RBC/UKQCD update on hadronic light-by-light scattering

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Muon g-2 Theory Initiative Workshop

Irène Joliot-Curie Lab, Orsay

September 11, 2025

Outline

- RBC setup and previous results
- 2.36 GeV, 64^3 lattice update
- Pion pole contribution at long distance
- Sign of disconnected contribution to the pion form factor

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Hadronic light-by-light contribution to the muon anomaly from lattice QCD with infinite volume QED at physical pion mass

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(RBC and UKQCD Collaborations)

$$a_\mu^{\mathrm{HLbL}} \times 10^{10} = 12.47(1.15)_{\mathrm{stat}}(0.95)_{\mathrm{syst}}[1.49], \qquad (63)$$

RBC setup and previous results

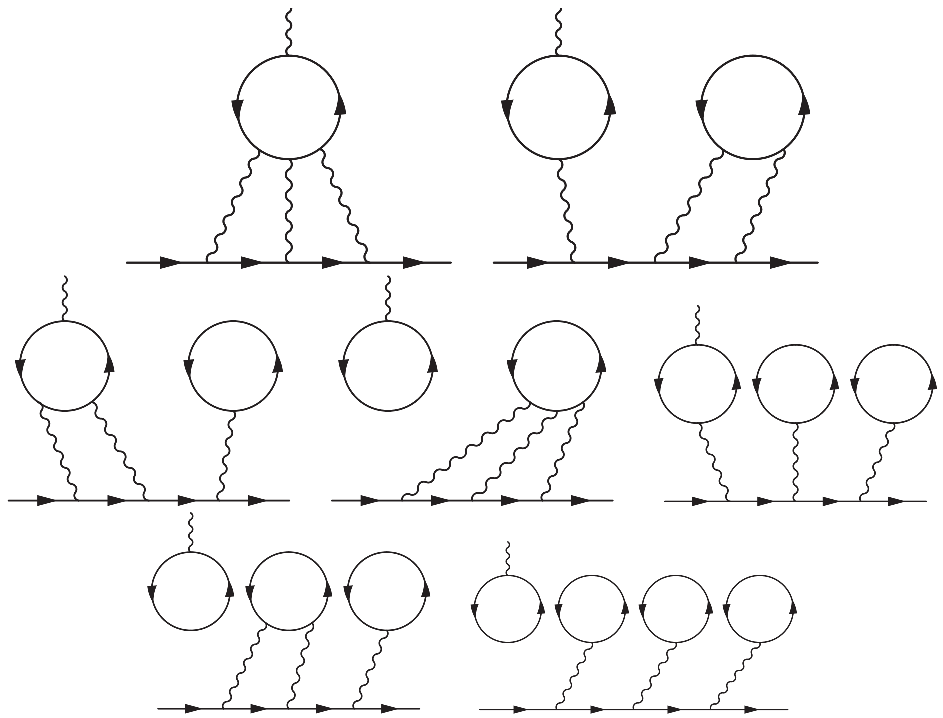


FIG. 2. Diagrams contributing to the muon anomaly.

$$a_\mu^{\mathrm{HLbL}} = \frac{2me^2}{3}\frac{1}{VT}\sum_{x_{\mathrm{op}}}\sum_{x,y,z}\frac{1}{2}\epsilon_{i,j,k}(x_{\mathrm{op}} - x_{\mathrm{ref}}(x,y,z))_j \times (6e^4)\mathcal{H}_{k,\rho,\sigma,\lambda}(x_{\mathrm{op}},x,y,z)\mathcal{M}_{i,\rho,\sigma,\lambda}(x,y,z) \quad (8)$$

where

$$\mathcal{M}_{i,\rho,\sigma,\lambda}(x,y,z) = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{6}i^3\mathcal{G}_{\rho,\sigma,\lambda}(x,y,z)\Sigma_i\right]. \quad (9)$$

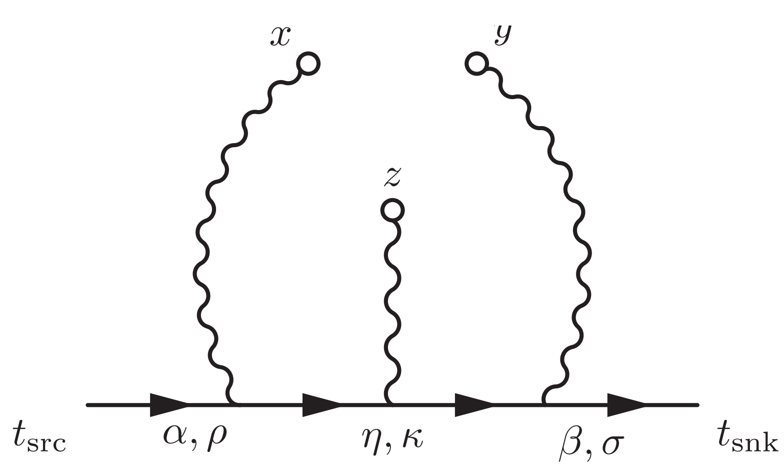


FIG. 3. Diagrammatic representation of the QED weighting function defined in Eq. (9), following Ref. [46].

Subtracted weight function reduces FV, discretization errors

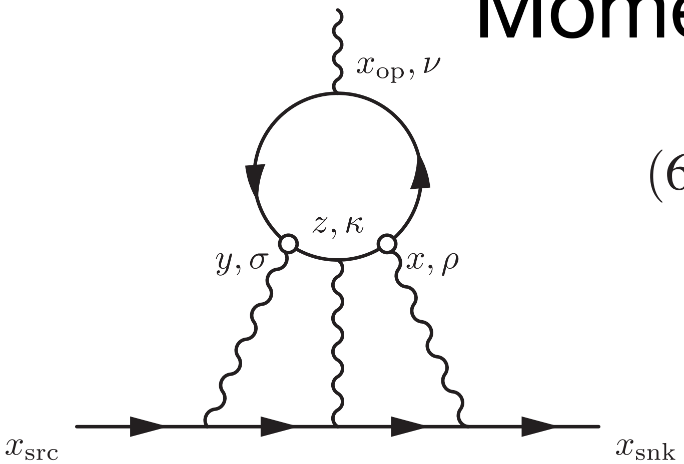
$$\mathfrak{G}^{(2)}_{\sigma,\kappa,\rho}(y,z,x) = \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,x) - \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(z,z,x) - \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,z). \quad (12)$$

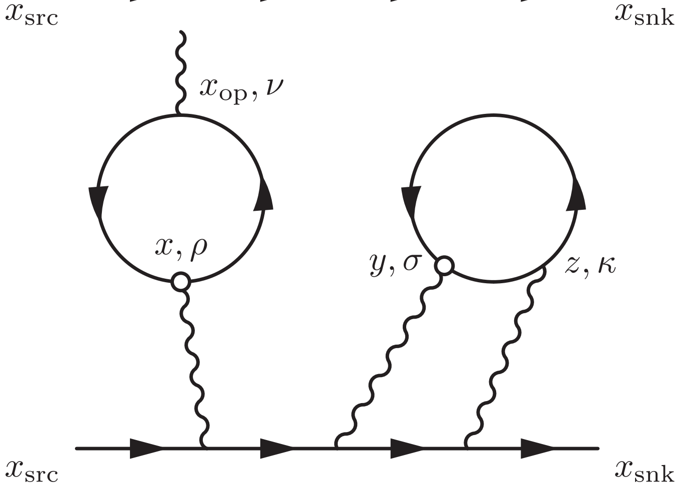
$$i^3\mathcal{G}_{\rho,\sigma,\kappa}(x,y,z) = \mathfrak{G}^{(2)}_{\rho,\sigma,\kappa}(x,y,z) + \mathfrak{G}^{(2)}_{\sigma,\kappa,\rho}(y,z,x) + \mathfrak{G}^{(2)}_{\kappa,\rho,\sigma}(z,x,y) + \mathfrak{G}^{(2)}_{\kappa,\sigma,\rho}(z,y,x) + \mathfrak{G}^{(2)}_{\rho,\kappa,\sigma}(x,z,y) + \mathfrak{G}^{(2)}_{\sigma,\rho,\kappa}(y,x,z). \quad (13)$$

Infinite volume, continuum QED weighting function

Mainz, RBC/UKCD

Moment method based on two point-source propagators





$$(6e^4)\mathcal{H}_{k,\rho,\sigma,\lambda}(x_{\rm op}, x, y, z) = \langle T J_k(x_{\rm op}) J_\rho(x) J_\sigma(y) J_\lambda(z)\rangle_{\rm QCD} \tag{15}$$

$$J_\nu(x) = \sum_{q=u,d,s,c} e_q Z_V \bar{\psi}_q(x)\gamma_\nu \psi_q(x) \tag{16}$$

$$\begin{aligned} &6e^4\mathcal{H}^{\rm con}_{\nu,\rho,\sigma,\kappa}(x_{\rm op}, x, y, z) \\ &\Rightarrow 6e^4\mathcal{H}^{\text{con-no-perm}}_{\nu,\rho,\sigma,\kappa}(x_{\rm op}, x, y, z) \\ &= -6\Big\langle {\rm Re} \sum_{q=u,d,s} e_q^4 {\rm Tr}(\gamma_\nu S_q(x_{\rm op}, x)\gamma_\rho S_q(x, z) \\ &\quad \times \gamma_\kappa S_q(z, y)\gamma_\sigma S_q(y, x_{\rm op}))\Big\rangle_{\rm QCD}, \end{aligned} \tag{18}$$

$$\begin{aligned} &6e^4\mathcal{H}^{\rm discon}_{\nu,\rho,\sigma,\kappa}(x_{\rm op}, x, y, z) \\ &\Rightarrow 6e^4\mathcal{H}^{\text{discon-no-perm}}_{\nu,\rho,\sigma,\kappa}(x_{\rm op}, x, y, z) \\ &= 3\Big\langle \sum_{q'=u,d,s} e_{q'}^2 {\rm Tr}(\gamma_\nu S_{q'}(x_{\rm op}, x)\gamma_\rho S_{q'}(x, x_{\rm op})) \\ &\quad \times \sum_{q=u,d,s} e_q^2 {\rm Tr}(\gamma_\kappa S_q(z, y)\gamma_\sigma S_q(y, z) \\ &\quad - \langle \gamma_\kappa S_q(z, y)\gamma_\sigma S_q(y, z)\rangle_{\rm QCD})\Big\rangle_{\rm QCD}, \end{aligned} \tag{19}$$

