HVP update from Fermilab Lattice, HPQCD, and MILC

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Fermilab-HPQCD-MILC Acknowledgements

Collaboration Members

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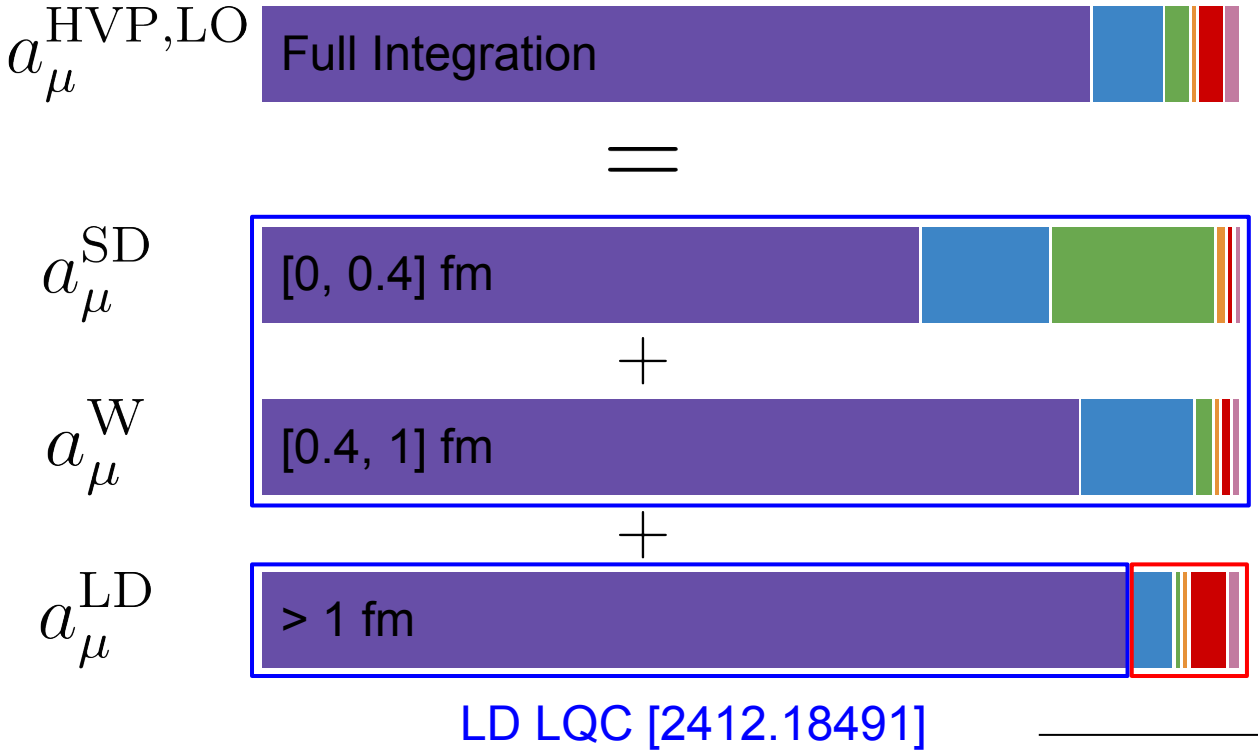
(Fermilab Lattice, HPQCD, and MILC Collaborations)

Talk Outline

- Overview
- Completed Analyses
- Remaining Contributions
- Upcoming Projects
- Summary and Outlook

Overview

Fermilab/HPQCD/MILC HVP Progress



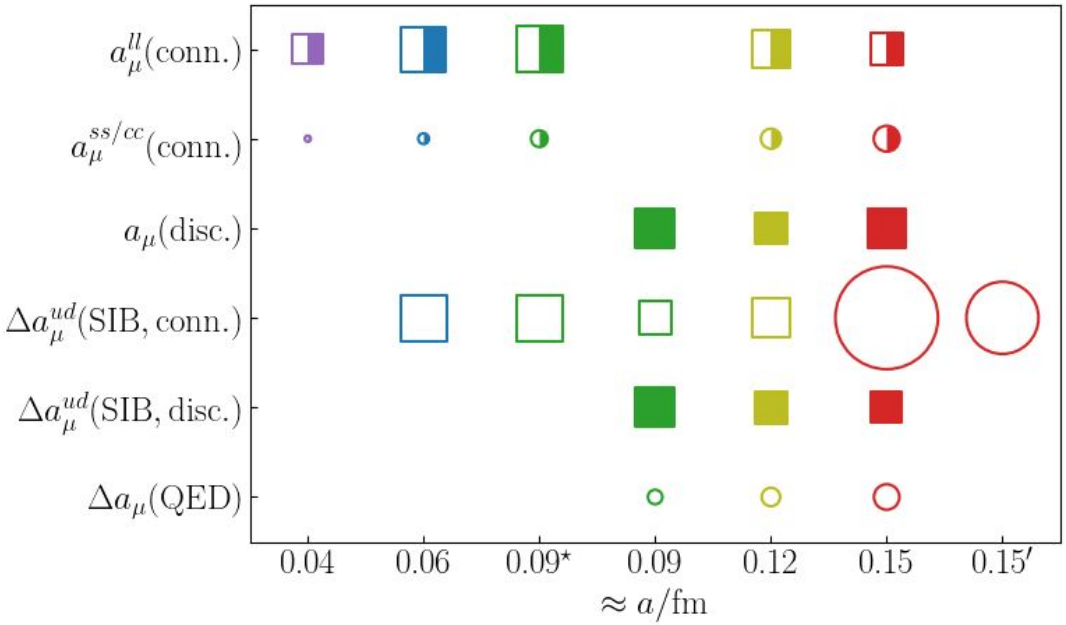
Light Conn. ~ 88%
Strange Conn. ~ 7%
Charm Conn. ~ 2%
b, t Conn. < 0.05%
Disc. ~ 2%
Iso-Breaking ~ 1%

SD & W
[2411.09656]

Upcoming paper
on last ~5%

SD+W+LD cross-checked with Full Integration

$N_f = 2 + 1 + 1$ HISQ Physical-Mass Ensembles



- All tuned within ~4% of physical meson masses [1]
- $M_\pi L = 3.7 - 4.1$ [2]

- Open: Local vector current
- Solid: One-link vector current
- Squares: Low-mode averaged
- Size ~ Statistics $(n_{\text{cfg}} \times n_{\text{src}})$
- Colors: Lattice spacing

- Other contributions:
  - Bottom contribution from HPQCD [2101.08103]
  - QED disconnected and sea in early stages

[1] Except original 0.09 fm (~5.7%)
[2] Except 0.15 fm ($M_\pi L = 3.4$)

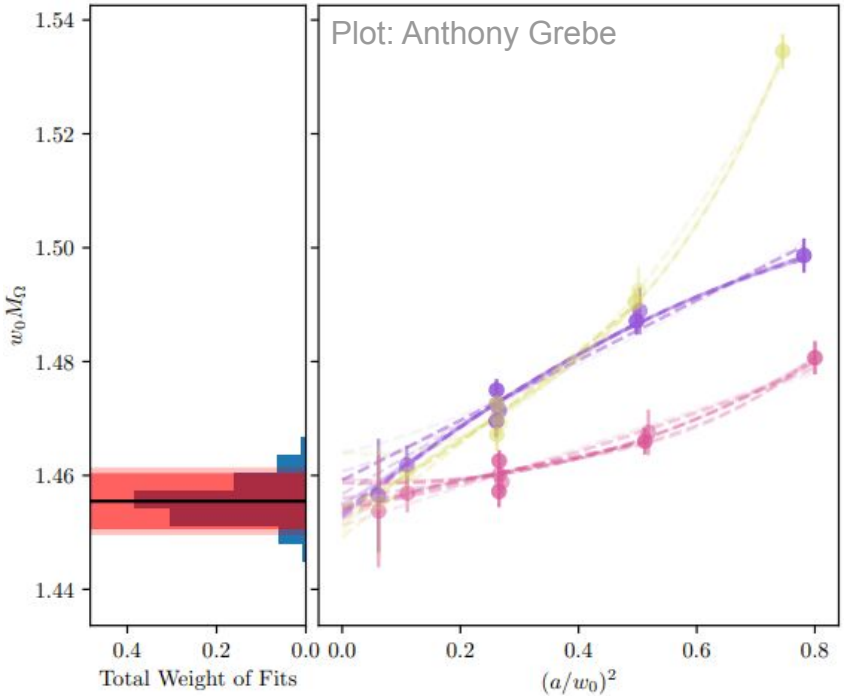
Updated Scale Setting

- New precision results for w_0 (fm) (4 per mil) and M_Ω ($\lesssim$2 per mil with aM_Ω)[1] scales

$$w_0 = 0.17187(68)\ \text{fm}$$
$$M_\Omega = 1671.01(43)\ \text{MeV}$$

- QED corrections included
- Quantification of systematics including quark-mass dependence, FV, …

- Continued cross-checks with f_π scale for HVP



"High-Precision Scale Setting with the Omega-Baryon Mass and Gradient Flow" [2509.XXXXX]

[1] This precision for a~0.04 fm in progress

Analysis Strategy

- Each analysis independently blinded
  - Multiplicative blinds for all observables (secondary additive blind for SIB conn.)
  - “Reblinding” of strange and charm with different w_0/a scheme
- Lattice artifacts
  - Correlator reconstruction for LD tail
  - FV, mass mistuning, TB corrections from EFT and EFT-inspired models (χPT, CM, MLLGS, HP)
  - (Chiral) continuum fit variations (local and one-link currents used where available)
- Uncertainty quantification
  - Systematic uncertainties and covariances estimated using Bayesian model averaging (BMA) with Bayesian Akaike information criterion (BAIC) weights
  - Error propagation cross-checked with global bootstrap

Recap of Completed Analyses

Complete SD & W and Full Light Quark Connected

	Results*	Rel. Err.	Largest Errs.
a_μ^{SD}	$69.05(1)(21)[21] \times 10^{-10}$	0.31%	$a \to 0$
a_μ^{W}	$236.45(17)(83)[85] \times 10^{-10}$	0.36%	FV, $a \to 0$, scale
$a_\mu^{ll,\mathrm{LD}}(\mathrm{conn.})$	$400.2(2.3)(3.7)[4.3] \times 10^{-10}$	1.1%	$a \to 0$, stat, (scale, FV)
$a_\mu^{ll,\mathrm{SD+W+LD}}(\mathrm{conn.})$	$655.2(2.3)(3.9)[4.5] \times 10^{-10}$	0.69 %	

*Errors: (stat)(syst)[total]

Upcoming Improvements:

- New strange and charm data: 0.042 fm and one-link current
- Lattice determination of QED corrections
- Additional low mode averaged data for Conn. SIB
- Ongoing data generation: statistics at 0.042 fm, large volume (0.09 fm), 0.06 fm disc, ...

See [2411.09656, 2412.18491] for details

Correlator Reconstruction: Fit vs Bounding Methods

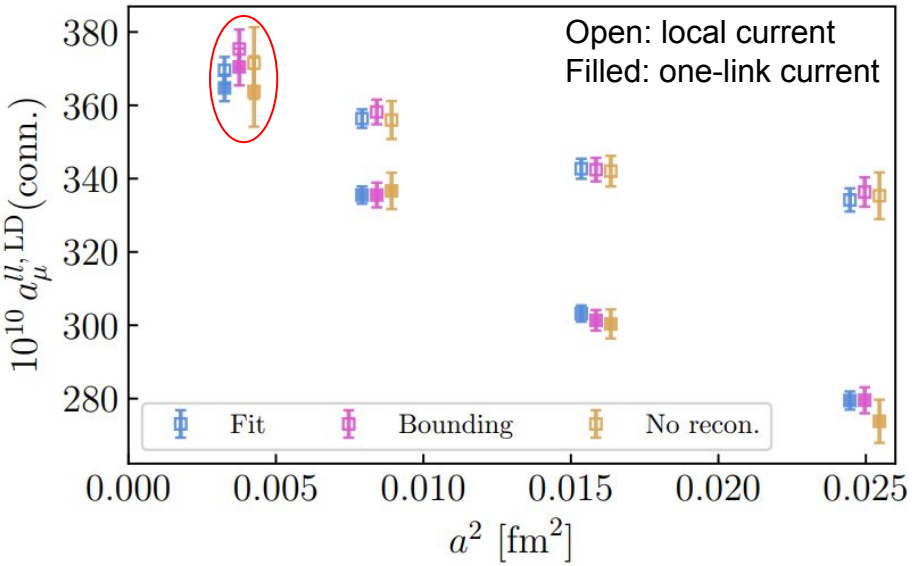
Fit Method

- Fit ansatz from spectral decomposition for amplitudes and energies
- Replace data with reconstruction after $t^\star$ (chosen to maximize data usage and minimize impact of noise)

Bounding Method

- Ground state energy bounded between $E_{\pi\pi}$ and $E_{0,\mathrm{fit}}$ ⇒ bound on correlator
- Replace data after bounds meet at t_c (amplitude set by $C(t_c)$)
- Systematic assigned by varying t_c

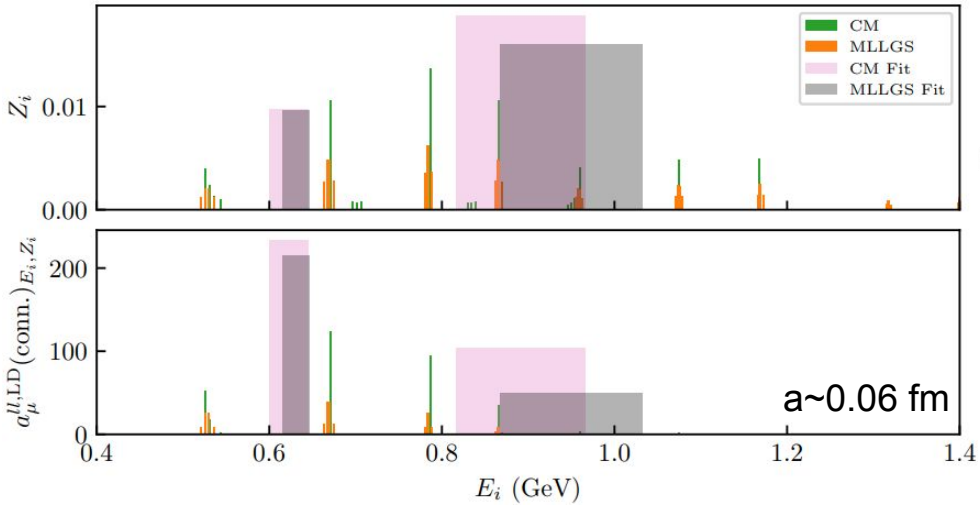
Tension between two methods:



