HLbL: Update on hQCD

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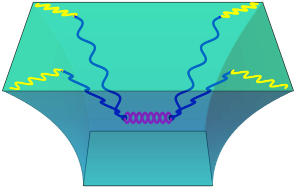

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Strongly coupled QFT $\leftrightarrow$ Theories of gravity in higher dimensions

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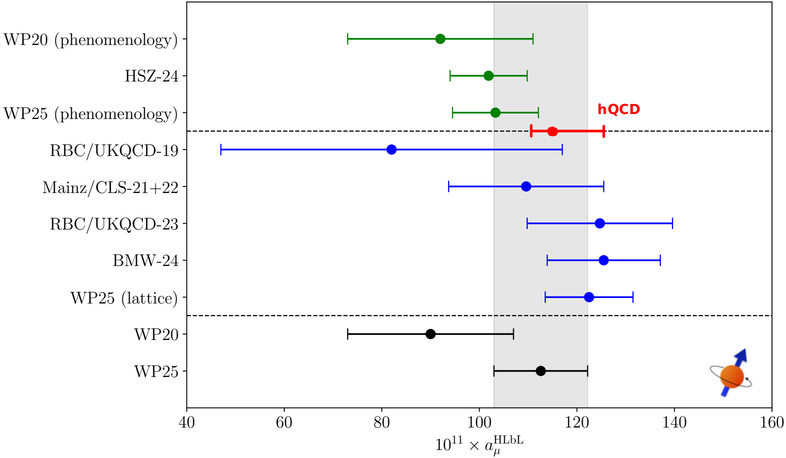
WP2: $\sim 400\%$ error for **tensor contribution** of -1×10^{-11}

CLMR 2501.09699, 2501.19293: still underestimated! (tensor effective poles!)



hQCD: completed by negative contribution from pseudoscalar boxes (dispersive)

[Eq. (5.52) in WP2]

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  - Excited states in Soft Wall models

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- Tensor mesons
  - TFF and nonzero $\mathcal{F}_3^T \to$ opposite sign a_μ for groundstate when compared to quark model
  - Excited states: short distance constraints and tensor effective poles
  - $\hat{\Pi}_i$ from model vs. from 4-point kinematic disp. approach
  - hQCD and RχT

History on hQCD for HLbL and a_μ contributions

- 2010, 2019: In chiral limit π^0, TFF asymptotics $\checkmark$

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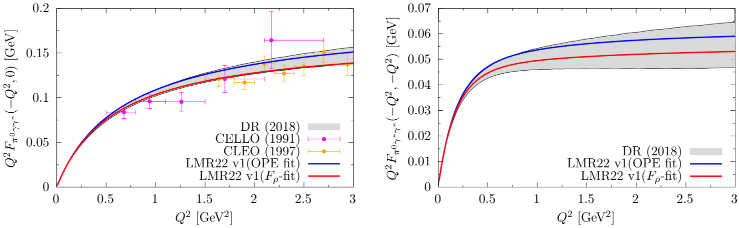

π^0 TFF comparison of best guess hQCD model with dispersive results from Hoferichter et al.[1808.04823] (Fig. 53 in WP2).

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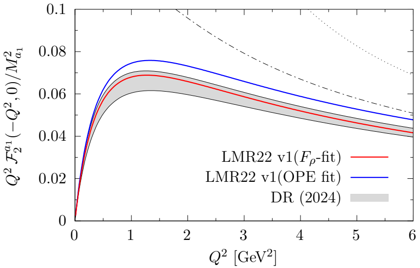

a_1 TFF comparison of best guess hQCD model with dispersive results from Lüdtke et al.[2410.11946] (Fig. 55 in WP2).

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- 2022-2024: m_s, $U(1)_A$ anomaly, $\eta, \eta', ..., f_1, f_1', ...$; scalars
  [LMR 2211.16562, 2411.10432; CCD 2110.05962]

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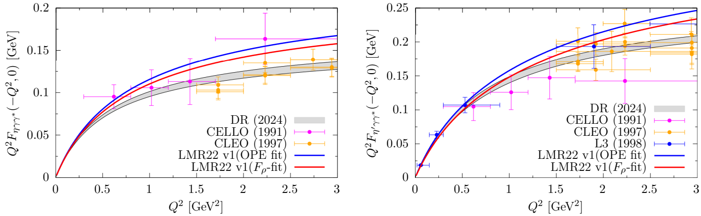

Singly virtual η, η' TFF comparison of best guess hQCD model with dispersive results from Holz et al.[2411.08098] (Fig. 54 in WP2).

