

Pion vector form factor with explicit inelasticities

A dispersive approach

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Motivation

Current situation:

- Tension in the 2π data
 - Differences in datasets are almost exclusively contained in inelastic contributions [Stoffer et al. \(2023\)](#)
 - Inelastic effects are combined as a multiplicative factor in the current dispersive treatment [Leplumey and Stoffer \(2025\)](#)
 - No unified fits available for multi-channel treatments [Achasov and Kozhevnikov \(2013\)](#)
-

Two-potential model:

[Hanhart \(2012\)](#)

- Based on unitarity and analytic properties of the S-matrix
- Inelastic channels built explicitly and fit to data

Two-potential model: diagrammatic representation

Background T-matrix:

$$T_B = V_B + V_B \Pi T_B$$

Resonance T-matrix:

$$T_R = g^T \frac{1}{m^2-s} g + g^T \frac{1}{m^2-s} g \Sigma T_R$$

Form factor:

$$F = a \frac{1}{m^2-s} g + a \frac{1}{m^2-s} g \Pi T$$

Two-potential model

$$V_B(s)_{ab} = f_{0,ab} \quad \Longrightarrow \quad T_B = (1 - V_B \Pi)^{-1} V_B$$

$$\text{with } \Pi(s) = \frac{s}{\pi} \int_{s_{\text{thr}}}^{\infty} \frac{ds'}{s'} \frac{\rho(s') \xi(s')^2}{s' - s - i\epsilon} + \text{const.}$$

$$\gamma(s) = 1 + T_B(s) \Pi(s) = \exp \left(\frac{s}{\pi} \int_{s_{\text{thr}}}^{\infty} \frac{ds'}{s'} \frac{\delta_B(s')}{s' - s - i\epsilon} \right)$$

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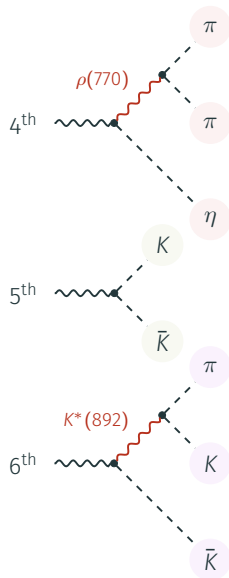
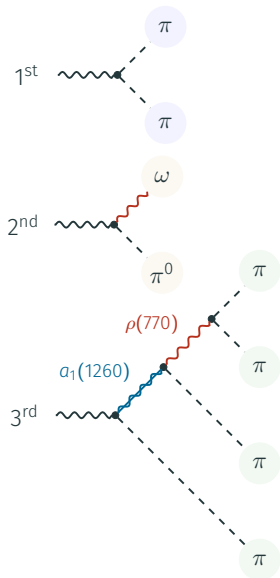
$$V_R(s)_{ab} = \sum_{k=1}^{n_R} \frac{g_a^k g_b^k}{m_k^2 - s} \quad \Longrightarrow \quad T_R = (1 - V_R \Sigma)^{-1} V_R$$

$$\text{with } \Sigma(s) = \frac{s}{\pi} \int_{s_{\text{thr}}}^{\infty} \frac{ds'}{s'} \frac{\rho(s') \xi(s')^2 \gamma(s')^2}{s' - s - i\epsilon} + \text{const.}$$

$$\boxed{F = \gamma (1 - V_R \Sigma)^{-1} P} \quad \text{with } P(s)_b = \sum_{k=1}^{n_R} \frac{g_b^k a_k}{m_k^2 - s}$$

Application to $\sigma(500)$, $\rho(770)$: Heuser et al. (2024).

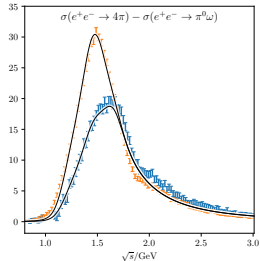
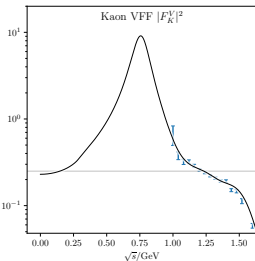
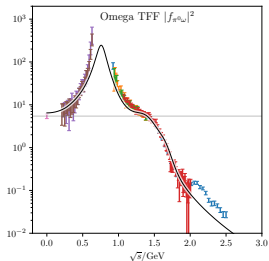
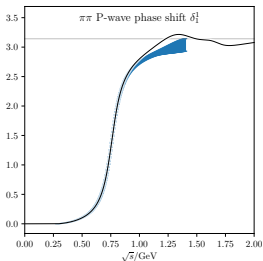
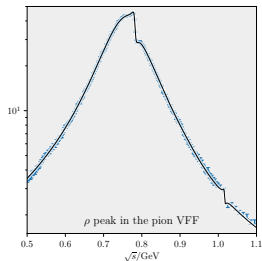
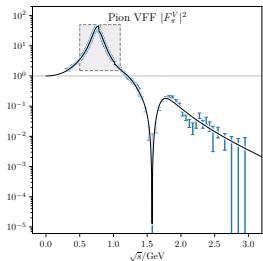
Included channels ($J = 1, l = 1$)



