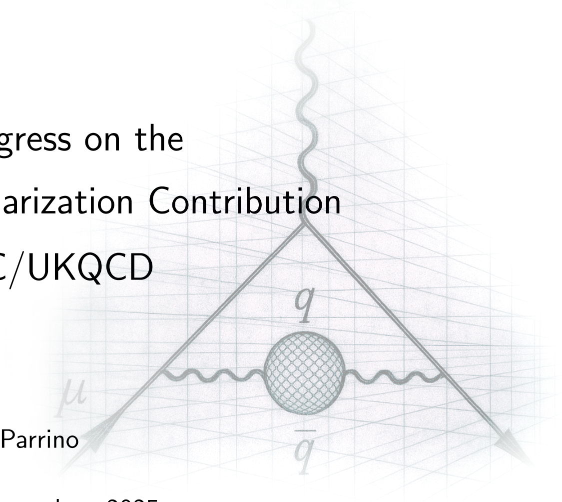
Recent Progress on the Hadronic Vacuum Polarization Contribution from RBC/UKQCD



Julian Parrino

Universität Regensburg

Orsay, 10th September, 2025

The RBC & UKQCD collaborations

Boston University
Nobuyuki Matsumoto

BNL and BNL/RBRC
Peter Boyle
Taku Izubuchi
Christopher Kelly
Shigemi Ohta (KEK)
Amarji Soni
Masaaki Tomii
Xin-Yu Tuo
Shuhei Yamamoto

University of Cambridge
Nelson Lachini

CERN
Matteo Di Carlo
Felix Erben
Andreas Jüttner (Southampton)
Tobias Tsang

Columbia University
Norman Christ
Sarah Fields
Ceran Hu
Yikai Huo
Joseph Karpie (JLab)
Erik Lundstrum
Bob Mawhinney
Bigeng Wang (Kentucky)

University of Connecticut
Tom Blum
Jonas Hildebrand
Luchang Jin
Vaishakhi Moningi
Anton Shcherbakov
Douglas Stewart
Joshua Swaim

DESY Zeuthen
Raoul Hodgson

Edinburgh University
Luigi Del Debbio
Vera Gülpers
Maxwell T. Hansen
Nils Hermansson-Truedsson
Ryan Hill
Antonin Portelli
Azusa Yamaguchi

Johannes Gutenberg University of Mainz
Alessandro Barone

Liverpool Hope/Uni. of Liverpool
Nicolas Garron

LLNL
Aaron Meyer

Autonomous University of Madrid
Nikolai Husung

University of Milano Bicocca
Mattia Bruno

Nara Women's University
Hiroshi Ohki

Peking University
Xu Feng
Tian Lin

University of Regensburg
Andreas Hackl
Daniel Knüttel
Christoph Lehner
Julian Parrino
Sebastian Spiegel
Andreas Völklein

RIKEN CCS
Yasumichi Aoki

University of Siegen
Matthew Black
Anastasia Boushmelev
Oliver Witzel

University of Southampton
Bipasha Chakraborty
Ahmed Elgaziari
Jonathan Flynn
Joe McKeon
Rajnandini Mukherjee
Callum Radley-Scott
Chris Sachrajda

Stony Brook University
Fangcheng He
Sergey Syritsyn (RBRC)

RBC/UKQCD Roadmap (1/2)

- 2018: First QCD+QED calculation of all components with total uncertainty of $\delta a_\mu^{\mathrm{HVP}} = 19 \times 10^{-10}$; introduced windows [Blum et al., PRL121(2018)022003]
- 2023: Update QCD+QED intermediate distance window to uncertainty of 0.8×10^{-10} and light-quark connected short-distance window to 0.7×10^{-10} [Blum et al., PRD108(2023)054507]
- 2024: Scrutinize continuum-limit of light-quark connected short-distance window (consistent result, consolidate at 0.7×10^{-10}) [Spiegel, Lehner, arxiv:2410.17053; PRD111(2025)114517]
- 2024: First calculation of light-quark connected long-distance window; reduced uncertainty of light-quark connected isospin symmetric result to 5×10^{-10} [Blum et al., arxiv:2410.20590; PRL134(2025)201901]
- Target: 1.5×10^{-10} total uncertainty to match FNAL E989

RBC/UKQCD Roadmap (2/2)

- Next (2025/26): reduce uncertainty of quark-disconnected and QED+SIB corrections; aim at target precision for all components apart from the long-distance isospin symmetric window
- 1.9.2025: Long-distance reconstruction of QED corrections to the HVP [Lehner, Parrino, Völklein, arxiv:2508.21685]
  → Talk by Christoph Lehner

Long-distance reconstruction of QED corrections to the hadronic vacuum polarization for the muon g-2

C. Lehner,[1,*] J. Parrino,[1] and A. Völklein[1]

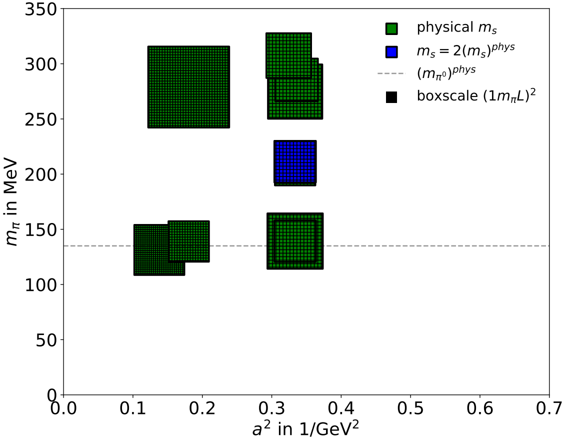
[1] *Fakultät für Physik, Universität Regensburg, Universitätsstraße 31, 93040 Regensburg, Germany*

