Dispersive approach to hadronic light-by-light scattering in four-point kinematics

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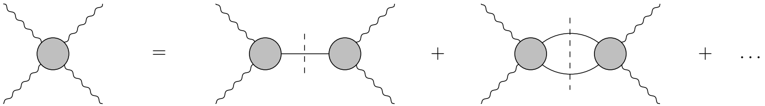
mainly based on

[M. Hoferichter, P. Stoffer, M. Zillinger, JHEP 02 (2025) 121 and work in progress]

Dispersive treatment of HLbL in four-point kinematics



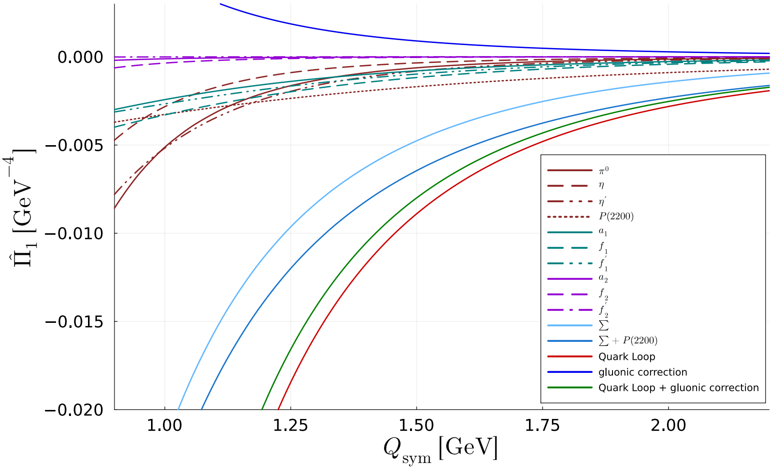
- Organized in terms of hadronic intermediate states
- Scalar functions $\check{\Pi}_i$ free from kinematic singularities in Mandelstam variables $\rightarrow$ enables dispersive treatment in s, t, u (four-point kinematics) [Colangelo, Hoferichter, Procura, Stoffer, JHEP 04 (2017) 161]
- Presence of kinematic singularities in q_i^2 is inevitable
- Residues of singularities vanish due to set of sum rules, but narrow resonances (apart from pseudoscalars) do not fulfill sum rules individually
- New tensor basis for HLbL in four-point kinematics enables evaluation of axial-vector states for the first time $\rightarrow$ still kinematic singularities for $J \geq 2$
- Kinematic singularities can be avoided by working in triangle kinematics $\rightarrow$ all redundancies of the BTT set disappear in the $g-2$ limit [Lüdtke, Procura, Stoffer, JHEP 04 (2023) 125] $\rightarrow$ see talk by Emilis Kaziukenas

Matching strategy for HLbL

- Use momenta variables Q_1, Q_2, Q_3 for matching procedure
- Introduce matching scale $Q_0 \in [1.2, 2.0]\,\mathrm{GeV}$ which separates low-energy part from high-energy part of HLbL tensor
- Symmetric region:
  - If $Q_1, Q_2, Q_3 > Q_0$: Use quark loop and two-loop gluonic correction with $\mu = Q_0$ in $\alpha_{\mathrm{QCD}}(\mu)$ [Bijnens et al., JHEP 04 (2021) 240]
  - If $Q_1, Q_2, Q_3 \leq Q_0$: Use description in terms of hadronic intermediate states
- Mixed region:
  - If $Q_1, Q_2 \gg Q_3 > Q_0$ (+ crossed) already well described in terms of quark loop and gluonic corrections
  - If $Q_1, Q_2 > Q_0$, $Q_3 \leq Q_0$ & $Q_3^2 \leq r\frac{Q_1^2+Q_2^2}{2}$ with $r \in [1/8, 1/2]$ (+ crossed): can relate HLbL tensor to the VVA correlator
  - Relation to VVA requires knowledge of longitudinal and transverse form factors $w_{L,T}(q^2) \to$ dedicated dispersive analysis for $w_{L,T}^{(3)}(q^2)$ [Lüdtke, Procura, Stoffer, JHEP 04 (2025) 130]