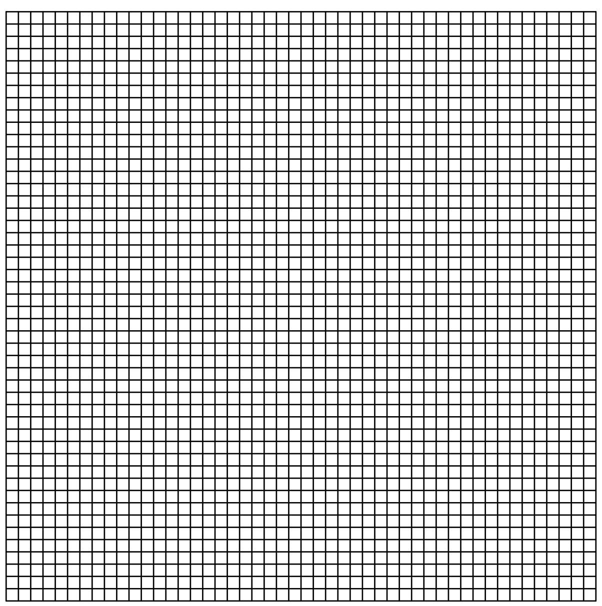
$$\begin{aligned} &x_{\rm ref}(x, y, z) \\ &= x_{\text{ref}-\text{far}}(x, y, z) \\ &= \begin{cases} x & \text{if } |y - z| < \min(|x - y|, |x - z|) \\ y & \text{if } |x - z| < \min(|x - y|, |y - z|) \\ z & \text{if } |x - y| < \min(|x - z|, |y - z|) \\ \frac{1}{3}(x + y + z) & \text{otherwise} \end{cases} \end{aligned} \tag{14}$$

$$x_{\text{ref-discon}} = x. \tag{21}$$

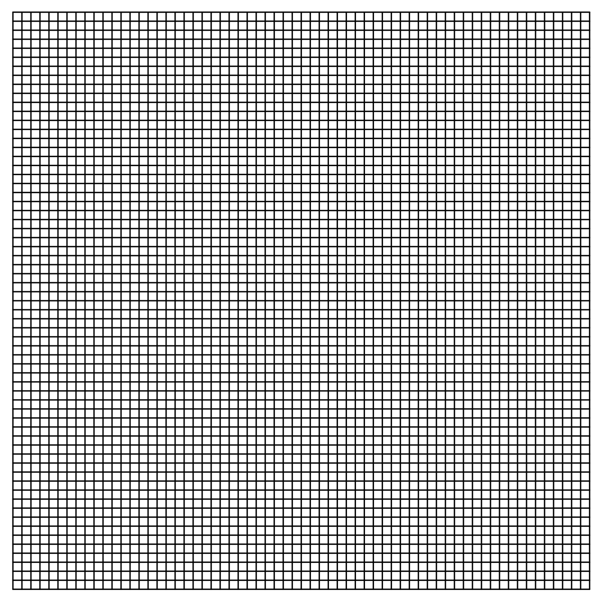
RBC setup and previous results

- 2+1 flavors of MDWF fermions, Iwasaki gluons, generated by RBC/UKQCD collaborations

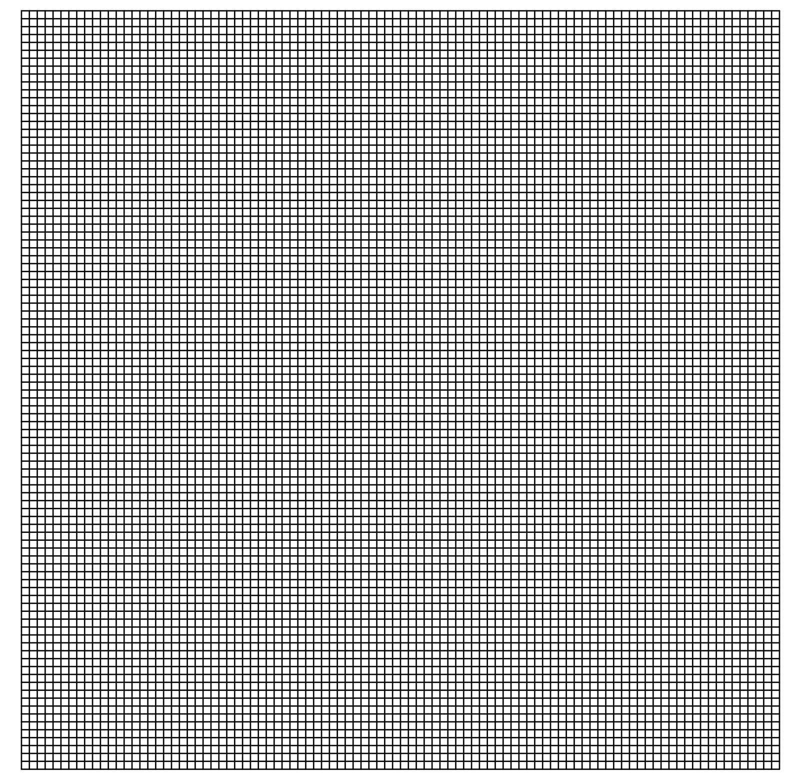
- Physical masses



a^{-1}=1.73 GeV (0.114 fm)
L=5.47 fm, $48^3 \times 96$
Complete

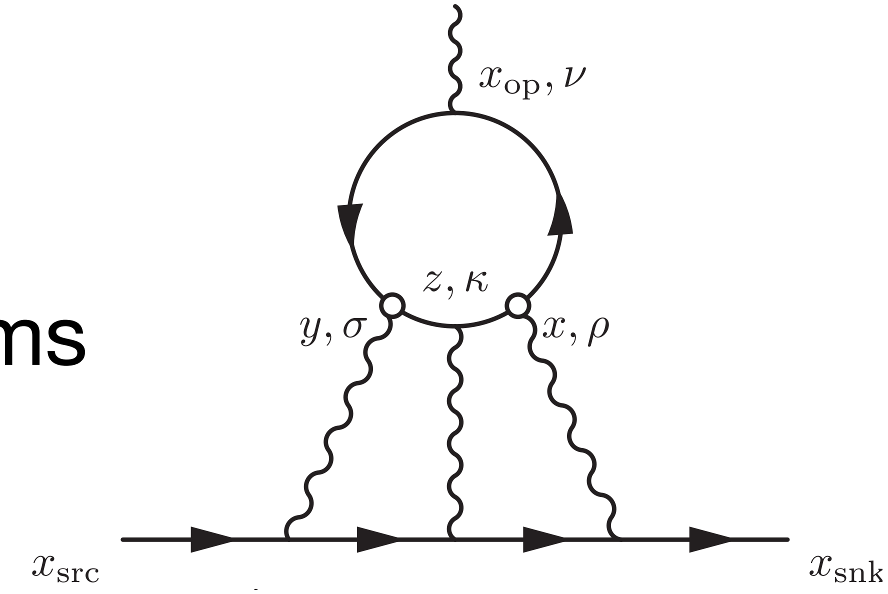


a^{-1}=2.36 GeV (0.084 fm)
L=5.38 fm, $64^3 \times 128$
Analysis underway



a^{-1}=2.69 GeV (0.073 fm)
L=7.0 fm, $96^3 \times 192$
Planned

Sampling strategy for connected diagrams



probability $p(r)$, which is a function of the distance between the two points, $r = |x - y|$,

$$p(r) = \begin{cases} \frac{N_{\rm psrc}-1}{2(L^3T-1)} & \text{if } r = 0 \\ 1 & \text{if } 8 \geq r > 0 \\ \frac{1}{(r/8)^3} & \text{if } L \geq r > 8 \\ 0 & \text{if } r > L \end{cases}, \tag{22}$$

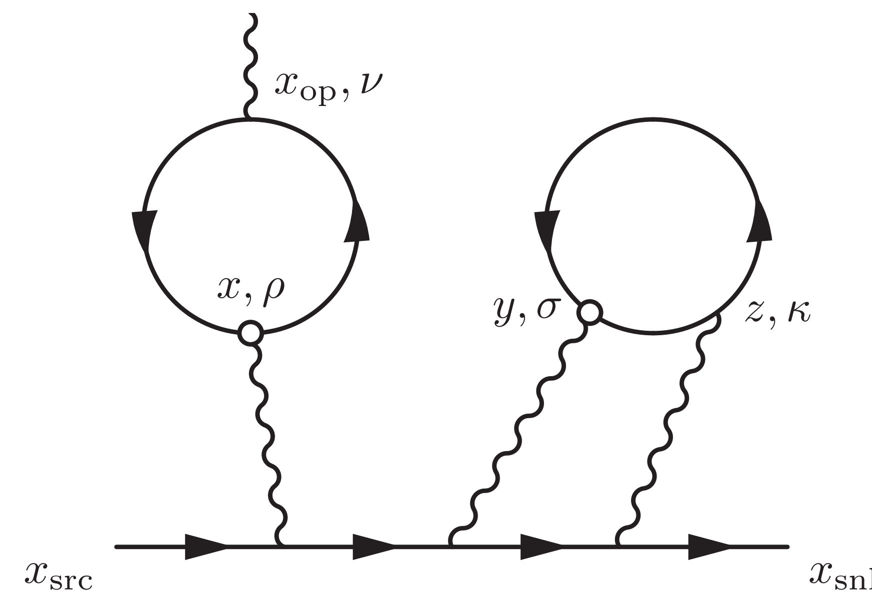
$$w(r) = \begin{cases} w_0 & \text{if } r = 0 \\ w_0/p(r) & \text{if } r > 0 \end{cases}, \tag{23}$$

$$w_0 = \sum_x p(|x|). \tag{24}$$

- $48^3 \times 96$, 113 configurations, 2-step AMA using Z-Möbius approximation
- 2048 light quark point-source props, uniformly distributed over sites
- Subset of 1024 for strange quarks
- $2048 \times (2048 - 1)/2 + 2048$ = 2,098,176 combi's,
  Sample ~57,000 pairs with probability *p*(*r*) and weight with *w*(*r*)
  Loop contribution exponentially smaller for large *r*=|*x*-*y*|
- Uniformly, randomly choose (sparsen) 1/16 of all sink points ($z, x_{\rm op}$)

Sampling strategy for disconnected diagrams

- Noise not suppressed with $r=|x-y|$, sum over all (x,y) pairs with $r < 48$
- Sum over x_{op}
- z depends on x,y: adaptively sparsen sum according to norm of loop: $n(z,y)$, $p_y(z)$ for $t_0 = 5\times10^{-5}$ since loop is suppressed with $|y-z|$
- 17,000 z locations in sum. Checked noise not enhanced with $t_0 = 1\times10^{-3}$



$$n(z,y) = \sum_{\kappa,\sigma} |\mathrm{Tr}(\gamma_\kappa S_q(z,y)\gamma_\sigma S_q(y,z) - \langle \gamma_\kappa S_q(z,y)\gamma_\sigma S_q(y,z)\rangle_{\rm QCD})|^2$$