(Dated: September 1, 2025)

- XX.9.2025: Isospin-breaking effects in inclusive hadronic τ data for the muon $(g-2)$ from first principles → Talk by Mattia Bruno
- Focus of this talk: Calculation of isospin breaking correction to the HVP

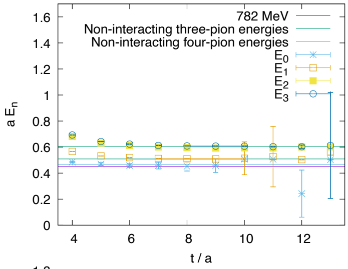
Ensemble overview

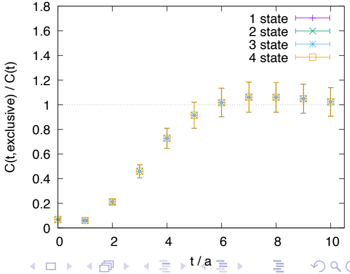
- $N_f = 2 + 1$ Möbius domain wall ensembles
- 4 ensembles at physical pion mass in use so far
- Ensembles up to volumes of $m_\pi L \approx 8$; also new physical pion ensemble with $L^3 = (11\mathrm{fm})^3$
- Generation of new finer physical pion mass ensemble $(128^3 \times 288)$: a^2 GeV$^2 = 0.08$ and $m_\pi L = 4.9$; see talk by Christoph at KEK workshop



Quark-disconnected reconstruction

- Reconstruct long-distance contribution again exactly from the finite-volume exclusive state contributions.
- Relate disconnected contribution to $I = 0$:
  $C^{I=0} = (Q_u + Q_d)^2(c/2 - d)$, $C^{I=1} = (Q_u - Q_d)^2 c/2$
  $\Rightarrow$
  $C^{\mathrm{conn}} = (Q_u^2 + Q_d^2)c$,
  $C^{\mathrm{disc}} = (Q_u + Q_d)^2(-d) = C^{I=0} - C^{\mathrm{conn}}/10$
  with connected (c) and disconnected (d) diagram.
- Study three-pion operators with lowest momenta; ω state seems to dominate strongly over three-pion FV states
- Status: complete data generation over ensembles; then: blind analysis with multiple analysis groups





Tadpole fields

- For quark-disconnected diagram (d) but also for many QED diagrams, we need estimators of the tadpole field

$$T_x^{(f)} = \langle \psi_f(x)\bar{\psi}_f(x)\rangle$$

with quark flavor $f \in \{u, d, s\}$.

- Employ method of our first quark-disconnected physical pion mass paper [Blum et al., PRL116(2016)232002]:
  - Compute $T^{(ud)} - T^{(s)} = T^{(ud,low)} + T^{(ud,high)} - T^{(s)}$ with $T^{(ud,high)} = T^{(ud)} - T^{(ud,low)}$
  - Use multi-grid Lanczos [Clark, Jung, Lehner, arxiv:1710.06884] to compute exact low-mode field $T_x^{(ud,low)}$ over 2000-5000 lowest preconditioned Dirac modes
  - Estimate $T^{(ud,high)} - T^{(s)}$ using a random sparse Z_2 grid

Isospin breaking corrections: Scheme definition

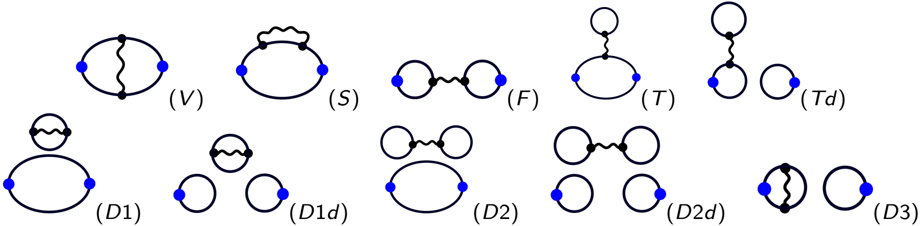
- Use RM123 approach: Exapnd around isospin symmetric QCD
- Parameters for $N_f = 1 + 1 + 1$: $\boldsymbol{g} = (g_s, m_u, m_d, m_s, \alpha_{\mathrm{QED}})$
- Use physical hadron mass shifts masses to set scale PDG 2024 [PhysRevD.110.030001]

$$\begin{aligned} m_{\pi^0} &= 134.9768(5)\ \mathrm{MeV}, \\ m_{K^0} &= 497.611(13)\ \mathrm{MeV}, \\ m_{K^0} - m_{K^+} &= 3.9340(2)\ \mathrm{MeV}, \\ m_{\Omega} &= 1672.45(29)\ \mathrm{MeV} \end{aligned}$$

- Electromagnetic coupling does not renormalize at leading order

$$(\alpha_{\mathrm{QED}})^{-1} = 137.035999177(21)$$

Computational strategy: Wick contractions



- For Wick contractions: Lattice operator toolkit (http://github.com/jparrino/lotk, see Backup)
- Names (V,S,...) refer to topology without minus sign, charge- or symmetryfactor

$$G^{1\gamma^*}(z) = 2\pi\alpha_{\text{QED}}\Big\{ -\frac{17}{81}(2\mathrm{V}+4\mathrm{S}) + \frac{25}{81}(2\mathrm{F}+\mathrm{D1}) + \frac{7}{81}(4\mathrm{T}+4\mathrm{D3}) - \frac{5}{81}(4\mathrm{T_d}+\mathrm{D1_d}+\mathrm{D2}) + \frac{1}{81}(\mathrm{D2_d})\Big\}$$