Comparison of OPE expressions and hadronic states for $\hat{\Pi}_1$

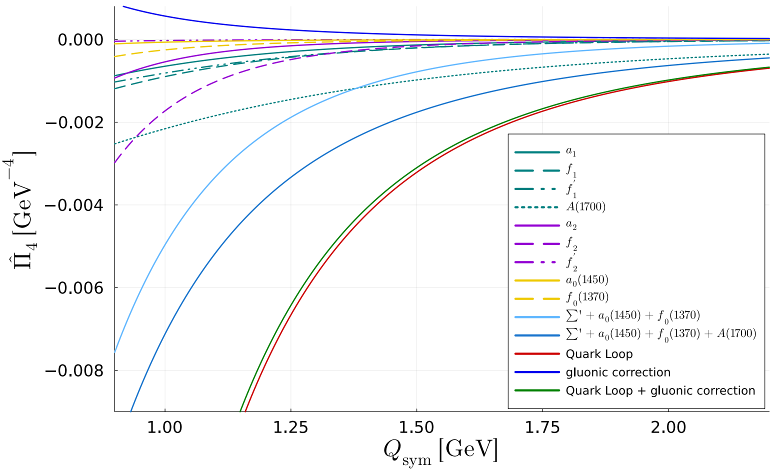
- $\hat{\Pi}_1(Q_{\text{sym}}) := \hat{\Pi}_1(Q_{\text{sym}}, Q_{\text{sym}}, Q_{\text{sym}}),\ \ \alpha_{\text{QCD}}(\mu)$ with $\mu = 1.5\,\text{GeV}$



- Pseudoscalar and axial-vector states most relevant in the asymptotic regime for the longitudinal component

Comparison of OPE expressions and hadronic states for $\hat{\Pi}_4$

- $\hat{\Pi}_4(Q_{\text{sym}}) := \hat{\Pi}_4(Q_{\text{sym}}, Q_{\text{sym}}, Q_{\text{sym}})$, $\alpha_{\text{QCD}}(\mu)$ with $\mu = 1.5\,\text{GeV}$



- Tensor contribution quite relevant in the asymptotic region for the transverse component

Effective Poles

- Finite set of hadronic states cannot satisfy the SDCs exactly in four-point kinematics → Only infinite tower of hadronic states can fulfill the SDCs
- Introduce effective poles in triangle kinematics to capture the impact of missing states in the low-energy region
- Pseudoscalar pole for longitudinal component in triangle kinematics

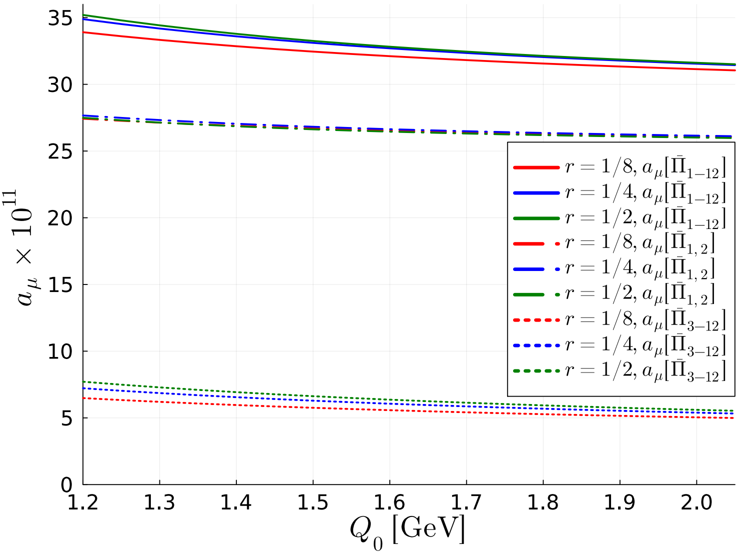
$$\hat{\Pi}_1^{\text{eff}} = \frac{F_{P\gamma^*\gamma^*}(q_1^2, q_2^2) F_{P\gamma^*\gamma^*}(M_P^2, 0)}{q_3^2 - M_P^2}$$