Preliminary results

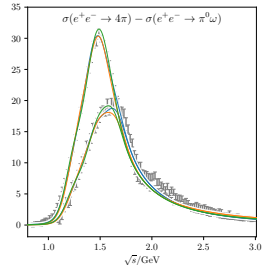
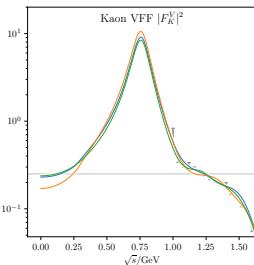
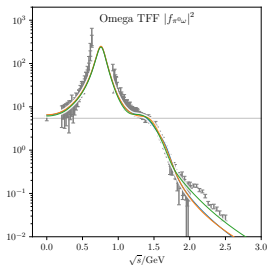
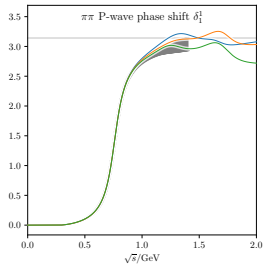
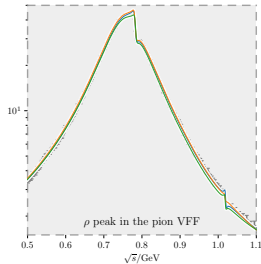
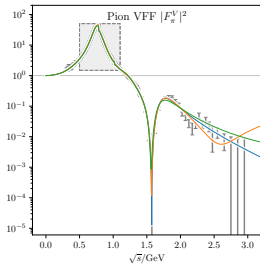
Example fit (4 resonances)

[data references in backup slides]

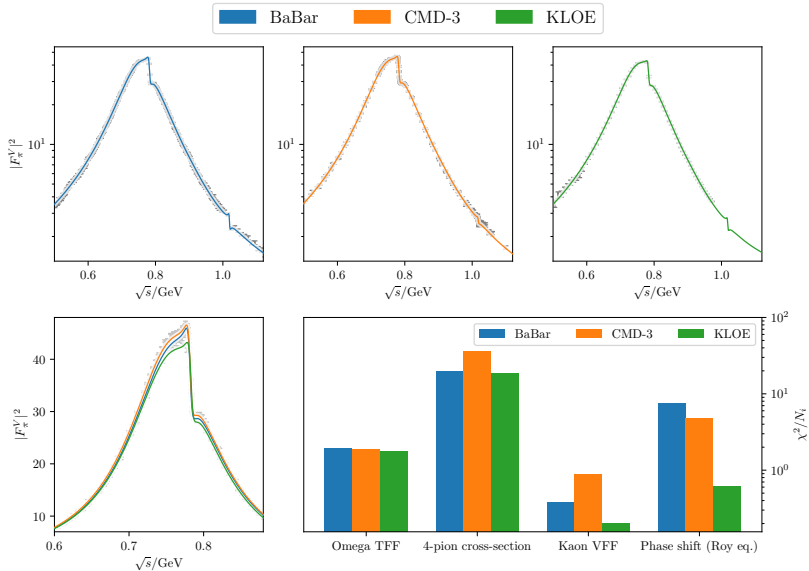


Fit comparison for $\pi\pi$ datasets (4 resonances)

■ BaBar
 ■ CMD-3
 ■ KLOE



Fit comparison for $\pi\pi$ datasets (4 resonances)



What about errors? Next slide!

To do before drawing conclusions:

- More datasets (*rest of the $\pi\pi$ data*, tail from the *τ decay data*)
- Avoid local minima by *unifying initial fit parameters*
- Generate *correlation matrix* for χ^2 contributions
- Avoid bias via *d'Agostini procedure*
- Incorporate *energy rescaling*
- Generate systematic uncertainties by varying e.g. *number of resonances* (ask me about the backup slide with 5 resonances)

Stay tuned for the arXiv submission!