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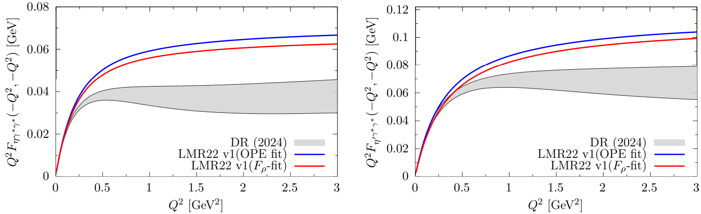

Doubly virtual η, η' TFF comparison of best guess hQCD model with dispersive results from Holz et al.[2411.08098] (Fig. 54 in WP2).

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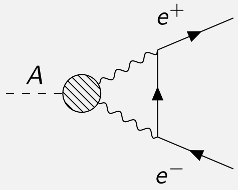

Improved f_1, f_1' mixing angles through Superconnections :

$$B^{f_1}_{e^+e^-} = 3.25 \times 10^{-9}$$
$$B^{f_1'}_{e^+e^-} = 1.79 \times 10^{-9}$$

(SND: $B^{f_1}_{e^+e^-} = 5.1^{+3.7}_{-2.7} \times 10^{-9}$)

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- 2025: IR, mixed and UV splits, tensor mesons

Recap of Pseudoscalars

Using the LMR22(F_ρ) model [2212.05547]:*

	Dispersive [WP2]	hQCD
π^0	$63.0^{+2.7}_{-2.1}$	63.4(2.7)
η	14.7(9)	17.6(1.7)
η'	13.5(7)	14.9(2.0)
Sum	$91.2^{+2.9}_{-2.4}$	95.9(3.8)

For pseudoscalars and axial vectors:
Holographic a_μ formula for $\hat{\Pi}_i$ $\equiv$ Dispersive pole a_μ formula for $\hat{\Pi}_i$

*In units of 10^{-11}

Excited pions

Excited pions in HW models [2108.12345]: $\sum_n a_\mu^{\pi^{*n}} \simeq (0.8^\dagger \ldots 1.8) \times 10^{-11}$

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Soft-wall model [Bari group, 2301.06456 & 2402.07579]:
Besides excessive contribution from π^0 ($a_\mu^{\pi^0} = 75.2 \times 10^{-11}$)
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m	$a_\mu \times 10^{11}$	m	$a_\mu \times 10^{11}$
2.080	1.706	8.032	0.233
3.023	0.633	8.536	0.238
3.821	0.428	9.026	0.244
4.538	0.329	9.503	0.254
5.200	0.282	9.967	0.266
5.820	0.255	10.422	0.279
6.409	0.241	10.866	0.295
6.971	0.233	11.302	0.314
7.511	0.231	11.729	0.334

$\rightarrow \sum_n a_\mu^{\pi^{*n}} = \infty$

similarly for tower of axials! **Therefore excluded from hQCD results for WP2**
(could be repaired with scalar-extended CS term and in modified soft-wall-like models)

Axial vectors and SDCs overview

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hQCD model [LMR22] prediction considerably larger:†

$$a_\mu^{\text{axials+MV-SDC+}P^*} = 32.7^{+3.0}_{-0.0} \times 10^{-11}$$

†Range of models considered in LMR24 around "best-guess" model LMR22(F_ρ-fit)

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Confirmed beautifully by Hoferichter et al. [HSZ 2412.00178 w/o $\{S,T\}_{\text{IR}}$]:

$$a_{\mu,\text{dispersive}}^{\text{axials+SDC+}P^*} = 36.4(4.6) \times 10^{-11}$$

