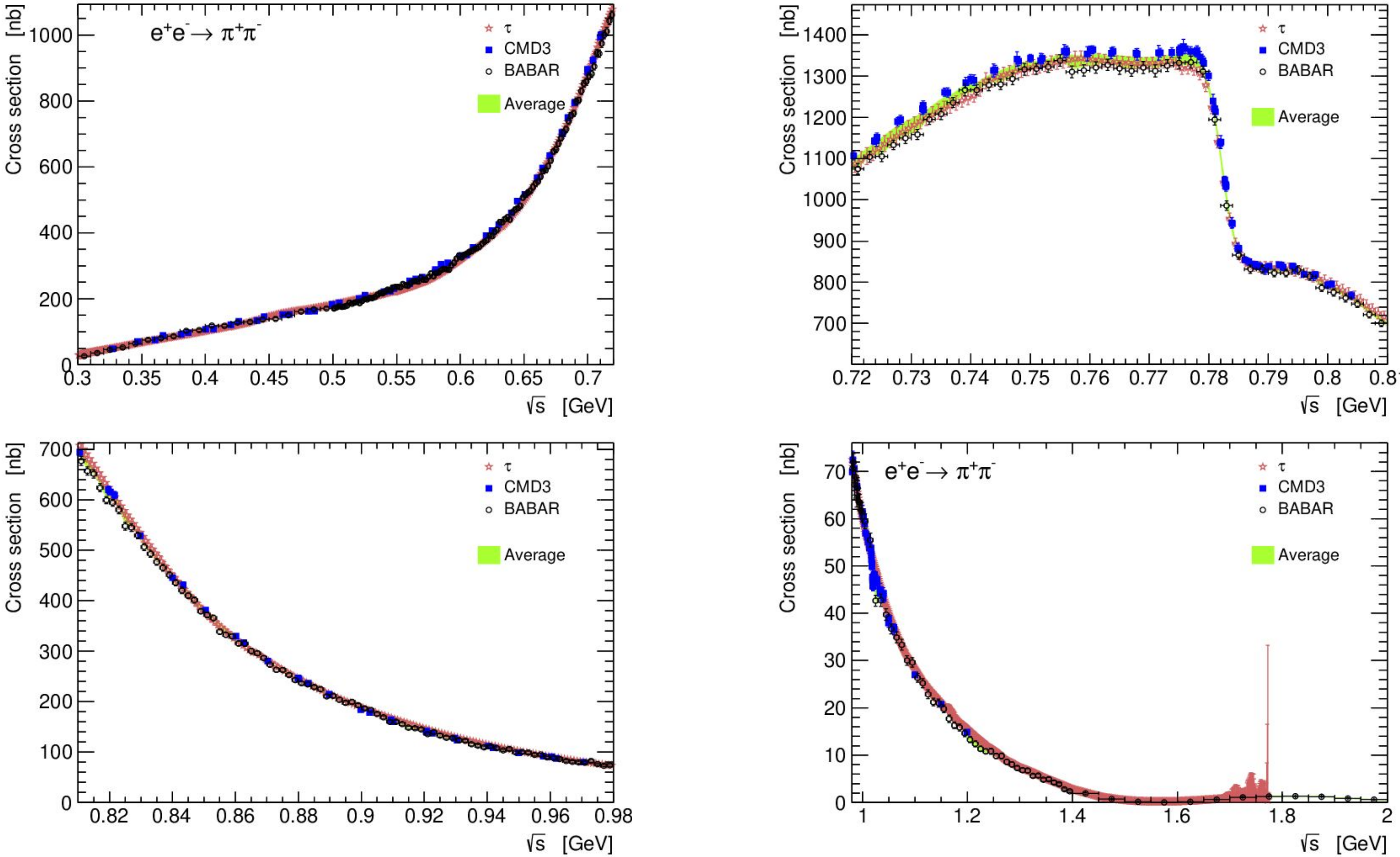
→ Scale uncertainties in bins with χ^2 /ndof > 1 (PDG)



→ Level of agreement significantly better than the one observed for $e^+e^- \to \pi^+\pi^-$ data

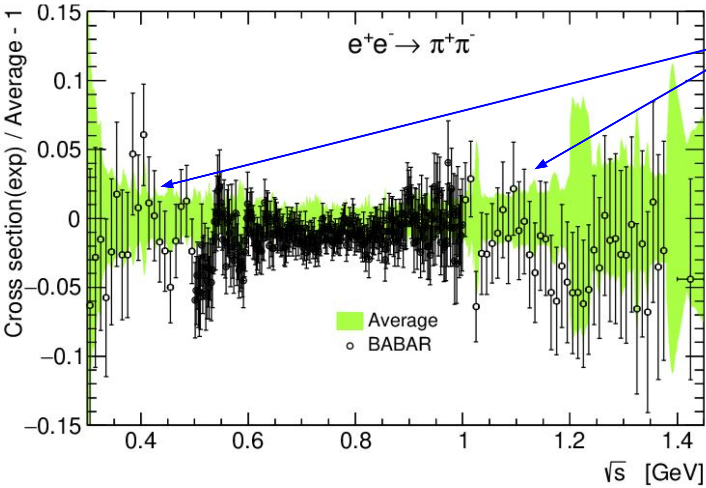
Combining the $e^+e^- \to \pi^+\pi^-$ data, BaBar & CMD3 & Tau(+IB)