See [2412.18491] for details

Fake-Data Tests

- Generate spectrum from EFT and EFT-inspired models (CM, MLLGS)
- Draw samples using (real) data covariance
- Reconstruct correlator with fit or bounding method



$$C_{\mathrm{fit}}(t) = \sum_{n=0}^{N_{\mathrm{states}}} Z_n^2 e^{-E_n t} + (-1)^t \sum_{m=0}^{M_{\mathrm{osc.\ states}}} Z_m^2 e^{-E_m t}$$

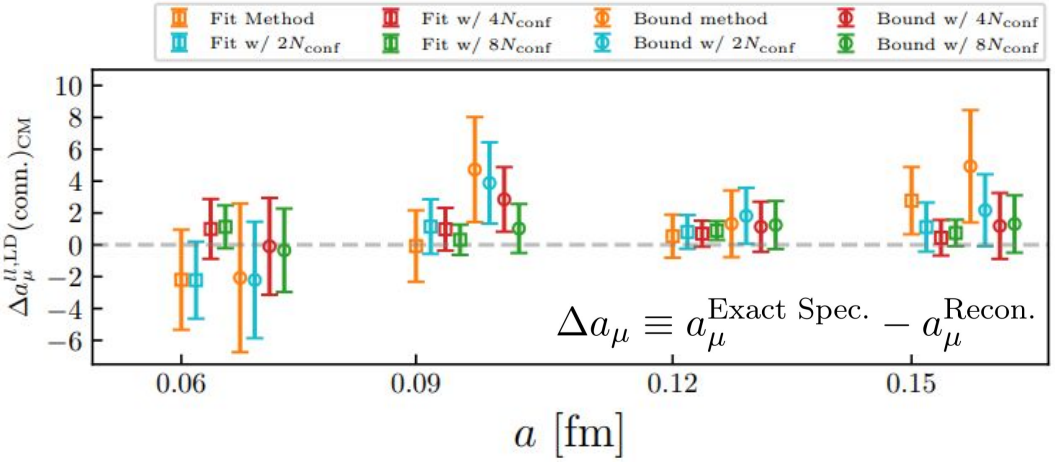
- Example of 2 state fit
- Note “imperfect” spectrum can still give reliable LD correlator reconstruction

See [2412.18491] for details

Fake-Data Tests

- Vary sample size to test convergence to model truth
- Results qualitatively consistent with varying random seeds

- Fit method consistent with truth (even with “imperfect” 2 state approximation)
- Bounding method less robust to stat fluctuations

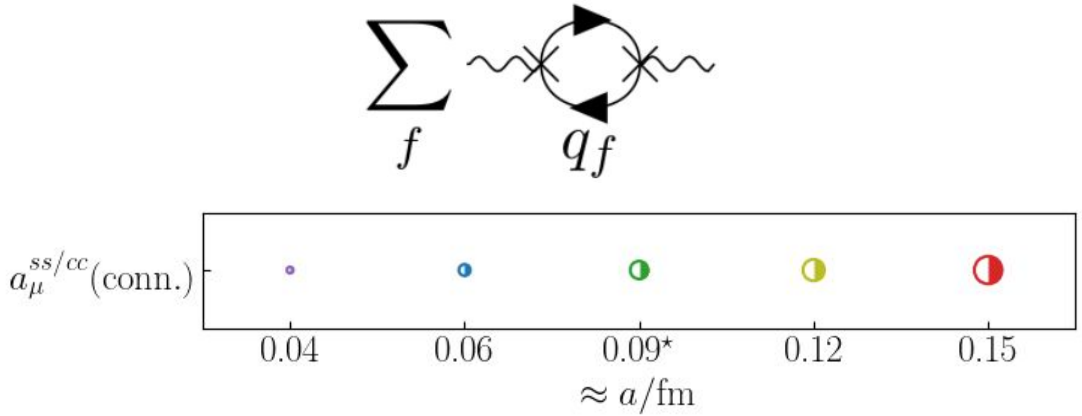


- We proceed with the fit method, with continued cross-checks

See [2412.18491] for details

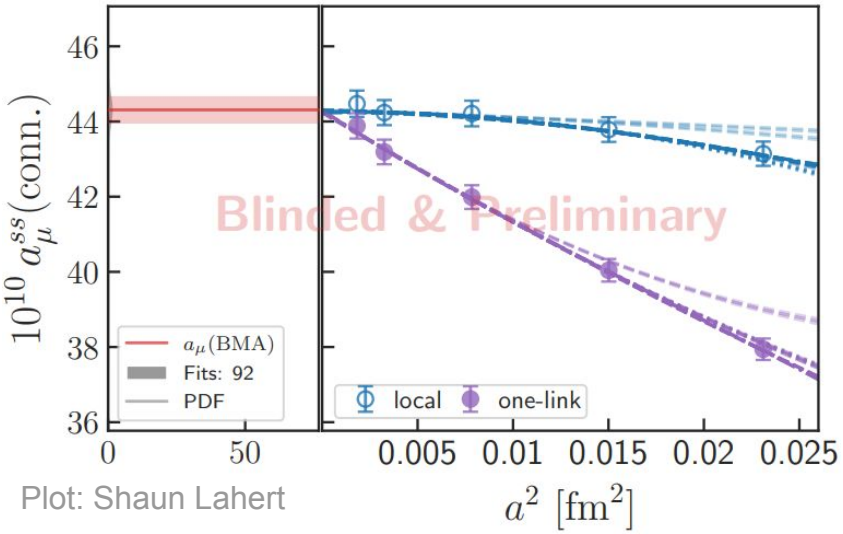
Status of Remaining Contributions

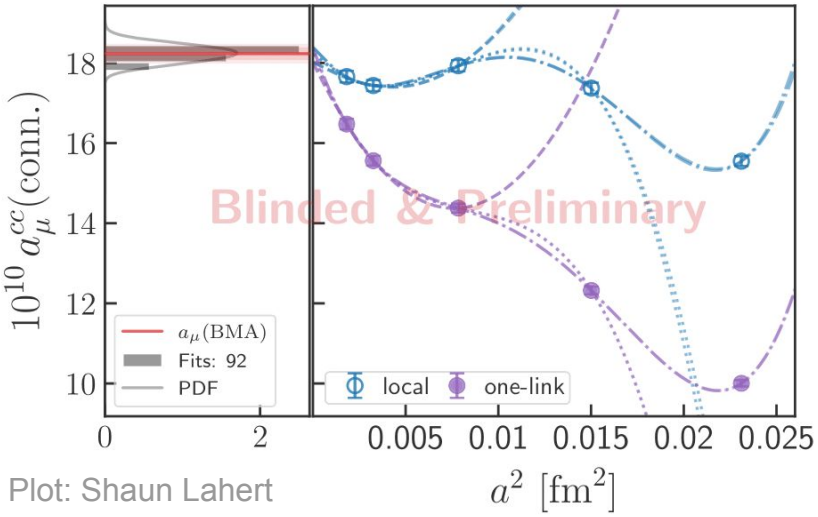
Strange and Charm Contributions



- New data: 0.042 fm, one-link current

Strange and Charm Contributions

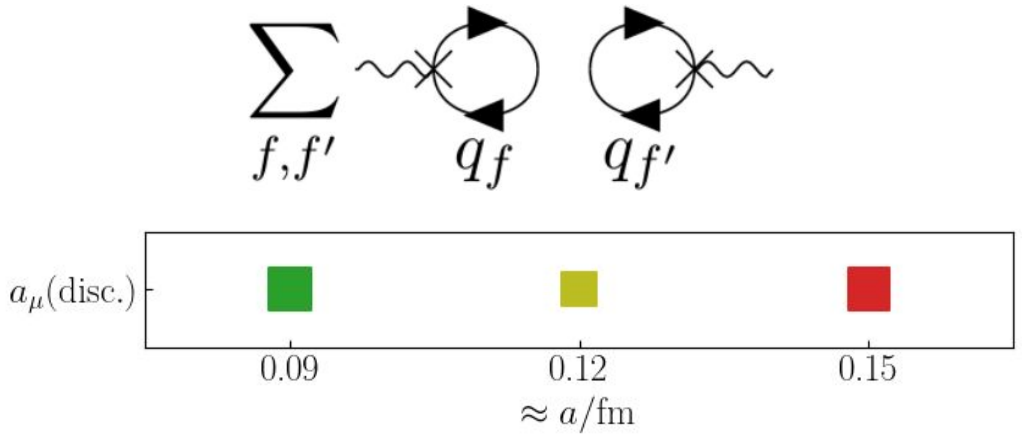




- Full Integration extrapolations shown
- Largest errors from scale setting and charm continuum limit

Other windows in backups

Disconnected Contribution



- Light and strange contributions

Data generation at 0.06 fm ongoing

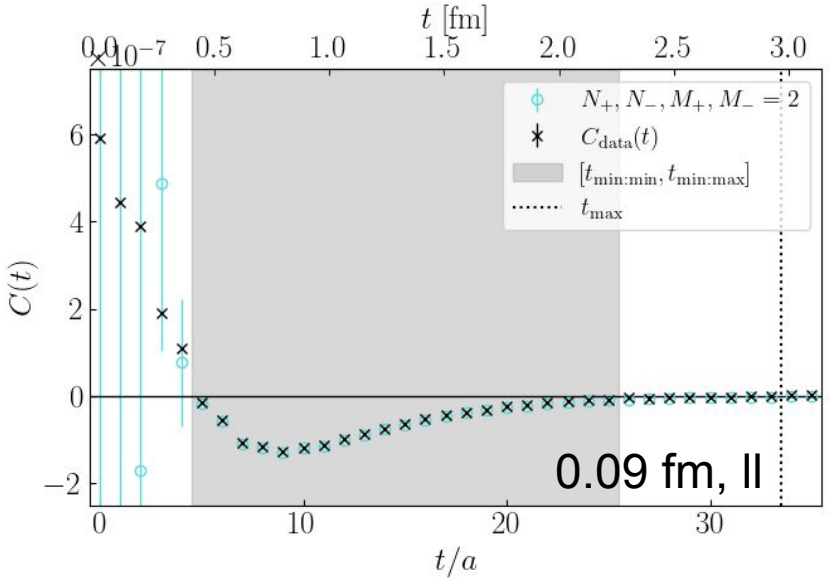
Disconnected Correlator Fits

- Non-positive definite correlator

$$C_{\text{disc}}(t) \sim C_{I=0}(t) - C_{I=1}(t)$$
$$\sim C_{\omega}(t) - C_{\rho}(t)$$

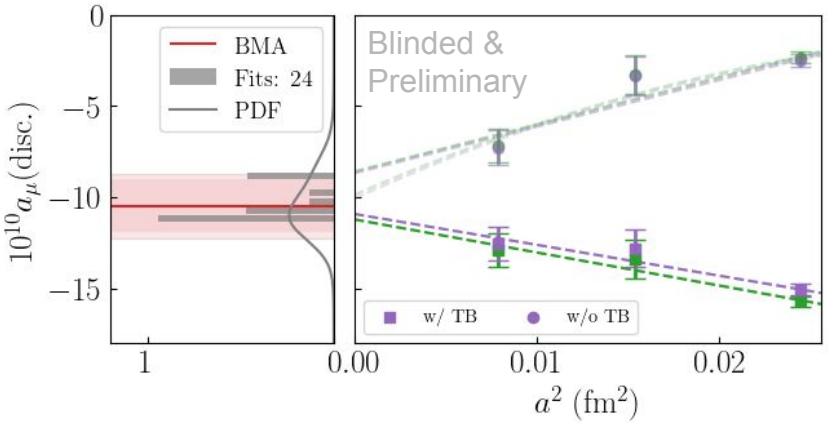
- Modified fit ansatz

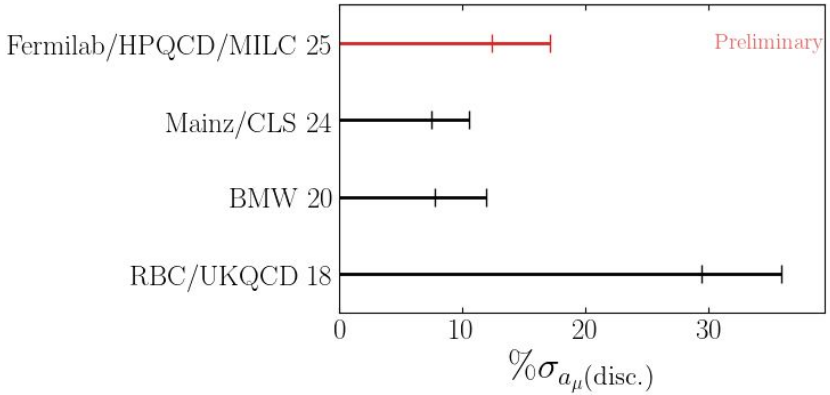
$$C_{\text{fit}}^{\text{disc}}(t) = \text{const} + \left[\sum_{n=0}^{N_+} Z_{n,+}^2(\cdot) + (-1)^t \sum_{m=0}^{M_+} Z_{m,+,\text{osc}}^2(\cdot)\right] - \left[\sum_{n=0}^{N_-} Z_{n,-}^2(\cdot) + (-1)^t \sum_{m=0}^{M_-} Z_{m,-,\text{osc}}^2(\cdot)\right]$$



- Cross-check with bounding method (on I = 0) underway

Disconnected Results





- Full Integration results shown
- Largest errors from statistics and continuum limit

- Relative error comparisons for Full Integration
- Inner error bars are statistics