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Axials and SDCs

Region	$a_\mu^{\cdots} \times 10^{11}$	hQCD$_{[\text{LMR22}(F_\rho\text{-fit})]}$	dispersive$_{[\text{HSZ}]}$
IR	a_1	4.2	3.8(7)
	$f_1 + f_1'$	8.9	8.4(1.4)
	AV^*	0.7	
	PS^*	1.7	
	eff.poles		2.0
	Sum	15.4	14.2(1.6)
Mixed	a_1	2.4	
	$f_1 + f_1'$	7.1	
	AV^*	1.9	
	PS^*	-0.04	
	Sum	$11.4^\dagger$	15.9(1.7)
UV		$5.7^\dagger$	$6.2^{+0.2}_{-0.3}$
IR+Mixed	Sum	26.8	30.1(1.9)

$\dagger$: $11.4 \to 13.5$, $5.7 \to 6.3$ once tensors are included (see below)

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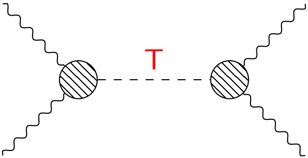
$$a_\mu(\text{QM}) = -2.5 + \underline{0}, \quad a_\mu(\text{hQCD}) = +2.9 + \underline{5.6}$$

(in units of 10^{-11})

Tensor mesons

T$= f_2(1270), a_2(1320), f_2'(1430), ..$



TFF decomposition:

$$\mathcal{M}^{\mu\nu\alpha\beta}(q_1^2, q_2^2) = \sum_{i=1}^{5} \frac{\mathcal{F}_i^T(q_1^2, q_2^2)}{m_T^{n_i}} T_i^{\mu\nu\alpha\beta},$$

Tensor mesons

hQCD describes tensors via metric fluctuations h_{MN} [Katz et al. 0510388]

$$S = -2k_T \int d^5x \sqrt{g} (\mathcal{R} + 2\Lambda) + \frac{1}{2g_5^2} \mathrm{tr} \int d^5x \sqrt{g} F_{MN} F^{MN}$$

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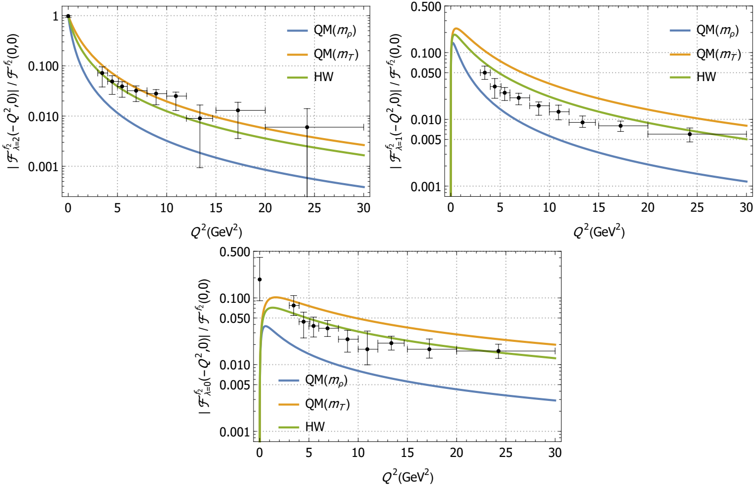
Only 1 new parameter k_T compared to LMR22. Fitted using symmetric LSDC (see later). Non-zero $\mathcal{F}_i^T$: [CLMR 2501.09699]

$$\mathcal{F}_1^T(-Q_1^2, -Q_2^2) = -m_T \frac{1}{g_5^2} \mathrm{tr}\mathcal{Q}^2 \int \frac{dz}{z} h_n(z) \mathcal{J}(z, Q_1) \mathcal{J}(z, Q_2),$$

$$\mathcal{F}_3^T(-Q_1^2, -Q_2^2) = -m_T^3 \frac{1}{g_5^2} \mathrm{tr}\mathcal{Q}^2 \int \frac{dz}{z} h_n(z) \frac{\partial_z \mathcal{J}(z, Q_1)}{Q_1^2} \frac{\partial_z \mathcal{J}(z, Q_2)}{Q_2^2}.$$

$\mathcal{F}_1^T$ and $\mathcal{F}_3^T$ originate both from $5d$ covariant $F_{MN} F^{MN}$.
Quark model ansatz (QM) only has nonzero $\mathcal{F}_1^T$.
Neither QM nor hQCD tensor TFFs reproduce the LC asymptotics

- $m_T = 1235$ MeV (experimentally: 1275.4(8) MeV)
- $\Gamma_{\gamma\gamma} = 2.3$ keV (experimentally: 2.65(45) keV)



Nice agreement with singly virtual data but no sensitivity to $\mathcal{F}_3^T$.