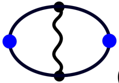
Computational strategy: Stochastic sampling

Lattice QCD implementation using gpt library (http://github.com/lehner/gpt)

- We need to calculate correlators of type

$$\int_{x,y} G_{\nu\rho}(x,y)\langle\pi^+(\boldsymbol{p},t)|T\{j^{em}_\nu(y)j^{em}_\rho(x)\}|\pi^-(\boldsymbol{p},0)\rangle_{QCD}$$

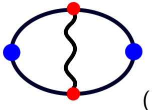
$$\supset \int_{x,y,\boldsymbol{z}} G_{\nu\rho}(x,y)\Big\langle \mathrm{Tr}\Big[\gamma_5 S(z,x)\gamma_\rho S(x,(\mathbf{0},t))\gamma_5 S((\mathbf{0},t),y)\gamma_\nu S(y,z)\Big]\Big\rangle_U =$$



(V)

- Challenge: $V^2 \times L^3$ sum has to be computed
- Calculating all-to-all propagators is not feasible
  $\rightarrow$ RBC/UKQCD approach: Evaluate integrals stochastically

Computational strategy: Stochastic sampling



(*V*)

- Propagators are calculated on stochastic subset of points (Introduced in calculation of hadronic light-by-light contribution [Blum et al. , arxiv:1510.07100])
- Propagator from N source points to M sink points are saved on disc
- No extra inversion needed to calculate diagram
- Can be contracted with different versions of the photon propagator

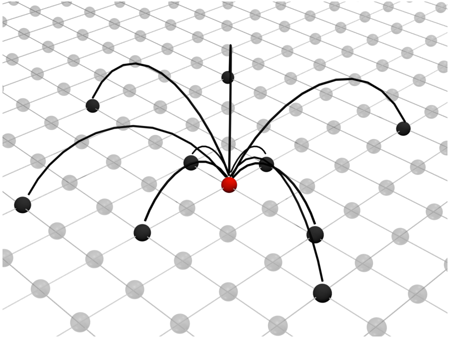
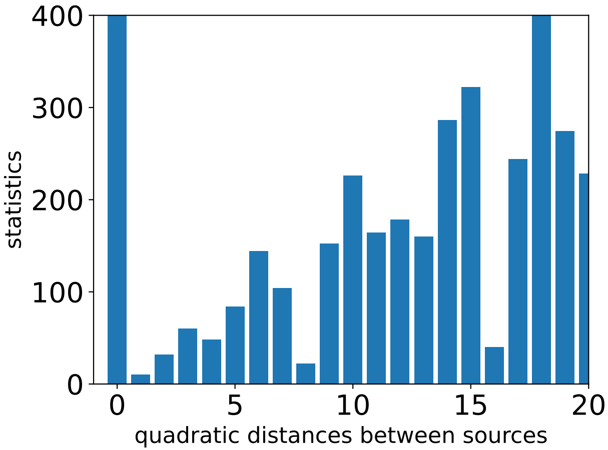


Fig. 1: Propagator for (red) source point to several (black) sink points

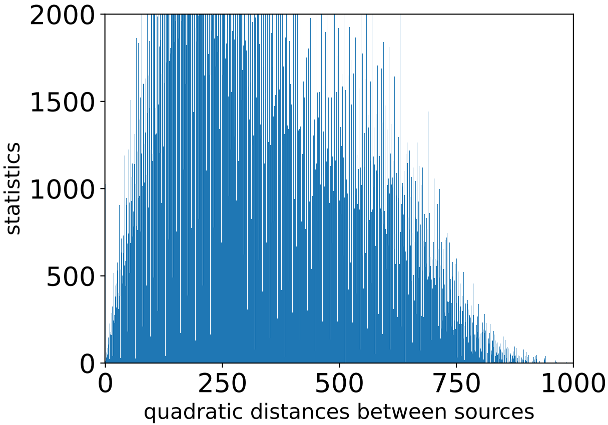
Computational strategy: Stochastic sampling

- Histogram of point source separations on $24^3 \times 48$ ensemble
- Zero-distance is sampled with high statistics
- $O(10^3)$ source positions
- $O(10^6)$ sink positions
- Number of points to sample from can be chosen on the fly



Computational strategy: Stochastic sampling

- Histogram of point source separations on $24^3 \times 48$ ensemble
- Short-distance regime with more statistics
- $O(10^3)$ source positions
- $O(10^6)$ sink positions
- Number of points to sample from can be chosen on the fly



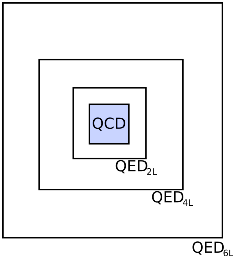
Computational strategy: Photon propagator

To address systematic uncertainties, consider several photon regularizations:

- QED_L [Hayakawa et al., arxiv:0804.2044]:] $G(x,y) = \sum_{k \in (\frac{2\pi}{L}\mathbb{Z})^4} \frac{e^{ik(x-y)}}{(2\pi)^4} \frac{1}{\hat{k}^2}(1 - \delta_{\boldsymbol{k},0})$

- QED_r [Di Carlo et al., arxiv:2501.07936]: $G(x,y) = \sum_{k=\frac{2\pi n}{L}, n \in \mathbb{Z}} \frac{e^{ik(x-y)}}{(2\pi)^4} \frac{(1+\delta_{|\boldsymbol{n}|,1}/6)}{\hat{k}^2}(1 - \delta_{\boldsymbol{k},0})$