- Axial-vector pole for transverse component in triangle kinematics

$$\hat{\Pi}_4^{\text{eff}} = \frac{(q_1^2 + q_3^2 - m_A^2)\mathcal{F}_2^A(m_A^2, 0)\left[2\mathcal{F}_1^A(q_1^2, q_3^2) + \mathcal{F}_3^A(q_1^2, q_3^2)\right]}{2m_A^4(q_2^2 - m_A^2)} + \left(q_1^2 \leftrightarrow q_2^2\right)$$

- Couplings of effective poles determined such that SDCs are fulfilled by sum of hadronic states and effective poles in the symmetric limit
- Matching looks reasonable in the asymmetric directions

Stability Plot of the a_μ integral under variation of Q_0 and r



- Very mild dependence on Q_0 and r

Results and uncertainty estimate

$$a_\mu^{\mathrm{HLbL}}\Big|_{\mathrm{subleading}}[\bar{\Pi}_{1,2}] = 26.9(2.1)_{\mathsf{exp}}(1.0)_{\mathsf{match}}(3.7)_{\mathsf{sys}}(3.2)_{\mathsf{eff}}[5.4]_{\mathsf{total}} \times 10^{-11}$$
$$a_\mu^{\mathrm{HLbL}}\Big|_{\mathrm{subleading}}[\bar{\Pi}_{3\text{–}12}] = 6.3(1.5)_{\mathsf{exp}}(1.4)_{\mathsf{match}}(0.2)_{\mathsf{sys}}(2.2)_{\mathsf{eff}}[3.0]_{\mathsf{total}} \times 10^{-11}$$
$$a_\mu^{\mathrm{HLbL}}\Big|_{\mathrm{subleading}}[\bar{\Pi}_{1\text{–}12}] = 33.2(3.3)_{\mathsf{exp}}(2.2)_{\mathsf{match}}(4.6)_{\mathsf{sys}}(3.9)_{\mathsf{eff}}[7.2]_{\mathsf{total}} \times 10^{-11}$$

- Experimental error propagated from two-photon decay widths of heavy scalars, tensors and axialvector TFFs
- Matching uncertainty stems from varying $Q_0 \in [1.2, 2.0]\,\mathrm{GeV}$ and $r \in [1/8, 1/2]$
- Systematic uncertainties: Reflect a 30% error due the use of $U(3)$ relations for axial-vector states
- Added a 100% uncertainty for the tensor contribution in $a_\mu[\bar{\Pi}_{1\text{–}12}]$ to protect against the strong cancellation observed between $a_\mu[\bar{\Pi}_{1,2}]$ and $a_\mu[\bar{\Pi}_{3\text{–}12}]$
- $a_\mu^{\mathrm{HLbL}}\Big|_{\mathrm{total}} = a_\mu^{\mathrm{HLbL}}\Big|_{\mathrm{disp}} + a_\mu^{\mathrm{HLbL}}\Big|_{\mathrm{subleading}} + a_\mu^{\mathrm{HLbL}}\Big|_{\mathrm{charm}} = 101.9(7.9) \times 10^{-11}$

Anomalous magnetic moment of electron and τ lepton

- Same matching procedure can be applied to electron and τ
- For the electron numerical instabilities arise in the kernel functions $T_i(Q_1, Q_2, \tau, m_\ell)$ entering the master formula for a_e $\rightarrow$ can be solved by appropriate expansion or higher internal precision
- The different masses imply a different scaling of regions in the light-by-light integral
- Most important contribution for e: $P = \pi^0, \eta, \eta'$
- Most important contribution for τ: pQCD and OPE

$$a_e^{\mathrm{HLbL}} = 3.51(23) \times 10^{-14}$$
$$a_\tau^{\mathrm{HLbL}} = 3.77(29) \times 10^{-8}$$

- Relative precision comes out almost identical for all three leptons:

$$\{7, 8, 8\}\% \quad \text{for} \quad a_\ell^{\mathrm{HLbL}}, \ell \in \{e, \mu, \tau\}$$

