Quark-disconnected contribution to the pion transition form factor (Mainz ’19)

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8th Plenary workshop of the Muon g-2 Theory Initiative - Orsay

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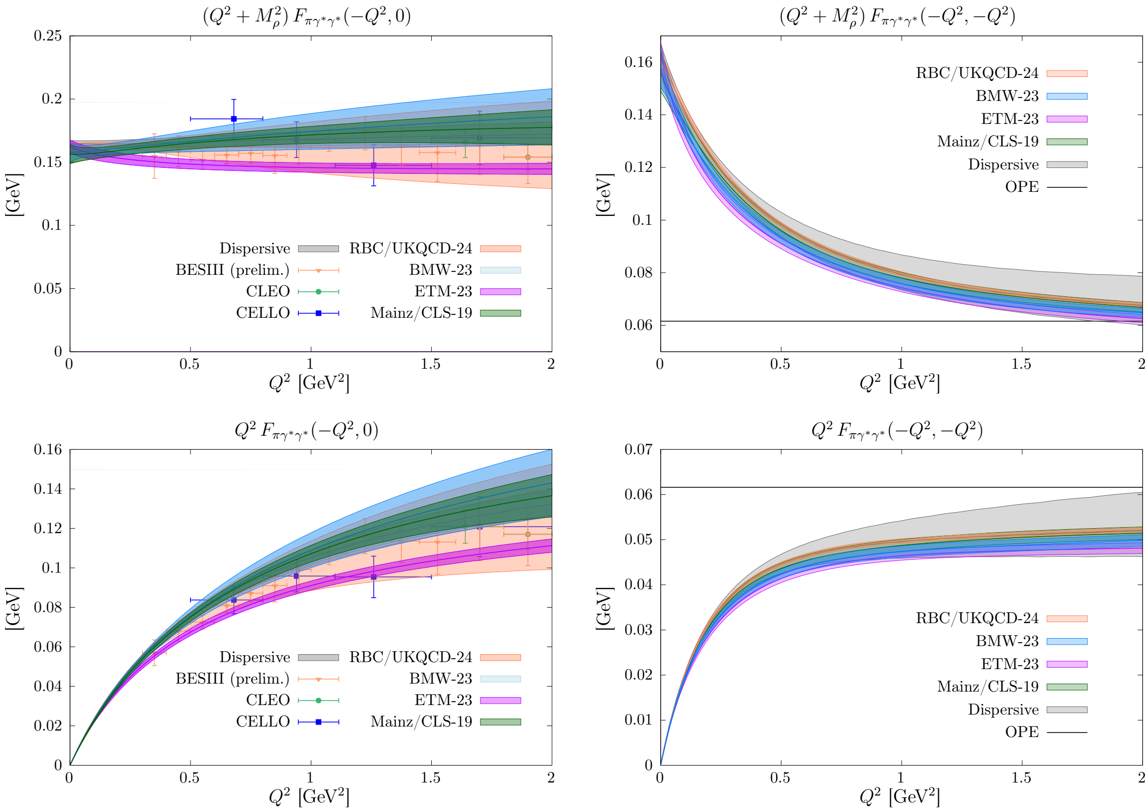
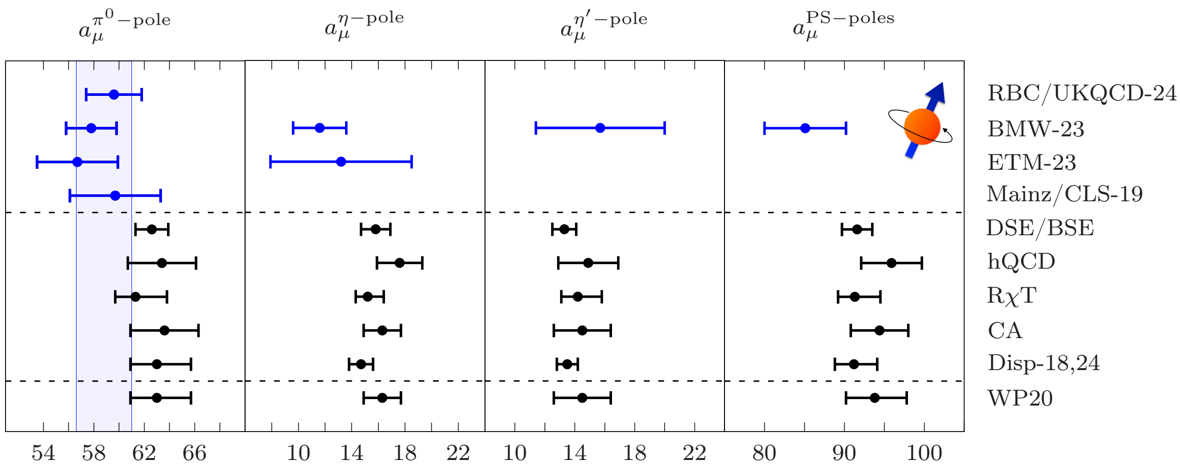
CPT Centre de Physique Théorique