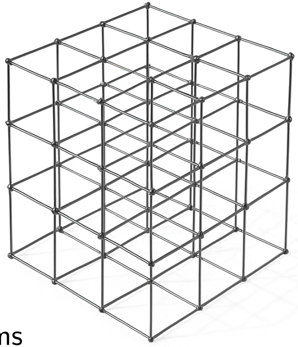
- QED_∞: [Blum et al., arxiv:1801.07224] $G(x,y) = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{1}{\hat{k}^2} e^{ikx}$
  - Use subsequent version of QED_L
  - E.g. QED_{2L},QED_{4L}, QED_{6L}, QED_{NL}
  - Ensures same UV-behaviour
  - Take the limit $N \to \infty$



Crosschecks: Reference implementation

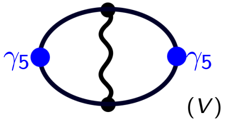
Crosscheck for implementation

- Do calculation on $4^3 \times 8$ box
- Compute exact all-to-all propagators
- One gauge configuration
- Full statistics $< 4\text{GB} \rightarrow$ can be run on laptop
- Simple implementation of all diagrams using Numpy code
- Check for all operator insertions and photon implementations
- Crosscheck implementation of stochastic sampling for all diagrams



Crosschecks: Reference implementation

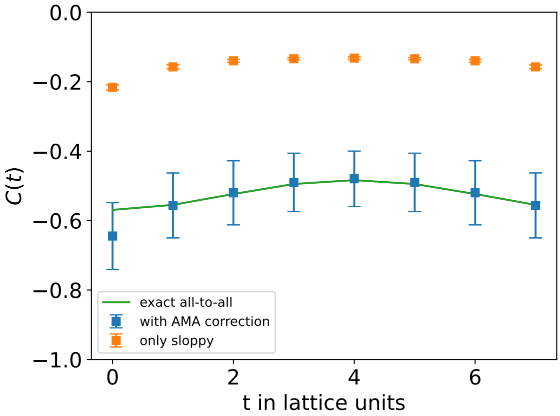
- Crosscheck diagram V for QED_L



- Between reference implementation and lattice QCD code using gpt

  [http://github.com/lehner/gpt]

- Intentionally large AMA correction to check for correctness
- Statistical error from point-sources sampling



Crosschecks: Reference implementation

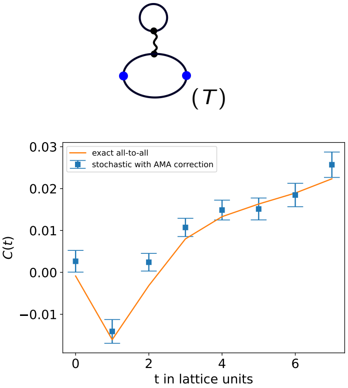


Fig. 2: (T) in QED_{4L} for $(\gamma_3\gamma_5, \gamma_5)$

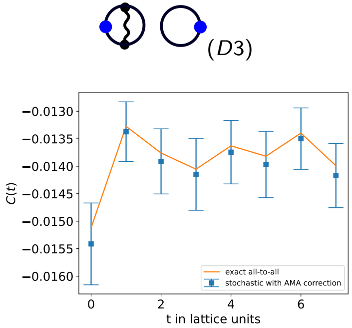


Fig. 3: (D3) in QED_{6L} for (γ_2, γ_2)

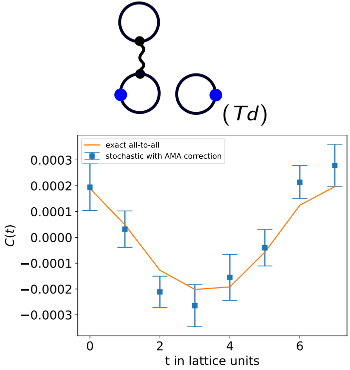
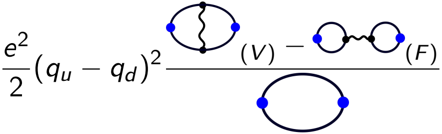


Fig. 4: (Td) in QED_r for (γ_1, γ_1)

- For all 14 diagrams, checks have been performed for several external operator insertions and implementations of the photon propagator

Preliminary results: Pion mass shift

- $24^3 \times 48$ ensemble at $m_\pi = 275$ MeV
- Extract mass shift $\Delta m_\pi = m_{\pi^+} - m_{\pi^0}$ from fit to



- Obtained Δm_π with $\sim 2\%$ uncertainty

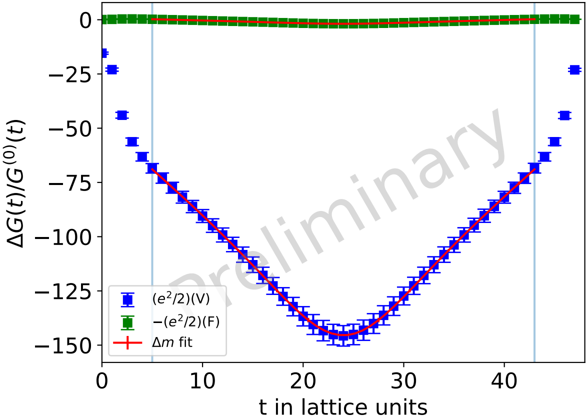
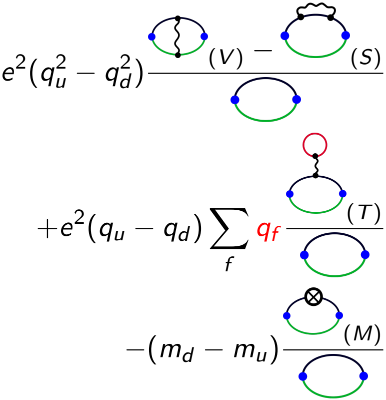