Tensors and $g-2$ in 4pt. kinematics

k_T	M_T [GeV]	$\Gamma_{\gamma\gamma}$ [keV]	IR	Mixed	a_μ $[10^{-11}]$
F_ρ fit	1.235	2.3+0.8+0.2	2.93	0.23	3.17
OPE fit	1.235	2.6+0.9+0.2	3.28	0.25	3.55
by $\Gamma_{\gamma\gamma}$	1.2754(8)	2.65(45)	2.28	0.16	2.4(4)
	1.3182(6)	1.01(9)	0.85	0.05	0.9(1)
	1.5173(24)	0.08(2)	0.06	0.003	0.06(2)
	$f_2 + a_2 + f_2'$		3.19	0.21	3.4(4)

For hQCD model at low energies (IR: $Q_i < 1.5$ GeV):

$$a_{\mu\ \mathrm{IR}}^{f_2+a_2+f_2'} = +2.9(4) \times 10^{-11}$$

Quark model (only $\mathcal{F}_1^T \neq 0$) as in HSZ:

$$a_{\mu\ \mathrm{IR}}^{f_2+a_2+f_2'}[\mathrm{QM}(m_\rho)] = -2.5(8) \times 10^{-11}$$

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[Melnikov-Vainshtein 2003], [Bijnens et al. 2019]

$$\mathcal{C}_{\mathrm{MV}} = \lim_{Q_3\to\infty} \lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q, Q, Q_3) = -\frac{2}{3\pi^2},$$

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hQCD axials contribution after resummation (in OPE fit):

$$\mathcal{C}^A_{\mathrm{MV}} = \mathcal{C}_{\mathrm{MV}}$$
$$\mathcal{C}^A_{\mathrm{sym}} = 0.81\,\mathcal{C}_{\mathrm{sym}}$$

Excited tensor states

Tensor contributions can be resummed similarly

$$\bar{\Pi}_1(Q,Q,Q_3) = -\frac{4}{k_T}\left(\frac{\mathrm{tr}\mathcal{Q}^2}{g_5^2}\right)^2 \iint_0^{z_0} \frac{dzdz'}{zz'} \times \mathcal{J}(z,Q)\mathcal{J}(z',Q)\frac{\partial_{z'}\mathcal{J}(z',Q_3)}{Q_3^2}\partial_{z'}G(z,z';0).$$

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Fitting k_T s.t. tensors supply missing 19% also yields $\Gamma_{\gamma\gamma}$ in superb agreement with data.
In F_ρ fit (best guess), both constraints are satsified at $\sim 90\%$ level in accordance with α_s corrections at large but finite Q.

Transverse SDCs (Preliminary)

Symmetric limit of $\hat{\Pi}_i$ compared to the quark loop (QL):‡

$$\lim_{Q\to\infty} Q^{a_i}\hat{\Pi}_i(Q,Q,Q):$$

$\hat{i}$	A	T	A/QL	T/QL	sum
1	−0.03657	−0.00846	81.22	18.78	100 %
4	−0.0061	−0.01144	37.50	70.40	108 %
7	−0.0061	−0.00256	75.00	31.55	107 %
17	+0.0061	+0.00034	59.04	3.33	62 %
39	+0.0183	+0.00769	68.80	28.94	98 %

Tensors improve all transverse SDCs significantly (except $\hat{\Pi}_{17}$).

‡ $a_i = 4$ for $i = 1, 4$ and $a_i = 6$ else.

Effective tensor pole contributions to the $g-2$

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- Excited f_2 in IR $a_\mu \sim 2.26 \times 10^{-11}$.

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large excited state contributions within 4pt. kinematics dispersive approach plausible!