→ Motivated by the previous findings, combine τ, BABAR and CMD-3 spectra

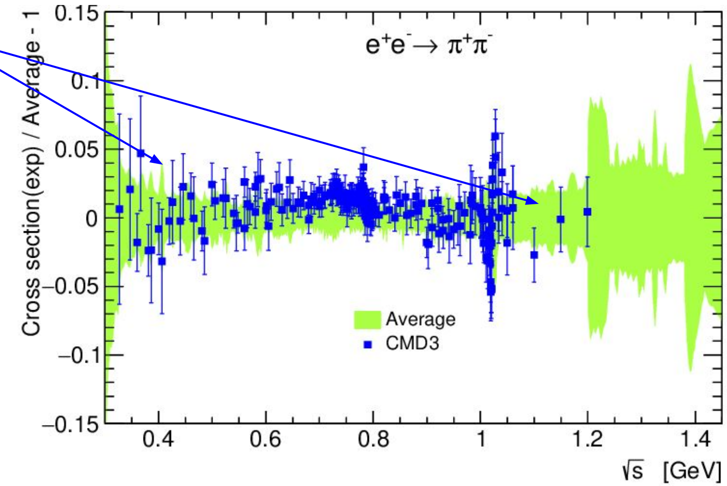


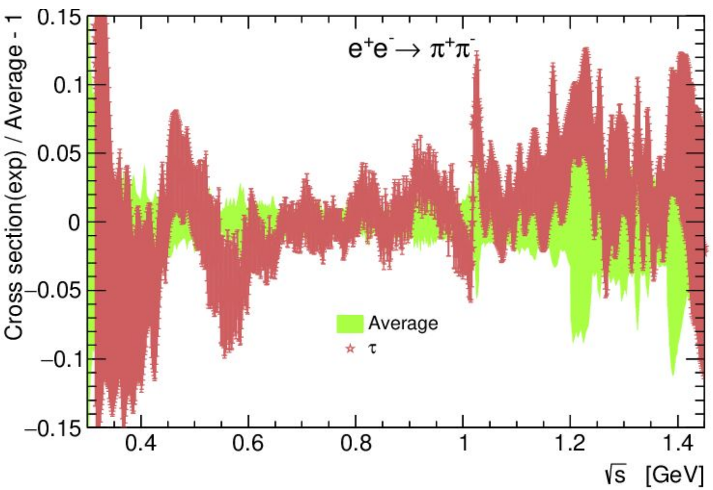
Back

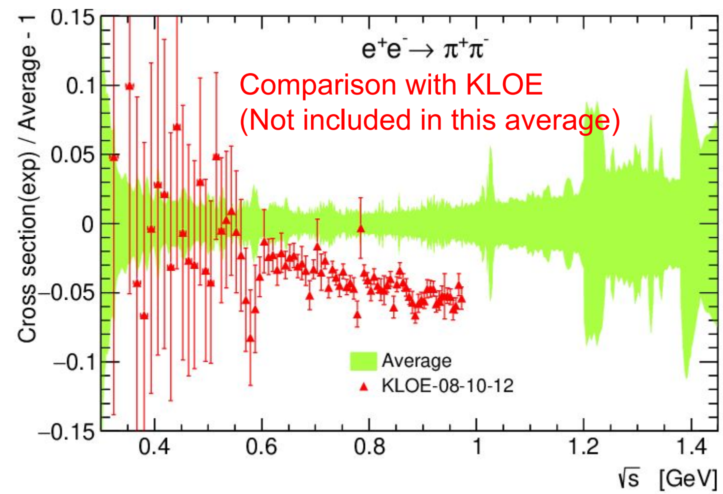
Combining the $e^+e^- \to \pi^+\pi^-$ data, BaBar & CMD3 & Tau(+IB)



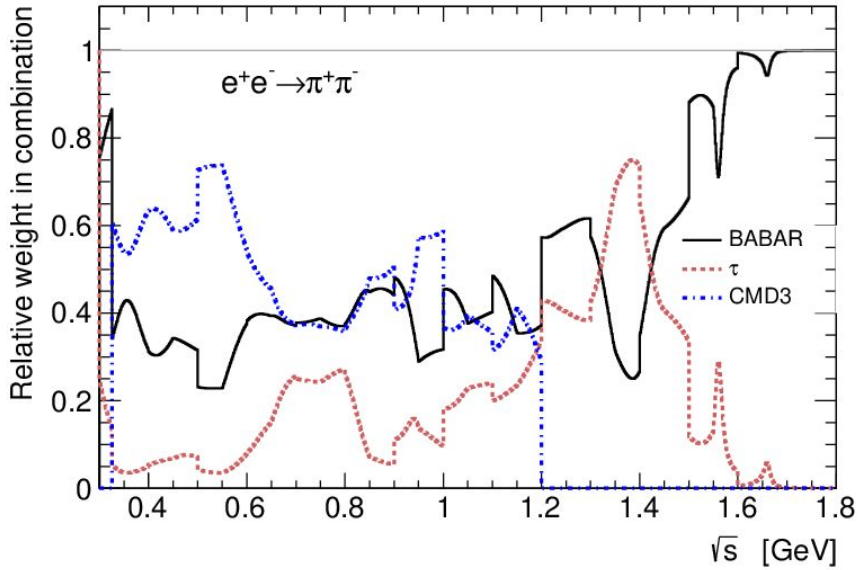
Reasonable BABAR/CMD3 agreement at low & high E

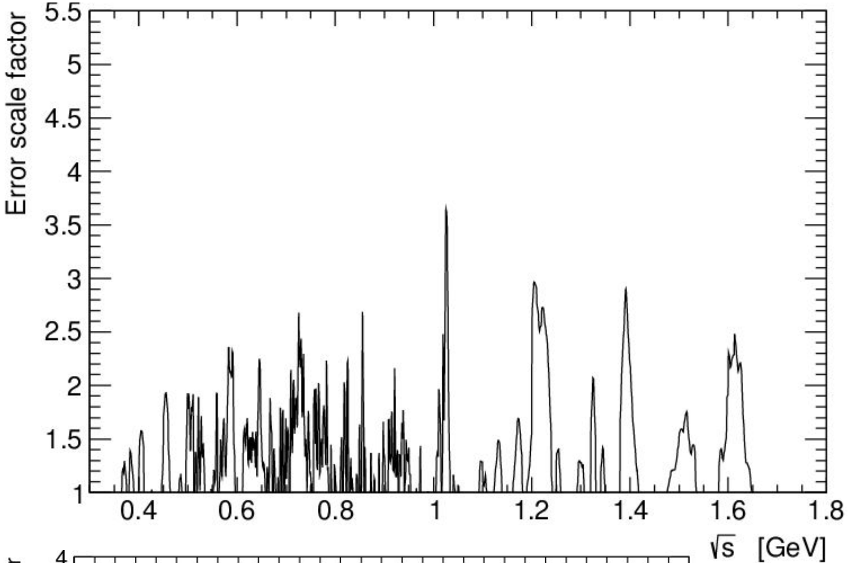


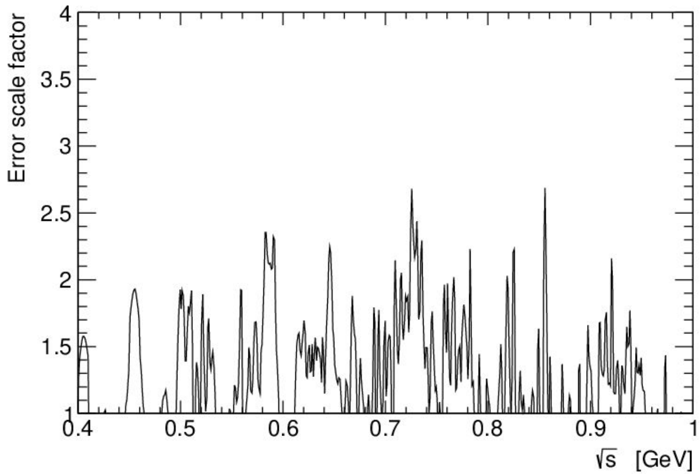




Combining the $e^+e^- \to \pi^+\pi^-$ data, BaBar & CMD3 & Tau(+IB)







→ Average dominated by BaBar and CMD3;
BaBar and τ cover full energy range

→ Some tension between BaBar & CMD3 in the ρ region

→ Much larger tension (slope and shift) when comparing KLOE with the BABAR + CMD-3 + τ combination

Combining the $e^+e^- \to \pi^+\pi^-$ data, BaBar & CMD3 & Tau(+IB)

x 10^{-10}

a_μ [0.3 ; 1.8 GeV] = 519.8 ±3.3 (±1.3(stat) ±3.1 (syst))

Without applying the χ^2 /ndof rescaling of uncertainties:
a_μ [0.3 ; 1.8 GeV] = 519.8 ±2.5 (±1.0 (stat) ±2.3 (syst))

→ Coherent with DHLMZ value (2312.02053, EPJ C) obtained from average of BaBar, CMD3 and τ integrals:
518.0 ±3.3 (after uncertainty rescaling x1.5)
(Different e+e- combination to complete CMD3 energy range;
Using fit of τ data to complete their integral for [Thr.;0.36 GeV])

a_μ^{win} [0.3 ; 1.8 GeV] = 148.5 ±1.0 (±0.4(stat) ±0.9 (syst))

Without applying the χ^2 /ndof rescaling of uncertainties:
a_μ^{win} [0.3 ; 1.8 GeV] = 148.5 ±0.7 (±0.3 (stat) ±0.6 (syst))

a_μ [0.6 ; 0.9747 GeV] = 394.6 ±2.5 (±1.3 (stat) ±2.2 (syst))

Without applying the χ^2 /ndof rescaling of uncertainties:
a_μ [0.6 ; 0.9747 GeV] = 394.6 ±1.8 (±0.9(stat) ±1.6 (syst))

a_μ^{win} [0.6 ; 0.9747 GeV] = 127.4 ±0.8 (±0.4 (stat) ±0.7 (syst))

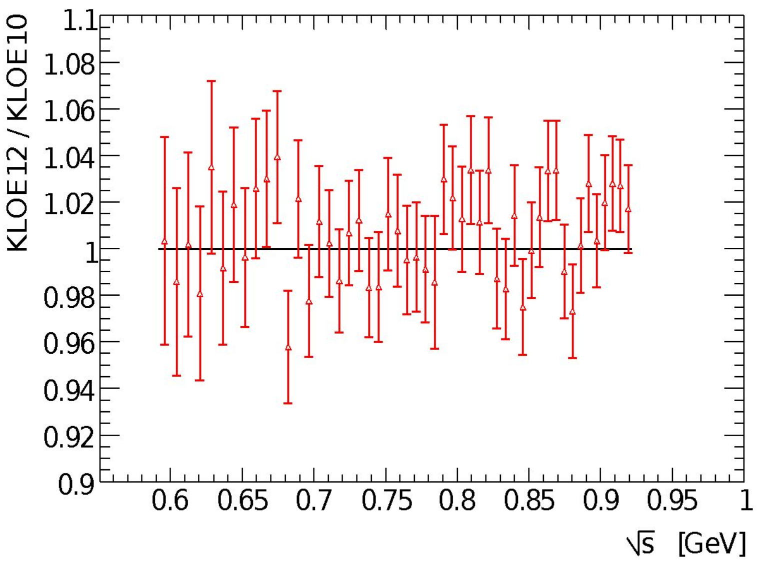
Without applying the χ^2 /ndof rescaling of uncertainties:
a_μ^{win} [0.6 ; 0.9747 GeV] = 127.4 ±0.6 (±0.3(stat) ±0.5 (syst))

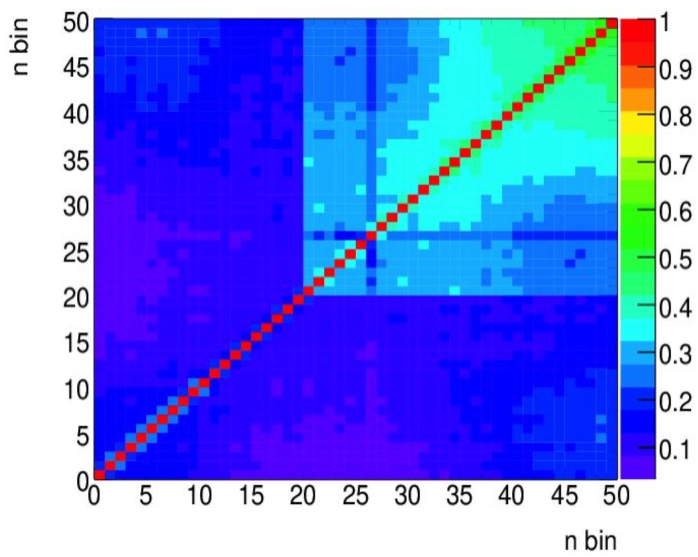
→ Still non-negligible effect of uncertainty enhancement through the local χ^2 /ndof rescaling;
In addition, an extra uncertainty accounting for systematic deviations between measurements has to be added, as done for DHMZ'19

Ratios between measurements

→ Compute ratio between pairs of KLOE measurements

→ Full propagation of uncertainties and correlations using pseudo-experiments (agreement with analytical linear uncertainty propagation)





→ Good agreement between KLOE 10 and KLOE 12

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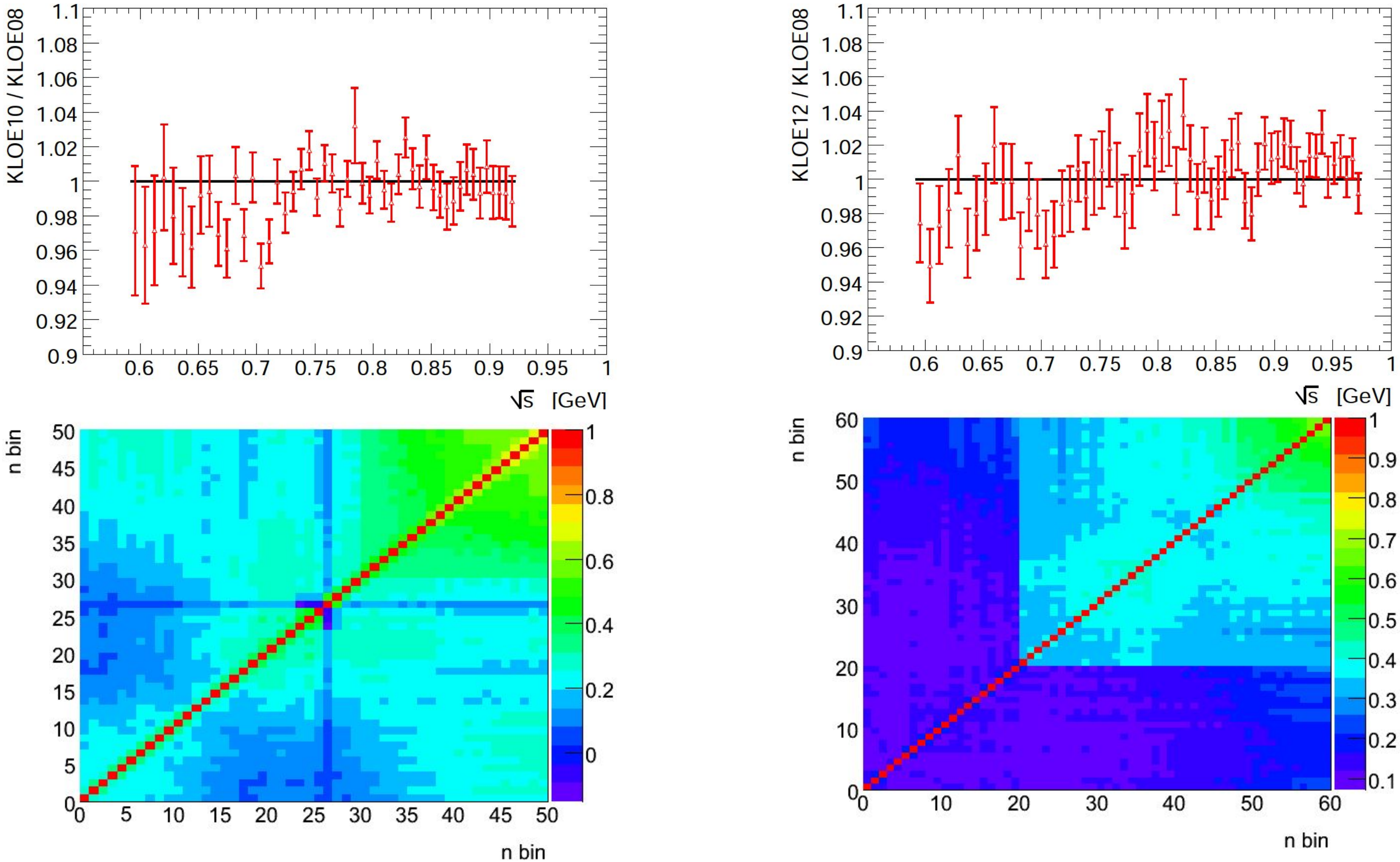
Ratios between measurements



Direct comparison of the 3 KLOE measurements

→ Quantitative comparison between the ratios and unity, taking into account correlations

KLOE 10 / KLOE 08

χ^2 [0.35;0.85] GeV^2 : 79.0 / 50(DOF)
p-value= 0.0056

χ^2 [0.35;0.58] GeV^2 : 46.2 / 23(DOF)
p-value= 0.0028

χ^2 [0.58;0.85] GeV^2 : 29.7 / 27(DOF)
p-value= 0.33

χ^2 [0.64;0.85] GeV^2 : 20.7 / 21(DOF)
p-value= 0.47

KLOE 12 / KLOE 08

χ^2 [0.35;0.95] GeV^2 : 73.7 / 60(DOF)
p-value= 0.11

χ^2 [0.35;0.58] GeV^2 : 21.8 / 23(DOF)
p-value= 0.53

χ^2 [0.35;0.64] GeV^2 : 27.5 / 29(DOF)
p-value= 0.55

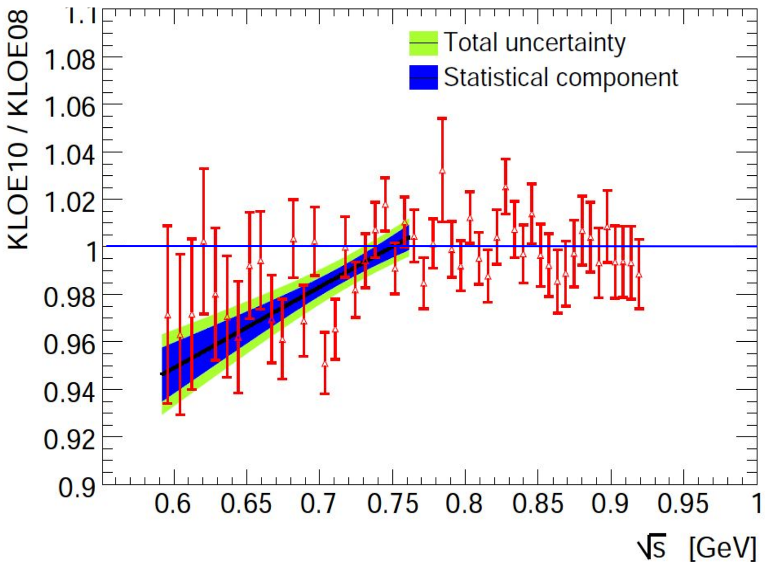
χ^2 [0.64;0.95] GeV^2 : 39.4 / 31(DOF)
p-value= 0.14

Quantitative comparisons of the KLOE measurements

→ Quantitative comparison between the ratios and unity, taking into account correlations

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results: $\pm1\sigma$ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit



Comparison with Unity:

χ^2 [0.35;0.85] GeV^2 : 79.0 / 50(DOF)

p-value= 0.0056

χ^2 [0.35;0.58] GeV^2 : 46.2 / 23(DOF)

p-value= 0.0028

χ^2 [p0 + p1√s]: 36.1 / 21(DOF)

p-value= 0.02

p0 : 0.745 ± 0.085

p1 : 0.341 ± 0.117

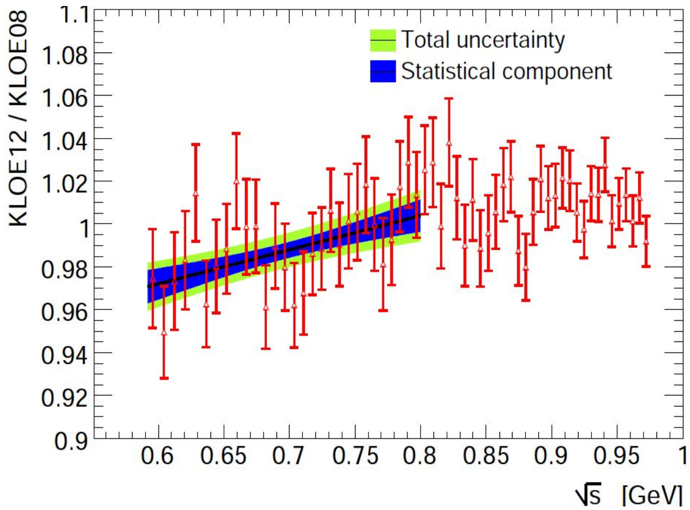
→ Significant shift & slope (~2.5-3σ) at low √s, no significant shift at high √s
Similar shift & slope for KLOE 12 / KLOE 08 (*see below*)

→ Should motivate conservative treatment of uncertainties and correlations in combination

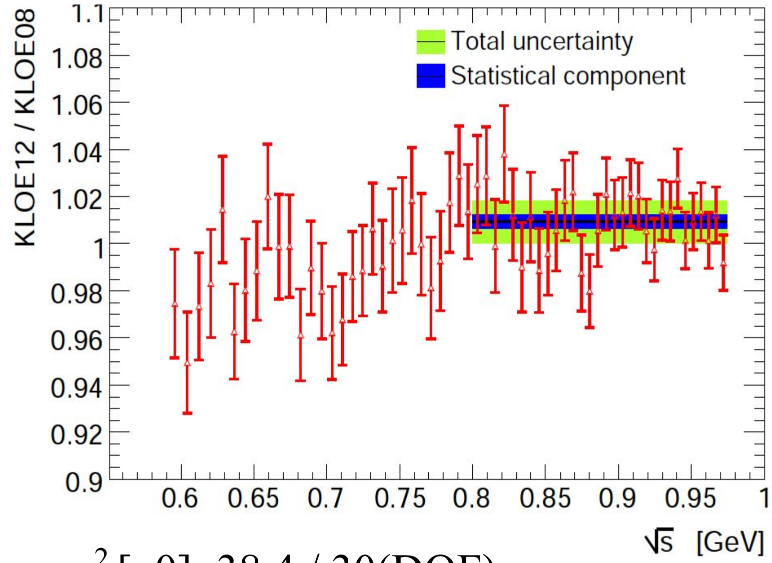
Direct comparison of the 3 KLOE measurements

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results: ±1σ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit



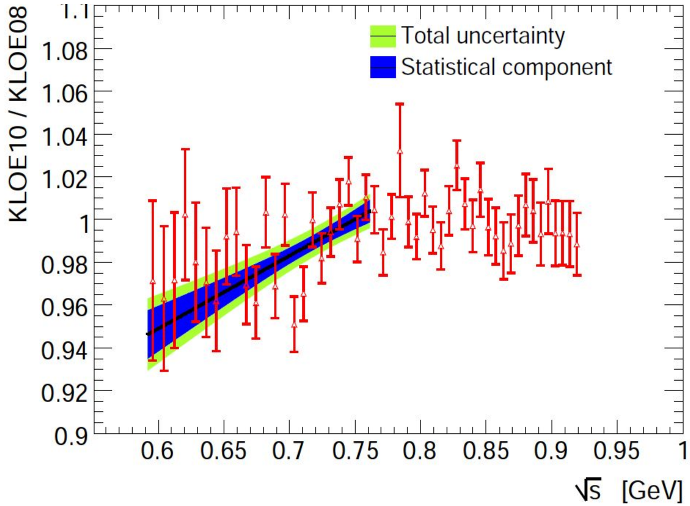
χ^2 [p0 + p1√s]: 20.7 / 27(DOF)
p-value= 0.80
p0 : 0.876 ± 0.056
p1 : 0.159 ± 0.081



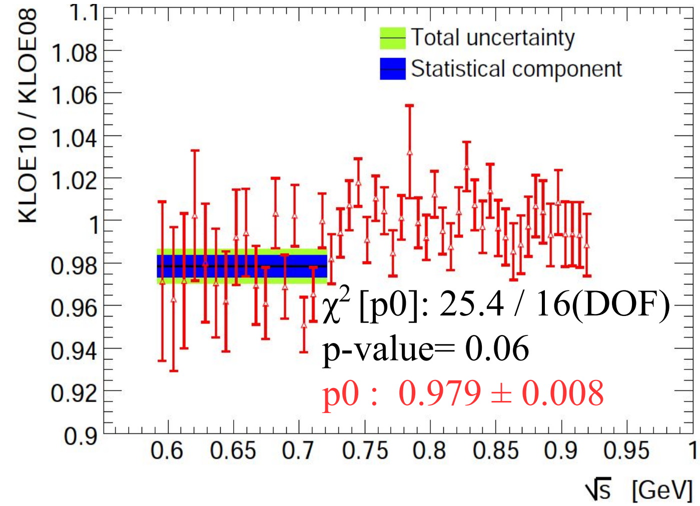
χ^2 [p0]: 38.4 / 30(DOF)
p-value= 0.14
p0 : 1.009 ± 0.009

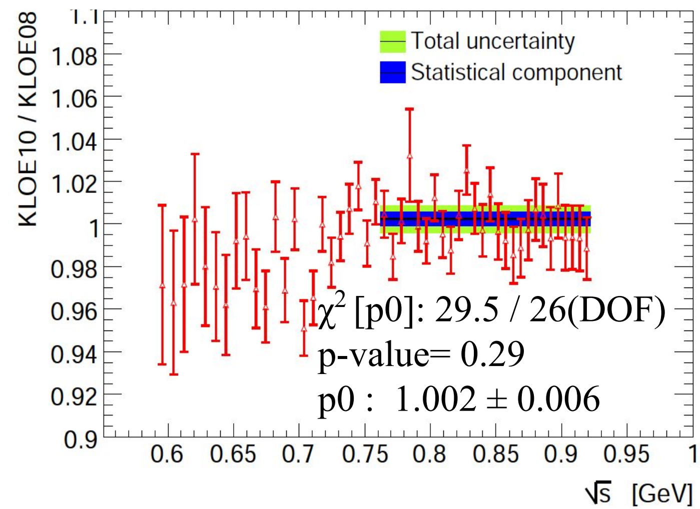
→ Significant shift and slope (~2σ) at low √s, no significant shift at high √s

Direct comparison of the 3 KLOE measurements



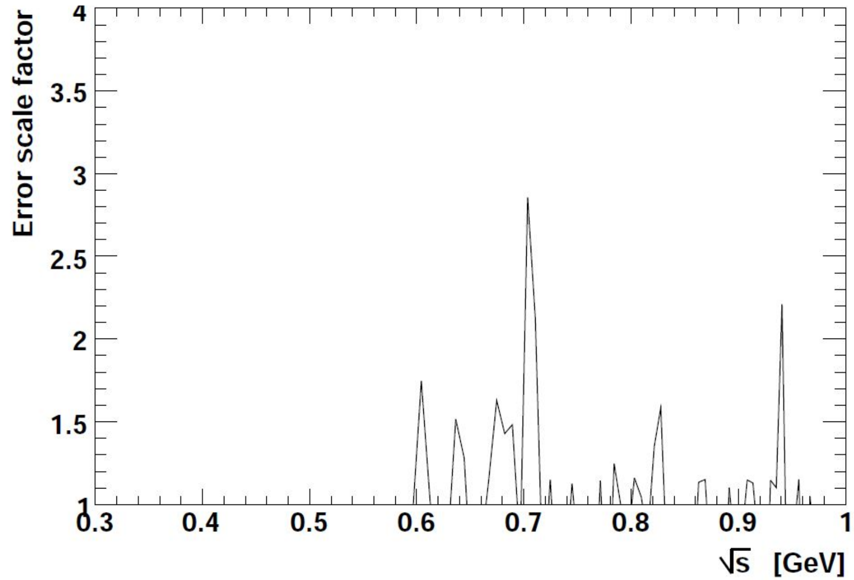
χ^2 [p0 + p1√s]: 36.1 / 21(DOF)
p-value= 0.02
p0 : 0.745 ± 0.085
p1 : 0.341 ± 0.117





→ Significant shift and slope (~2.5-3σ) at low √s, no significant shift at high √s

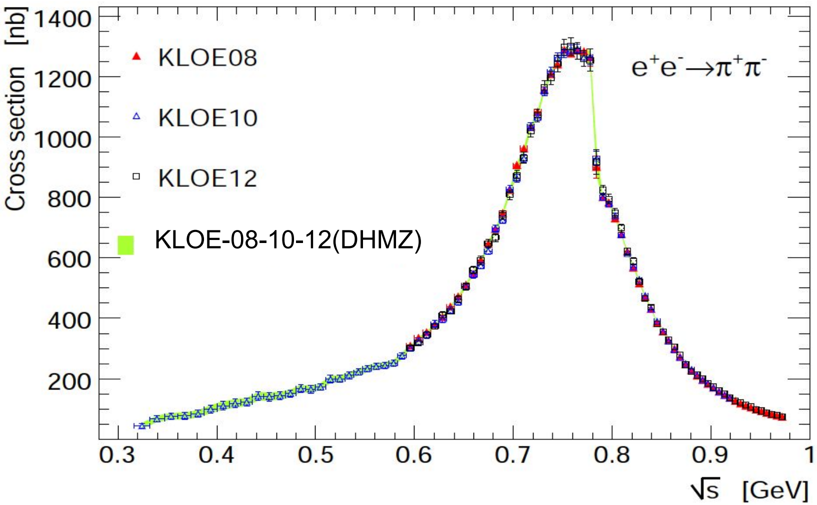
Local comparison of the 3 KLOE measurements