Fig. 5: Pion mass shift in QED_L

Preliminary results: Kaon mass shift

- Extract mass shift
  $\Delta m_K = m_{K^+} - m_{K^0}$ from fit to



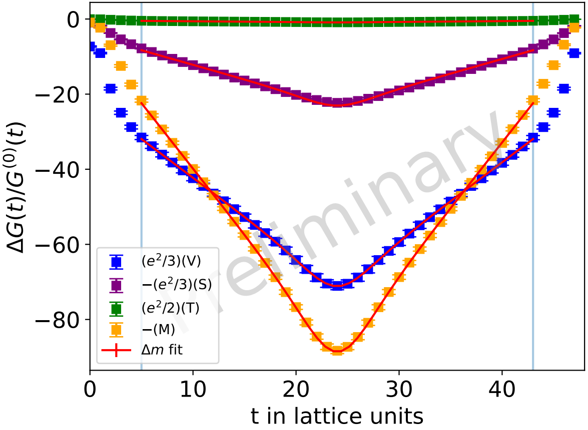
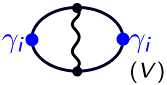


Fig. 6: Kaon mass shift in QED_L

Preliminary results: HVP integrand

- HVP integrand for diagram V on $24^3 \times 48$ ensemble at $m_\pi = 275$ MeV

$$\tilde{f}(t, m_\mu)\ \frac{1}{3}\sum_{i=1}^{3}$$



- For now: Use blinded HVP kernel

$$\tilde{f}(t, m_\mu) = t^4$$

- Good signal to noise ratio

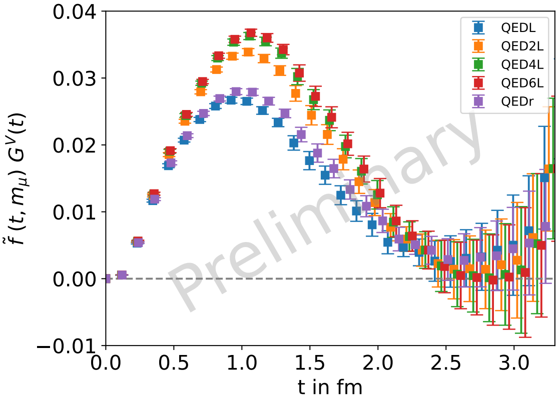
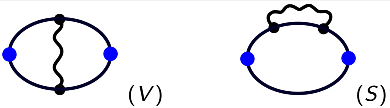


Fig. 7: Integrand for the EM contribution to the HVP (diagram V)

Preliminary results: Connected contribution

- Connected contribution in QED_L with blinded kernel
- Tail is affected by signal-to-noise problem
  $\rightarrow$ Use long-distance reconstruction (Christoph Lehners Talk)



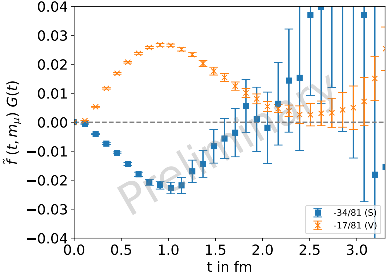


Fig. 8: Fully connected diagrams

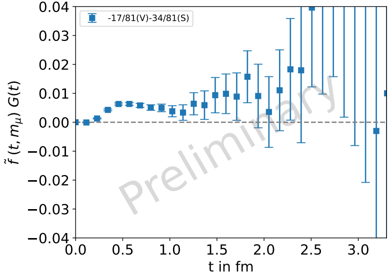
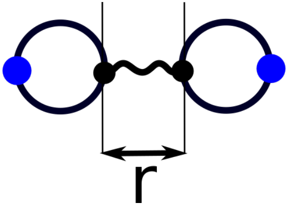


Fig. 9: Combined contribution

Preliminary results: Diagram F

- Contribution from diagram F is finite
- For QED_L, VEV subtraction not necessary, but useful for noise reduction
- Calculate diagram for several separations between internal vertices



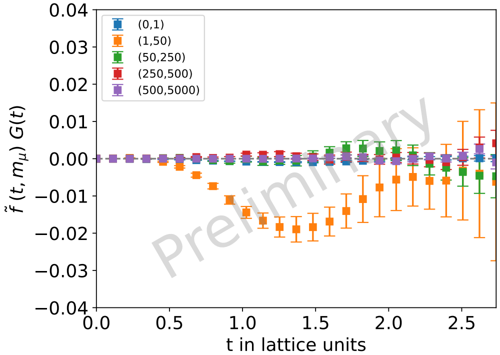
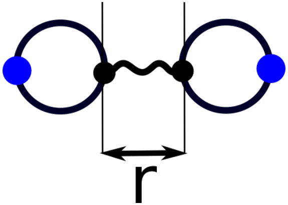


Fig. 10: Diagram F in QED_L for different separations (r_1^2, r_2^2)

Preliminary results: Diagram F

- Contribution from diagram F is finite
- For QED_L, VEV subtraction not necessary, but useful for noise reduction
- Calculate diagram for several separations between internal vertices