Recent work on tensor meson transition form factors

Tensor mesons in dispersive approach to HLbL

- New basis:
  - $P, S, A \to \check{\Pi}_i^{\text{new}}$ no kinematic singularities ✓
  - $T \to \check{\Pi}_i^{\text{new}}$ still kinematic singularities!
- Lorentz decomposition of the amplitude $T \to \gamma^*\gamma^*$ yields 5 form factors

$$M^{\mu\nu\alpha\beta} = \sum_{i=1}^{5} T_i^{\mu\nu\alpha\beta} \frac{1}{m_T^{n_i}} \mathcal{F}_i^T(q_1^2, q_2^2)$$

- No kinematic singularities if only $\mathcal{F}_{1,3}^T$ or only $\mathcal{F}_{2,3}^T$ are present
- Dispersive approach for HLbL: tensor mesons included via simple quark model

$$\frac{\mathcal{F}_1^T(q_1^2, q_2^2)}{\mathcal{F}_1^T(0,0)} = \left(\frac{M_\rho^2}{M_\rho^2 - q_1^2 - q_2^2}\right)^2, \qquad \mathcal{F}_{2,3,4,5}^T(q_1^2, q_2^2) = 0$$

- Simple quark model features:
  - correct normalization
  - correct scaling for asymptotic behavior in doubly-virtual limit
  - realistic mass scale set by M_ρ

Situation for tensors in WP2

Region		Dispersive	hQCD	Regge	DSE/BSE
$Q_i > Q_0$		$6.2^{+0.2}_{-0.3}$	$6.3(7)$	$4.8(1)$	$2.3(1.5)$
Mixed	A, S, T	$3.8(1.5)$			
	OPE	$10.9(0.8)$			
	Effective pole	1.2			
	Sum	$15.9(1.7)$	$13.5(2.4)$	$12.8(5)$	$10.1(3.0)$
$Q_i < Q_0$	$A = f_1, f_1', a_1$	$12.2(4.3)$	$13.1(1.5)$	$10.9(1.0)$	$8.6(2.6)$
	$S = f_0(1370), a_0(1450)$	$-0.7(4)$			$-0.8(3)$
	$T = f_2, a_2$	$-2.5(8)$	$2.9(4)$		
	Other	2.0	$8.0(9)$	$3.2(6)$	$2.8(6)$
	Sum	$11.0(4.4)$	$24.0(2.8)$	$14.1(1.2)$	$10.6(2.7)$
Sum		$33.2(4.7)$	$43.8(5.9)$	$31.7(1.6)$	$23.0(7.4)$

- Difference between dispersive approach and hQCD significant for tensors
- hQCD states that $\mathcal{F}_1^T$ and $\mathcal{F}_3^T$ appear at a comparably important level

$\rightarrow$ Inclusion of $\mathcal{F}_3^T$ leads to the sign change in the evaluation of tensor mesons

$\mathcal{F}_3^T$ transition form factor

- Questions:
  - What is so special about $\mathcal{F}_3^T$?
  - Is it a coincidence that our new basis allows for the evaluation if only $\mathcal{F}_1^T$ and $\mathcal{F}_3^T$ are present?

- $\mathcal{F}_3^T$ does not appear in the on-shell decay rate or helicity fraction

$$\Gamma_{\gamma\gamma} = \frac{\pi\alpha^2 m_T}{5}\left(|\mathcal{F}_1^T(0,0)|^2 + \frac{1}{24}|\mathcal{F}_2^T(0,0)|^2\right)$$

$$r_h = \frac{|\mathcal{F}_{\lambda=0}^T(0,0)|^2}{|\mathcal{F}_{\lambda=2}^T(0,0)|^2} = \frac{|\mathcal{F}_2^T(0,0)|^2}{24|\mathcal{F}_1^T(0,0)|^2}$$

- $\mathcal{F}_{4,5}^T$ contribute to singly-virtual kinematics, but $\mathcal{F}_3^T$ only contributes to the doubly virtual case where no data are available yet $\rightarrow$ hard to probe $\mathcal{F}_3^T$