- 4 collaborations have published their results : Mainz'19 , ETMC'23, BMW'23 and RBC/UKQCD'24
- All are based on different discretizations of the fermion action



Summary plot from WP'25



- White paper '25 consensus :

$$a_\mu^{\pi^0\text{-pole}} = a_\mu^{\pi^0\text{-pole,conn}} \pm \left| a_\mu^{\pi^0\text{-pole,disc}} \right| = 58.8(1.1)_{\text{stat}}(1.1)_{\text{syst}}(1.6)_{\text{disc}}[2.2]_{\text{tot}} \times 10^{-11}$$

→ not all collaborations agree on the sign of the quark-disconnected contribution

→ in their most recent calculation, RBC-UKQCD found an opposite sign compared to other collaborations

→ disconnected contribution $\sim 1\%$ at the level of the TFF

- This talk : update on the Mainz'19 analysis

- New analysis of disconnected data compared to Mainz'19 publication [1903.09471]

  → new data set for the disconnected contribution (both 2-point and 1-point functions).

  (data generated by Konstantin Ottnad in Mainz)

  → analysis performed by Jonna Koponen - using an independent analysis code

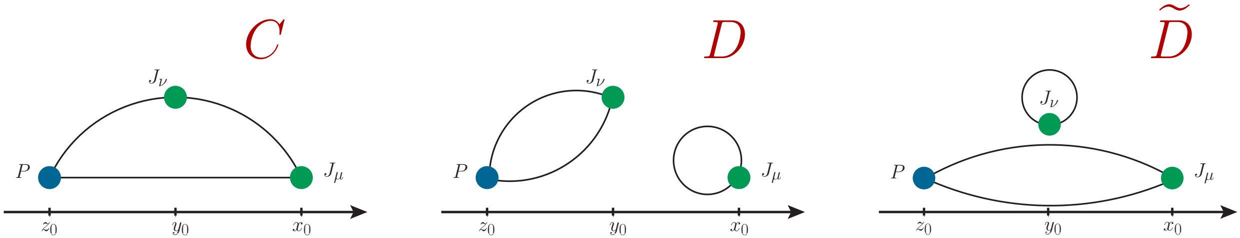
- Connected and disconnected contributions have been computed using two different codes

  → check conventions between codes : sign of the Fourier transform, γ matrix basis, ...

  → we have re-computed some two-point functions using both codes to check that everything is consistent

  → so far, no mistake or bug has been found

- Can we check the sign of the disconnected contribution independently ?