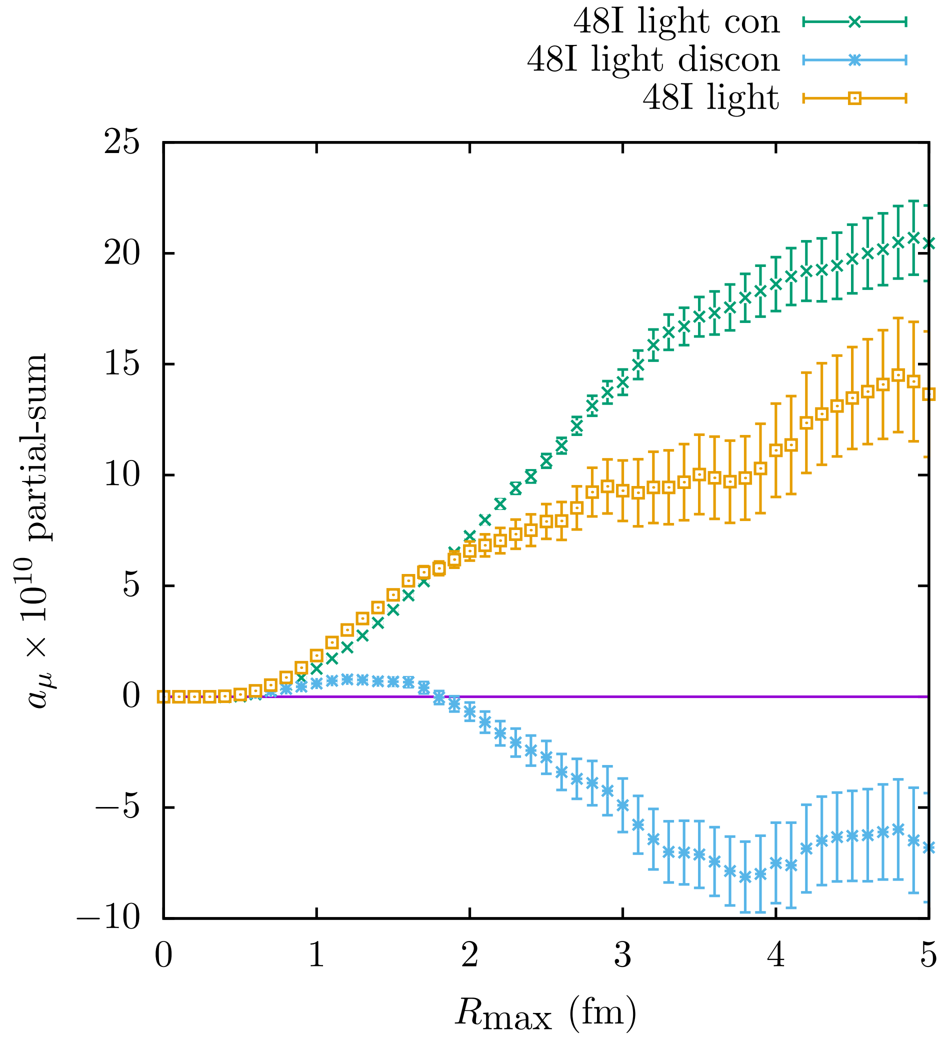
$$p_y(z) = \begin{cases} 1 & \text{if } n(z,y) \geq t_0^2 \quad \text{and} \quad |z-y| \leq L \\ \sqrt{n(z,y)}/t_0 & \text{if } n(z,y) < t_0^2 \quad \text{and} \quad |z-y| \leq L\,, \\ 0 & \text{if } |z-y| > L \end{cases} \tag{25}$$

$$R_{\max} = \max(|x-y|, |y-z|, |x-z|), \qquad (27)$$

Long distance dominated by pion pole:

$$\lim_{R\to\infty} \frac{a_\mu^{\text{discon}}(R_{\max} > R)}{a_\mu^{\text{con}}(R_{\max} > R)} = -\frac{1}{2} \cdot \frac{(e_u^2 + e_d^2)^2}{e_u^4 + e_d^4} = -\frac{25}{34}. \qquad (28)$$

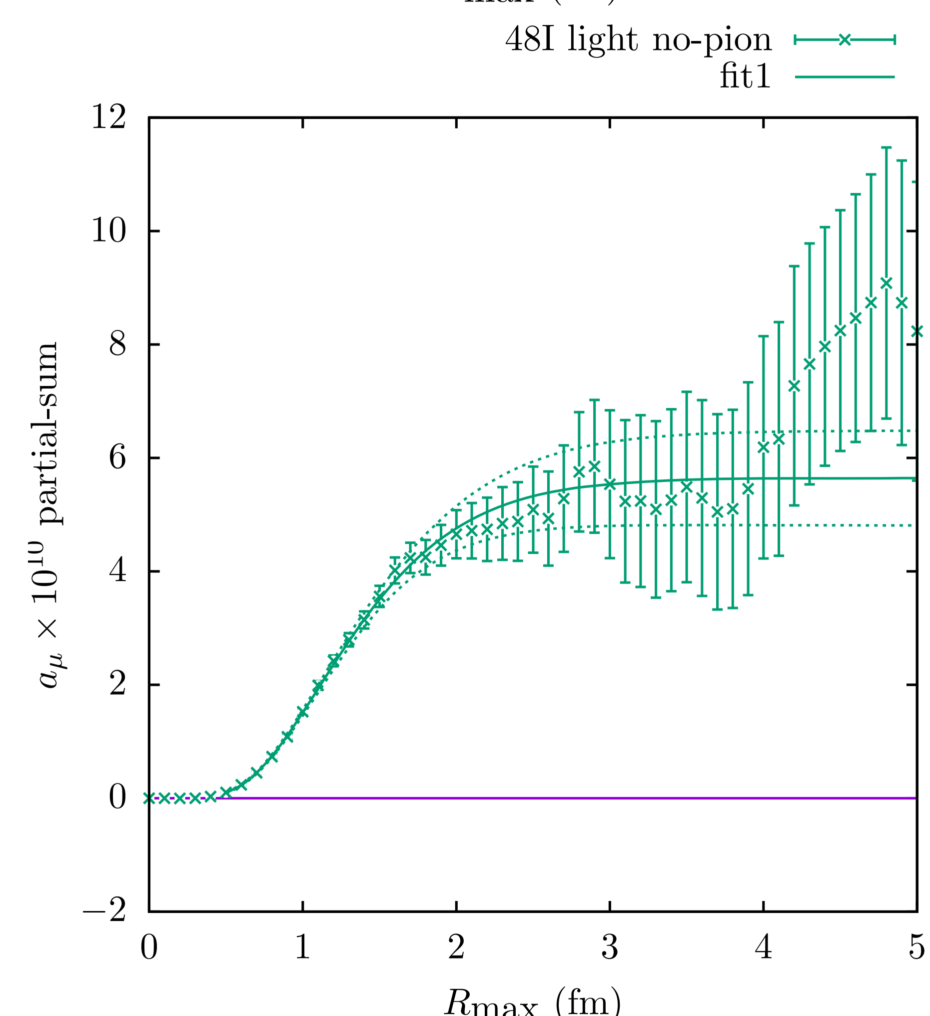
$$a_\mu^{\text{no-pion}} = a_\mu^{\text{discon}} + \frac{25}{34} a_\mu^{\text{con}}, \qquad (29)$$



No-pion piece plateaus much sooner

Fit no pion piece

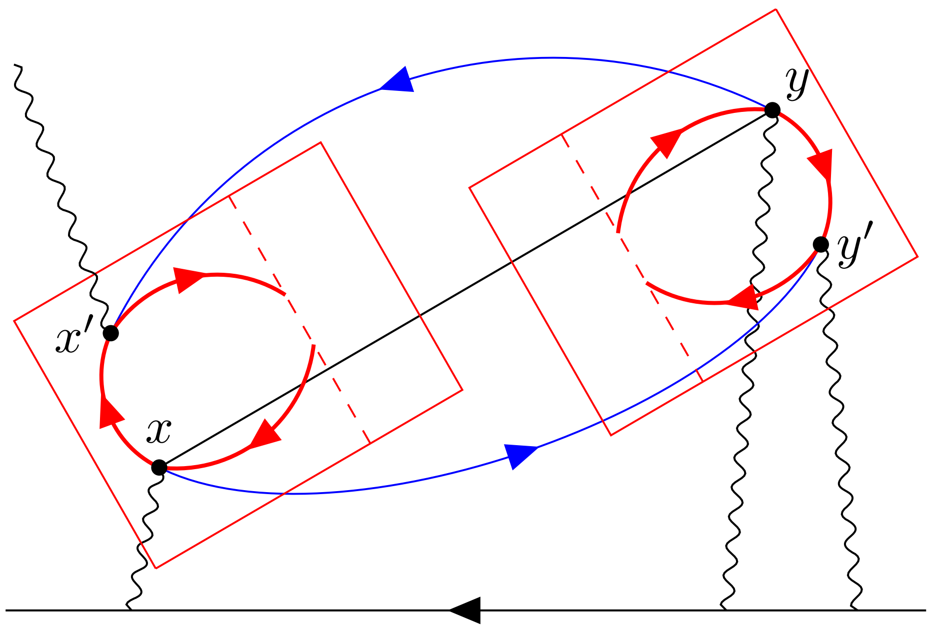
$$f(R_{\max}) = A/\text{fm}^4 \frac{R_{\max}^6}{R_{\max}^3 + (C\,\text{fm})^3} e^{-BR_{\max}/(\text{fm}\cdot\text{GeV})} \qquad (30)$$



Total is more precise

$$a_\mu^{\text{total}} = a_\mu^{\text{no-pion}} + \frac{9}{34} a_\mu^{\text{con}}. \qquad (32)$$

Long distance pion pole contribution (use for R_{max} > 4 fm)



$$6e^4\mathcal{H}^{\pi^0}_{\mu',\mu,\nu',\nu}(x',x,y',y)=\mathcal{F}_{\mu',\mu}\left(\tilde{x},-i\frac{\partial}{\partial x_\mu}\right)\mathcal{F}_{\nu',\nu}\left(\tilde{y},-i\frac{\partial}{\partial y_\mu}\right)\times D_{\pi^0}(x-y). \quad \text{(B9)}$$

$$\langle TJ_{\mu'}(x')J_\mu(x)J_{\nu'}(y')J_\nu(y)\rangle \approx D_{\pi^0}(x-y)\times\mathcal{F}_{\mu',\mu}\left(x'-x, im_\pi\frac{x-y}{|x-y|}\right)\times\mathcal{F}_{\nu',\nu}\left(y'-y, im_\pi\frac{y-x}{|y-x|}\right). \quad (42)$$

- Don’t assume functional form of pion form factor, calculate directly on lattice in Euclidean time and use *O*(4) rotations

- Same QED weight function as before, randomly choose *x*,*y* pairs

- Infinite volume π^0 propagator $D_{\pi^0}(x-y)$

- Approximation: expand derivatives in (B9) in 1/|*x*-*y*|, keep leading

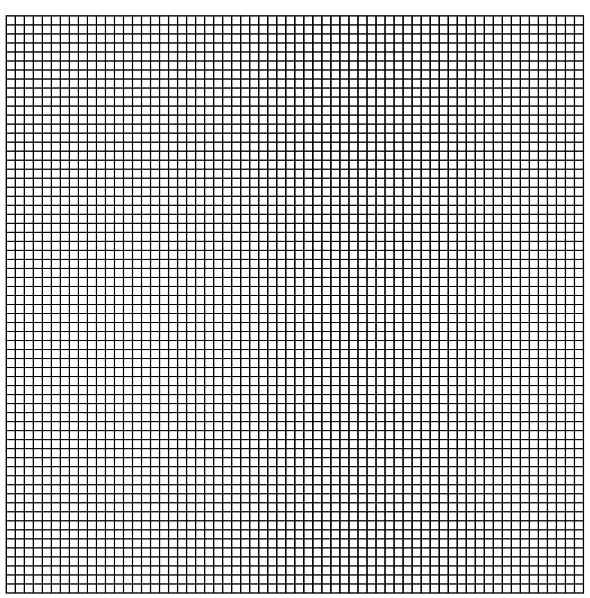
Total

- No π^0 (<2.5 fm) + connected (<4.0 fm) 10.32 (99)(31)[1.04] $\times 10^{-10}$
- π^0 pole long distance (> 4.0 fm) 2.00 (11)(28)[30] $\times 10^{-10}$
- strange disconnected(< 2.5 fm) -0.004 (223)(29)[225] $\times 10^{-10}$
- charm 0.28 (0)(5)[5] $\times 10^{-10}$ (Mainz)
- Sub-leading disconnected 0.00 (0)(7)[7] $\times 10^{-10}$ (Mainz)
- FV -0.47 (0)(11) $\times 10^{-10}$
- pion mass retuning 0.35 (7)(17) $\times 10^{-10}$

$$a_\mu^{\mathrm{HLbL}} \times 10^{10} = 12.47(1.15)_{\mathrm{stat}}(0.95)_{\mathrm{syst}}[1.49], \qquad (63)$$