Summary, outlook

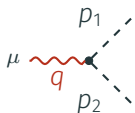
- Based on dispersion relations, a **unified description of the dominant inelastic channels** was constructed
 - Consistent with unitarity and analyticity
 - Preliminary results show the **difference in quality of the inelastic description**, when comparing different 2π datasets
 - Further study is required to make statements about **statistical significance** of these differences
-

Related projects:

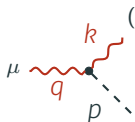
- Resonance pole positions $(\rho, \rho', \rho'', \dots)$ → **Leon Heuser** (poster)
- $\omega \rightarrow 3\pi$ Dalitz plot parameters → **Miriam Penners** (poster)
- QCD sum rules → **Yaxuan Huang**

Backup slides

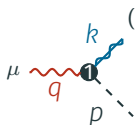
Vertex structures



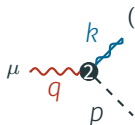
$$\sim \xi_{VPP}^\mu = (p_1 - p_2)^\mu,$$



$$\sim \xi_{VVP}^{(\lambda)\mu} = \epsilon^{\mu\nu\alpha\beta} n_\nu^{(\lambda)} p_\alpha q_\beta,$$



$$\sim \xi_{VAP,1}^{(\lambda)\mu} = (q \cdot k) n^{(\lambda)\mu} - (n^{(\lambda)} \cdot q) k^\mu,$$



$$\sim \xi_{VAP,2}^{(\lambda)\mu} = q^2 k^2 n^{(\lambda)\mu} + (q \cdot k)(n^{(\lambda)} \cdot q) q^\mu - k^2 (n^{(\lambda)} \cdot q) q^\mu.$$

Semistable channels

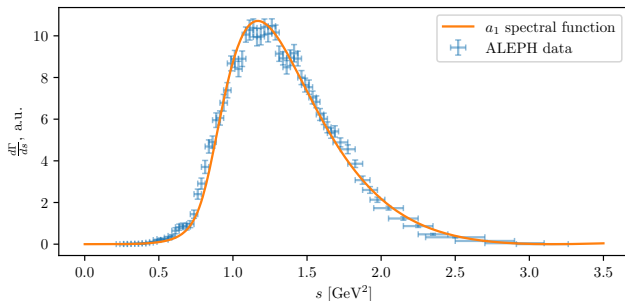
Spectral function for an unstable state A ,

$$\sigma_A(q^2) = -\frac{1}{\pi} \text{Im} (G_A(q^2)) , \quad \text{where} \quad G_A(q^2) = \frac{1}{q^2 - M_A^2 + iM_A\Gamma_A(q^2)}$$

$$\text{Disc}\Pi_{\pi\pi\eta}(s) = 2i \int_{(2M_\pi)^2}^{(\sqrt{s}-M_\eta)^2} dq_2^2 \sigma_\rho(q_2^2) \rho(s, q_2^2, M_\eta^2) \xi_{VVP}^2(s, q_2^2, M_\eta^2)$$

$$\text{Disc}\Pi_{\pi KK}(s) = 2i \int_{(M_\pi+M_K)^2}^{(\sqrt{s}-M_K)^2} dq_3^2 \sigma_{K^*}(q_3^2) \rho(s, q_3^2, M_K^2) \xi_{VVP}^2(s, q_3^2, M_K^2)$$

Semistable channels: 4π and the a_1 spectral function

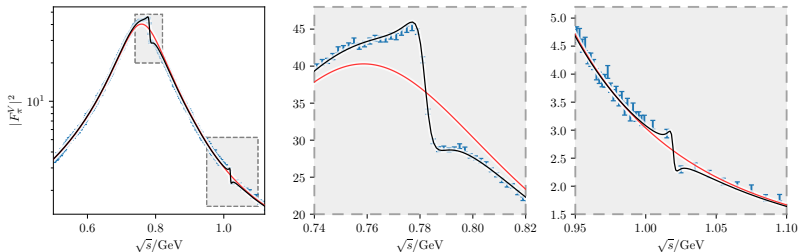


$$\text{Disc}\Pi_{4\pi}(s) = 2i \int_{(3M_\pi)^2}^{(\sqrt{s}-M_\pi)^2} dq_1^2 \sigma_{a_1}(q_1^2) \rho(s, q_1^2, M_\pi^2)$$

$$\text{Disc}\Pi_{\rho\pi}(q_1^2) = 2i \int_{(2M_\pi)^2}^{(\sqrt{q_1^2}-M_\pi)^2} dq_2^2 \sigma_\rho(q_2^2) \rho(q_1^2, q_2^2, M_\pi^2) \xi_{VVP}^2(q_1^2, q_2^2, M_\pi^2)$$