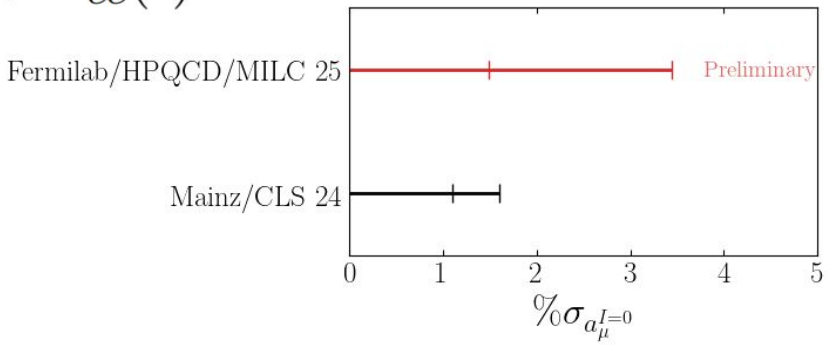
Isoscalar Channel

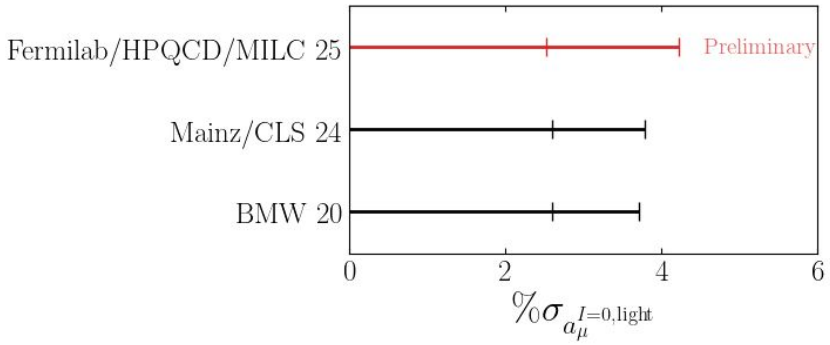
$$C^{I=0}(t) \equiv \underbrace{\frac{1}{10}C_{ll}(t) + C_{\text{disc}}(t)}_{\equiv C^{I=0,\text{light}}} + C_{ss(t)} + C_{cc}(t)$$

- Cancellation of dominant I = 1 systematic effects
- Full I = 0 channel currently only data on coarsest ensembles (0.09–0.15 fm, one-link)
  ⇒ Large discretization uncertainty
- I = 0,light cross-check for disc. underway

$$a_\mu(\text{disc.}) = a_\mu^{I=0,\text{light}} - \frac{1}{10}a_\mu^{ll}(\text{conn.})$$

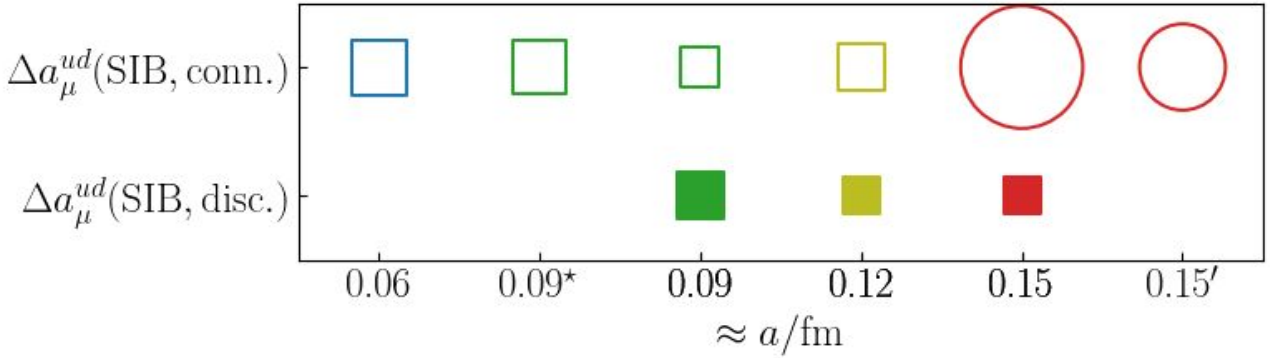
- Relative error comparisons for Full Integration





Mainz/CLS 24: ss/cc subtracted/added neglecting correlations
BMW 20: value read from figure

Strong Isospin Breaking Corrections

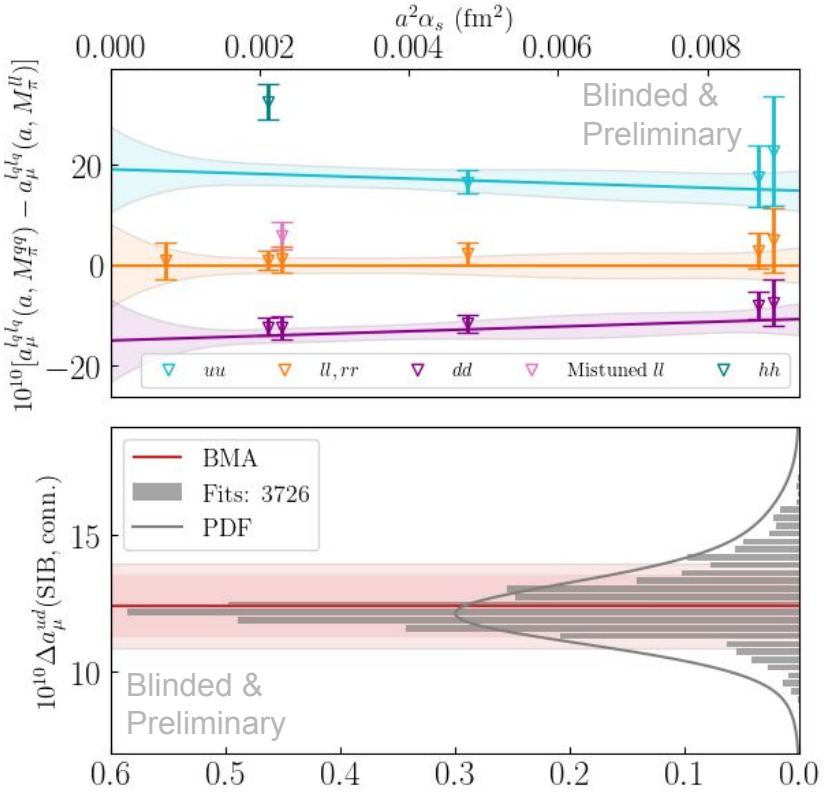


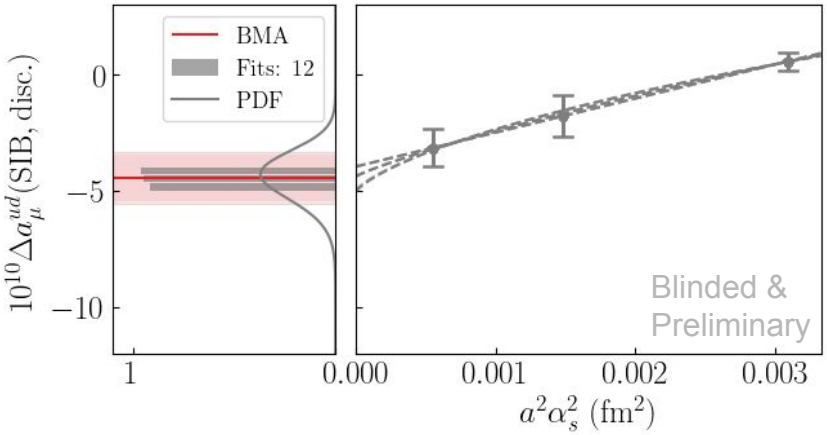
$$\Delta a_\mu^{ud}(\mathrm{SIB}) = \Delta a_\mu^{ud}(\mathrm{SIB,conn.}) + \Delta a_\mu^{ud}(\mathrm{SIB,disc.})$$
$$= a_\mu^{uu} + a_\mu^{dd} - a_\mu^{ll} = a_\mu^{ud} - a_\mu^{ll}$$

- Conn SIB: Chiral continuum extrapolation, compute correction in continuum
- Disc SIB: Continuum extrapolation of correction

LMA for SIB conn. a~0.15 ongoing

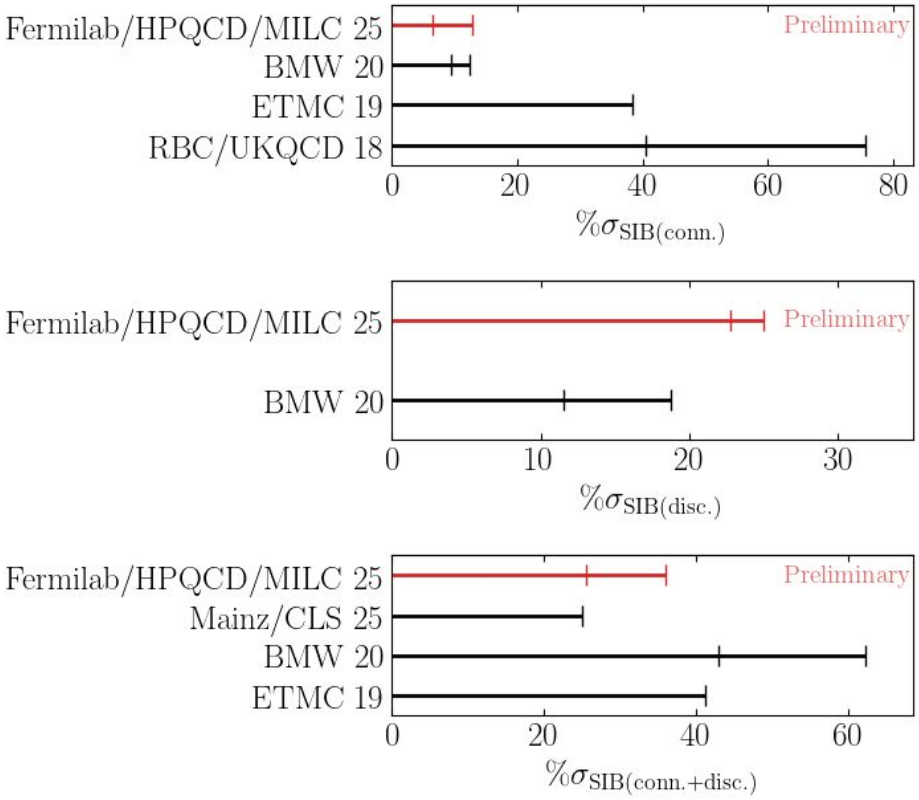
SIB Corrections





- Full Integration extrapolations shown
- Largest errors from statistics (SIB disc.) and continuum limit (SIB conn.)

SIB Relative Error Comparisons

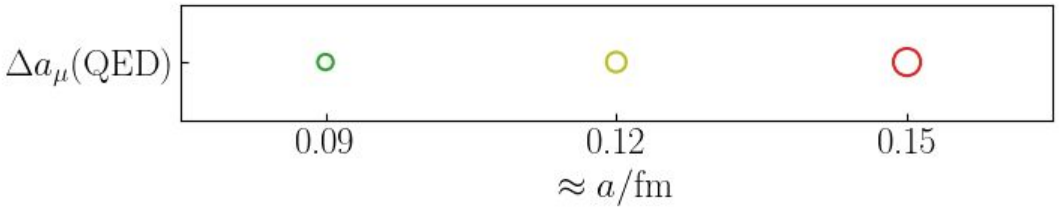


- Comparisons for Full Integration
- Inner error bars are statistics

- To account for blinding in conn+disc in the comparison, we assume central values from
  - Our previous conn SIB [1710.11212]
  - BMW 20 disc SIB

Mainz/CLS 25 see 2505.24344
ETMC 19 disc+conn assumes 15% systematic uncertainty for disc SIB (cf. 1901.10462)

QED Corrections

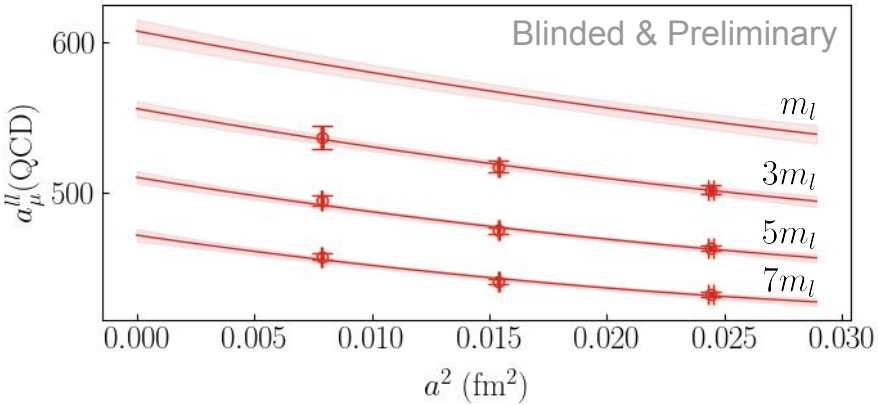


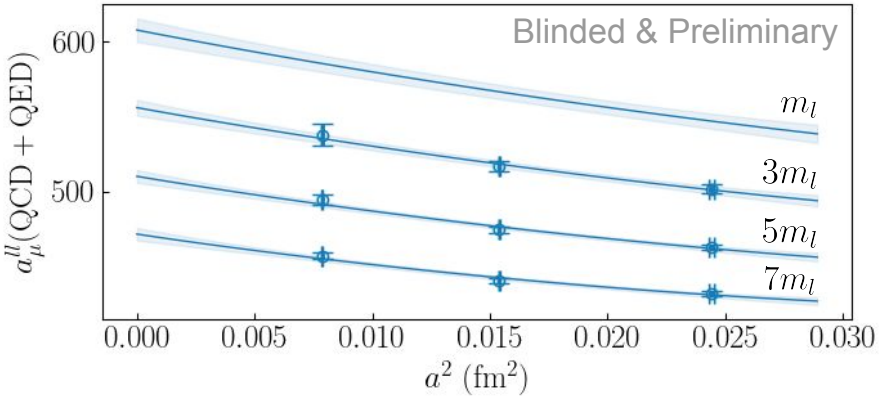
$$\Delta a_\mu^{ll/ss}(\mathrm{QED}) = a_\mu^{ll/ss}(\mathrm{QCD}+\mathrm{QED}) - a_\mu^{ll/ss}(\mathrm{QCD})$$

- Light and strange connected, valence QED corrections

QED disc. and sea project in early stages

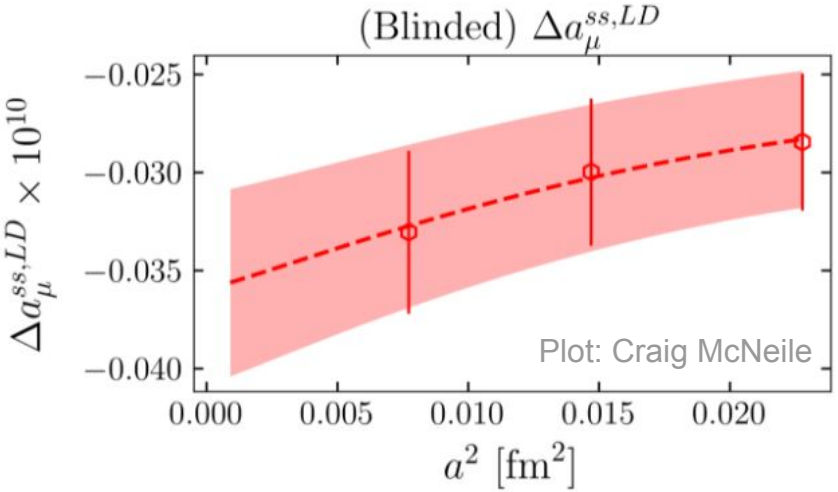
QED Light Corrections





- Full Integration extrapolations shown
- Chiral continuum extrapolation of 3/5/7m_l
- Cross-check of extrapolating differences underway

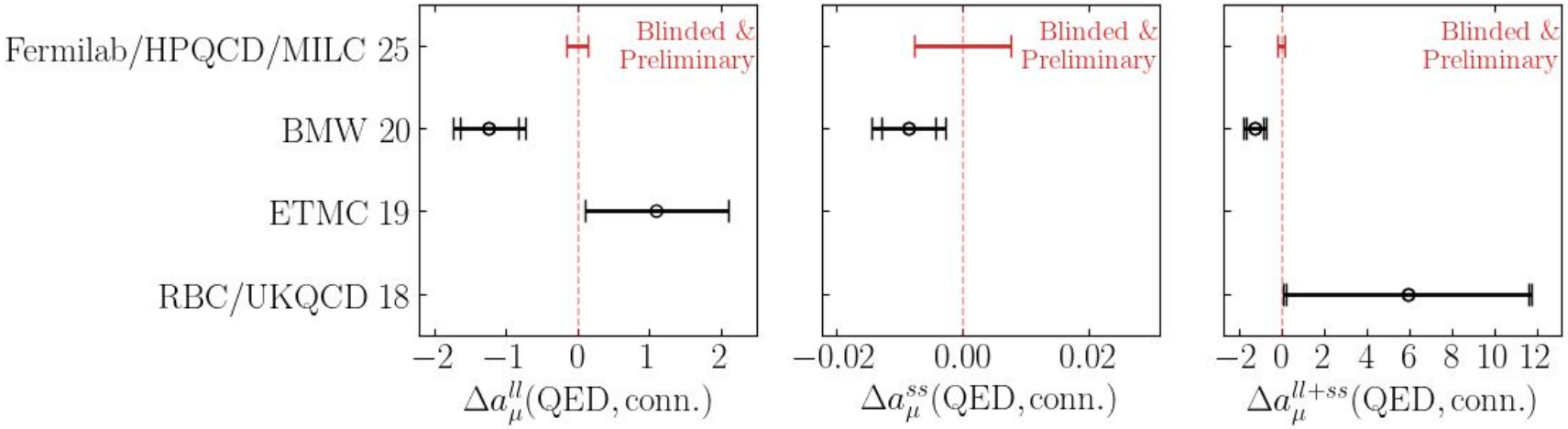
QED Strange Corrections



$$\Delta a_\mu^{ss} = a_\mu^{ss}(\mathrm{QCD} + \mathrm{QCD}) - a_\mu^{ss}(\mathrm{QCD})$$

- LD extrapolation shown
- Data evaluate at physical strange mass
- Sea mass mistuning corrected using η_s scheme (consistent with FLAG24)

QED Preliminary Error Comparisons



- Comparisons for Full Integration
- Blinded absolute errors shown, central values set to zero
- For sums of flavors and windows for our results in the comparison:
  - Conservatively assume 100% correlations
  - Errors inflated to cover blinding factors
  - Small visual effect

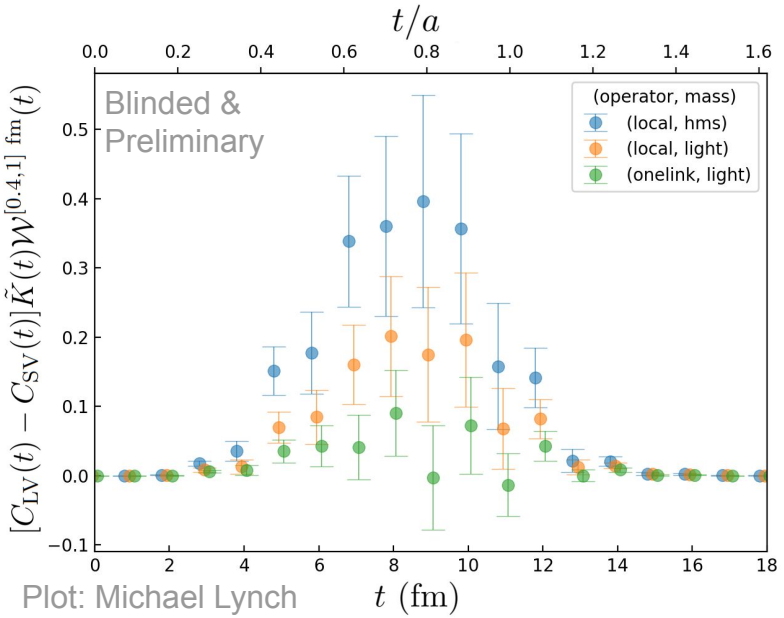
Summary and Outlook

- Complete SD and W and SD+W+LD light quark connected results published
- Analysis of LD contributions for strange, charm, disc, I = 0, SIB, QED in progress
- Preliminary errors on track for determination of LO HVP to ~1% precision

Ongoing and Upcoming Projects

Finite Volume Study

- FV effects currently estimated with EFT and EFT-inspired models (χPT, CM, MLLGS, HP)
- Upcoming data-driven FV determination with large volume (L~11 fm) ensemble at a~0.09 fm
- Data generation ongoing: light (local and one-link), “harmonic mean square” mass (local)

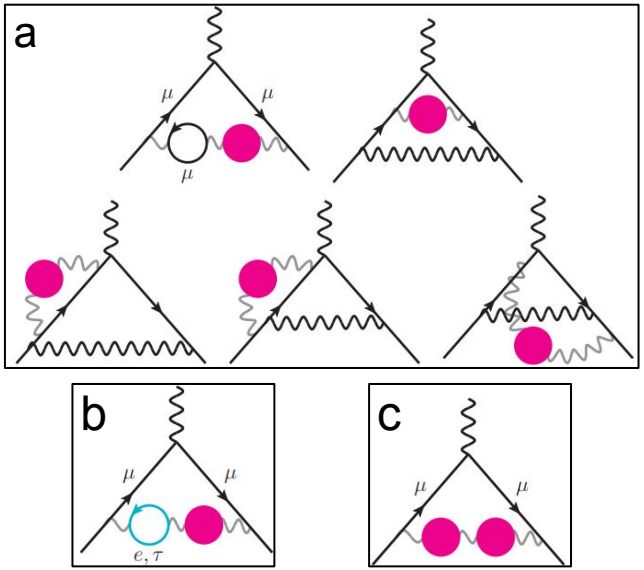


- Light quark connected integrand on W
- Current number of configs (LMA):
  - ~70 large volume (LV)
  - ~1000 small volume (SV)

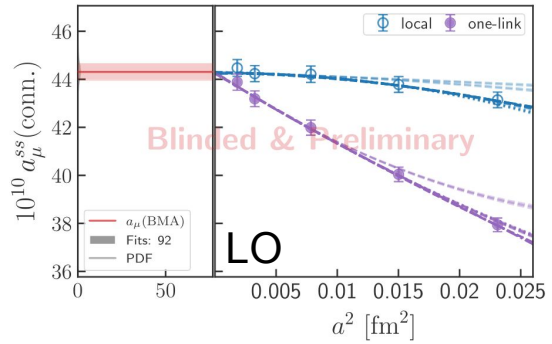
Other Ongoing Data Generation:

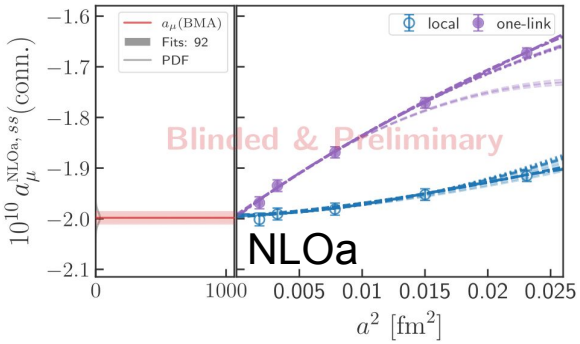
- Light-quark conn. statistics at a ~ 0.042 fm
- Disc. at a ~ 0.06 fm
- Two-pion for a < 0.15 fm

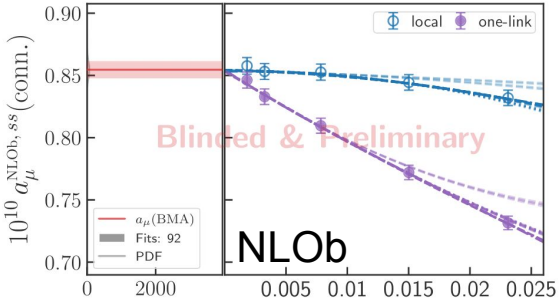
NLO HVP

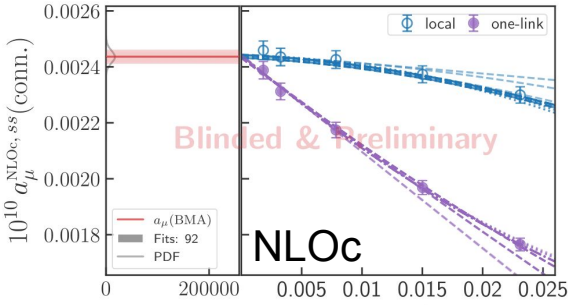


NLO diagrams
[1806.08190]









Plots: Shaun Lahert

Preliminary (N)LO Strange Extrapolations

Hadronic Contribution to EM Running Coupling ($\Delta\alpha$)

Lead analyst: Adrián del Pino (advised by Alejandro Vaquero)

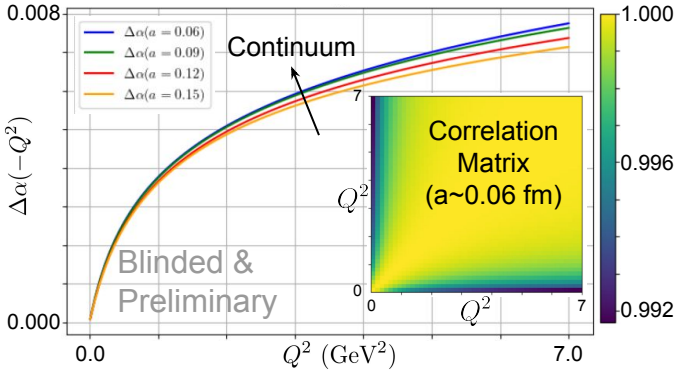
$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha(Q^2)}$$

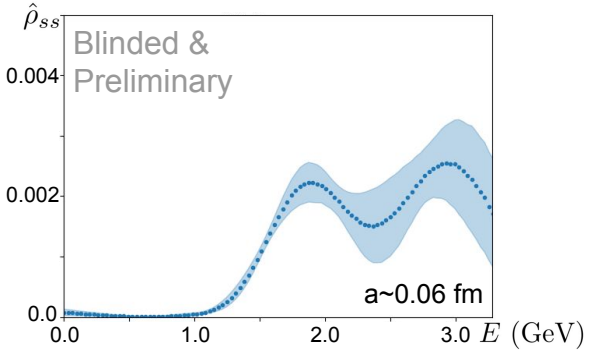
$$\Delta_{\text{had}}\alpha(Q^2) = 4\pi\alpha\hat{\Pi}^{\gamma\gamma}(Q^2)$$

- Extrapolations at different Q^2 highly correlated
  ⇒ Requires careful treatment of numerics
- Alternatively, extrapolate (smeared) spectral function and convolve with kernel in the continuum

$$\hat{\Pi}(Q^2) = \int \frac{\mathrm{d}\omega}{2\pi}\hat{\rho}(\omega)\bar{K}(\omega, Q^2)$$

⇒ Inverse problem (e.g., HLT)





Similar analysis for $\sin^2\theta_{\text{W}}(Q^2)$ also underway

Summary and Outlook

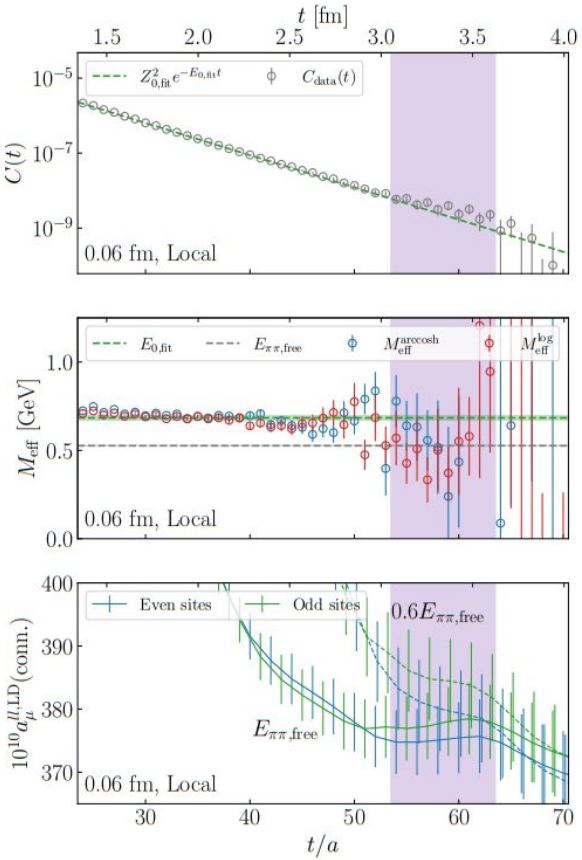
- Complete SD and W and SD+W+LD light quark connected results published
- Analysis of LD contributions for strange, charm, disc, I = 0, SIB, QED in progress
- Preliminary errors on track for determination of LO HVP to ~1% precision