2.36 GeV, 64^3 lattice update

- 2+1 flavors of MDWF, Iwasaki gluons. Physical masses
- 119 Configurations
- New adaptive sampling method for each configuration
  - On average, ~2048 point-sources and ~ 1/32 fraction of sinks
  - $\sim 337{,}000$ pairs used per config



a^{-1}=2.36 GeV (0.084 fm)
L=5.38 fm, $64^3 \times 128$

$$w(x) = \frac{1}{4}\left(2 + w_l(x) + w_s(x)\right) \tag{1}$$

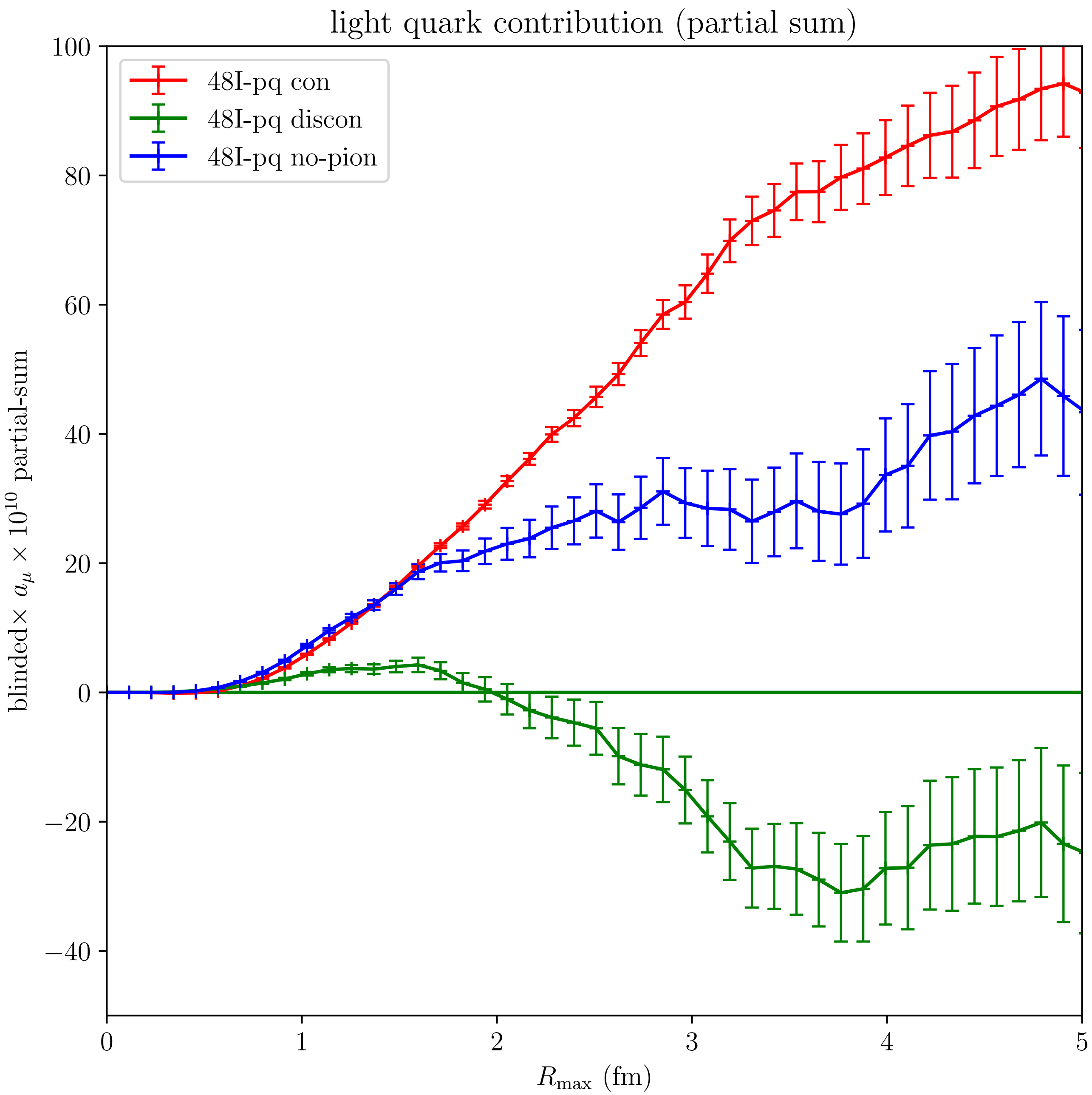
$$w_l(x) = \frac{1}{N_t}\sum_t \frac{|S_l(x;t)|^2}{\langle |S_l(x;t)|^2\rangle_{\text{config-avg}}} \tag{2}$$

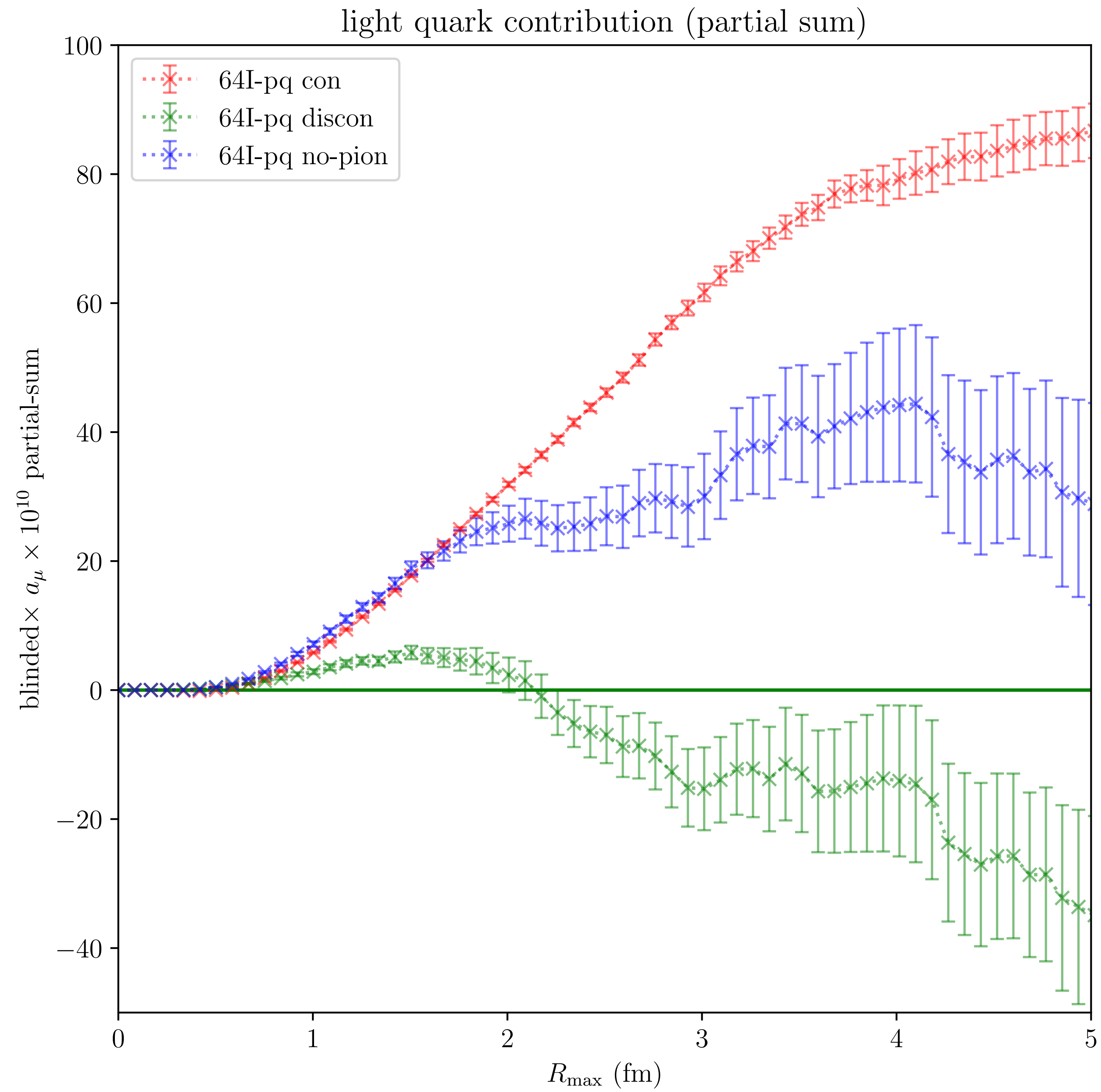
$$w_s(x) = \frac{1}{N_t}\sum_t \frac{|S_s(x;t)|^2}{\langle |S_s(x;t)|^2\rangle_{\text{config-avg}}} \tag{3}$$

$$\langle |S_l(\vec{x}, t_x; t)|^2\rangle_{\text{config-avg}} = \frac{1}{VN_t}\sum_{\vec{x}, t_0} |S_l(\vec{x}, t_x - t + t_0; t_0)|^2$$