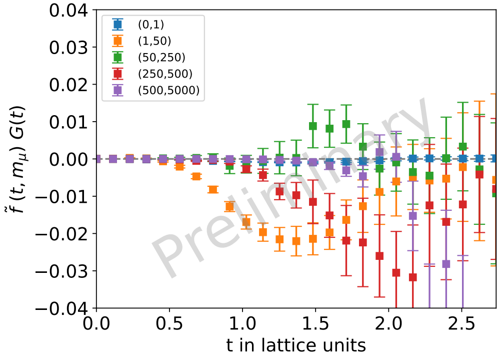


Fig. 11: Diagram F in QED_{6L} for different separations (r_1^2, r_2^2)

Preliminary results: Diagram F

- Dominant part of $G^{1\gamma^*}(z)$ comes from

$$-\frac{17}{81}(\mathrm{V}+2\mathrm{S})+\frac{25}{81}\mathrm{F}$$

- Comparison between diagram F and connected (blinded kernel)
- Noise is comparable between (F) and (V)
- Strong cancellation between connected part and diagram F

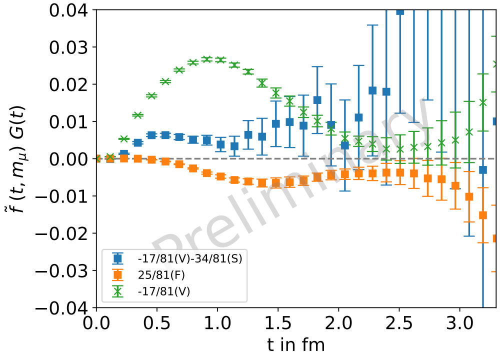
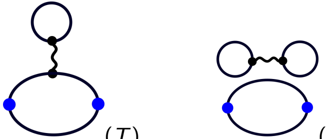


Fig. 12: Comparison of diagram F and connected contribution in QED_L

Preliminary results: Tadpole diagrams

- Reuse tadpole fields from leading order disconnected calculation
- Tadpole diagrams T and D2



- Absolute size of tadpole contributions well below 1/10 of connected part
- Long-distance reconstruction can also be applied here

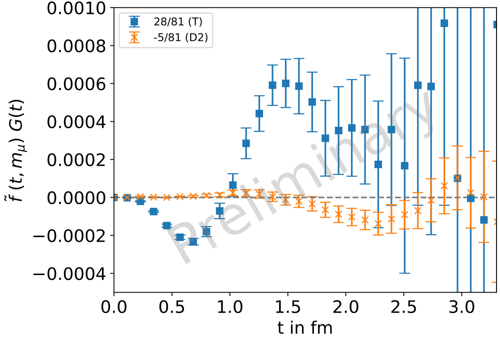
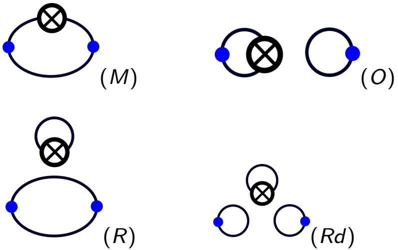


Fig. 13: Tadpole diagrams in QED_L

Preliminary results: Strong isospin breaking

- Computation with scalar insertion
- Global fit with different valence masses



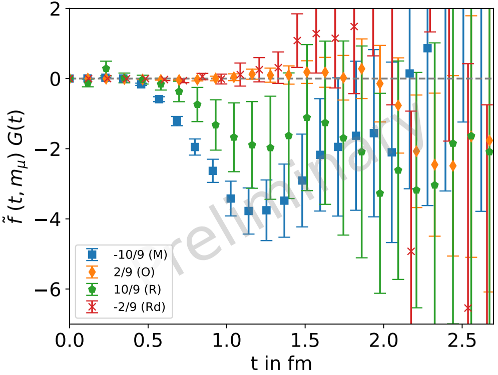
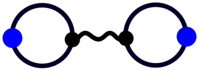


Fig. 14: Strong isospin breaking corrections

Very preliminary results: Physical point



(F)

- Computation of diagram F on two physical point ensembles,
  $m_\pi = 139.32(30)$ MeV,
  $a^{-1} = 1.7312(28)$ GeV
- With different physical volume,
  $m_\pi L = 3.9$ (48I),
  $m_\pi L = 5.2$ (Ca)
- Large finite-size effects for QED_L

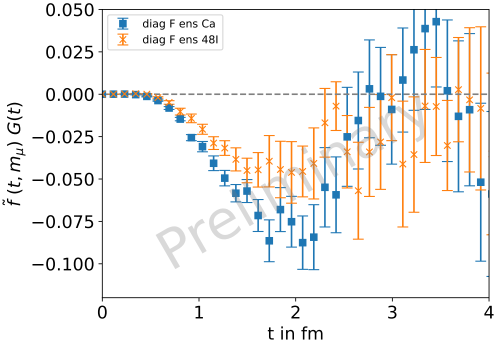
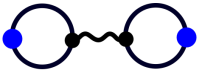


Fig. 15: QED_L

Very preliminary results: Physical point



(F)

- Computation of diagram F on two physical point ensembles, $m_\pi = 139.32(30)$ MeV, $a^{-1} = 1.7312(28)$ GeV
- With different physical volume, $m_\pi L = 3.9$ (48I), $m_\pi L = 5.2$ (Ca)
- Large finite-size effects for QED_L
- Finite-size effects reduced, when volume of QED box $L \to \infty$