Resonance χPT

- In hQCD, leading contribution with respect to $1/N_c$ comes from $\mathcal{F}_1^T$ and $\mathcal{F}_3^T$
  - $\rightarrow$ $\mathcal{F}_{2,4,5}^T$ subleading in hQCD

Resonance χPT

- In hQCD, leading contribution with respect to $1/N_c$ comes from $\mathcal{F}_1^T$ and $\mathcal{F}_3^T$
  - $\rightarrow$ $\mathcal{F}_{2,4,5}^T$ subleading in hQCD
- **Can we observe the same in RχT?**
- Use standard counting with symmetric tensor field $T_{\mu\nu}$
- At LO, the only operator contributing to the process $T \rightarrow \gamma^*\gamma^*$ is

$$\mathcal{L}^{(0)} = c_1^{(0)} \langle T_{\mu\nu}\{u^\mu, u^\nu\}\rangle$$

- Calculated chiral loops, which generate contributions to all 5 TFFs ($\mathcal{F}_1^T$ UV divergent, the others finite)
- Divergent expression for $\mathcal{F}_1^T$ requires a counter term at NLO:

$$\mathcal{L}^{(1)} = c_1^{(1)} \langle T_{\mu\nu} f_{+\ \alpha}^{\mu} f_+^{\nu\alpha}\rangle$$

- $\mathcal{L}^{(1)}$ only generates a contribution to $\mathcal{F}_1^T$

Resonance χPT

- Loops are subdominant and yield too small values for $\Gamma_{\gamma\gamma}$ and r
  - $\Gamma_{\gamma\gamma}^{\text{loop}}(f_2(1270)) = 0.3\,\text{keV}$ $\quad \Gamma_{\gamma\gamma} = 2.6(5)\,\text{keV}$ PDG
  - $r_h^{\text{loop}}(f_2(1270)) = 0.019$ $\quad r_h = 0.095(20)$ Dai, Pennington (2014)

- Phenomenological agreement for $\Gamma_{\gamma\gamma}$ can be enforced by means of $c_1^{(1)}$

- At NNLO one can write down 13 additional contact operators which produce contributions to all $\mathcal{F}_i^T$ $\rightarrow$ From RχT perspective in “standard"counting only $\mathcal{F}_1^T$ seems to be special

- Is it possible to construct a minimal basis? Counting ambiguous because of derivatives $\rightarrow$ see talk by Jonas Mager

The process $\gamma^*\gamma^* \to \pi\pi$

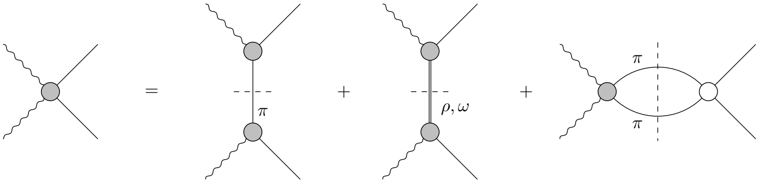
- To improve on the simple quark model we have to go back to this work

> **Dispersion relations for $\gamma^*\gamma^* \to \pi\pi$: helicity amplitudes, subtractions, and anomalous thresholds**
>
> **Martin Hoferichter[a] and Peter Stoffer[b]**

- Consider the D-wave of $\gamma^*\gamma^* \to \pi\pi$
- The $f_2(1270)$ can be understood as an effect of the $\pi\pi$ final-state rescattering
- Expressions for helicity amplitudes already worked out
- Inclusion of higher left-hand cuts ($V = \rho, \omega$) necessary to reproduce observed $f_2(1270)$ resonance peak in the on-shell process $\gamma\gamma \to \pi\pi$
- Anomalous thresholds in left-hand cuts can be handled numerically