- Ongoing data generation to reach ~5 per mil level
  - Large volume (L ~ 11 fm) at a ~ 0.09 fm
  - Light-quark conn. statistics at a ~ 0.042 fm
  - Disc. at a ~ 0.06 fm
  - Two-pion for a < 0.15 fm
- NLO HVP underway
- Data being repurposed for other projects (e.g., $\Delta\alpha$)

Thank you

Backup Slides

Further Evidence for Bias in the Bounding Method

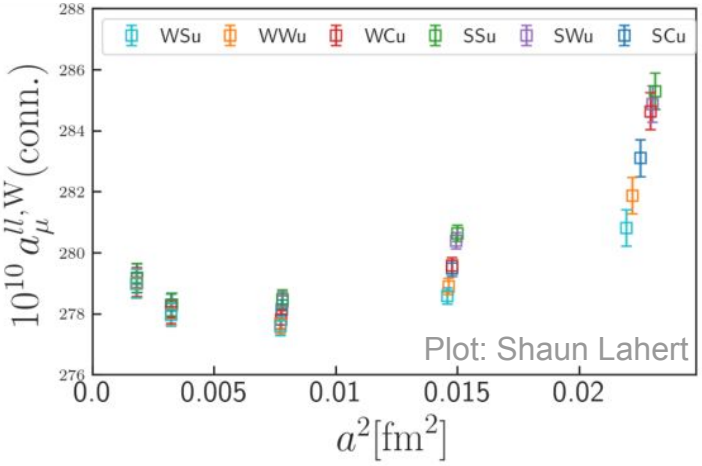


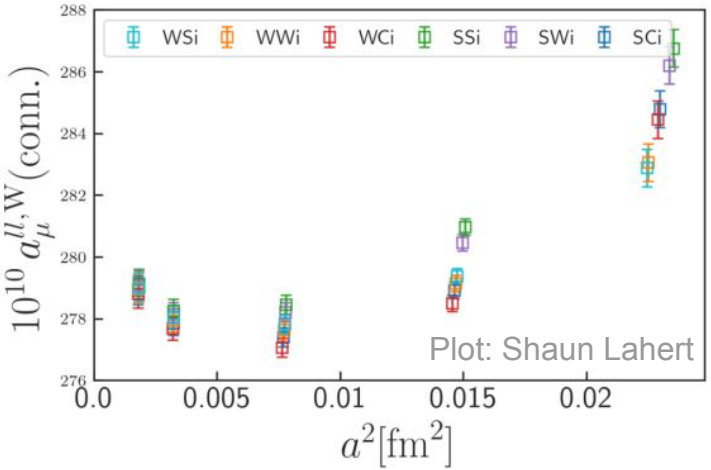
- Data deviates from single-exponential behavior in region where bound begins (purple band)
- “Missing” state is unphysically light (below free two pion energy)
  ⇒ Statistical fluctuation manifests as spurious state
- Correlator upper bound not monotonic at (physical) energy lower bound

- By comparison, the fit method is insensitive to these fluctuations, and thus is less biased than the bounding method (even with the bound’s larger errors)

See [2412.18491] for details

Strange and Charm “Reblinding”





- Reblinding idea: Take advantage of different discretization effects of different w_0/a schemes
- SD/W 2411.09656: v1: “WCu”, v2: aM_Ω
- Current analysis (w/ 0.042 fm data): “WWi” + new blinding factor

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u = unimproved, i = improved (in recent publications, these have been changed to o = original, c = corrected, respectively)

Scheme Definition

Pure QCD schemes (FLAG24):

$$\text{(isospin-symmetric pure-QCD world)} \Rightarrow \begin{cases} M_\Omega & \equiv 1671.26 \text{ MeV}, \\ M_{\pi^+} & \equiv 135.0 \text{ MeV}, \\ M_K & \equiv 494.6 \text{ MeV}, \\ M_{D_s^+} & \equiv 1967 \text{ MeV}. \end{cases}$$

$$\text{(pure-QCD world)} \Rightarrow \begin{cases} M_\Omega & \equiv 1671.26 \text{ MeV}, \\ M_{\pi^+} & \equiv 135.0 \text{ MeV}, \\ M_{K^+} & \equiv 491.6 \text{ MeV}, \\ M_{K^0} & \equiv 497.6 \text{ MeV}, \\ M_{D_s^+} & \equiv 1967 \text{ MeV}. \end{cases}$$

Isobroken-light mesons:

- Pure QCD up-down ratio form [1807.05556]
- Error inflated to cover FLAG24 kaon splitting

$$m_u/m_d = 0.455(18)$$

$$M_\pi^{uu} = M_{\pi^+}\sqrt{\frac{2}{1+m_d/m_u}} = 106.7(1.5) \text{ MeV},$$

$$M_\pi^{dd} = M_{\pi^+}\sqrt{\frac{2}{1+m_u/m_d}} = 158.25(98) \text{ MeV}.$$

Isospin-symmetric QCD+QED scheme implicit from FLAG24+PDG

See [2411.09656] for details

Continuum Extrapolations

- E.g., light quark connected:

$$a_\mu^{qq}(a,\{M_A\}) = a_\mu^{qq}\left(1 + F^{\text{disc.}}(a) + F^{M}(\{M_A\})\right)$$

$$F^{\text{disc.}}_{\text{local}}(a) = C_{a^2,n}\left[(a\Lambda)^2\alpha_s^n\right] + \sum_{k=2}^{4} C_{a^{2k}}(a\Lambda)^{2k}, \quad \text{where } n = \{0\}, 1, 2$$

$$F^{\text{disc.}}_{\text{one-link}}(a) = \{C_{a^2\log(a)}(a\Lambda)^2\log(a\Lambda)\} + \sum_{k=1}^{4} C_{a^{2k}}(a\Lambda)^{2k}$$

$$F^{M}(\{M_A\}) = C_{\text{sea}}\sum_{A=\pi,K}\delta M_A^2\,, \qquad \delta M_A^2 = \frac{M^2_{A,\,\text{phys.}} - M^2_{A,\,\text{latt.}}}{M^2_{A,\,\text{phys.}}}$$

- Local and one-link currents used for all single flavor (ll,ss,cc) connected quantities
- Terms in braces only for SD (log-enhancement)
- Discretization terms and scale Λ from Empirical Bayes analysis
- Chiral-continuum fits (e.g., SIB conn.) have χPT-motivated pion mass dependent terms
- See [2411.09656, 2412.18491] for details

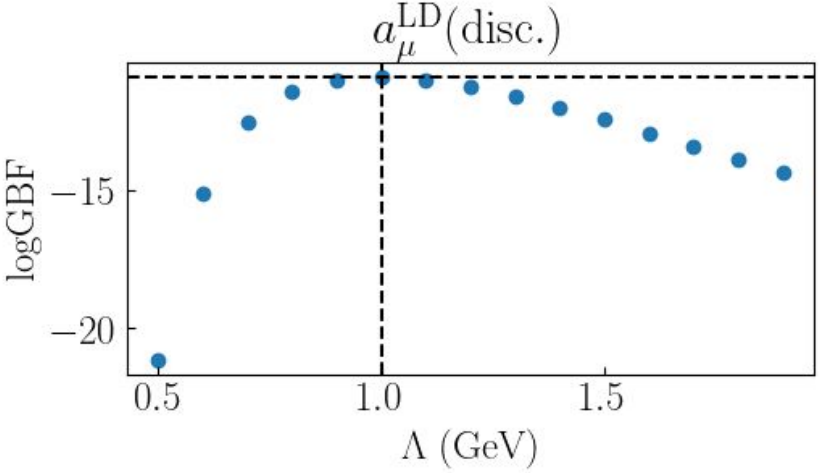
Empirical Bayes Tuning

- Continuum extrapolation prior widths depend on scale Λ
- Symanzik EFT

$$a_\mu(a) = a_\mu \left(1 + c_{20}(a\Lambda)^2 + c_{40}(a\Lambda)^4 + c_{60}(a\Lambda)^6 + \ldots \right.$$
$$+\alpha_s(a)\left(c_{21}(a\Lambda)^2 + c_{41}(a\Lambda)^4 + c_{61}(a\Lambda)^6 + \ldots\right)$$
$$+\alpha_s^2(a)\left(c_{22}(a\Lambda)^2 + c_{42}(a\Lambda)^4 + c_{62}(a\Lambda)^6 + \ldots\right)$$
$$\left. + \ldots\right)$$

- Coefficients $\mathcal{O}(1)$ with appropriate Λ
  ⇒ Impose priors ~ 0(1) and choose Λ to maximize log Gaussian Bayes Factor (log GBF)
- Prior widths doubled in final BMAs
- Independent tuning for each observable on each window

- Tuning also informs important discretization terms for fit
- Analogous for chiral continuum extrapolations with (staggered) χPT



See also [hep-lat/0110175]

BMA Procedure

BMA:

$$\langle a_\mu \rangle = \sum_{n=1}^{N_M} \langle a_\mu \rangle_n \, \mathrm{pr}\left(M_n \mid D\right)$$

BMA Systematic

$$\sigma^2_{a_\mu} = \sum_{n=1}^{N_M} \sigma^2_{a_\mu,n} \mathrm{pr}\left(M_n \mid D\right) + \sum_{n=1}^{N_M} \langle a_\mu \rangle^2_n \, \mathrm{pr}\left(M_n \mid D\right) - \langle a_\mu \rangle^2$$

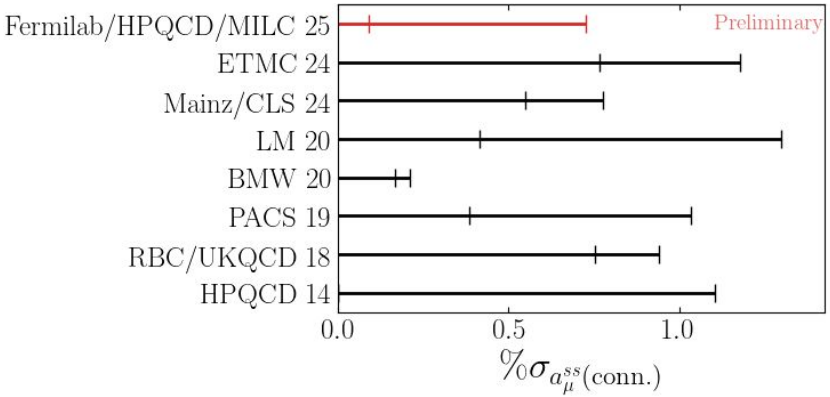
Bayesian Akaike Information Criteria (BAIC):

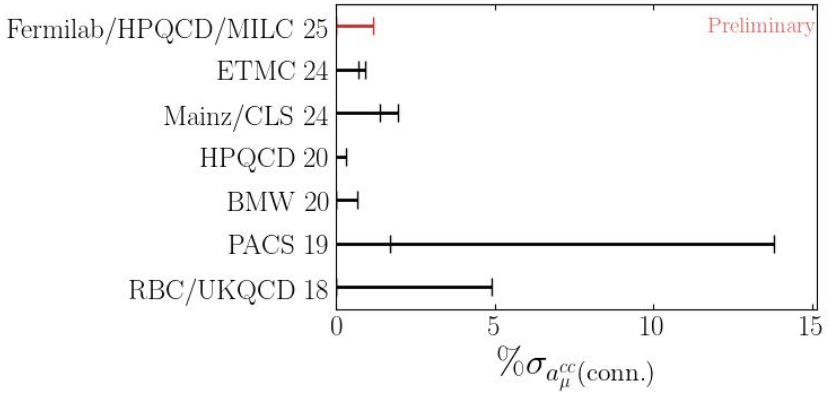
$$\mathrm{pr}(M \mid D) = \mathrm{pr}(M) \exp\left[-\frac{1}{2}\left(\chi^2_{\mathrm{data}}\left(\mathbf{a}^\star\right) + 2k + 2N_{\mathrm{cut}}\right)\right]$$

Correlations between contributions:

$$\mathrm{Cov}[a,b] = \sum_{m=1}^{N_{MA}} \sum_{n=1}^{N_{MB}} \mathrm{Cov}_{mn}[a,b] \, \mathrm{pr}(M_m^A, M_n^B|D) + \sum_{m=1}^{N_{MA}} \sum_{n=1}^{N_{MB}} \langle a \rangle_m \langle b \rangle_n \, \mathrm{pr}(M_m^A, M_n^B|D) - \langle a \rangle \langle b \rangle$$

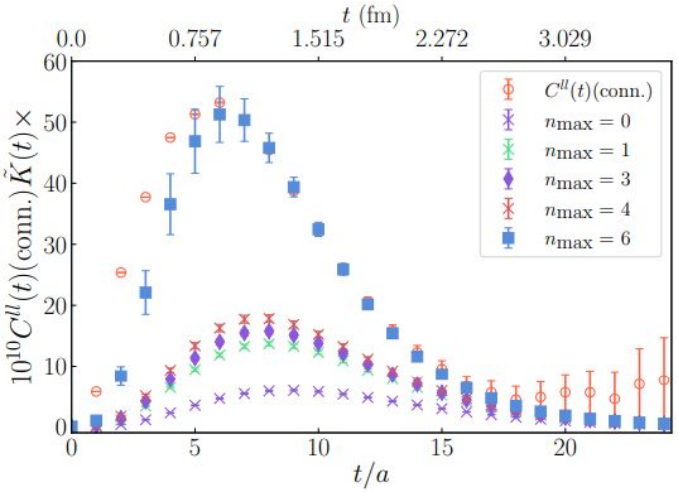
Strange and Charm Relative Error Comparisons

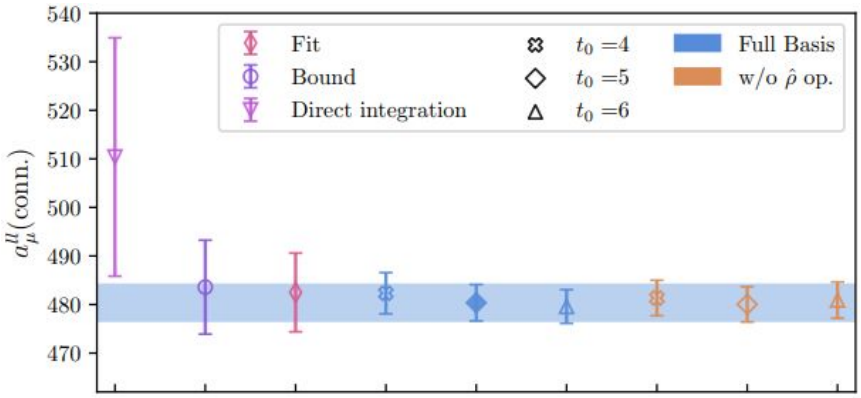




- Relative error comparisons for Full Integration
- Inner error bars are statistics

Reconstruction with Staggered Two-Pion Spectrum





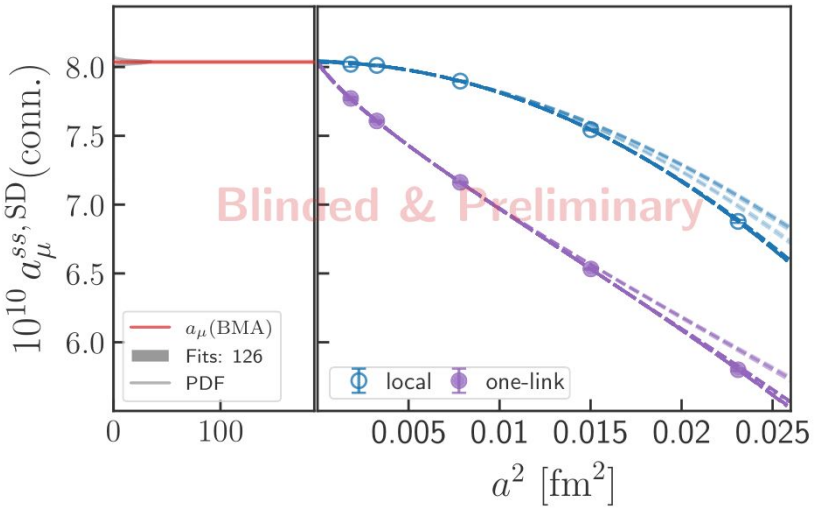
- Two-pion contribution compute on coarsest (a~0.15 fm) HISQ ensemble
- Stat. error reduced by over half compared to other correlator reconstruction methods
- Study on finer spacings underway

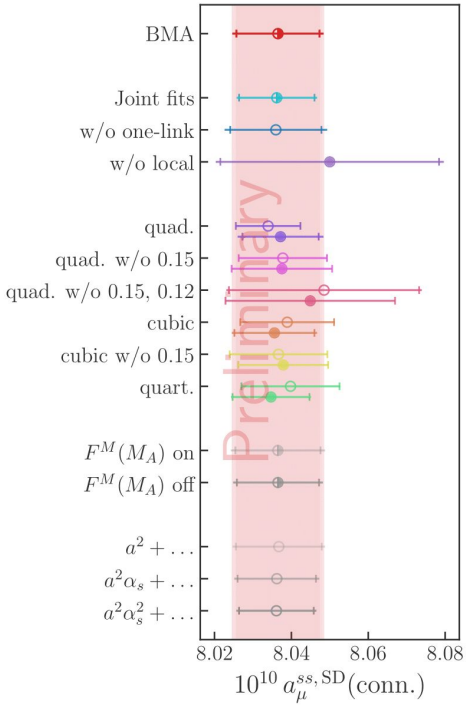
See [2409.00756] for details

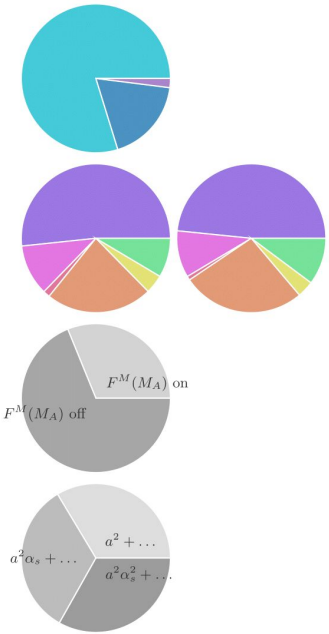
BMA Plots

- Left plots are (chiral) continuum BMAs
- Right plots are subset averages, where pie charts give subset probabilities
- cf. [2411.09656, 2412.18491] for more plot information