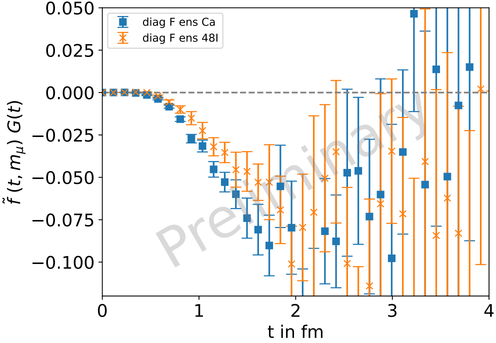
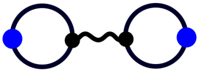


Fig. 16: QED_{2L}

Very preliminary results: Physical point



(F)

- Computation of diagram F on two physical point ensembles,
  $m_\pi = 139.32(30)$ MeV,
  $a^{-1} = 1.7312(28)$ GeV
- With different physical volume,
  $m_\pi L = 3.9$ (48I),
  $m_\pi L = 5.2$ (Ca)
- Large finite-size effects for QED_L
- Finite-size effects reduced, when volume of QED box $L \to \infty$

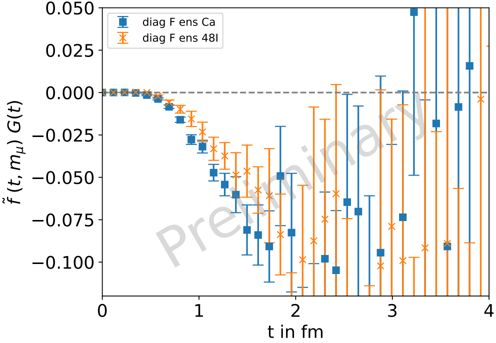
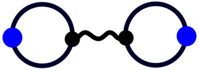


Fig. 17: QED_{4L}

Very preliminary results: Physical point



(F)

- Computation of diagram F on two physical point ensembles,
  $m_\pi = 139.32(30)$ MeV,
  $a^{-1} = 1.7312(28)$ GeV
- With different physical volume,
  $m_\pi L = 3.9$ (48I),
  $m_\pi L = 5.2$ (Ca)
- Large finite-size effects for QED_L
- Finite-size effects reduced, when volume of QED box $L \to \infty$

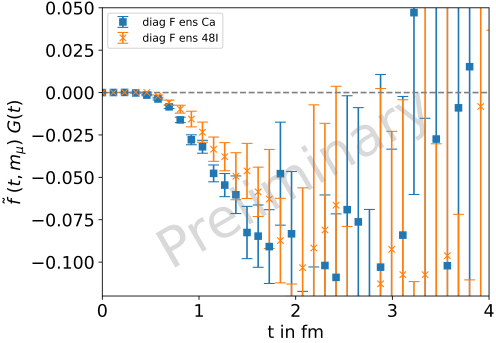
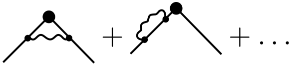


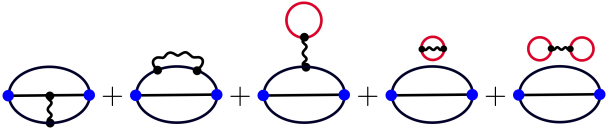
Fig. 18: QED_{6L}

Outlook

- Ongoing production of correlators on physical point ensembles with different lattice spacing for all diagrams
- Global analysis of blindend correlators and extrapolation to physical point defined by the $m_\pi, m_K, \Delta m_K, m_\Omega$
- QED corrections to renormalization factor for local vector current Z_V



- QED corrections to Omega Baryon mass m_Ω for lattice scale setting



Backup: Wick contractions

For Wick contractions: Lattice operator toolkit (http://github.com/jparrino/lotk)

- Lightweight easy to use symbolic manipulation of operators and contractions
- Operators are written in ASCII format
- Algebraic operations and simplifications
- Automatic generation of Wick contractions in LaTeX and graphically
- Algebraic operations and simplifications
- Can look at arbitrary quark charges, to study individual diagrams

```
import lotk
[...]

o1=kaonMinusOperator("x")
o2=kaonPlusOperator("y")
o3=2*o1*o2-o2*o1

print(o3.simplify())


```

Listing: $O^{K^-}(x)O^{K^+}(y)$ operator

Backup: Wick contractions

For Wick contractions: Lattice operator toolkit (http://github.com/jparrino/lotk)

- Lightweight easy to use symbolic manipulation of operators and contractions
- Operators are written in ASCII format
- Algebraic operations and simplifications
- Automatic generation of Wick contractions in LaTeX and graphically
- Algebraic operations and simplifications
- Can look at arbitrary quark charges, to study individual diagrams

```
FACTOR 1.0 0.0
UBAR x
GAMMA 5
S x
SBAR y
GAMMA 5
U y



```

Listing: $O^{K^-}(x)O^{K^+}(y)$ operator

Backup: Wick contractions

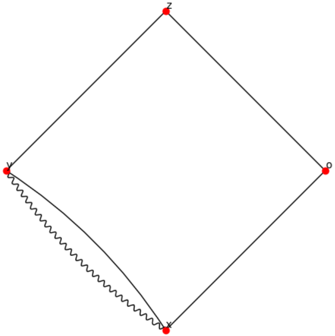
```python
import lotk
[...]
import lotk.diagrams.toLatex as tl
import lotk.diagrams.drawDiagrams as dd

fourptf = kaonNeutralOperator("z")
    *vectorCurrent(["U","D","S"],"y","NU")
    *vectorCurrent(["U","D","S"],"x","MU")
    *kaonBarNeutralOperator("o")
    *photonPropagator("x","y")

contraction = TraceContraction.contract(
    fourptf).simplify()
latex = tl.diagrams_to_latex(str(
    contraction[0]))
dd.generate_feynman_diagram(str(
    contraction[0]))
```