The process $\gamma^*\gamma^* \to \pi\pi$

- **Topologies in the Omnès solution**

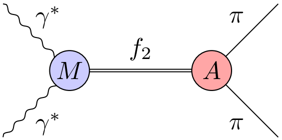


- Off-shell behavior described by pion VFF $F_\pi^V(q^2)$ and vector meson transition form factors $F_{\omega\pi}(q^2)$
- ρ should be described in terms of the full 2π spectral function $\to$ P-wave amplitude $f_1(s, q^2)$ for $\gamma^* \to 3\pi$

Matching of helicity amplitudes onto form factors

- Consider $f_2(1270)$ as a narrow resonance and match expressions for helicity amplitudes $h_{2,i}(s)$ onto the form factors $\mathcal{F}_i^T$
- $H^{J=2,\pi\pi}_{\lambda_1\lambda_2}(s,z) = (2\cdot 2+1) d^{J=2}_{m0}(z) h_{J=2,\lambda_1\lambda_2}(s)$
- Matching procedure for helicity amplitudes in the limit $s \to M^2_{f_2}$:

$$iH^{J=2,\pi\pi}_{\lambda_1\lambda_2}(s,z) \quad = $$



$$\lim_{s\to M^2_{f_2}} iH^{J=2,\pi\pi}_{\lambda_1\lambda_2}(s,z) = \varepsilon^{\lambda_1}_\mu(q_1)\varepsilon^{\lambda_2}_\nu(q_2) \lim_{s\to M^2_{f_2}} \left(iM^{\mu\nu\alpha\beta}(p,q_1,q_2) \frac{is^T_{\alpha\beta\gamma\delta}}{s-M^2_{f_2}+iM_{f_2}\Gamma_{f_2}} iA^{\gamma\delta}(p,p_1,p_2) \right)$$

$$\lim_{s\to M^2_{f_2}} \left(\frac{s-M^2_{f_2}+iM_{f_2}\Gamma_{f_2}}{s-4M^2_\pi} h_{J=2,\lambda_1\lambda_2}(s) \right) = \sum_{i=1}^{5} C_i(q_1^2,q_2^2,M^2_{f_2}) \mathcal{F}_i^T(q_1^2,q_2^2)$$

- Consistency check: z dependence in $H^{J=2,\pi\pi}_{\lambda_1\lambda_2}(s,z)$ drops out

Modified Omnès representation

$$h_{2,i}(s) = N_{2,i}(s) + \frac{\Omega_2(s)}{\pi}\Bigg\{\int_{-\infty}^{0} ds'\,\frac{1}{\Omega_2(s')}K_{ij}(s,s')\,\mathrm{Im}\,h_{2,j}(s') + \int_{4M_\pi^2}^{\infty} ds'\,\frac{\sin\delta_2(s')}{|\Omega_2(s')|}K_{ij}(s,s')N_{2,j}(s')\Bigg\}$$

- $N_{2,i}(s)$: only Born term as inhomogeneity
- Only imaginary part required for higher left hand cuts $V = \rho, \omega$
- $K_{ij}(s,s')$: integration kernels from the full 5×5 D-wave Roy-Steiner system
- The imaginary part manifestly cancels in $\left(\frac{s - M_{f_2}^2 + iM_{f_2}\Gamma_{f_2}}{s - 4M_\pi^2} h_{J=2,\lambda_1\lambda_2}(s)\right)$ such that form factors $\mathcal{F}_i^T(q_1^2, q_2^2)$ are real in the spacelike region

Narrow-width approximation for $h_{2,i}(s)$

- Full solution for $\delta_2(s)$ and $\Omega_2(s)$ available, but use narrow-width approximation for the matching to form factors
- Narrow-width approximation for phase shift $\delta_2(s)$ and $\Omega_2(s)$

$$\delta_2(s) = \arctan\left(\frac{M_{f_2}\Gamma_{f_2}}{M_{f_2}^2 - s}\right) + \pi\theta\big(s - M_{f_2}^2\big), \quad \Omega_2(s) = \frac{M_{f_2}^2}{M_{f_2}^2 - s - iM_{f_2}\Gamma_{f_2}}$$

- Phase-shift-dependent fraction in right-hand cut integral reduces to a constant [Stamen et al., Eur.Phys.J.C 83 (2023) 6, 510]

$$\frac{\sin\delta_2(s)}{|\Omega_2(s)|} = \frac{\Gamma_{f_2}}{M_{f_2}}$$

- Poor approximation inside the right-hand cut integral $\rightarrow$ stick to full solution or use energy-dependent width $\Gamma_{f_2}(s)$
- On-shell point $q_1^2 = q_2^2 = 0$ investigated with preliminary result: r_h in the same ballpark as expected from chiral loops
- How robust is the phenomenological extraction of r_h? New input from BESIII?

Summary & Outlook

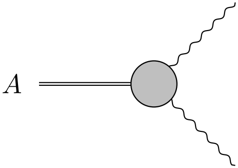
- Moderate dependence on matching parameters Q_{match} and r
- Effective poles introduced to fulfill SDCs exactly
- Need improvement for TFFs of tensors to reduce the systematic uncertainty
- Introduced matching procedure for helicity amplitudes onto form factors $\rightarrow$ Check on-shell point $q_1^2 = q_2^2 = 0$ and compute $\Gamma_{f_2\gamma\gamma}$ as well as the helicity fraction r_h
- Later proceed to doubly-virtual kinematics where input for vector meson transition form factors $F_{V\pi}(q^2)$ is needed and address their asymptotic behavior $F_{V\pi}(Q^2) \sim 1/Q^4$
- Does the correct asymptotic behavior for vector mesons imply the correct asymptotic behavior for tensor TFFs?
- Compare different scenarios for vector meson transition form factors with the available data from BESIII

Appendix

Input for Matching (1): Axial-vector TFFs (VMD)

- Matrix element for $A \to \gamma^*\gamma^*$

  $\mathcal{M}^{\lambda_1\lambda_2;\lambda_A}\left(A \to \gamma^*\gamma^*\right) = e^2\epsilon_\mu^{\lambda_1}\epsilon_\nu^{\lambda_2}\epsilon_\alpha^{\lambda_A}\mathcal{M}^{\mu\nu\alpha}(q_1,q_2)$



- BTT decomposition [Hoferichter, Stoffer JHEP 05 (2020) 159]

$$\mathcal{M}^{\mu\nu\alpha}(q_1,q_2) = \frac{i}{m_A^2}\sum_{i=1}^{3} T_i^{\mu\nu\alpha}\mathcal{F}_i(q_1^2,q_2^2)$$

- TFFs free of kinematic singularities and zeros $\to$ dispersive treatment
- Experimental constraints analyzed and implemented within a VMD model for $A = f_1(1285)$ [Hoferichter, Kubis, Zanke JHEP 08 (2023) 209]
- Relate $f_1(1420)$ and $a_1(1260)$ via $U(3)$ symmetry
- Inclusion of 3 multiplets for VMD model:
  - (ρ,ρ',ρ'') for isovector part
  - $(\omega,\omega',\omega'')$ and (ϕ,ϕ',ϕ'') for isoscalar part

Input for Matching (1): Axial-vector TFFs (asymptotic)

- Supplement VMD model with asymptotic piece → ensures correct doubly-virtual behavior [Hoferichter, Stoffer JHEP 05 (2020) 159], [Zanke, Hoferichter, Kubis, JHEP 07 (2021) 106]

$$\mathcal{F}_2^{\text{asym}}(q_1^2, q_2^2) = -F_A^{\text{eff}} m_A^3 \frac{\partial}{\partial q_1^2}\left[\frac{1}{\pi^2}\int_{s_0}^{\infty} dx \int_{s_0}^{\infty} dy \frac{\rho^{\text{asym}}(x,y)}{(x-q_1^2)(y-q_2^2)}\right] + \mathcal{O}(1/q_i^6)$$

$$\rho^{\text{asym}}(x,y) = 3\pi^2 xy\delta''(x-y)$$

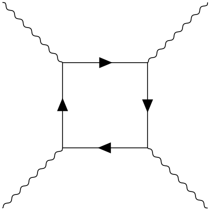
- Use linear combination of asymptotic pieces with free parameter α to also ensure correct singly virtual behavior

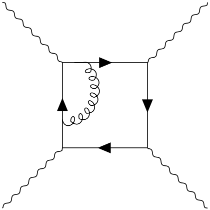
$$\tilde{\mathcal{F}}_2^{\text{asym}}(q_1^2, q_2^2) = \frac{3F_A^{\text{eff}} m_A^3}{1+\alpha}\left(\int_{s_0}^{\infty} dx \left[\frac{q_2^2 x}{(x-q_1^2)^3(x-q_2^2)} - \frac{3q_1^2 x}{(x-q_1^2)^4(x-q_2^2)}\right] + \alpha \int_{s_0}^{\infty} dx \frac{q_2^2(x+q_1^2)}{(x-q_1^2)^3(x-q_2^2)^2}\right)$$

- Still correct normalization: $\mathcal{F}_2^{\text{asym}}(0,0) = 0$, but can choose α such that $\mathcal{F}_2^{\text{asym}}(q^2, 0) = \frac{3F_A^{\text{eff}} m_A^3}{q^4} + \mathcal{O}(1/q^6)$ with correct coefficient
- Later: include mass effects → might be sizeable for axial-vector mesons

Input for Matching (2)

- Input for SDC3 ($Q_1^2 \sim Q_2^2 \sim Q_3^2 \gg \Lambda_{\mathrm{QCD}}^2$)
  - Usage of Operator Product Expansion (OPE) with background photon field
    [Bijnens, Hermansson-Truedsson, Laub, Rodriguez-Sanchez JHEP 04 (2021) 240]
- LO contribution given by massless quark loop
- NLO contribution given by two-loop gluonic correction ($\sim 10\%$ correction to quark loop)





- Gluonic correction requires evaluation of $\alpha_{\mathrm{QCD}}(\mu)$
- Exact choice of μ is ambiguous
- Natural scale for avoiding large logarithms in the perturbative series amounts to setting $\mu \sim Q_{\mathrm{match}}$

Input for Matching (3)

- Input for SDC2 ($Q_1^2, Q_2^2 \gg Q_3^2,\ Q_1^2, Q_2^2 \gg \Lambda_{\mathrm{QCD}}^2$)
  - Standard OPE or OPE with background photon field can be used

    $\rightarrow$ see talk by N. Hermansson-Truedsson, [Bijnens et. al. JHEP 02 (2023) 167], [Colangelo et. al. JHEP 03 (2020) 101]
  - For standard OPE: q_3, q_4 much smaller than $\hat{q} = \frac{1}{2}(q_1 - q_2)$
- Leading term in OPE can be related to *VVA* correlator ($D = 3$)

$$\Pi_{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \frac{2i}{\hat{q}^2}\epsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha \int d^4x d^4y e^{-iq_3x} e^{iq_4y} \langle 0|T\{j_\lambda(x) j_\sigma(y) j_5^\beta(0)\}|0\rangle$$
$$= \frac{2}{\hat{q}^2}\epsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha W_{\lambda\sigma}{}^{\beta}(-q_3, q_4)$$

- Lorentz decomposition of $W^{(a)}_{\mu\nu\rho}(q_1, q_2)$ introduces the functions $w_L^{(a)}$ and $w_T^{(a)}$ which at one loop are fixed by the axial anomaly

$$w_L^{(a)}(q_3^2) = \frac{2N_c}{q_3^2}, \qquad w_T^{(a)}(q_3^2) = \frac{w_L^{(a)}(q_3^2)}{2}$$

- In the chiral limit: No perturbative or non-perturbative corrections for $w_L^{(3,8)}(q_3^2)$
- In the chiral limit: No perturbative corrections for $w_T^{(3,8)}(q_3^2)$
- OPE expressions for $\hat{\Pi}_i$ agree well with massless pQCD quark loop in the limit $Q_3^2 \gg \Lambda_{\mathrm{QCD}}^2$
- For $Q_3^2 \ll \Lambda_{\mathrm{QCD}}^2$ chiral corrections become large $\rightarrow$ Use CMV model, later full VVA