$$p(x) = \min\left(1, p_0 w(x)\right) \qquad r(x) \le p(x)$$

Preliminary, blinded, partial sums





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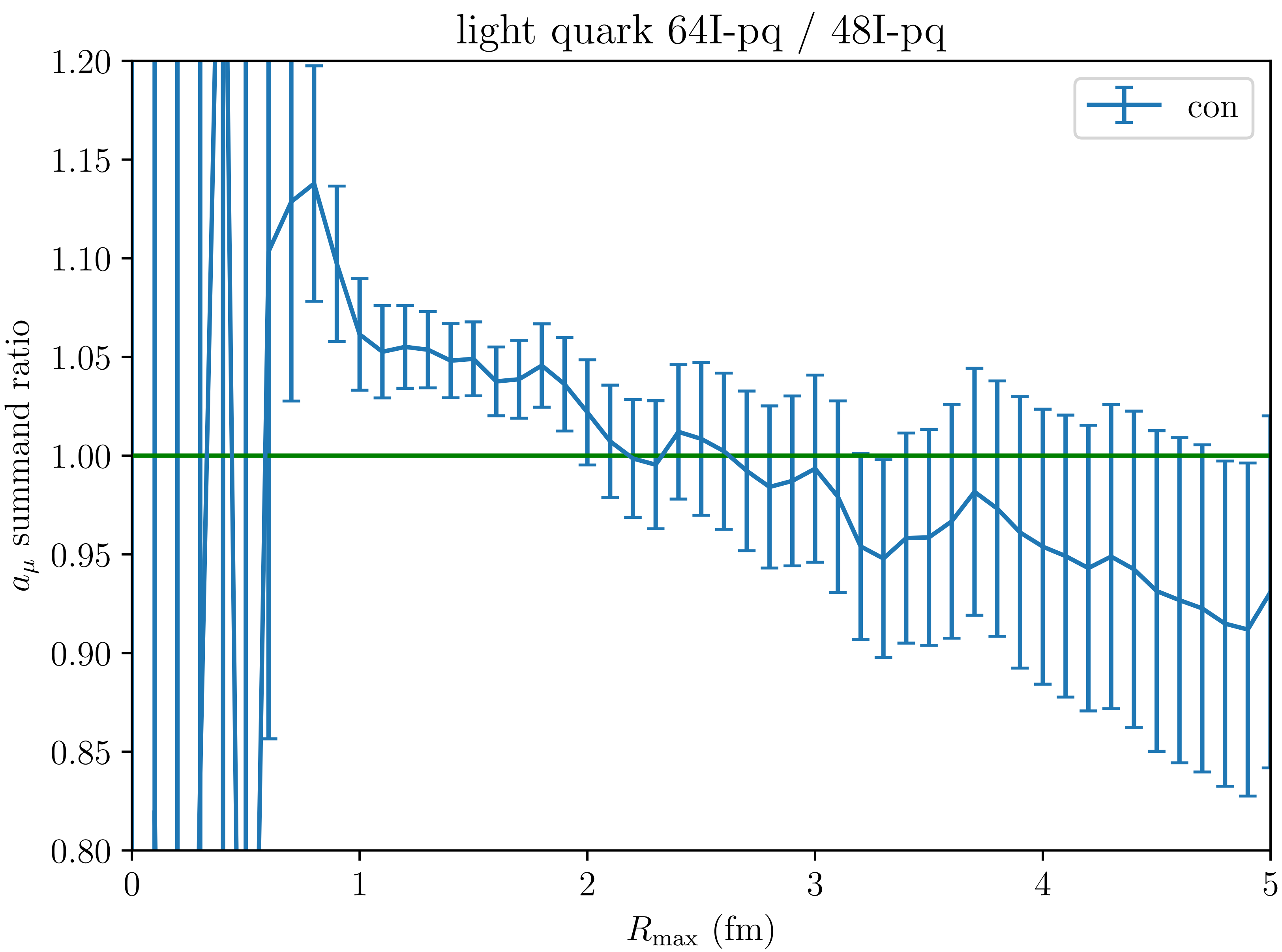
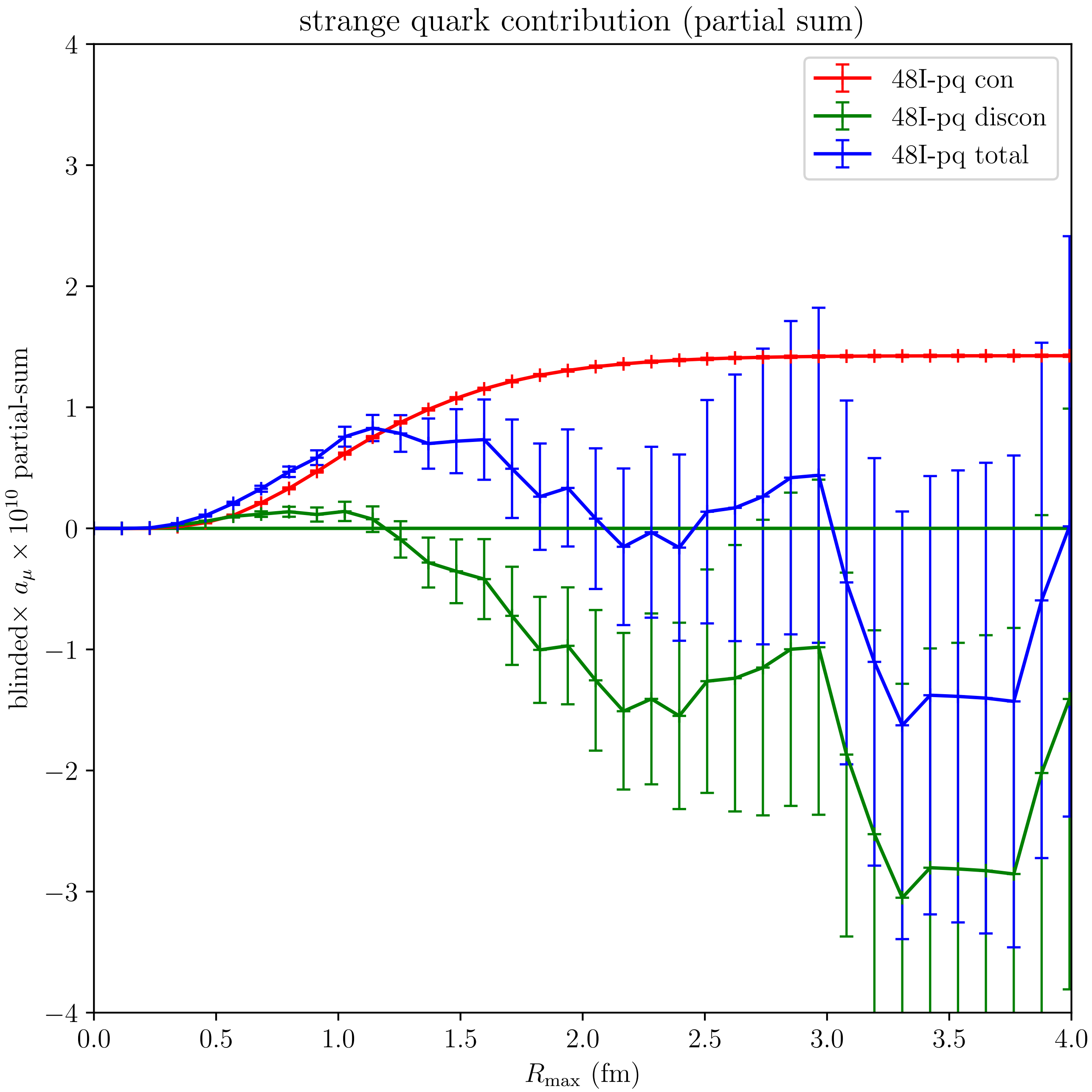
2.36 GeV, 64^3 lattice update

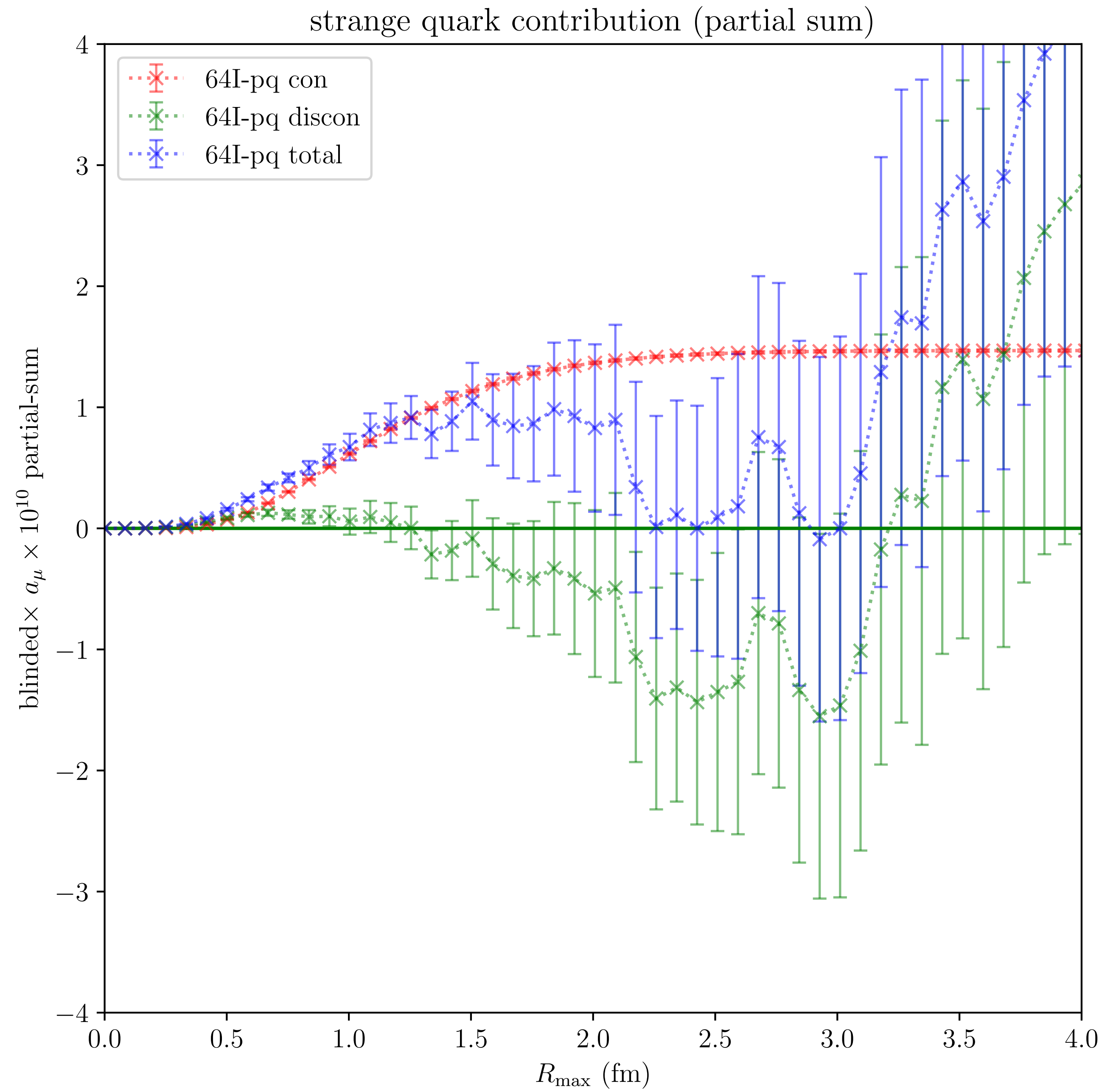
$\frac{64^3}{48^3}$ ratio, partial sum, connected only, for *u,d* quarks, preliminary



Lattice spacing dependence resolved for short(er) distance

Strange quark contribution, preliminary, blinded





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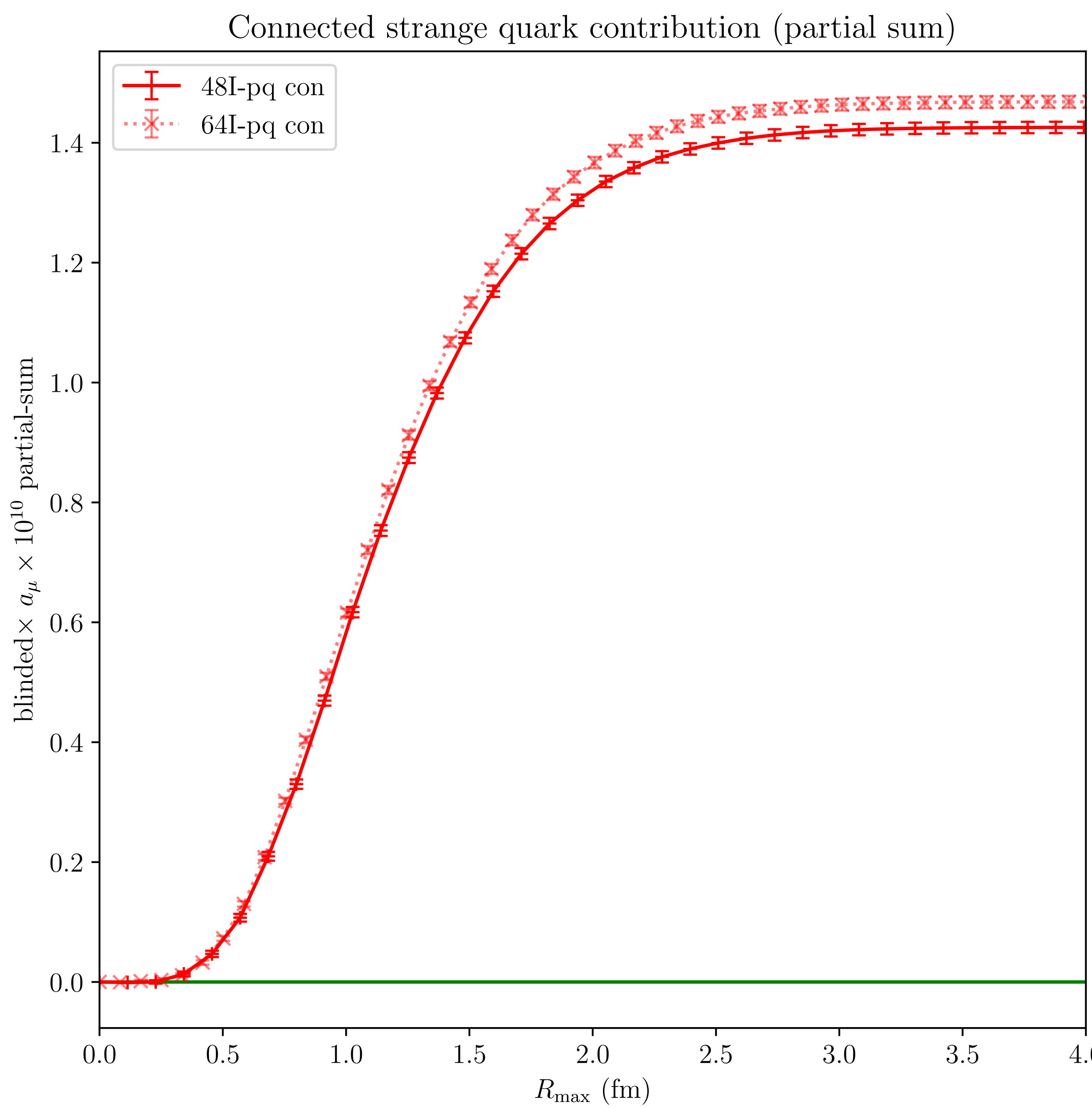
2.36 GeV, 64^3 lattice update

Strange discrepancy with BMW, roughly 2 standard deviations, or 5%.

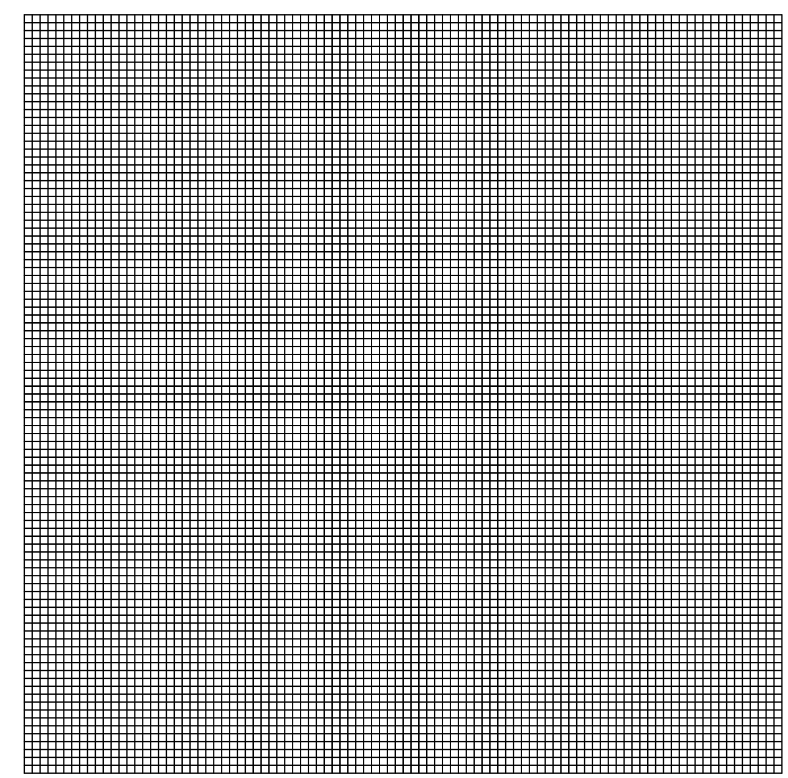
- In continuum limit (only two points, so could change)
- Need to check finite volume effects and isospin scheme too
- Interpolation of QED weights?



- 2+1 flavors of MDWF fermions, Iwasaki gluons.
- Physical masses

Goal: Control statistics, continuum limit and FV systematics < 5%

- Large set of point-source propagators already computed and saved
- Already enough to do strange
- Implement LMA for light quarks
- Continuum limit with 3 lattice spacings



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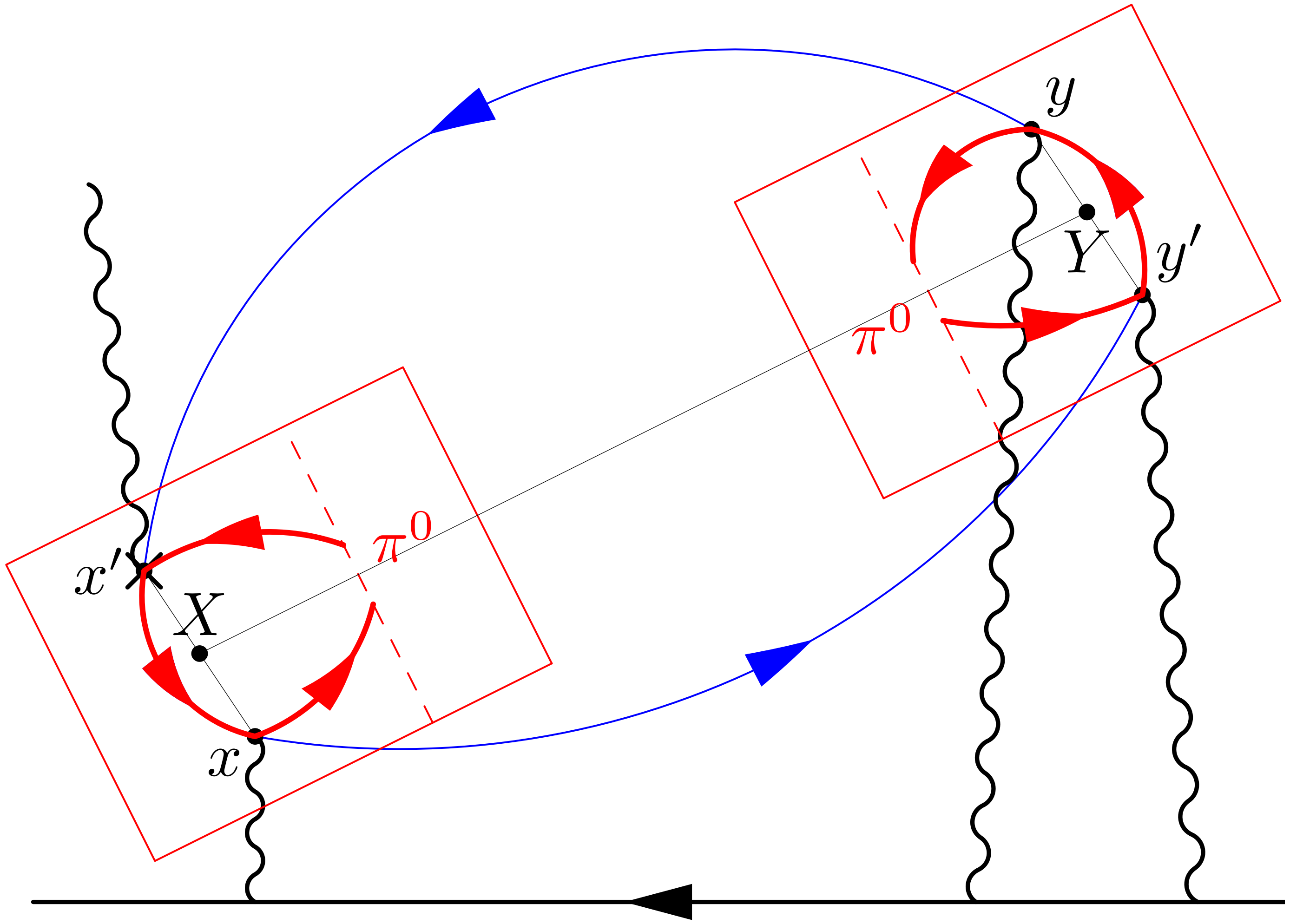