The correlation function of interest is

$$\mathcal{C}^{(3)}_{\mu\nu}(x_0, y_0, z_0; \vec{q}_1, \vec{p}) \equiv a^6 \sum_{\vec{x},\vec{z}} e^{-i\vec{q}_1(\vec{x}-\vec{y})+i\vec{p}\cdot(\vec{z}-\vec{y})} \langle\, j^{\mathcal{Q}}_\mu(x) j^{\mathcal{Q}'}_\nu(y) P^\dagger(z)\,\rangle\,.$$

In terms of Wick contractions (e.m. current : $\mathcal{Q}^{\text{e.m.}} = \frac{\lambda^3}{2} + \frac{1}{\sqrt{3}}\frac{\lambda^8}{2}$) :

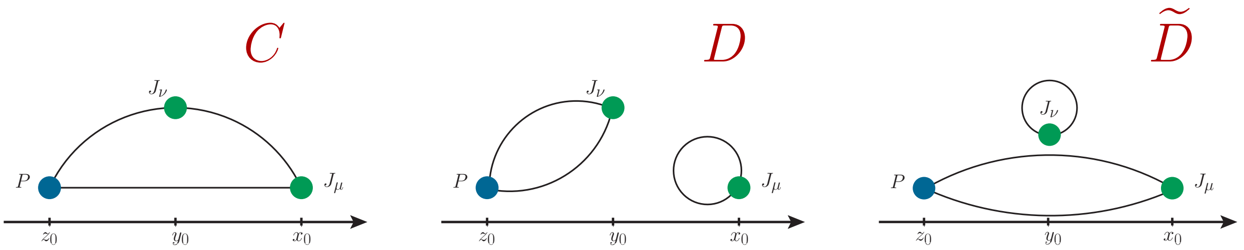
$$\mathcal{C}^{(3)}_{\mu\nu} = -\frac{1}{3\sqrt{2}}\left(2\,C_{\mu\nu} + D^{(l-s)}_{\mu\nu} + \widetilde{D}^{(l-s)}_{\mu\nu}\right).$$

In iso-symmetric QCD : one isovector ($I=1$) current and one isoscalar ($I=0$) current :

$$\mathcal{C}^{(3)}_{\mu\nu} = \mathcal{C}^{(3,sv)}_{\mu\nu} + \mathcal{C}^{(3,vs)}_{\mu\nu} \qquad \text{with} \qquad \begin{aligned} \mathcal{C}^{(3,sv)}_{\mu\nu} &= -\frac{1}{3\sqrt{2}}\left(C_{\mu\nu} + D^{(l-s)}_{\mu\nu}\right) \\ \mathcal{C}^{(3,vs)}_{\mu\nu} &= -\frac{1}{3\sqrt{2}}\left(C_{\mu\nu} + \widetilde{D}^{(l-s)}_{\mu\nu}\right) \end{aligned}$$

Thus, the following combination of disconnected diagrams has an interpretation within QCD :

$$\mathcal{C}^{(3,sv)}_{\mu\nu} - \mathcal{C}^{(3,vs)}_{\mu\nu} = -\frac{1}{3\sqrt{2}}\left[D^{(l-s)}_{\mu\nu} - \widetilde{D}^{(l-s)}_{\mu\nu}\right]$$



- In the kinetic regime $x_0 > y_0 > z_0$:

$$\mathcal{C}_{\mu\nu}^{(3,sv)} = -\frac{1}{3\sqrt{2}} \left(C_{\mu\nu} + D_{\mu\nu}^{(l-s)} \right)$$

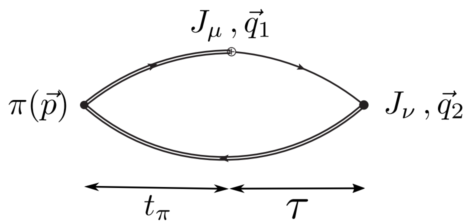
→ long-distance $\pi\pi$ contribution between y_0 and x_0 must cancel between C and D

$$\mathcal{C}_{\mu\nu}^{(3,vs)} = -\frac{1}{3\sqrt{2}} \left(C_{\mu\nu} + \widetilde{D}_{\mu\nu}^{(l-s)} \right)$$