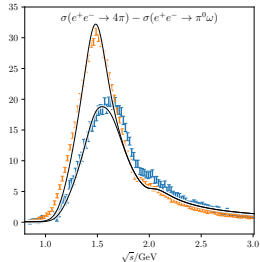
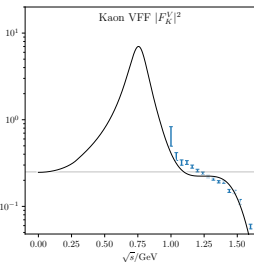
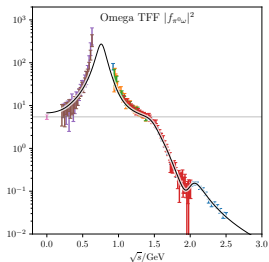
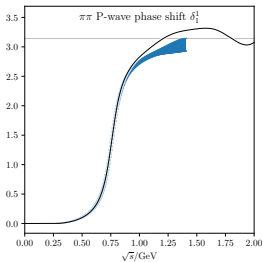
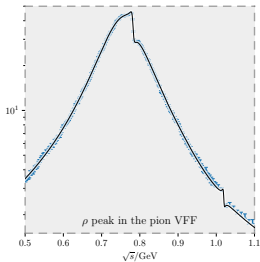
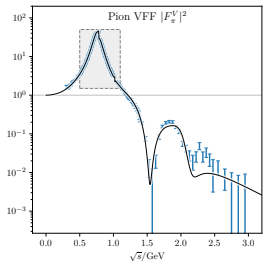
$$\Gamma_{a_1}(q_1^2) = \frac{g_{a_1\rho\pi}^2}{iM_{a_1}} \Pi_{\rho\pi}(q_2^2), \quad \Gamma_\rho(q_2^2) = \frac{g_{\rho\pi\pi}^2}{iM_\rho} \Pi_{\pi\pi}(q_2^2)$$

$\rho - \omega$ and $\rho - \phi$ mixing



$$F_{\pi}^V(s) \mapsto F_{\pi}^V(s) \left(1 + \kappa_{\omega} \frac{s}{s - m_{\omega}^2 + im_{\omega}\Gamma_{\omega}} + \kappa_{\phi} \frac{s}{s - m_{\phi}^2 + im_{\phi}\Gamma_{\phi}} \right)$$

Fit parameters: κ_{ω} , m_{ω} , Γ_{ω} , κ_{ϕ} , m_{ϕ} , Γ_{ϕ} .



Bias correction and energy rescaling

Bias correction (iterative procedure)

D'Agostini (1994)

$$\begin{aligned}\chi^2 &= (f(x_i) - y_i)^T \text{Cov}_{ij}^{-1} (f(x_j) - y_j) \\ \text{Cov}_{ij}^n &= \text{Cov}_{ij}^{\text{stat}} + \text{Cov}_{ij}^{\text{sys},n} \\ \text{Cov}_{ij}^{\text{sys},n} &= \frac{\text{Cov}_{ij}^{\text{sys}}}{y_i y_j} f^{n-1}(x_i) f^{n-1}(x_j)\end{aligned}$$

Energy rescaling

Colangelo et al. (2019)

$$\begin{aligned}\sqrt{s_i} &\mapsto \sqrt{s_i} + \xi_j (\sqrt{s_i} - \sqrt{s_{c,thr}}) \\ |F_c^j|_i^2 &\mapsto |F_c^j|_i^2 (1 + \xi_j A_c(s_i)) + \mathcal{O}(\xi_j^2) + \mathcal{O}(m_e^2)\end{aligned}$$

Data sources

$\pi^+\pi^-$ production: BaBar [Lees et al. (2012a)], KLOE [Anastasi et al. (2018)], CMD-3 [Ignatov et al. (2024)], Belle [Fujikawa et al. (2008)]

$\pi\pi$ P-wave phase shift: Bern [Colangelo (2004); Caprini et al. (2012)]

$\pi^0\omega$ production: BaBar [Lees et al. (2017)], CMD-2 [Akhmetshin et al. (2003)], SND [Achasov et al. (2000, 2016)]

ω decay: NA60 [Arnaldi et al. (2009, 2016)]

4π production: BaBar [Lees et al. (2012b, 2017)]

Kaon VFF from τ decay: BaBar [Lees et al. (2018)]

$\pi\pi\eta$ production: BaBar [Aubert et al. (2007)], CMD-2 [Akhmetshin et al. (2000)], SND [Aulchenko et al. (2015)]

$KK\pi$ production: BaBar [Aubert et al. (2008)]

a_1 spectral function from τ decay: ALEPH [Davier et al. (2014)]

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