New evaluation method

$$\tilde{x} = x' - x \quad X = \frac{x' + x}{2}$$

$$\tilde{y} = y' - y \quad Y = \frac{y' + y}{2}$$

$$R_\pi = X - Y$$

$$\tilde{x}_\mu = \Lambda_{\mu,\nu} \tilde{x}_\nu^{lat}$$

$$\tilde{y}_\mu = \Lambda_{\mu,\nu} \tilde{y}_\nu^{lat}$$

$$R_{\pi_\mu} = \Lambda_{\mu,\nu} R_{\pi_\nu}^{\text{lat}}$$

$$r_\pi = \max\{|x - x'|, |y - y'|\}$$

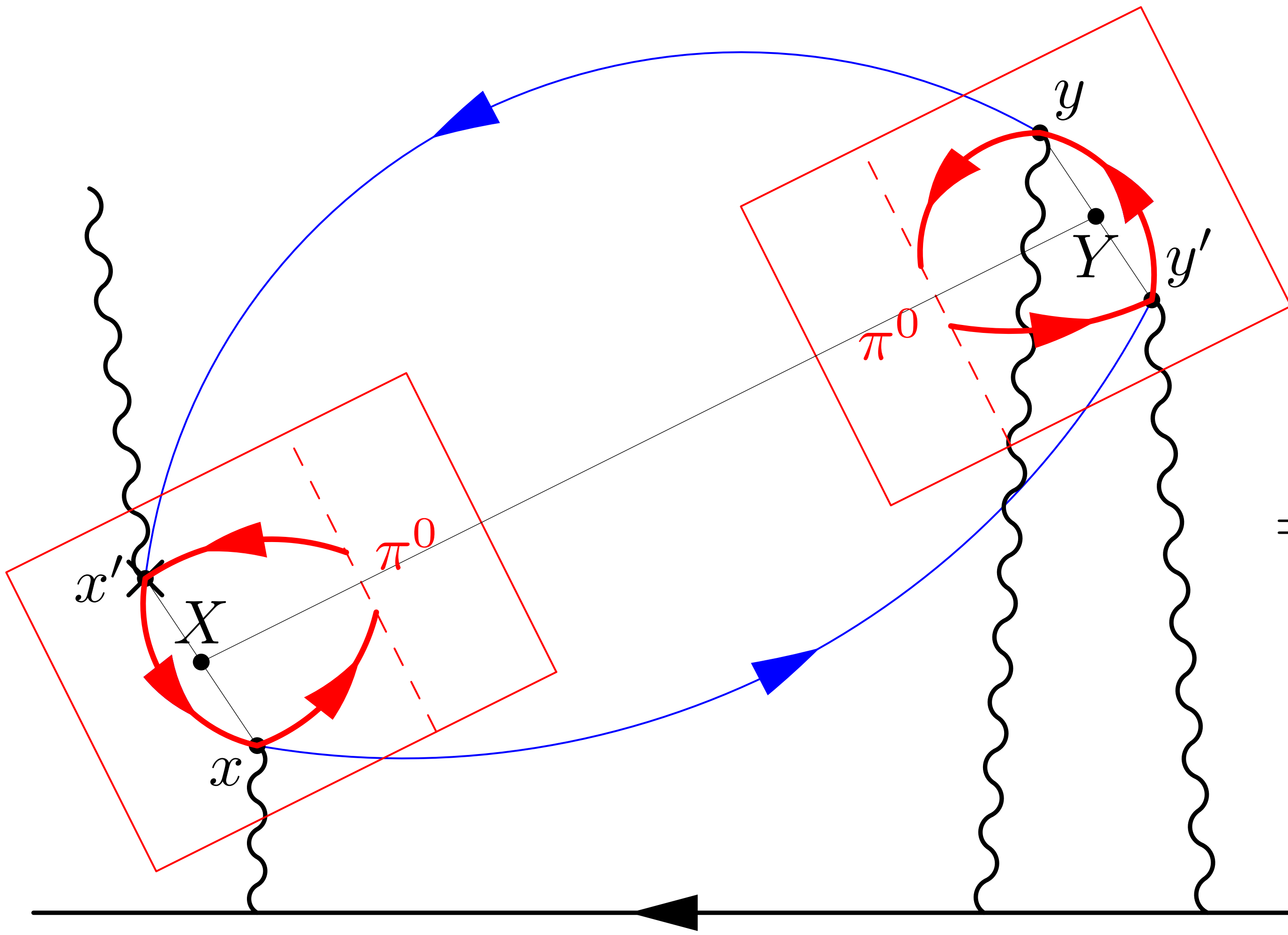
$$\tilde{\mathcal{F}}_{\mu'\mu}(x, q) = \epsilon_{\mu'\mu\alpha\beta} x_\alpha q_\beta \tilde{\mathcal{F}}_{\pi^0\gamma\gamma}(x^2, q \cdot x)$$

$$\tilde{\mathcal{F}}_{\pi^0\gamma\gamma}(x^2, q \cdot x) = \mathcal{F}_{\pi^0\gamma\gamma}(x^2, q \cdot x) e^{-iq\cdot\frac{x}{2}}$$

$$(6e^4)\mathcal{H}^{\pi_0}_{k,\rho,\sigma,\lambda}(\tilde{x}, \tilde{y}, R_\pi) = \epsilon_{k\rho\alpha\beta}\epsilon_{\sigma\lambda\gamma\zeta}\tilde{x}_\alpha \tilde{y}_\gamma \tilde{\mathcal{F}}^E_{\pi^0\gamma\gamma}(\tilde{x}^2, im_\pi \hat{R}_\pi \cdot \tilde{x})\tilde{\mathcal{F}}^E_{\pi^0\gamma\gamma}(\tilde{y}^2, -im_\pi \hat{R}_\pi \cdot \tilde{y}) B_{\beta\zeta}(R_\pi)$$

$$B_{\beta\zeta}(R_\pi) = \frac{m^2}{4\pi^2 R_\pi^2}\left[\left[(mR_\pi)K_1(mR_\pi)\right](\hat{R}_\pi)_\zeta(\hat{R}_\pi)_\beta - K_2(mR_\pi)(\delta_{\beta\zeta} - 4(\hat{R}_\pi)_\zeta(\hat{R}_\pi)_\beta)\right]$$

Pion pole contribution at long distance update [A. Shcherbakov]



New evaluation method

$$= \frac{1}{N_s} \sum_{s^{MC}}^{N_s} \frac{I(s^{MC}, R_{\max}^{cut}, r_\pi^{cut}, R_\pi^{cut})}{w(s^{MC})} \times w_{tot}$$

$$I(\tilde{x}, \tilde{y}, R_\pi, R_{\max}^{cut}, r_\pi^{cut}, R_\pi^{cut}) = \frac{me^2}{3} \mathfrak{Z}(\frac{R_\pi - \tilde{x}}{2} + C, -\frac{R_\pi - \tilde{y}}{2} + C, -\frac{R_\pi + \tilde{y}}{2} + C) \epsilon_{i,j,k} \tilde{x}_j (6e^4) \mathcal{H}^{\pi_0}_{k,\rho,\sigma,\lambda}(\tilde{x}, \tilde{y}, R_\pi)$$
$$\times \mathcal{M}_{i,\rho,\sigma,\lambda}(\frac{R_\pi - \tilde{x}}{2} + C, -\frac{R_\pi - \tilde{y}}{2} + C, -\frac{R_\pi + \tilde{y}}{2} + C)$$
$$\times \Theta(R_{\max} - R_{\max}^{cut}) \times \Theta(|R_\pi| - R_\pi^{cut}) \times \Theta(r_\pi^{cut} - r_\pi)$$

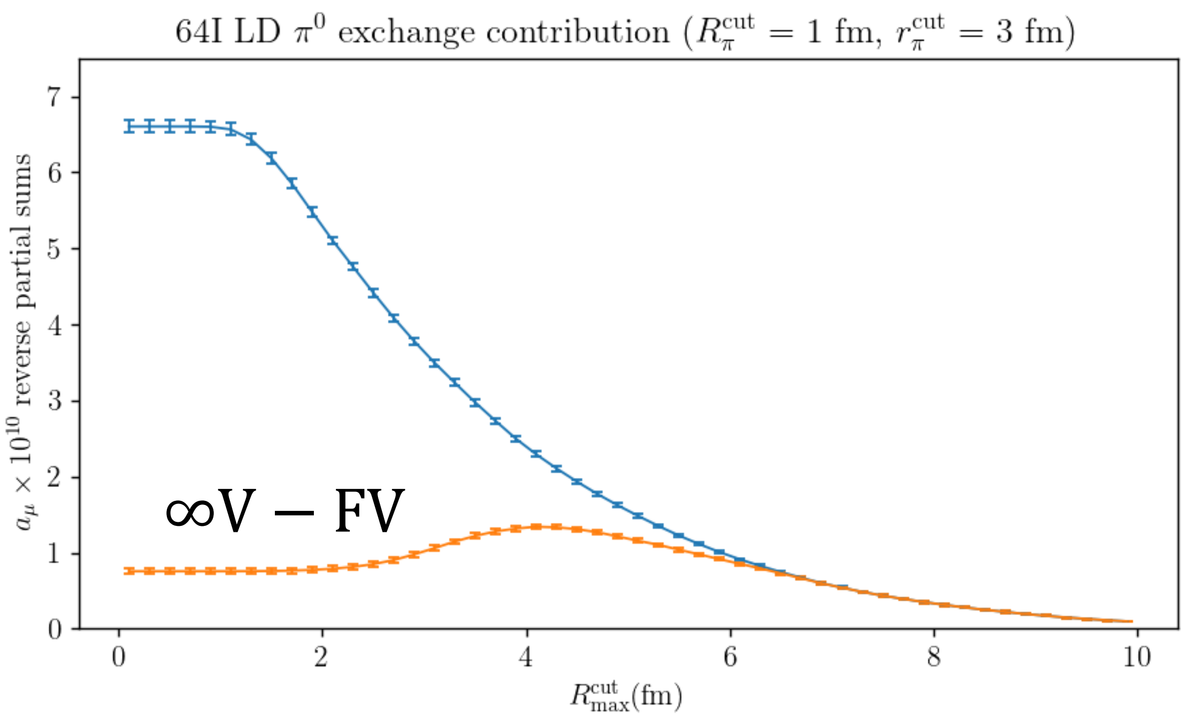
Importance sample:

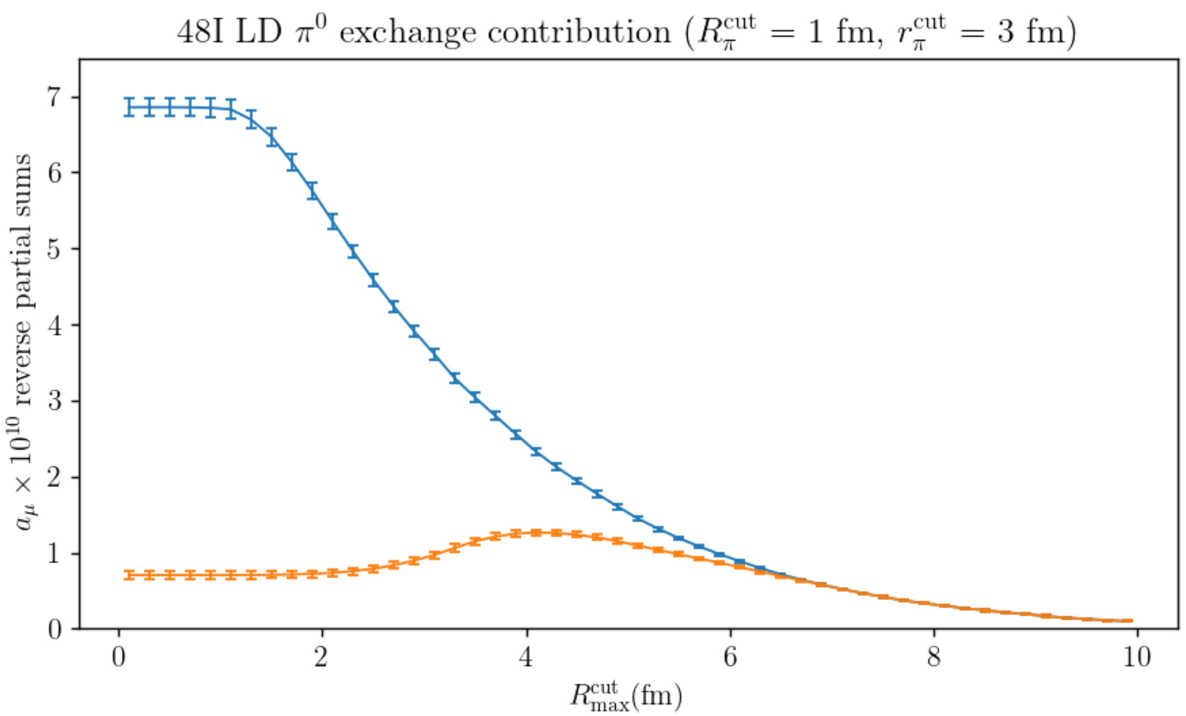
$$w(\tilde{x}, \tilde{y}, R_\pi) = \frac{1}{|\tilde{x}|^4 + \tilde{a}^2} \frac{1}{|\tilde{y}|^4 + \tilde{b}^2} \frac{e^{(-m|R_\pi|)}}{|R_\pi|^{3/2}}$$

$$w_{tot} = \int_{R_\pi} \sum_{\tilde{x}, \tilde{y}} w(\tilde{x}, \tilde{y}, R_\pi)$$