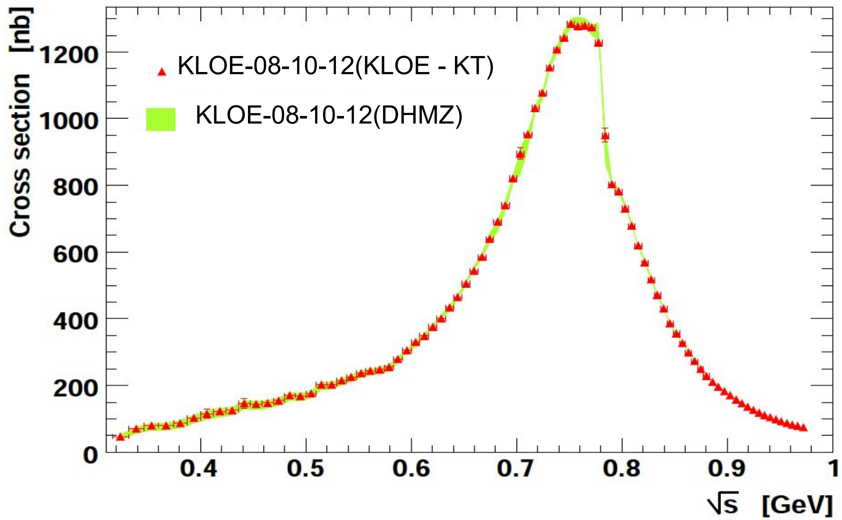


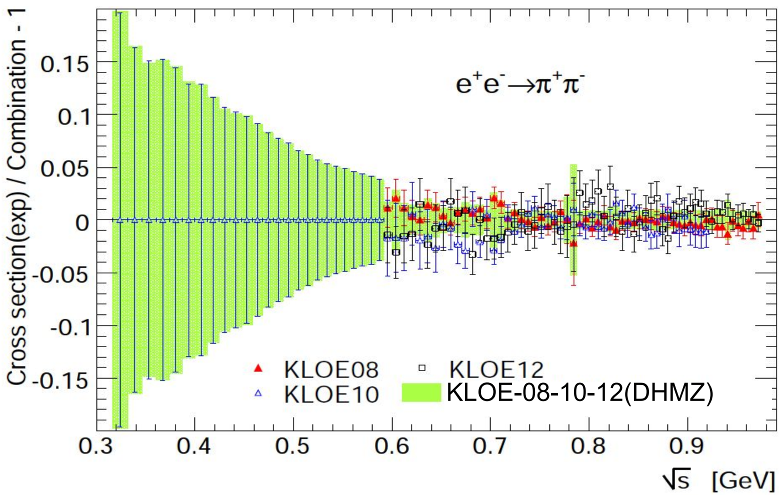
→ Local χ^2 /ndof test of the local compatibility between KLOE 08 & 10 & 12, taking into account the correlations: some tensions observed

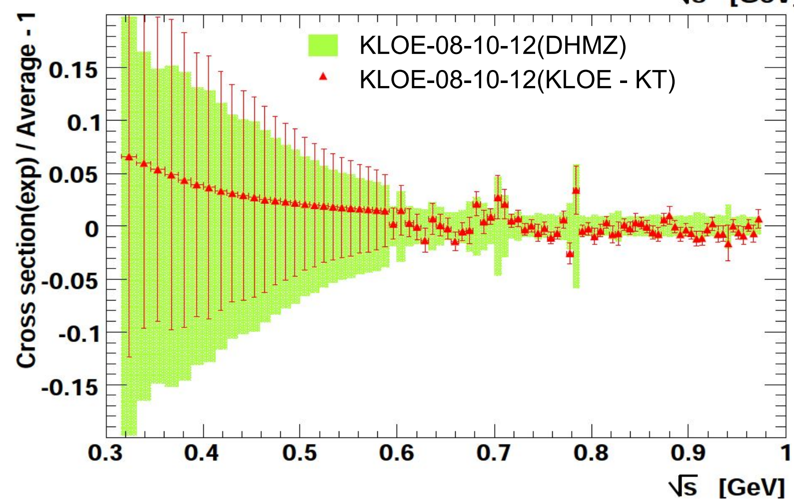
→ Does not probe general trends of the difference between the measurements (e.g. slopes in the ratio)

Combining the 3 KLOE measurements









Comparison of / consequences for combination methods

Analysis aspect	DHMZ	KNTW
Blinding	Not necessary (No ad-hoc choices to make)	Included for upcoming update
Binning	Fine ($\leq$ 1 MeV) final binning for average and integrals. Large (O(100 MeV) or less) common binning @ intermediate step: compare statistics of experiments coherently for deriving weights in fine bins.	Re-bin data into "clusters". Scans over cluster configurations for optimisation.
Closure test	Using model for spectrum: negligible bias. (since 2009)	Not performed
Additional constraints	Analyticity constraints for 2π channel.	None
Fitting	χ^2 minimisation with correlated uncertainties incorporated locally (in fine & large bins), for deriving weights. Full propagation of uncertainties & correlations.	χ^2 minimisation with correlated uncertainties incorporated globally.
Integration / interpolation	Av. of quadratic splines (3rd order polynomial), integral preservation in bins of measurements. Analyticity-based function for 2π ($<$ 0.6 GeV).	Trapezoidal for continuum, quintic for resonances.
Uncertainty inflation	Local χ^2 uncertainty inflation. (since 2009) Extra BABAR-KLOE systematic. (since 2019)	Local χ^2 uncertainty inflation. (adopted since 2017)
Inter-channel correlations	Taken into account. (since 2010)	Not included.
Missing channels	Estimated based on isospin symmetry. (since 1997 - ADH)	Adopted in subsequent updates

→ Large DHMZ/KNTW differences for the resulting uncertainties, as well as for the shapes of the combined spectra

WP TI	DHMZ19	KNT19
$a_\mu^{\text{HVP, LO}} \times 10^{10}$	694.0(4.0)	692.8(2.4)

→ CHKLS approach for 2π and 3π: Analyticity and global χ^2 fit

Uncertainties on uncertainties and on correlations

Topic of general interest, in other fields too

1908.00921(DHMZ), 2006.04822(WP Theory Initiative)

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Uncertainties on uncertainties and correlations

Numerous indications of uncertainties on uncertainties and on correlations, with a direct impact on combination fits

→ Shapes of systematic uncertainties *evaluated* in ~wide mass ranges with sharp transitions

→ One standard deviation is statistically not well defined for systematic uncertainties

→ Systematic uncertainties like acceptance, tracking efficiency, background etc. not necessarily fully correlated between low and high mass

→ Are all systematic uncertainty components fully independent between each-other? (e.g. tracking / trigger)

→ Y*ield uncertainties on uncertainties and on correlations*

→ Tensions between measurements (BABAR/KLOE/CMD3; 3 KLOE results etc.): *experimental indications of underestimated uncertainties*

→ *Statistical methods* (χ^2 with correlations, likelihood fits, ratios of measured quantities etc.) *should not over-exploit the information on the amplitude and correlations of uncertainties*