4pt. dispersive kinematics vs hQCD model a_μ

Dispersive reconstruction of $\hat{\Pi}_i$ vs (pole+non-pole) hQCD model $\hat{\Pi}_i$:

M_T [GeV]	$\Gamma_{\gamma\gamma}$ [keV]	a_μ^{pole} [10^{-11}]	a_μ^{full} [10^{-11}]
1.235	2.46	$7.43 - 5.12 = 2.31$	$9.33 - 3.24 = 6.09$
2.262	0.60	$2.17 + 0.57 = 2.74$	$1.30 - 0.22 = 1.08$
3.280	1.91	$0.96 + 0.30 = 1.26$	$0.54 - 0.11 = 0.43$
4.295	0.75	$0.42 + 0.14 = 0.56$	$0.22 - 0.04 = 0.19$
5.310	1.75	$0.25 + 0.09 = 0.34$	$0.13 - 0.02 = 0.11$
[1.2–5.3]		$11.23 - 4.02 = 7.21$	$11.52 - 3.63 = 7.89$
[1.2–10.4]		$11.62 - 3.87 = 7.76$	$11.71 - 3.66 = 8.05$
$[1.2\text{–}10.4]\|_{\text{IR}}$		$9.72 - 3.60 = 6.12$	$9.73 - 3.53 = 6.20$

Individual contributions§ reshuffled, overall the same.

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Reshuffling expected also in 3pt kinematics (*See talks by Emilis K. and Maximilian Z.*)
→ hQCD could be used to compare 3pt. and 4pt. kinematics.

§KLS value for k_T. Our value of k_T: multiply a_μ by ≈ 1.373 for F_ρ-fit

hQCD and RχT

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hQCD and RχT: An observation

Vector mesons can be modeled also with antisymmetric tensor fields $V_{\mu\nu}$.
See talk by Emilio E.

$$\text{On shell:} \quad V_\mu^{(n)} = \frac{1}{m_n} \partial^\nu V_{\nu\mu}^{(n)}$$

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Interpretation of this result:

- $\mathcal{F}_1^T$ and $\mathcal{F}_3^T$ are on same footing at low energy
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- Dimension of operator O_i generating $\mathcal{F}_i^T$ depends on model for vector mesons. Apparent importance is reshuffled! Not just limited to $\mathcal{F}_1^T, \mathcal{F}_3^T$, or tensor TFFs, it concerns all of RχT.

Open Issues

Tensor meson implementation in hQCD needs further improvements:

- Flavor multiplets of tensor mesons
- Tensor TFFs not agreeing with LC asymptotics (all 5 TFFs needed!)
- Transverse SDCs much improved, but still incomplete

Conclusions

- hQCD in excellent shape wrt. pseudoscalars, axial vectors and SDCs
- Opposite sign a_μ contribution for tensors compared to quark model
- Nonzero $\mathcal{F}_3^T$ responsible for sign flip, but no crosschecks with data possible at present
- Large tensor effective poles plausible in 4pt. kinematics

hQCD prediction for subleading contributions:

$$a_\mu^{\text{Axials+SDCs+P}^*\text{+Tensors+Tensor effective poles}} \sim (33 + 11) \times 10^{-11}$$

dispersive [HSZ]: $a_\mu^{\text{subleading}} = (35.7{-}2.5)(\pm 4.7) \times 10^{-11}$

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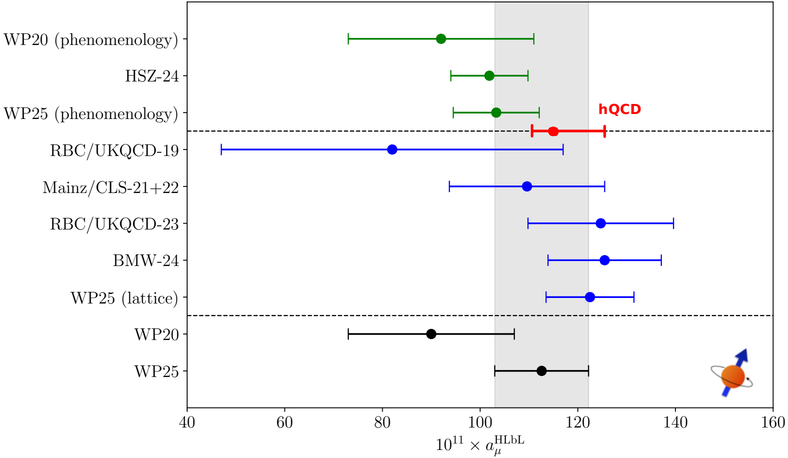

hQCD: completed by negative contribution from pseudoscalar boxes (dispersive)

• = LMR22(F_ρ-fit)+tensor contributions [CLMR25]