$\rightarrow$

$$-\frac{8}{9}\mathrm{Tr}[S_S(z,o)\gamma^5 S_L(o,x)\gamma^\nu S_L(x,y)\gamma^\mu S_L(y,z)\gamma^5]G_0(x,y)$$

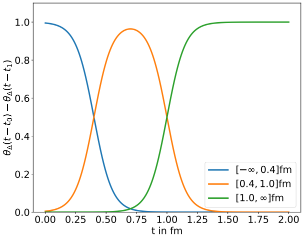


Backup: HVP contribution from lattice QCD

- Time-momentum representation,[Bernecker, Meyer , arxiv:1107.4388]

$$a_\mu^{\mathrm{hvp}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, f(t, m_\mu)\, \frac{1}{3} \sum_{\boldsymbol{x}} \sum_{i=1}^{3} \langle j_i(\boldsymbol{x}, t) j_i(0) \rangle$$

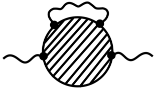
- Vector current $j_\mu^{em} = i\left(\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d + \dots\right)$
- For window quantities $a_\mu^{\mathrm{hvp}} = (a_\mu^{\mathrm{hvp}})^{\mathrm{SD}} + (a_\mu^{\mathrm{hvp}})^{\mathrm{ID}} + (a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$, window function also depends on lattice scale



Backup: QCD+QED: Electromagnetic corrections

- RM123 [Divitiis et al. , arxiv:1303.4896] approach: Expand around isospin symmetric theory

$$\begin{aligned}\langle T\{O(z)\}\rangle_{QCD+QED} &= \{O(z)\}\rangle_{QCD} + e^2 \frac{\partial}{\partial e^2}\langle T\{O(z)\}\rangle_{QCD+QED}\Big|_{e^2=0} + O(e^4) \\ &= \langle T\{O(z)\}\rangle_{QCD} \\ &\quad -\alpha_{\mathrm{EM}} 2\pi \int_{x,y} G_{\nu\rho}(x,y)\langle T\{O(z) j^{em}_{\nu}(y) j^{em}_{\rho}(x)\}\rangle_{QCD} + O(\alpha^2_{\mathrm{EM}})\end{aligned}$$



- With photon propagator in Feynman gauge: $G_{\nu\rho}(x,y) = \delta_{\nu\rho} G(x,y)$

- Strong isospin breaking corrections ($m_u \neq m_d$) are expressed via

$$\begin{aligned}\langle T\{O(z)\}\rangle_{m_f \neq \hat{m}} &= \langle T\{O(z)\}\rangle_{m_f = \hat{m}} \\ &\quad +(m_f - \hat{m})\frac{\partial}{\partial m_f}\langle T\{O(z)\}\rangle\Big|_{m_f=\hat{m}} + O((m_f - \hat{m})^2)\end{aligned}$$

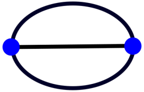
Backup: Finite volume correction

- Leading finite volume correction for charged pion mass in QED_L and QED_r ([Borsanyi et al., arxiv:1406.4088], [Di Carlo et al., arxiv:2501.07936])

$$\Delta m_\pi^2(L) = e^2 m_\pi^2 \Big[\frac{c_2}{4\pi^2 m_\pi L} + \frac{c_1}{2\pi (m_\pi L)^2} - \frac{c_0}{(m_\pi L)^3} \Big(\frac{\langle r_\pi^2 \rangle m_\pi^2}{3} + C \Big) + O(\frac{1}{(m_\pi L)^4}) \Big]$$

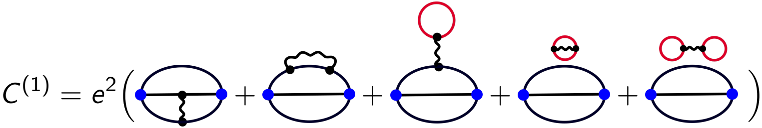
- With real coefficients c_0, c_1, c_2 from [Di Carlo et al., arxiv:2501.07936]
- For QED_r: $c_0 = 0$

Backup: QCD+QED: Omega baryon

- Omega mass can be obtained from correlation function $\langle \Omega^{\alpha}_{\mu}(z)\Omega^{\alpha}_{\mu}(0)\rangle$ of the operator $\Omega^{\alpha}_{\mu} = \epsilon_{ijk}(s_i^T C\gamma_\mu s_j)s_k^{\alpha}$ [Blum et al. , arxiv:1411.7017]
- At leading order: $C^{(0)}(t) =$ 
- Radiative corrections are small (previous electroquenched calculation by RBC/UKQCD in 2018 [Blum et al., arxiv:1801.07224])
- Only composed of strange quarks $\rightarrow$ no strong isospin breaking corrections at leading order
- Challenge: Multiple operators are needed for control of excited states, but in practice doable

Backup: QCD+QED: Omega baryon

- Omega mass can be obtained from correlation function $\langle \Omega^{\alpha}_{\mu}(z)\Omega^{\alpha}_{\mu}(0)\rangle$ of the operator $\Omega^{\alpha}_{\mu} = \epsilon_{ijk}(s_i^T C\gamma_{\mu} s_j)s_k^{\alpha}$ [Blum et al. , arxiv:1411.7017]
- Ongoing project: compute isospin breaking corrections including seaquark effects



- Extract first order effective mass from $\partial_t \frac{C^{(1)}(t)}{C^{(0)}(t)}$