SD Strange

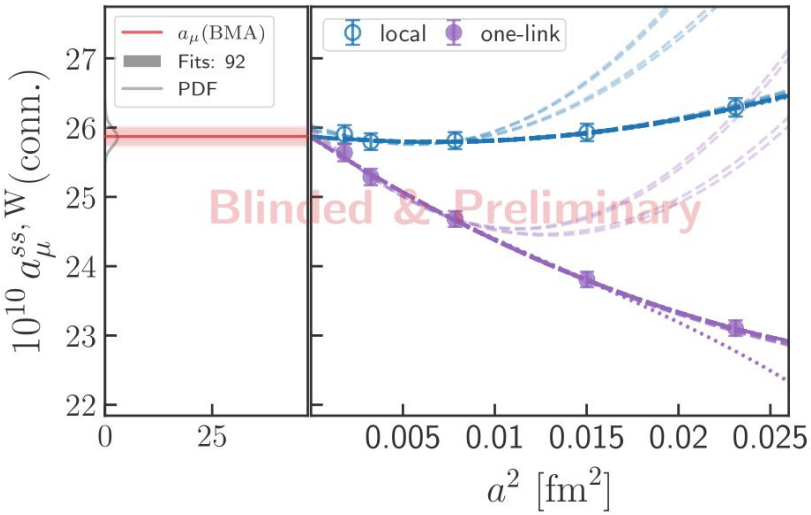


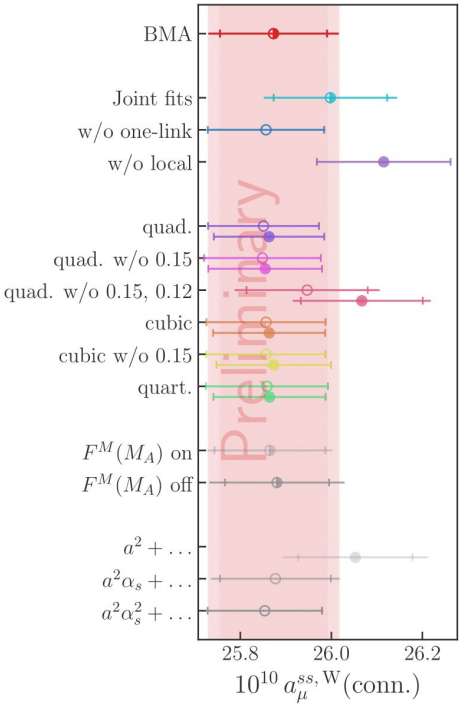


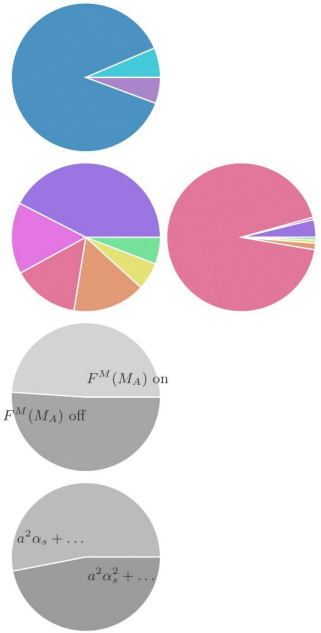


Plots from Shaun Lahert

W Strange

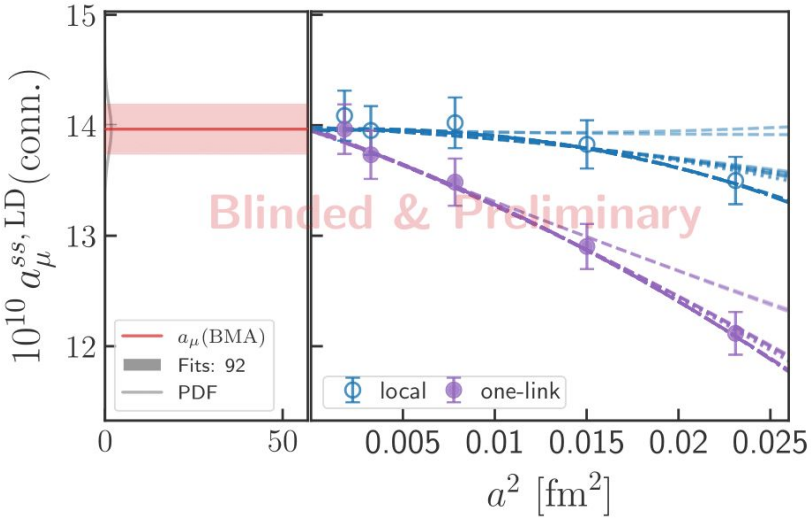


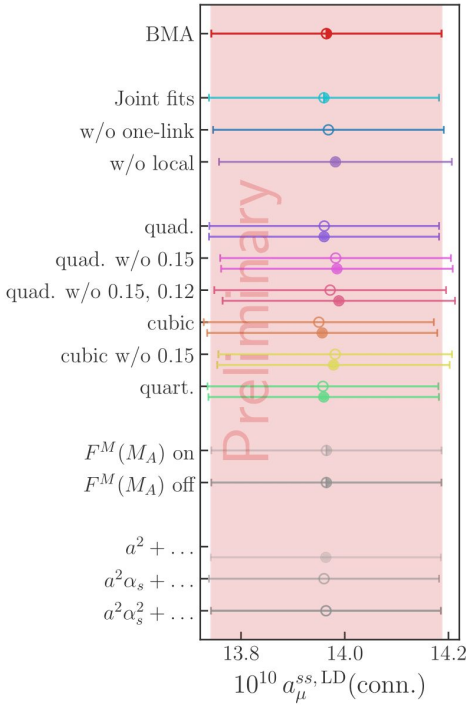


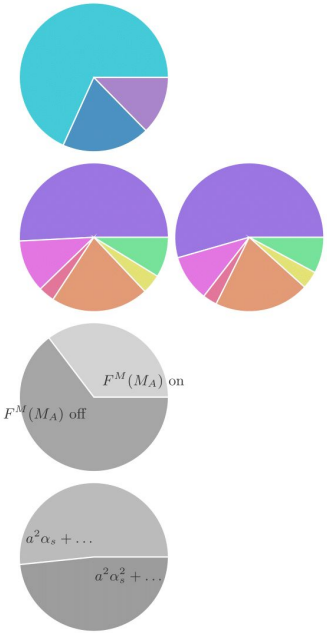


Plots from Shaun Lahert

LD Strange

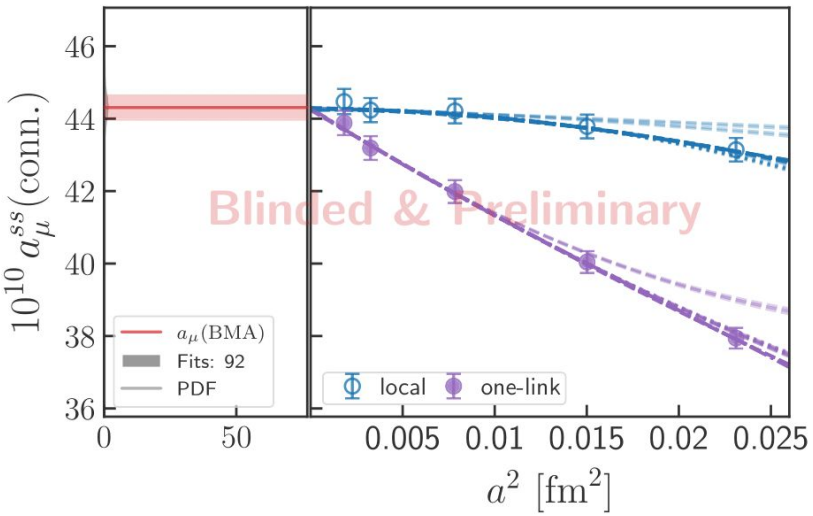


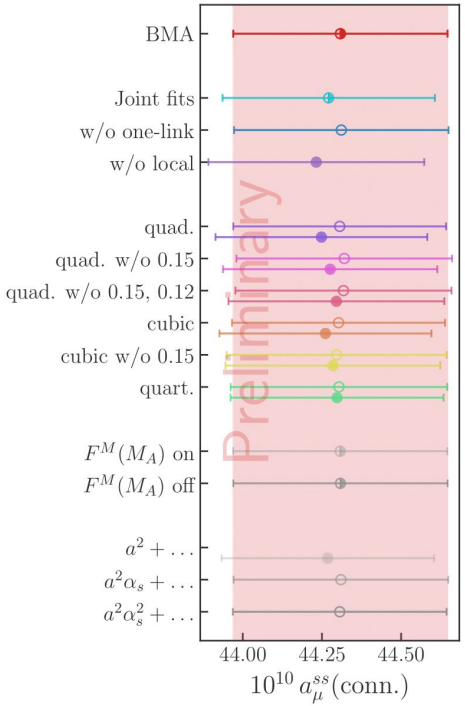


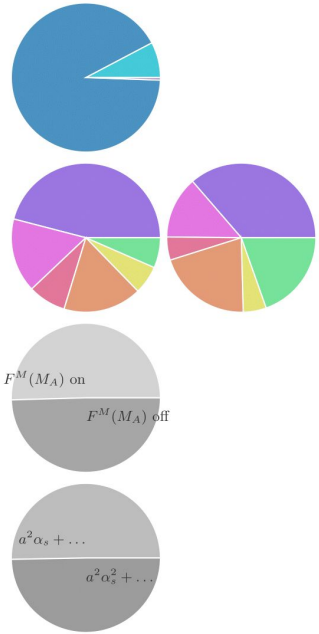


Plots from Shaun Lahert

Full Integration Strange

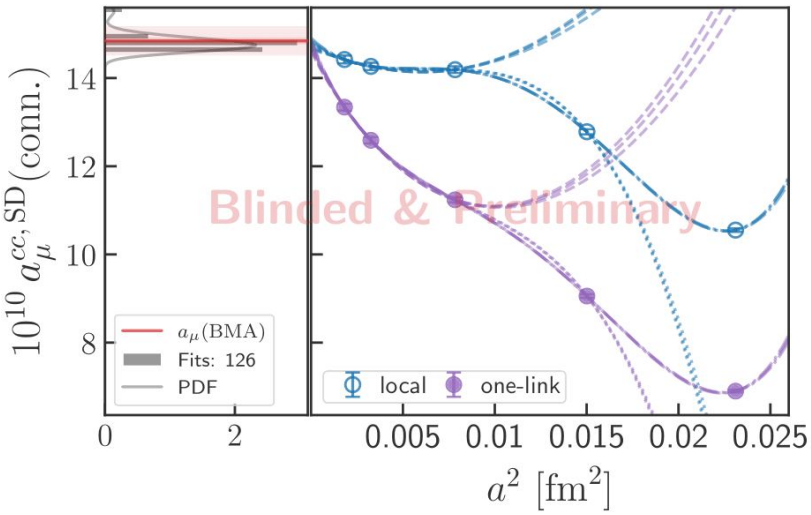


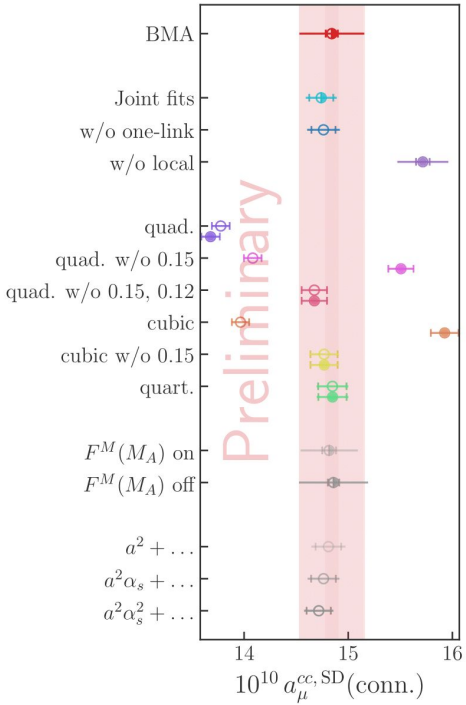


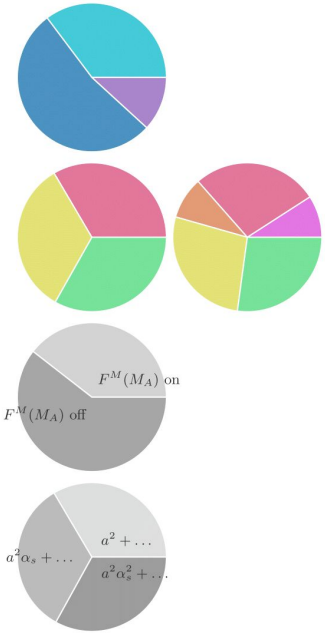


Plots from Shaun Lahert

SD Charm

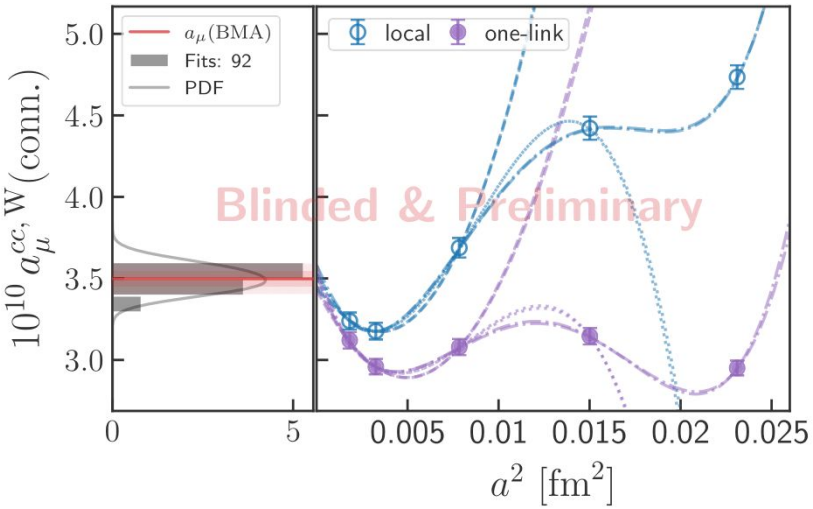


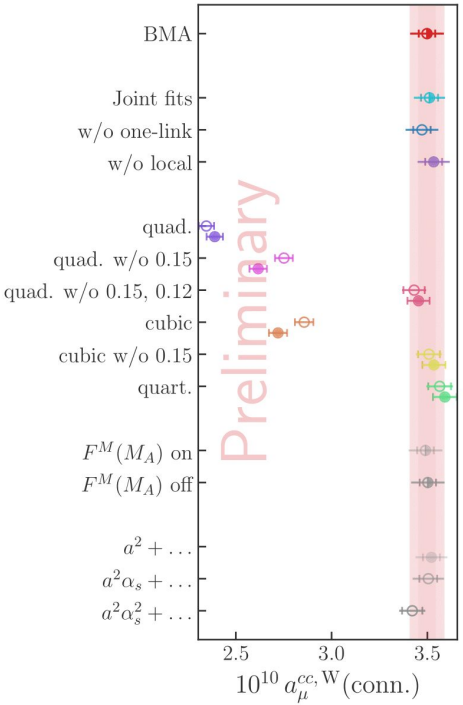


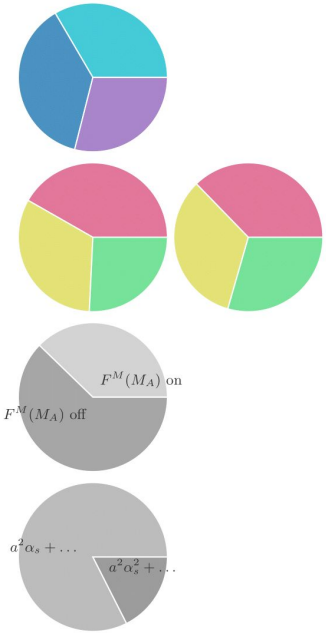


Plots from Shaun Lahert

W Charm

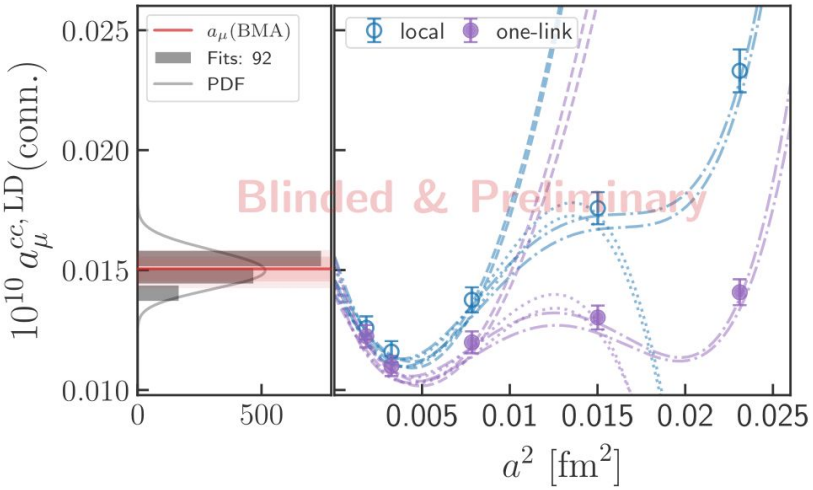


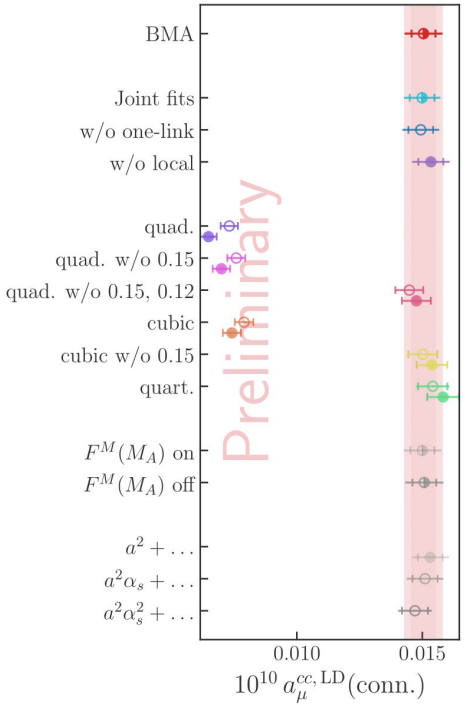