	# of configs	m_π(MeV)	a^{-1}(GeV)	N_s
48I	119	139	1.73	1500000
64I	112	139	2.359	1500000

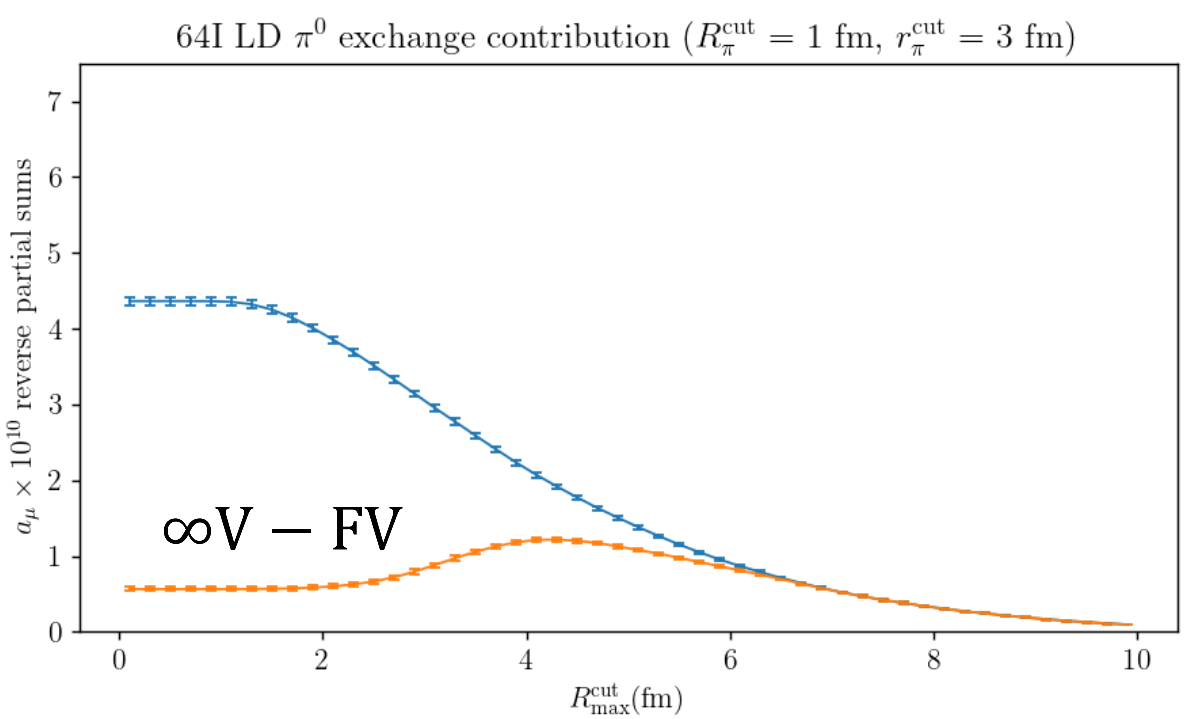
New

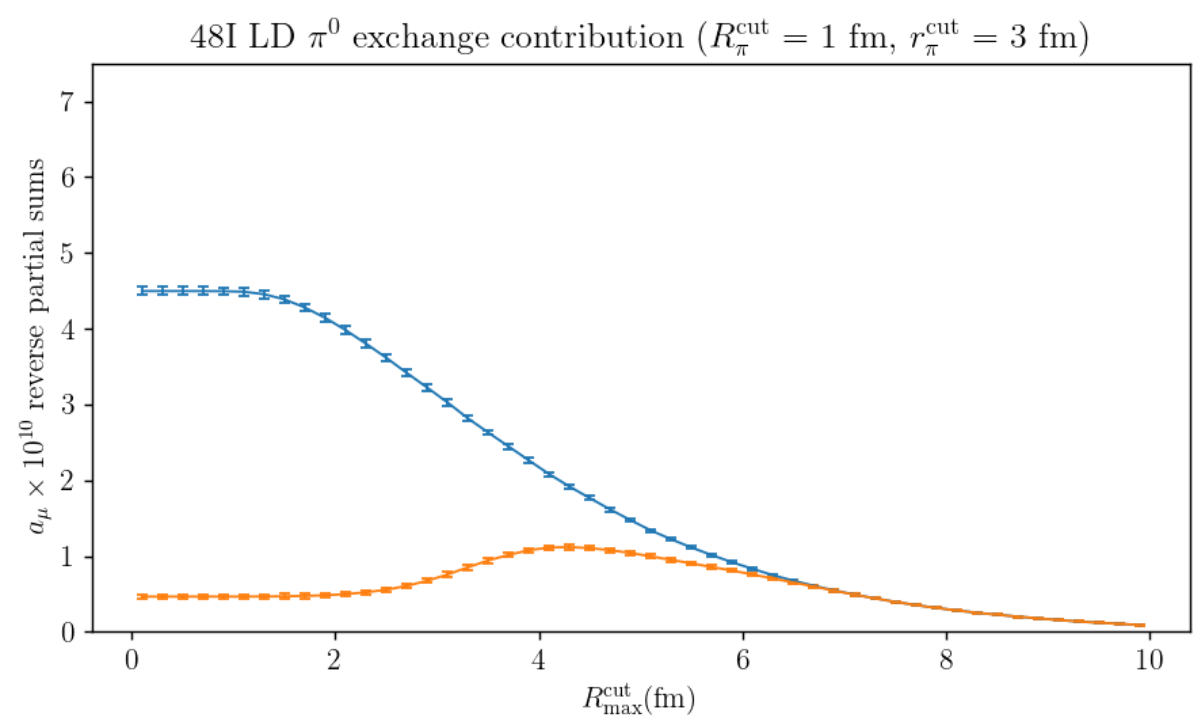




FV estimate from FV form of pion propagator
(FV correction is partial sum, not reverse)

Old





Difference at small $R_{\text{max}}^{\text{cut}}$ due to $1/|X\text{-}Y|$ expansion

Sign of the disconnected contribution to the Pion form factor

- Result checked with 4 independent codes/4 authors plus 1 automatic contractor
- Result checked against Mainz, BMW, and ETM setup (see Figure 1.)
- Less tension with experiment decay width

$$\pi^0 \to \gamma\gamma$$

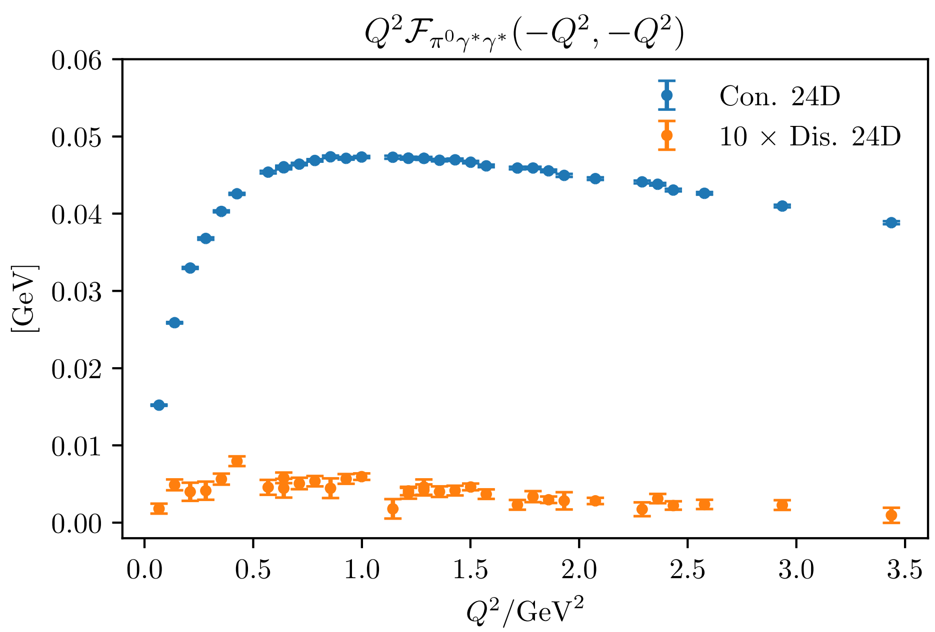
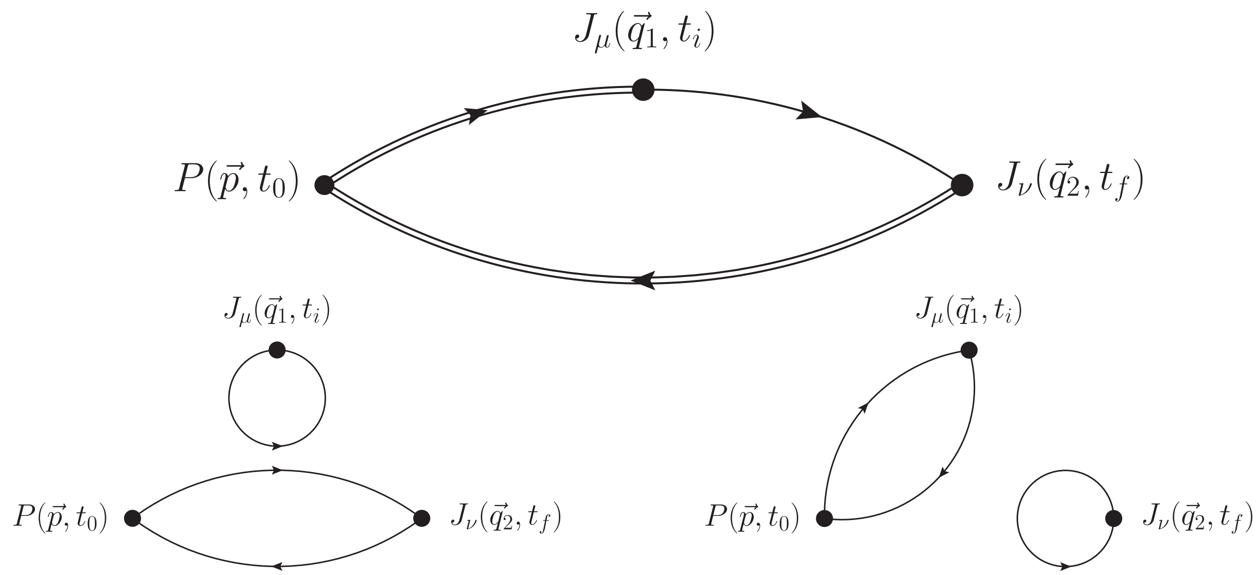


Figure 1: The connected and disconnected parts of the TFFs computed using the approach employed by the Mainz, BMW, and ETMC collaborations. The results shown here are compiled using the 24D ensemble as a representative example.



Figs. From Mainz Group [10.1103/PhysRevD.94.074507]

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