→ isoscalar contribution between y_0 and x_0 must cancel between C and $\widetilde{D}$

$$\mathcal{C}_{\mu\nu}^{(3)} = -\frac{1}{3\sqrt{2}} \left(2\, C_{\mu\nu} + \widetilde{D}_{\mu\nu}^{(l-s)} + D_{\mu\nu}^{(l-s)} \right)$$

$$\mathcal{C}_{\mu\nu}^{(3,sv)} - \mathcal{C}_{\mu\nu}^{(3,vs)} = -\frac{1}{3\sqrt{2}} \left[D_{\mu\nu}^{(l-s)} - \widetilde{D}_{\mu\nu}^{(l-s)} \right]$$



- We start with the following amplitude (on-shell pion)

$$\widetilde{A}^{(sv)}_{\mu\nu}(\tau,\vec{q}_1,\vec{p}) = \lim_{z_0\to-\infty} e^{E_{\vec{p}}(y_0-z_0)}\,\mathcal{C}^{(3,sv)}_{\mu\nu}(y_0+\tau,y_0,z_0;\vec{q}_1,\vec{p})$$
$$\widetilde{A}^{(vs)}_{\mu\nu}(\tau,\vec{q}_1,\vec{p}) = e^{-E_{\vec{p}}\tau}\,\widetilde{A}^{(sv)}_{\nu\mu}(-\tau,\vec{q}_2,\vec{p}).$$

- The form factor is then obtained by integrating over τ (the separation between the two currents)

$$\epsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta\,\mathcal{F}_{sv}(q_1^2,q_2^2) = i^{n_0}\frac{2E_{\pi,\vec{p}}}{Z_\pi}\int_{-\infty}^{\infty} d\tau\,e^{\omega_1\tau}\,\widetilde{A}^{(sv)}_{\mu\nu}(\tau,\vec{q}_1,\vec{p}) = i^{n_0}\,M^{\mathrm{E},sv}_{\mu\nu}(q_1,p),$$

where $q_1=(\omega_1,\vec{q}_1)$ and $q_2=(E_{\pi,\vec{p}}-\omega_1,\vec{p}-\vec{q}_1)$.

- On the lattice, easier to look a the level of the amplitude ($\sim$ correlation function)

$$\widetilde{A}^{(sv)}_{\mu\nu}(\tau,\vec{q}_1,\vec{p}) = \frac{Z_\pi}{4\pi\,E_{\pi,\vec{p}}}\int_{-\infty}^{\infty} d\tilde{\omega}\,e^{-i\tilde{\omega}\tau}\,M^{\mathrm{E},sv}_{\mu\nu}((i\tilde{\omega},\vec{q}_1),p).$$

- Strategy : use a model for $\mathcal{F}_{sv}(q_1^2,q_2^2)$ $\to$ compute $\widetilde{A}^{(sv)}_{\mu\nu}(\tau,\vec{q}_1,\vec{p})$ $\to$ compare with lattice data

$$\mathcal{C}^{(3,sv)}_{\mu\nu} - \mathcal{C}^{(3,vs)}_{\mu\nu} = -\frac{1}{3\sqrt{2}}\left[D^{(l-s)}_{\mu\nu} - \widetilde{D}^{(l-s)}_{\mu\nu}\right]$$

Starting point : dispersive formula in [Hoferichter et. al '18]

$$\mathcal{F}_{vs}(q_1^2, q_2^2) = \frac{1}{12\pi^2} \int_{4M_\pi^2}^{\infty} ds \frac{q_\pi(s)^3 \, F_\pi^V(s)^* f_1(s, q_2^2)}{s^{1/2} \, (s - q_1^2)}.$$

Preliminary results based on a simplified model for $F_\pi^V(s)$ and $f_1(s, q_2^2)$: