Dispersive Approach to Hadronic Light-by-Light in Triangle Kinematics

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FWF Austrian Science Fund
universität wien
Vienna Doctoral School in Physics

Overview

1. Hadronic puzzles in muon $g-2$
2. Tensors in HLbL: 3 pt. or 4 pt. kinematics?
3. Triangle kinematics: the road to spin 2
4. Soft-singular contribution to $\gamma^*\gamma^*\gamma \to \pi^+\pi^-$
5. Current work: the $\gamma^* T \to \pi^+\pi^-$ subprocess and tensor TFFs

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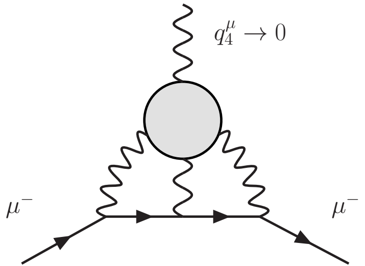

- **Small tension:** 1.5σ between data-driven and lattice average $\Rightarrow$ **more work TBD!**
- **Uncertainties:** large improvements from WP20, major part from $Q_i < 1.5$ GeV. $S + A + T + \ldots$ contribution $a_\mu^{\text{Low}} = 12.5(5.9) \times 10^{-11}$ vs data-driven $a_\mu^{\text{HLbL,LO}} = 103.3(8.8) \times 10^{-11}$:

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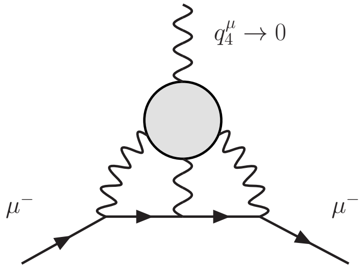

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  - **Axials** f_1, f_1', a_1 $\Rightarrow$**talk by Hannah 12/09**
  - **Tensors** f_2, a_2 - Quark model vs hQCD differ by a sign! $\Rightarrow$ **talks by Maximilian, Jonas**

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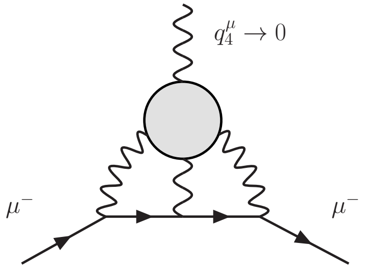

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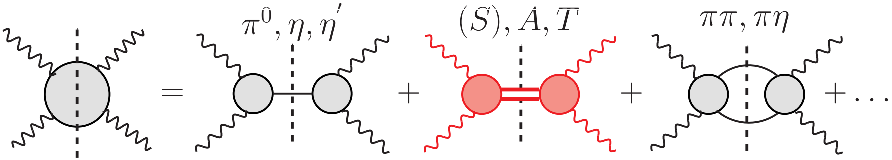
$\Rightarrow$ Reducing model dependence for tensors is particularly timely!

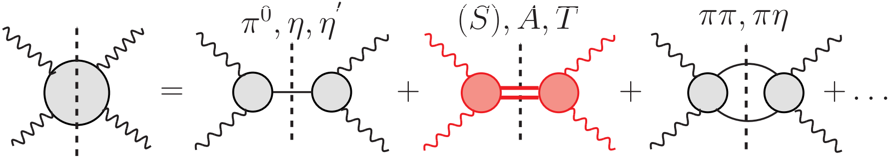
Overview

Tensors in HLbL: 4pt. kinematics



- **Two ways** to compute $(g-2)_\mu$: $q_4 \to 0$ at the end (4pt. kin.) or initially (triangle kin.). Different unitarity cuts.
- Original approach: **spurious singularities** in $\{q_i^2\}$ - cancel if Π_i satisfy **sum rules**.
- Hold for ∞-tower of intermediate states, **not guaranteed** for individual contributions.

Tensors in HLbL: 4pt. kinematics



- **Tensors: assume only TFFs** $\mathcal{F}^T_{1,3}$(or $\mathcal{F}^T_{2,3}$) $\neq 0$ then singularities vanish: *Hoferichter, Stoffer, Zillinger, JHEP 92 (2024)*. $\Rightarrow$**Uncertainties?**
- **Model results:** QM, hQCD, RChT - obtain $\mathcal{F}^T_1$ or $\mathcal{F}^T_{1,3}$ $\Rightarrow$ a^T_μ estimate.

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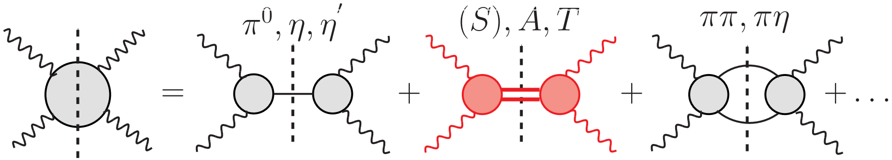

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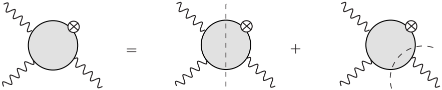
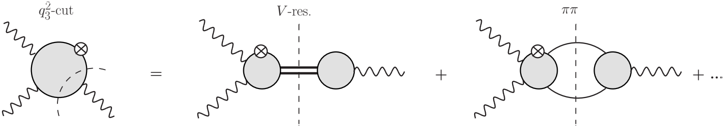
- **Model results:** QM, hQCD, RChT - obtain $\mathcal{F}^T_1$ or $\mathcal{F}^T_{1,3}$ ⇒ a^T_μ estimate.
- **General case:** kin. singularities. Circumvention in 4pt. kinematics (like for axials) - not understood yet.

⇒ **4 pt. kinematics - so far unclear how to include spin-2 resonances in a consistent, model-indep. way!**

Tensors in HLbL: Triangle kinematics

Lüdtke, Procura, Stoffer, JHEP 04 (2023) & JHEP 130 (2025)





- Take $q_4 \to 0$ immediately: disperse in q_i^2 - **manifestly free of kin. singularities!** Permits disp. description of spin-2 resonances.
- **Price to pay:** complicated unitarity cuts, new subprocesses $\gamma^*\gamma^*\gamma \to V$,$\gamma^*\gamma^*\gamma \to 2\pi$, $\pi\pi \to \gamma\pi\pi$.

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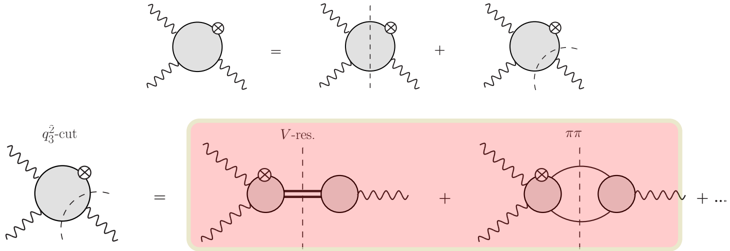

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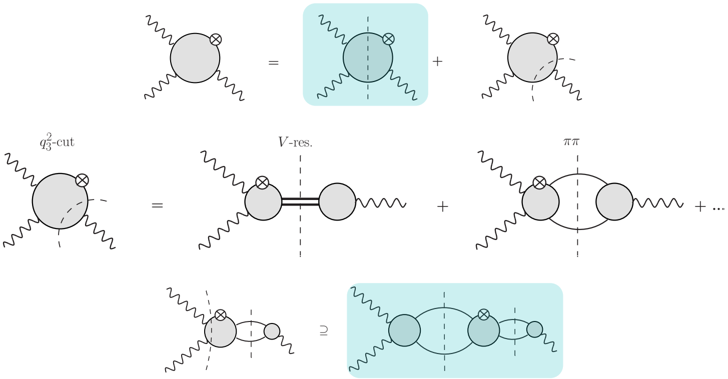

- **Soft-singular parts** exist, cancel only in the sum of two triangle kin. cuts. **Can not directly compare with 4 pt. kinematics (reshuffling!)**
- **Soft-regular part:** includes tensor resonances through e.g. D-wave of $\pi\pi$ scattering via $\pi\pi \to \gamma\pi\pi$ subprocess.

Tensors in HLbL: Triangle kinematics

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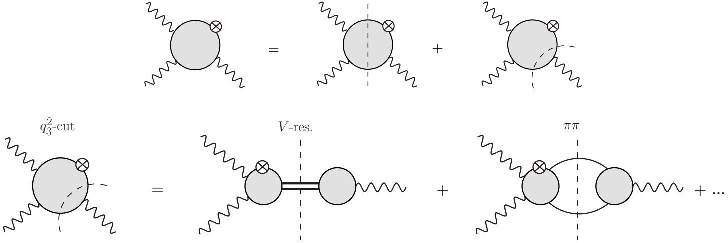

- **VVA insights:** triangle kin. may shed some light on **matching to SDCs.**

Triangle kinematics can include spin-2 effects in a_μ!
Roadmap?

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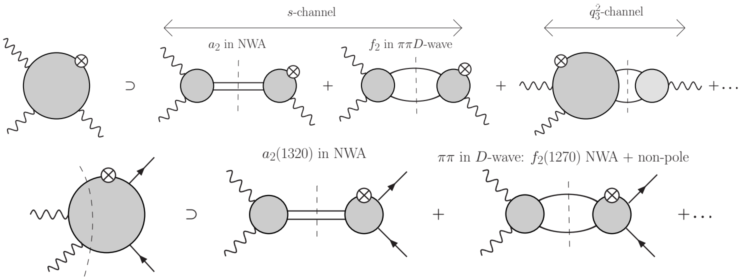

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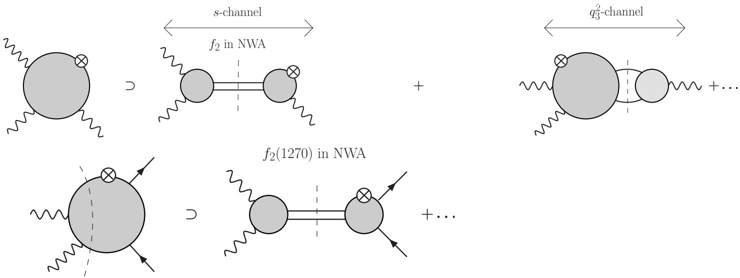

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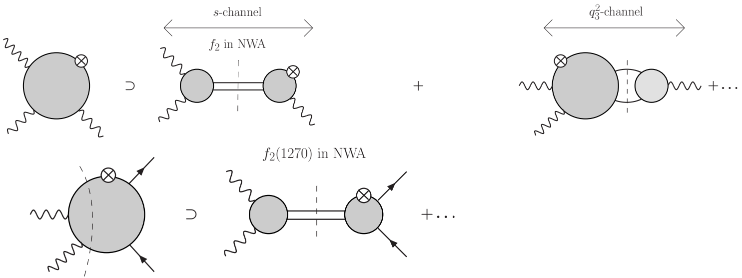

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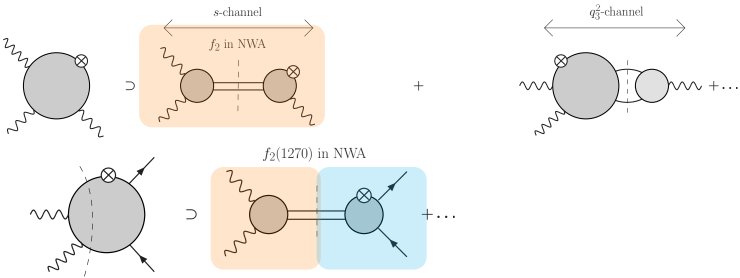

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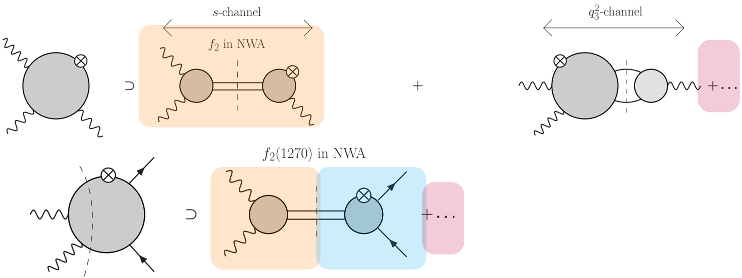

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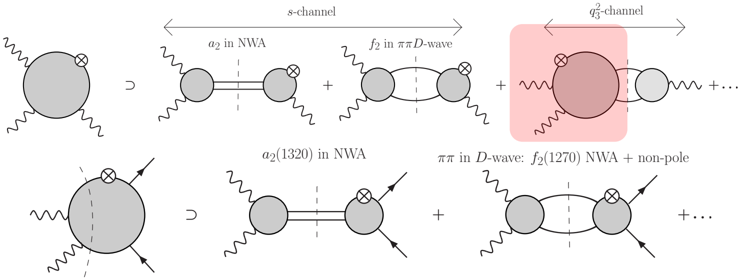

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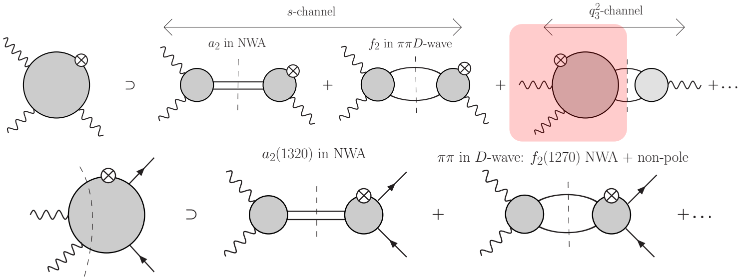

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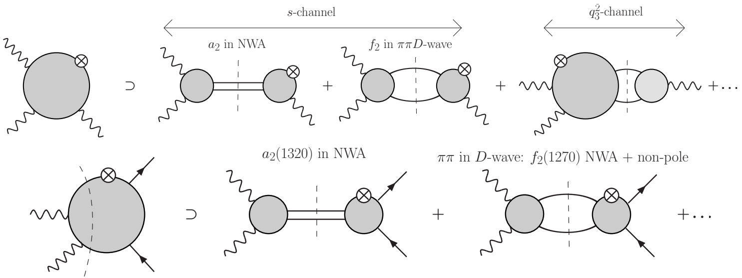

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5. **Re-shuffling** investigation? **combine** with **4 pt. kinematics** - reduced SDC uncertainty?

Overview

Motivation and goals

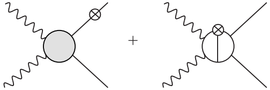
Goal (Dispersive soft-pole construction)

Construct an object that captures all soft singularities of $\gamma^*\gamma^*\gamma \to \pi^+\pi^-$ *in terms of* $\gamma^*\gamma^* \to \pi^+\pi^-$ *and VFF with:*

- *Unique* π*-pole im. parts;*
- *No kin. singularities;*
- *Symmetries: gauge invariance, crossing;*

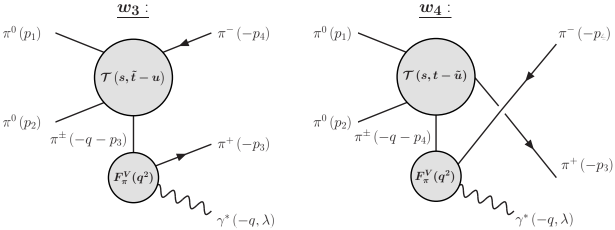
- Separation in $q_1 \to 0$:

$$C_i = C_i^{\text{poles}} + C_i^{\text{non-pole}} =$$



- **Why dispersive?** Alternatives (Low's thrm., Adler-Dothan) - only q_1^{-1}, q_1^0-terms in soft-q_1 expansion: remainder may be singular - **spoils disp. description of** $\boldsymbol{C_i^{\text{non-pole}}}$.
- **Complicated tensor structure of** $\boldsymbol{\gamma^*\gamma^*\gamma \to \pi^+\pi^-}$: 74 **BTT,** 38 Tarrach redundancies Δ_i - find simpler example first.
- **Starting point:** 5-particle subprocess $\pi^0\pi^0 \to \gamma^*\pi^+\pi^-$ (*J. Lüdtke, PhD thesis*) - use as analogy today. $\Rightarrow$ **Also Jan's presentation 12/09**
- **Applications to HVP:** radiative corrections - $\gamma^*\gamma^*\gamma \to \pi^+\pi^-$ with hard photons.

Poles of $\pi^0\pi^0 \to \gamma^*\pi^+\pi^-$

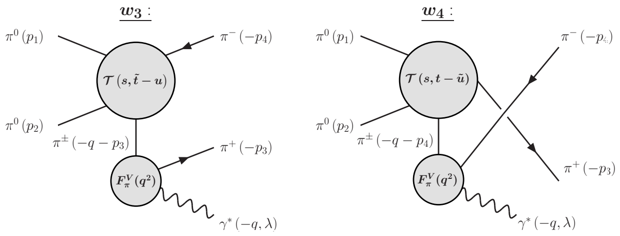


- Pion-pole im. part:

$$\mathrm{Im}^{\pi}_{w_3}\mathcal{M}^\mu = -\pi\,(2p_3 + q)^\mu\,\mathcal{T}(s, \tilde{t} - u)F^V_\pi(q^2)\delta((p_3 + q)^2 - M_\pi^2),$$
$$\mathrm{Im}^{\pi}_{w_4}\mathcal{M}^\mu = +\pi\,(2p_4 + q)^\mu\,\mathcal{T}(s, t - \tilde{u})F^V_\pi(q^2)\delta((p_4 + q)^2 - M_\pi^2),$$

- **Amplitude ansatz?**

Poles of $\pi^0\pi^0 \to \gamma^*\pi^+\pi^-$



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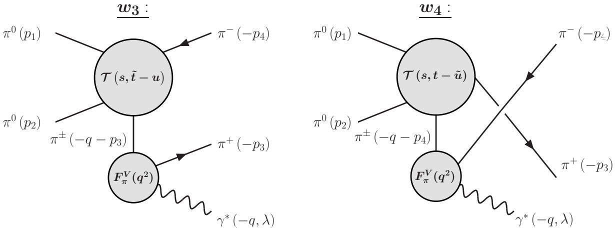
- **Amplitude ansatz?**

$$\mathcal{M}^\mu \overset{???}{=} F_\pi^V(q^2)\left(\frac{(2p_3+q)^\mu}{(p_3+q)^2-M_\pi^2}\mathcal{T}(s,\tilde{t}-u)-\frac{(2p_4+q)^\mu}{(p_4+q)^2-M_\pi^2}\mathcal{T}(s,t-\tilde{u})\right).$$

$\Rightarrow$ Symmetries? Gauge invariance?

Poles of $\pi^0\pi^0 \to \gamma^*\pi^+\pi^-$

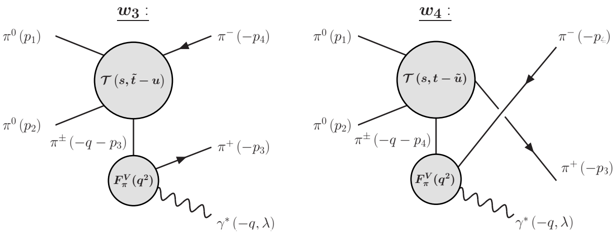


- Not gauge invariant! Additional term:

$$\mathcal{M}^\mu = F_\pi^V(q^2)\left(\frac{(2p_3+q)^\mu}{(p_3+q)^2 - M_\pi^2}\mathcal{T}(s,\tilde{t}-u) - \frac{(2p_4+q)^\mu}{(p_4+q)^2 - M_\pi^2}\mathcal{T}(s,t-\tilde{u}) \boldsymbol{-2\,(p_1 - p_2)^\mu\,\Delta\mathcal{T}}\right),$$

with $\Delta\mathcal{T} := \left(\mathcal{T}(s,\tilde{t}-u) - \mathcal{T}(s,t-\tilde{u})\right)/(\tilde{t} - u - t + \tilde{u})$. **Real, not singular!**

Poles of $\pi^0\pi^0 \to \gamma^*\pi^+\pi^-$



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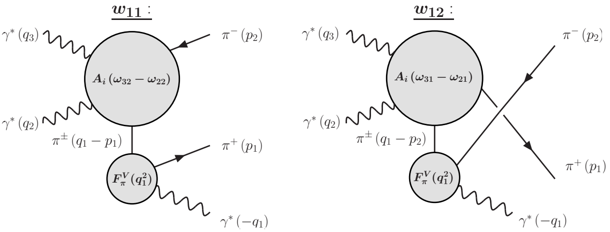
- For $3\gamma \to 2\pi$: $\mathcal{T} \overset{???}{\to} A_i \times T_i^{\nu\lambda}$

Try similar approach to $\gamma^*\gamma^*\gamma \to \pi^+\pi^-$ pole pieces

Poles of $\gamma^*\gamma^*\gamma \to \pi^+\pi^-$: direct construction



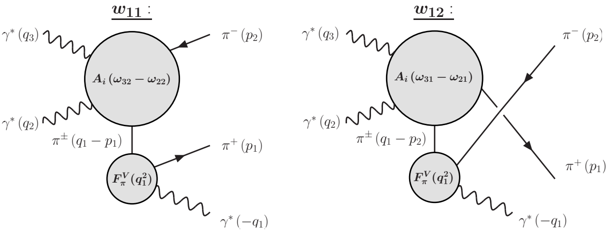
- Want result wrt VFF and $\gamma^*\gamma^* \to \pi^+\pi^-$.
- **Right imaginary parts** (note shift in q_1):

$$\mathcal{M}^{\mu\nu\lambda} = F_\pi^V(q_1^2)\left[\frac{(2p_1-q_1)^\mu}{(p_1-q_1)^2-M_\pi^2}W^{\nu\lambda}(p_1-q_1,p_2,q_2) - \frac{(2p_2-q_1)^\mu}{(p_2-q_1)^2-M_\pi^2}W^{\nu\lambda}(p_2-q_1,p_1,q_2) + \boldsymbol{R^{\mu\nu\lambda}}\right]$$

where $R^{\mu\nu\lambda}$ - **real, finite, very hard to construct.**

- $W^{\mu\nu} = \sum_{i=1}^5 T_i^{\mu\nu}A_i$ - split into 5 parts.

Poles of $\gamma^*\gamma^*\gamma \to \pi^+\pi^-$: direct construction



- First 2 functions $A_i, i = 1, 2$: $T_i^{\nu\lambda}$ indep. of soft-q_1 shift. Simplification:

$$\mathcal{M}_{A_i}^{\mu\nu\lambda} = F_\pi^V(q_1^2)\, T_i^{\nu\lambda}\left[\frac{(2p_1 - q_1)^\mu}{(p_1 - q_1)^2 - M_\pi^2} A_i^{11} - \frac{(2p_2 - q_1)^\mu}{(p_2 - q_1)^2 - M_\pi^2} A_i^{12} + R^\mu\right],$$

assumed $R^{\mu\nu\lambda} = T_i^{\nu\lambda} \times R^\mu$. Does it work?

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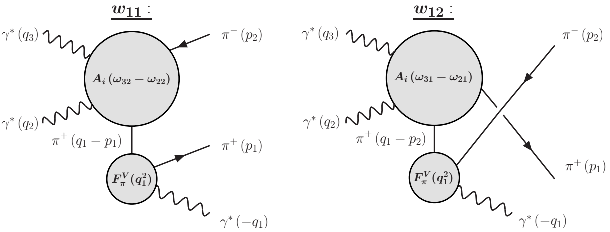

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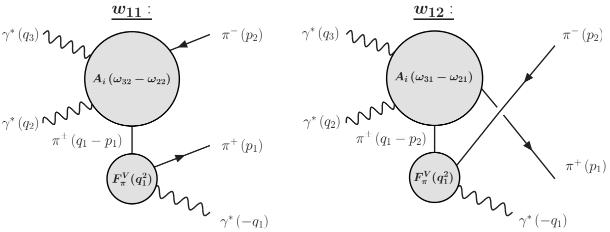
assumed $\boldsymbol{R^{\mu\nu\lambda} = T_i^{\nu\lambda}} \times \boldsymbol{R^\mu}$. Does it work?

- **Yes!**

$$\mathcal{M}_{A_i}^{\mu\nu\lambda} = F_\pi^V(q_1^2)T_i^{\nu\lambda}\left[\frac{(2p_1-q_1)^\mu}{(p_1-q_1)^2-M_\pi^2}A_i^{11} - \frac{(2p_2-q_1)^\mu}{(p_2-q_1)^2-M_\pi^2}A_i^{12} - 2(q_2-q_3)^\mu\frac{A_i^{11}-A_i^{12}}{W_1}\right].$$

Poles of $\gamma^*\gamma^*\gamma \to \pi^+\pi^-$: direct construction



- **Other functions** A_i, $i = 3, 4, 5$: soft-q_1 shift - $T_i^{\nu\lambda}$ differs in two channels.
- **Can prove:** Gauge inv. + pion crossing $\Rightarrow$ simple factorization $R^{\mu\nu\lambda} \sim T_i^{\nu\lambda} \times R^\mu$ **fails.**

Need a more systematic approach!

Poles of $\gamma^*\gamma^*\gamma \to \pi^+\pi^-$: systematic construction

Inspired by: J. Lüdtke, PhD thesis

1

Project $\mathrm{Im}\mathcal{M}^{\mu\nu\lambda}$ onto scalar funcs (no gauge-inv., symmetry issues). Disp. integral: $\mathcal{M}^{\mu\nu\lambda}$ singular, but right im. parts. **Singularities: only in $\mathrm{Re}\boldsymbol{\mathcal{M}}^{\mu\nu\lambda}$**.

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Tarrach $\boldsymbol{\Delta_i}$: write for $\mathrm{Im}C_i$ - only affect $\mathrm{Re}\mathcal{M}^{\mu\nu\lambda}$ $\Rightarrow$ **use to cancel kin. singularities.**

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Problems: $\#$ singularities < 38, Δ_i not unique $\Rightarrow$ **not a linear algebra problem.**
sQED toy: simplifying principles, some $\Delta_i \to 0$.

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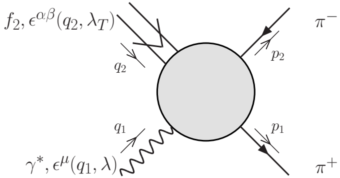
Status: pole pieces fully computed, tested with $SU(2)$ χPT at NLO.

Overview

The $\gamma^* T \to \pi^+\pi^-$ subprocess: BTT, pion pole



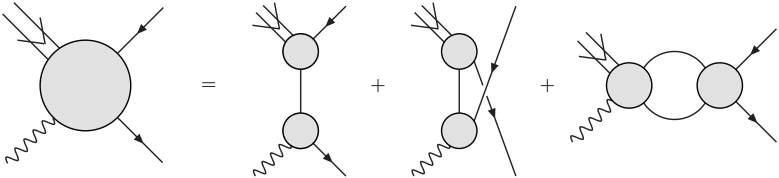
- Only $I = 1$-channel, pion crossing-odd amplitude, **8-indep. helicity amplitudes.**
- BTT decomp. - follows $T \to \gamma^*\gamma^*$: *Hoferichter, Stoffer, JHEP 05 (2020)*. Gauge inv. + polarization contr. + $\alpha \leftrightarrow \beta$ symm. $\Rightarrow$ **9 tensor structures with 1 Tarrach redundancy.**
- Pion crossing $\Rightarrow$ 8 non-redundant strucs. $\{T_i^{\mu\alpha\beta}\}_{i=1}^{8}$.
- Pion-pole part: in terms of VFF and $f_2 \to \pi^+\pi^-$ decay constant- $\mathcal{M}^{\alpha\beta} = q_5^\alpha q_5^\beta F_{f_2\pi\pi}$. Similar to $\gamma^*\gamma^* \to \pi^+\pi^-$:

$$\hat{B}_2 = -F_\pi^V(q_1^2) F_{f_2\pi\pi} \frac{1}{(t - M_\pi^2)(u - M_\pi^2)}, \quad \hat{B}_3 = F_\pi^V(q_1^2) F_{f_2\pi\pi} \left(\frac{1}{t - M_\pi^2} + \frac{1}{u - M_\pi^2} \right),$$

$$\hat{B}_5 = -\hat{B}_3, \qquad \hat{B}_6 = -\hat{B}_2 = -\frac{1}{2}\hat{B}_8, \qquad \hat{B}_1 = \hat{B}_4 = \hat{B}_7 = \hat{B}_9 = 0.$$

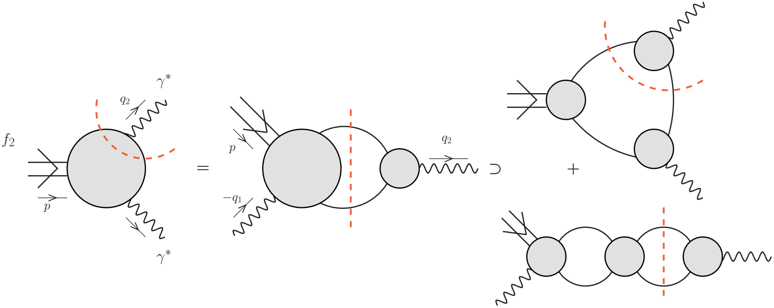
- Decay constant $F_{f_2\pi\pi}$ - e.g. from LO RChT: *Ecker, Zauner, Eur. Phys. J. C 52 (2007)*, estimate from $\Gamma(f_2 \to \pi^+\pi^-)$ - $F_{f_2\pi\pi} \simeq 3.3\ \text{GeV}^{-1}$.

The $\gamma^* T \to \pi^+\pi^-$ subprocess: Inhom. Omnès problem



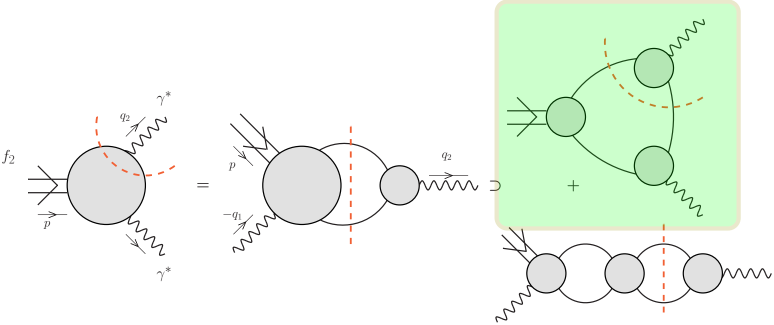
- Initially, take only pion-pole LHC into account.
- Analogous to $\gamma^*\gamma^* \to \pi^+\pi^-$ *:Hoferichter, Stoffer, JHEP 07 (2019)*.
- **All even waves vanish,** pure P-wave solution sufficient.
- Same coupling structure for $K_{ij}^{JJ'}(s, s')$: $J > J'$ - no coupling, $J = J'$ - indep. of hyperbolic param. a, $J < J'$ - polynomial in a, etc.
- **Triangle topology** in Omnès soltn. $\Rightarrow$ **anomalous discontinuity!**
- **Status:** DR system solved

Extraction of the $f_2(1280)$ TFFs: Introduction



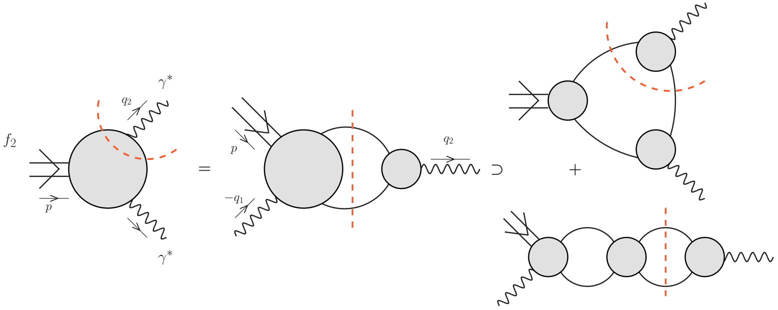
- Extract TFFs from q_2^2-cut of $T \to \gamma^*\gamma^*$.
- Pure P-wave $\gamma^* T \to \pi^+\pi^-$.
- Brodsky-Lepage: unsubtracted disp. rels. ($\mathcal{F}_1^T \sim 1/Q^4$, $\mathcal{F}_{i\neq 1}^T \sim 1/Q^6$).
- **Future work:** on top of $\pi\pi$ in q_2^2-cut include πV ($\sim$ V-res. in $\gamma^*\gamma^* \to \pi^+\pi^-$ LHC) $\Rightarrow$ **need $\pi V \gamma$ coupling, expect important effect**

Extraction of the $f_2(1280)$ TFFs: Single-variable DR



- **Pole pieces** - dual to VFF×RChT - exploit to compute pure π-triangle part of TFFs.
- Show contributions to **all 5 TFFs $\mathcal{F}_i^T \neq 0$**.
- DRs in q_i^2, including matching to asymptotic constraints: **need to impose crossing symmetry.**
- **Imaginary parts:** from π triangle, should cancel against 2π intermediates in HLbL.

Extraction of the $f_2(1280)$ TFFs: Comparison with data?



- **Matching to on-shell** $f_2 \to \gamma\gamma$ width: pure π triangle - significantly undershoots $\Gamma_{\gamma\gamma}$.
- **Matching to BL limit** - needs to be implement;
- **Belle:** only singly-virtual TFFs with $-Q^2 < 30\ \text{GeV}^2$ *(arXiv:1508.06757v2)* $\Rightarrow$ doubly-virtual region unavailable (results would be invaluable!)
- Cross-checking with other approaches: different disp. relations, but expect equivalent results (if same intermediates!) $\Rightarrow$ **see Maximilian's talk**

Conclusions

- **Triangle kinematics:** crucial for consistent and model indep. inclusion of spin-2 resonances in a_μ (especially when all TFFs known!)
- Obtained $\gamma^*\gamma^*\gamma(q \to 0) \to \pi^+\pi^-$. Disp. definition of soft-singular contributions.
- Reconstructing $\gamma^* T \to \pi^+\pi^-$, we compute **both TFFs and contribution of $f_2(1270)$ to a_μ in NWA.**

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Thank you for your attention!