Topic of general interest, in other fields too (e.g. ATLAS JES and Jet Xsec studies)

Two different approaches for combining (e^+e^-) data

DHMZ:

→ χ^2 computed locally (in each fine bin), taking into account correlations between measurements (see previous slides)

→ Used to determine the weights on the measurements in the combination and their level of agreement

→ Uncertainties and correlations propagated using pseudo-experiments or ±1σ shifts of each uncertainty component

KNTW:

→ χ^2 computed globally (for full mass range)

$$\chi_I^2 = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} \left(R_i^{(m)} - \mathcal{R}_m^{i,I}\right) \mathbf{C}_I^{-1}\left(i^{(m)}, j^{(n)}\right)\left(R_j^{(n)} - \mathcal{R}_n^{j,I}\right)$$

KNT (1802.02995)

$$\chi^2 = \sum_{i=1}^{195} \sum_{j=1}^{195} \left(\sigma^0_{\pi\pi(\gamma)}(i) - \bar{\sigma}^0_{\pi\pi(\gamma)}(m)\right) \mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right)\left(\sigma^0_{\pi\pi(\gamma)}(j) - \bar{\sigma}^0_{\pi\pi(\gamma)}(n)\right)$$

KLOE-KMT (1711.03085)

→ relies on description of correlations on long ranges

→ *One of the main sources of differences for the uncertainty on a_μ*

Evaluation of uncertainties and correlations (e^+e^-)

	$\sigma_{\pi\pi\gamma}$	$\sigma^0_{\pi\pi}$	F_π	$\Delta^{\pi\pi} a_\mu$
Reconstruction Filter	negligible			
Background subtraction	Tab. 1			0.3%
Trackmass	0.2%			
Pion cluster ID	negligible			
Tracking efficiency	0.3%			
Trigger efficiency	0.1%			
Acceptance	Tab. 2			0.2%
Unfolding	Tab. 3			negligible
L3 filter	0.1%			
$\sqrt{s}$ dependence of H	-	Tab. 4		0.2%
Luminosity	0.3%			
Experimental systematics				0.6%
FSR resummation	-	0.3%		
Radiator function H	-	0.5%		
Vacuum Polarization	-	0.1%	-	0.1%
Theory systematics				0.6%

$M^2_{\pi\pi}$ range (GeV^2)	Systematic error (%)
$0.35 \leq M^2_{\pi\pi} < 0.39$	0.6
$0.39 \leq M^2_{\pi\pi} < 0.43$	0.5
$0.43 \leq M^2_{\pi\pi} < 0.45$	0.4
$0.45 \leq M^2_{\pi\pi} < 0.49$	0.3
$0.49 \leq M^2_{\pi\pi} < 0.51$	0.2
$0.51 \leq M^2_{\pi\pi} < 0.64$	0.1
$0.64 \leq M^2_{\pi\pi} < 0.95$	-

KLOE 08 (0809.3950)

KLOE 10 (1006.5313)

	$\sigma_{\pi\pi\gamma}$	$\sigma^{\rm bare}_{\pi\pi}$	$\lvert F_\pi\rvert^2$	$\Delta a^{\pi\pi}_\mu$
	threshold ; ρ-peak			(0.1 - 0.85 GeV^2)
Background Filter	0.5% ; 0.1%			negligible
Background subtraction	3.4% ; 0.1%			0.5%
$f_0 + \rho\pi$ bkg.	6.5% ; negl.			0.4%
Ω cut	1.4% ; negl.			0.2%
Trackmass cut	3.0% ; 0.2%			0.5%
π-e PID	0.3% ; negl.			negligible
Trigger	0.3% ; 0.2%			0.2%
Acceptance	1.9% ; 0.3%			0.5%
Unfolding	negl. ; 2.0%			negligible
Tracking	0.3%			
Software Trigger (L3)	0.1%			
Luminosity	0.3%			
Experimental syst.				1.0%
FSR treatment	-	7% ; negl.		0.8%
Radiator function H	-	0.5%		
Vacuum Polarization	-	Ref. [34]	-	0.1%
Theory syst.				0.9%

→ Systematics *evaluated* in ~wide mass ranges with sharp transitions

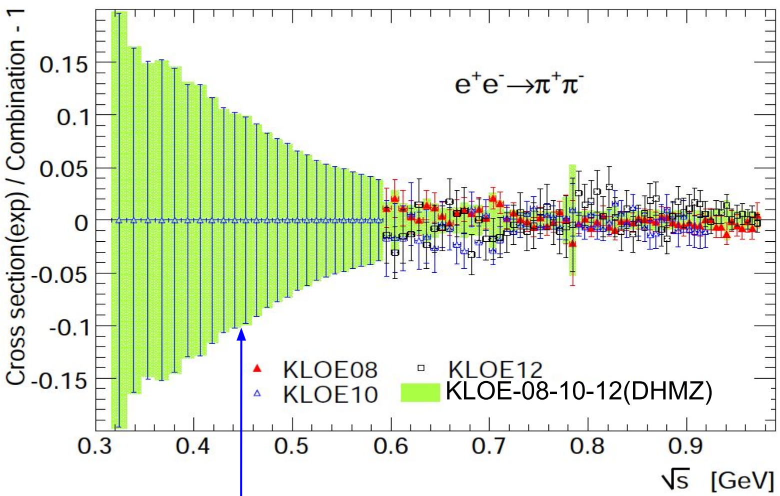
Evaluation of uncertainties and correlations (e^+e^-)

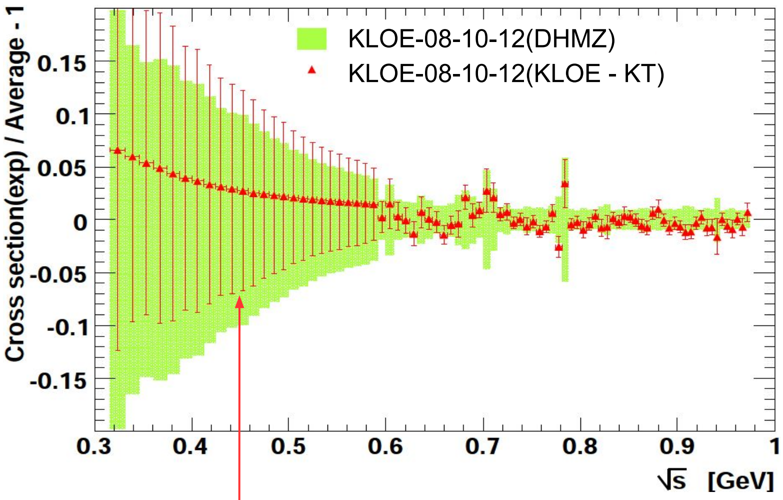
BABAR (1205.2228)