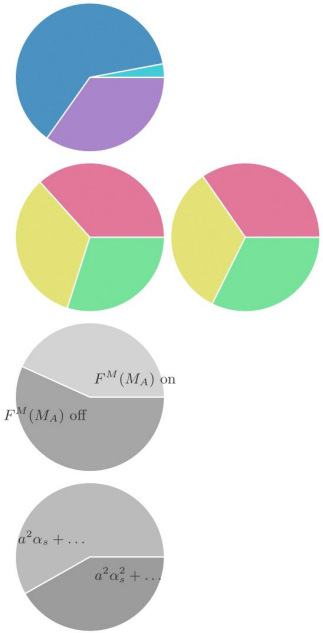


Plots from Shaun Lahert

LD Charm

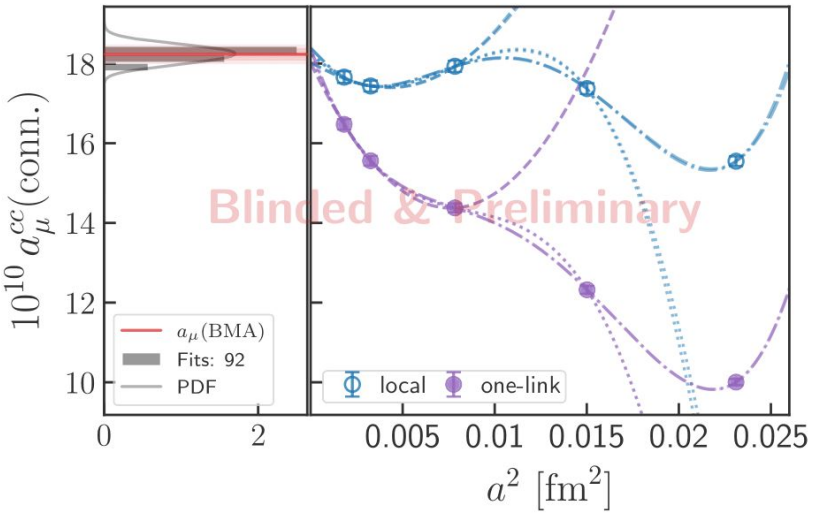


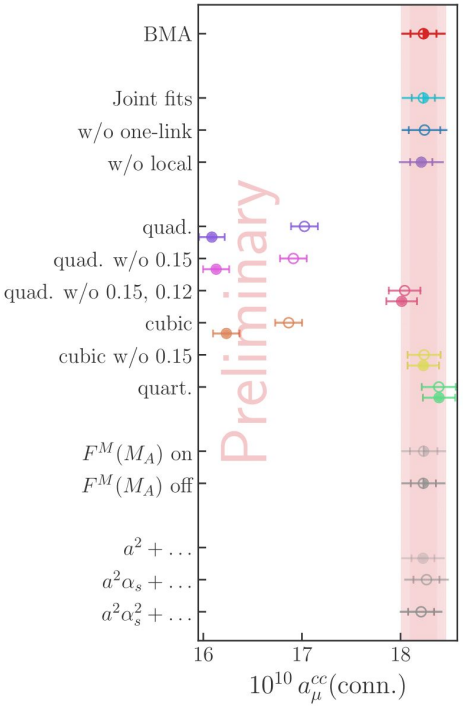


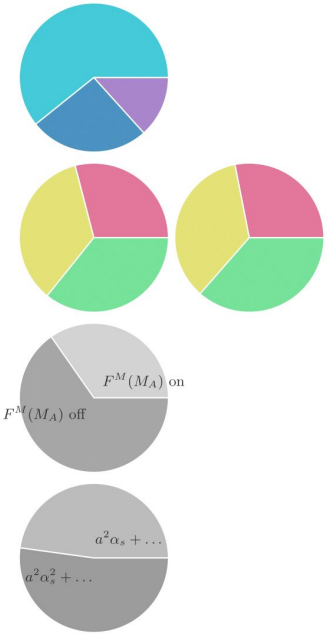


Plots from Shaun Lahert

Full Integration Charm

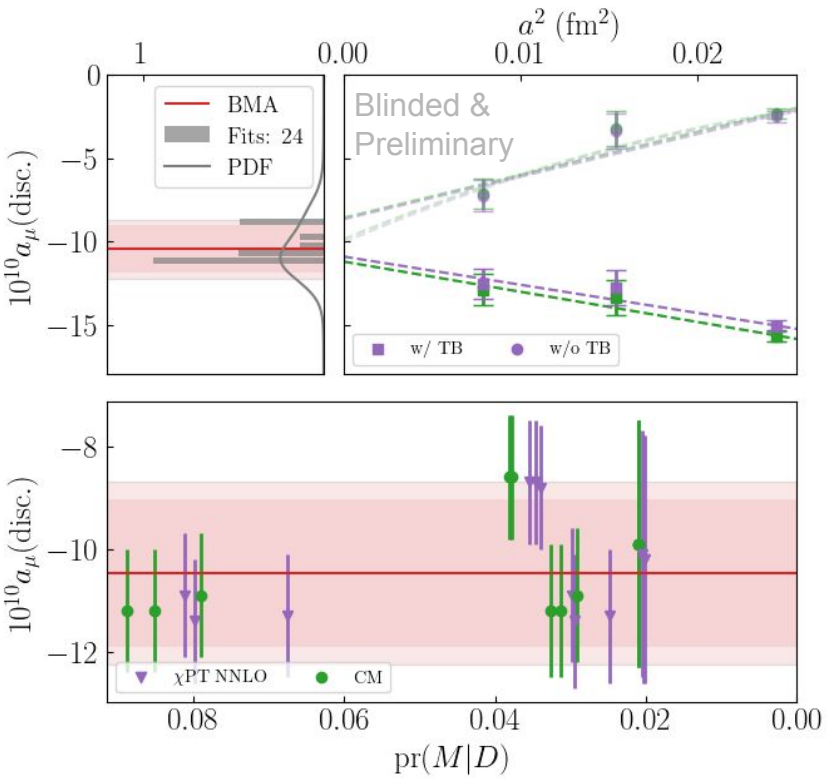


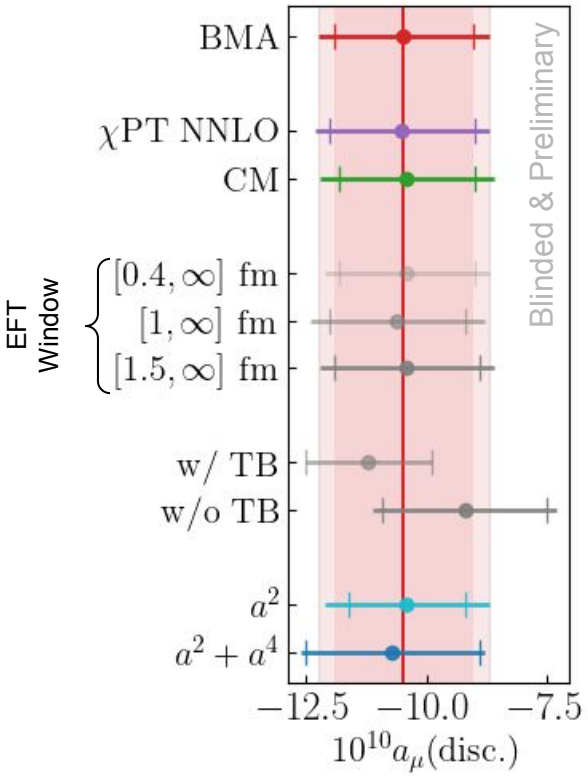


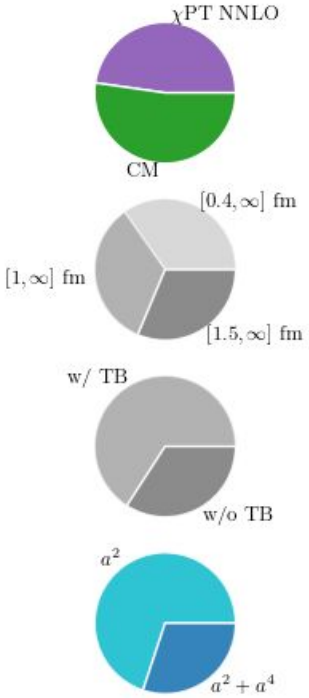


Plots from Shaun Lahert

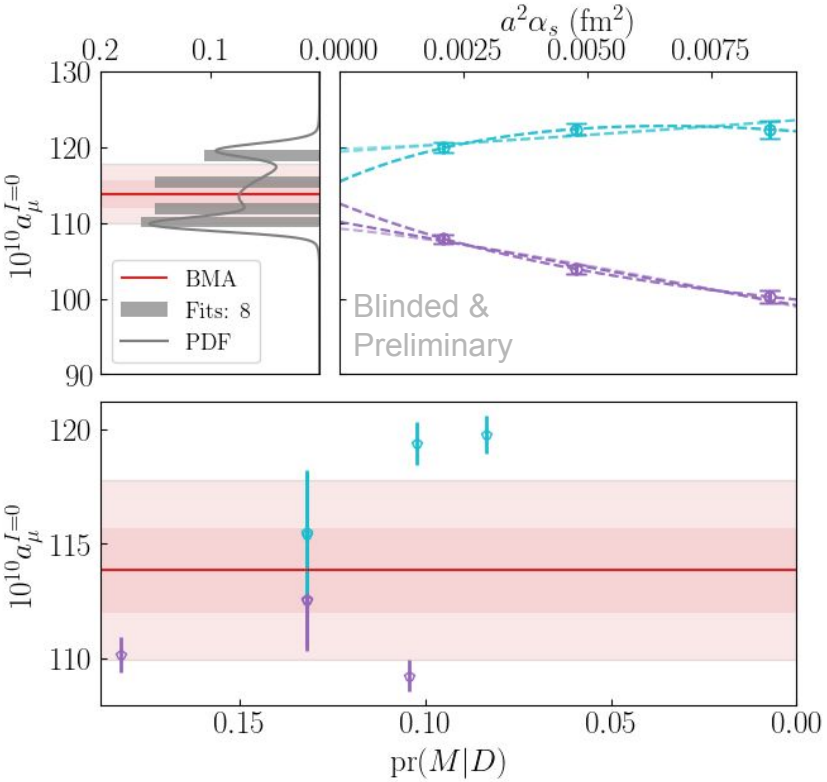
Full Integration Disconnected

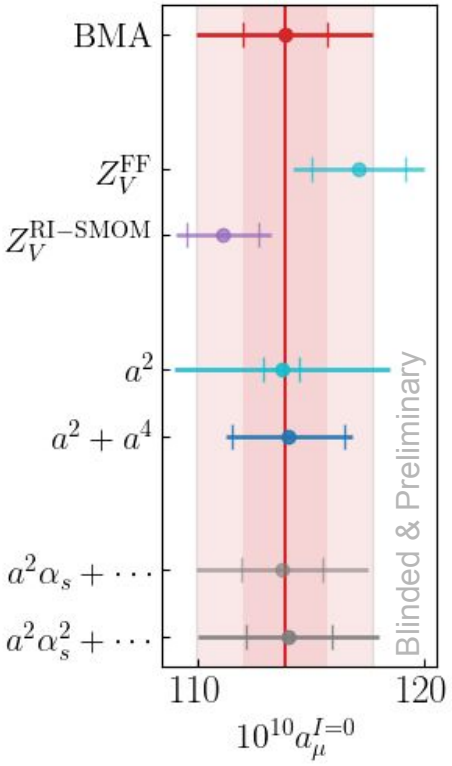


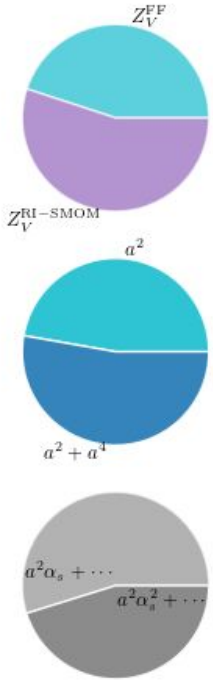




Full Integration I = 0







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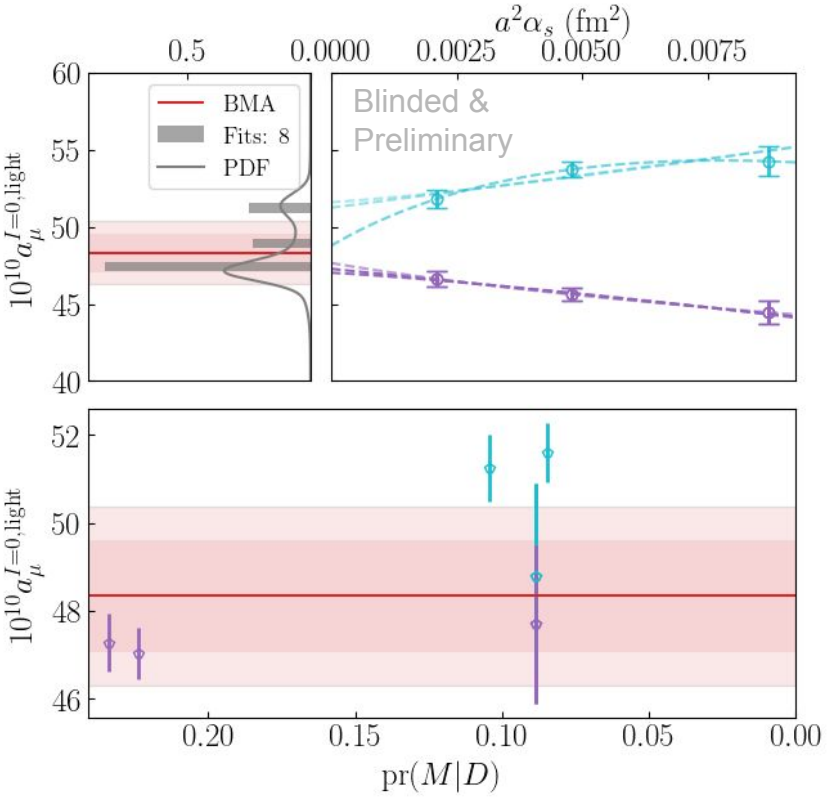
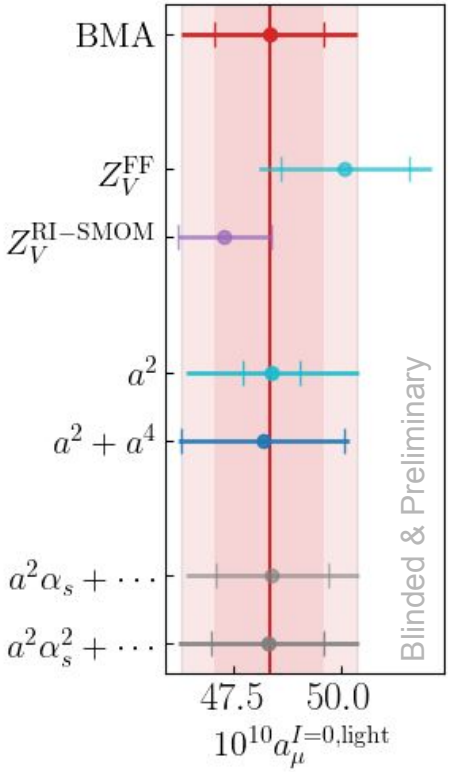
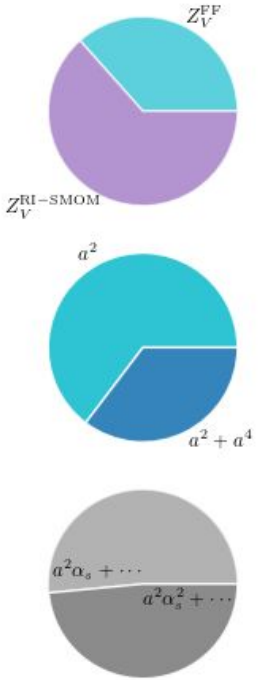
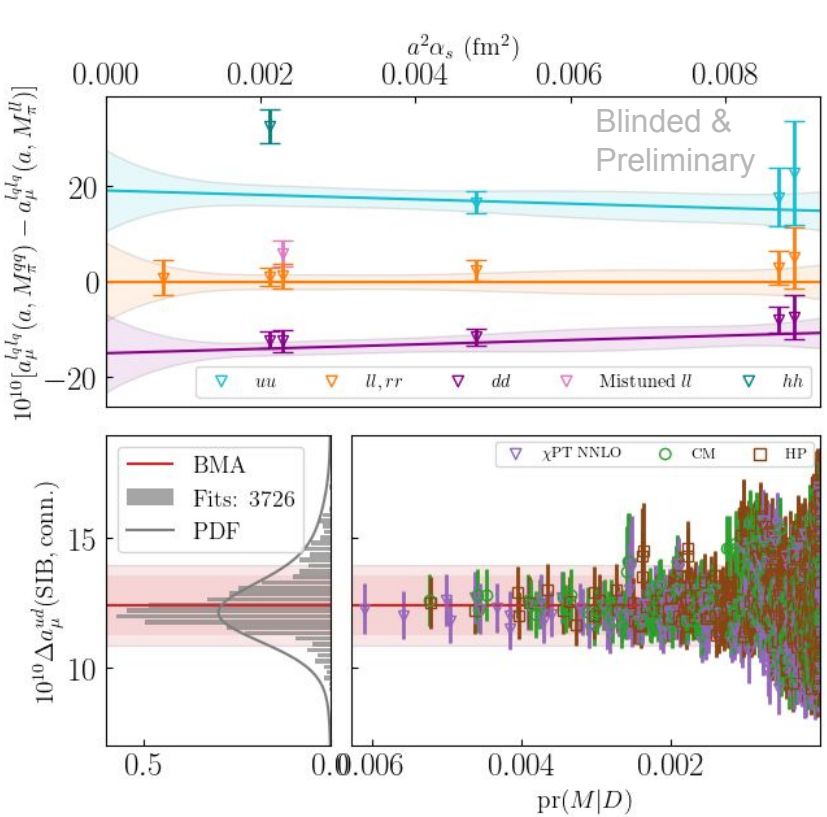
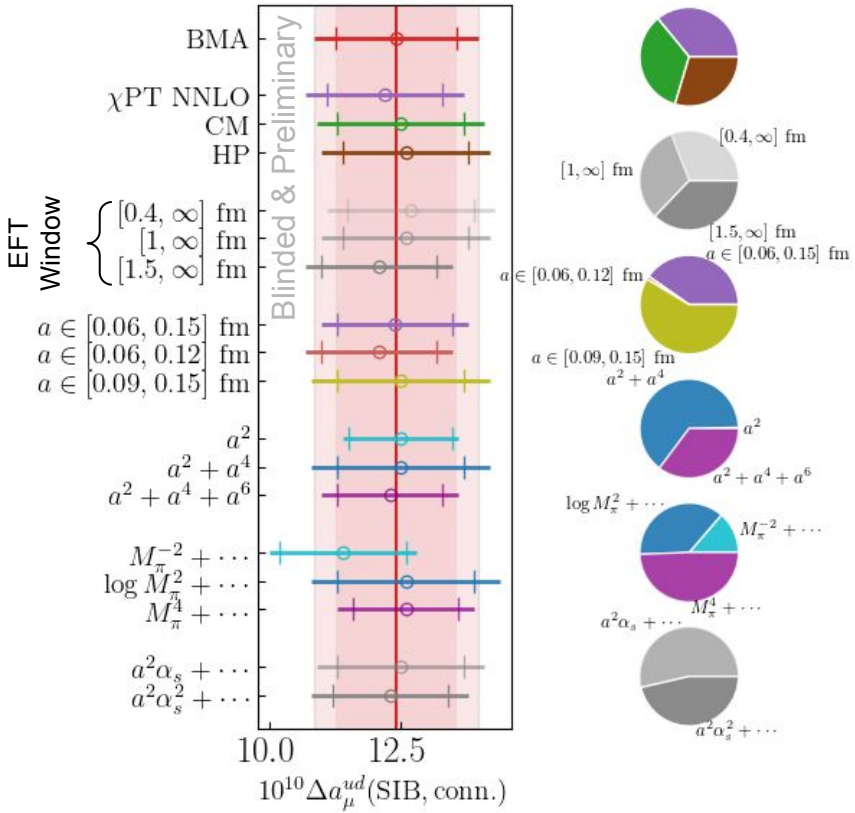






Full Integration SIB Connected





Full Integration SIB Disconnected