Sources	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.9	0.9-1.2	1.2-1.4	1.4-2.0	2.0-3.0
trigger/ filter	5.3	2.7	1.9	1.0	0.7	0.6	0.4	0.4
tracking	3.8	2.1	2.1	1.1	1.7	3.1	3.1	3.1
π-ID	10.1	2.5	6.2	2.4	4.2	10.1	10.1	10.1
background	3.5	4.3	5.2	1.0	3.0	7.0	12.0	50.0
acceptance	1.6	1.6	1.0	1.0	1.6	1.6	1.6	1.6
kinematic fit (χ^2)	0.9	0.9	0.3	0.3	0.9	0.9	0.9	0.9
correl $\mu\mu$ ID loss	3.0	2.0	3.0	1.3	2.0	3.0	10.0	10.0
$\pi\pi/\mu\mu$ non-cancel.	2.7	1.4	1.6	1.1	1.3	2.7	5.1	5.1
unfolding	1.0	2.7	2.7	1.0	1.3	1.0	1.0	1.0
ISR luminosity	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
sum (cross section)	13.8	8.1	10.2	5.0	6.5	13.9	19.8	52.4

→ Systematics *evaluated* in ~wide mass ranges with sharp transitions
(statistics limitations when going to narrow ranges)

Combining the 3 KLOE measurements





Local combination (DHMZ)

Information propagated between mass regions, through shifts of systematics - relying on correlations, amplitudes and shapes of systematics (KLOE-KT)

Combining the 3 KLOE measurements - $a_\mu^{\pi\pi}$ contribution

KLOE08 a_μ[0.6 ; 0.9] : 368.3 ± 3.2 [10^{-10}]
KLOE10 a_μ[0.6 ; 0.9] : 365.6 ± 3.3
KLOE12 a_μ[0.6 ; 0.9] : 366.8 ± 2.5

→ Correlation matrix:

	08	10	12
08	1	0.70	0.35
10	0.70	1	0.19
12	0.35	0.19	1

→ Amount of independent information provided by each measurement

→ KLOE-08-10-12(DHMZ) - a_μ[0.6 ; 0.9] : 366.5 ± 2.8 (Without χ^2 rescaling: ± 2.2)
→ Conservative treatment of uncertainties and correlations (*not perfectly known*) in weight determination

→ KLOE-08-10-12(KLOE-KT) - a_μ[0.6 ; 0.9]GeV : 366.9 ± 2.2 (Includes χ^2 rescaling)

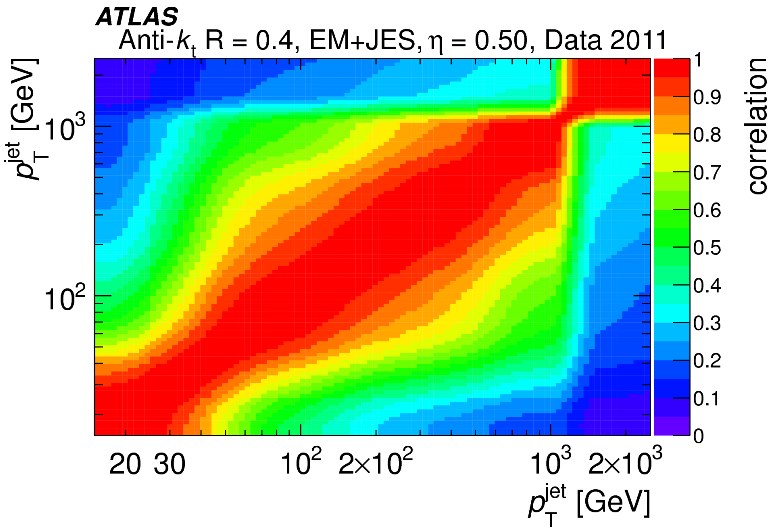
→ Assuming perfect knowledge of the correlations to minimize average uncertainty

Uncertainties on ATLAS JES correlations

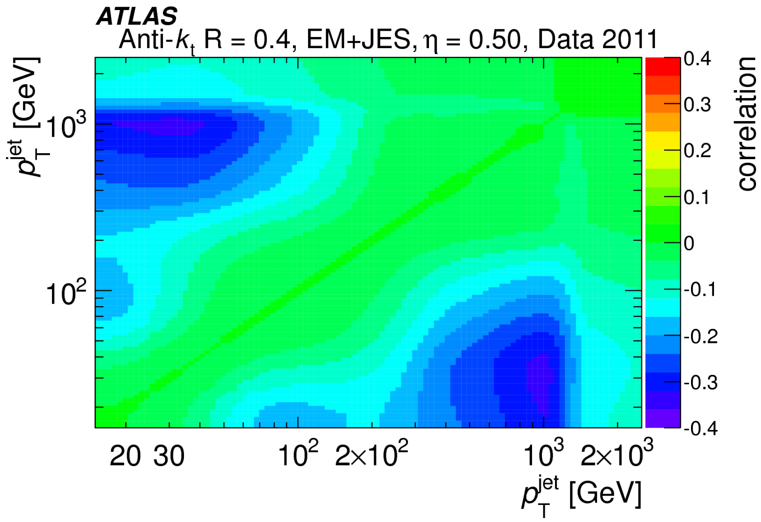
→ Derived two alternative configurations with stronger/weaker correlations w.r.t. nominal

(*Inspired remarks about uncertainties on uncertainties for combinations of hadronic spectra*)

Nominal correlation matrix



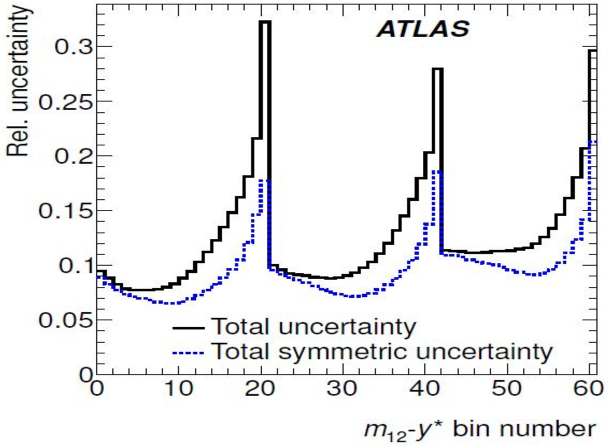
Strong - Weak correlation scenarios



χ^2 definitions and properties

$$\chi^2(\mathbf{d};\mathbf{t}) = \sum_{i,j} (d_i - t_i) \cdot \left[C^{-1}(\mathbf{t})\right]_{ij} \cdot (d_j - t_j) \qquad C_{ij} = C_{ij}^{stat} + \sum_k s_i^k \cdot s_j^k$$

$$\chi^2(\mathbf{d};\mathbf{t}) = \min_{\beta_a} \left\{ \sum_{i,j} \left[d_i - \left(1 + \sum_a \beta_a \cdot \left(\boldsymbol{\epsilon}_a^{\pm}(\beta_a)\right)_i \right) t_i \right] \cdot \left[C_{\mathrm{su}}^{-1}(\mathbf{t})\right]_{ij} \cdot \left[d_j - \left(1 + \sum_a \beta_a \cdot \left(\boldsymbol{\epsilon}_a^{\pm}(\beta_a)\right)_j \right) t_j \right] + \sum_a \beta_a^2 \right\},$$



→ Two χ^2 definitions, with systematic uncertainties included in covariance matrix or treated as fitted "nuisance parameters"

→ Equivalent for symmetric Gaussian uncertainties

(1312.3524 - ATLAS)

→ *Both approaches assume the knowledge of the amplitude, shape (phase-space dependence) and correlations of systematic uncertainties*

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