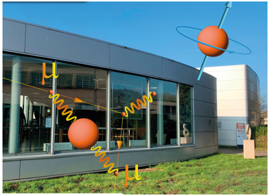
Short-distance constraints for HLbL



Johan Bijnens

Lund University

johan.bijnens@fysik.lu.se
https://www.particle-nuclear.lu.se/johan-bijnens

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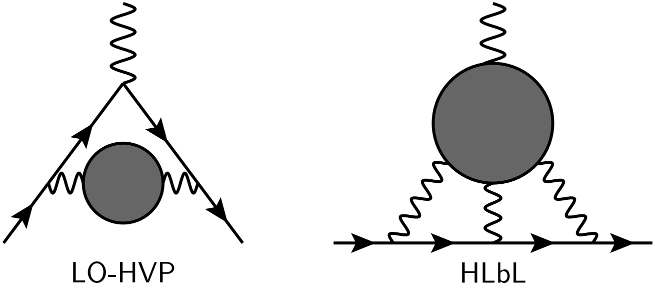
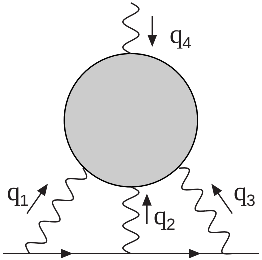
Overview

Hadronic contributions



- Muon and photon lines, representative diagrams
- The blobs are hadronic contributions: dominate theory uncertainties
- There are higher order contributions of both types: known accurately enough
- $a_\mu^{HVP} = 7045(61)\ 10^{-11}$ (LO+NLO+NNLO) (WP25)
- $a_\mu^{HLbL} = 115.5(9.9)\ 10^{-11}$ (LO+NLO) (WP25)
- $a_\mu^{HLbL} = 92(18)\ 10^{-11}$ (LO+NLO) (WP20)

Overview



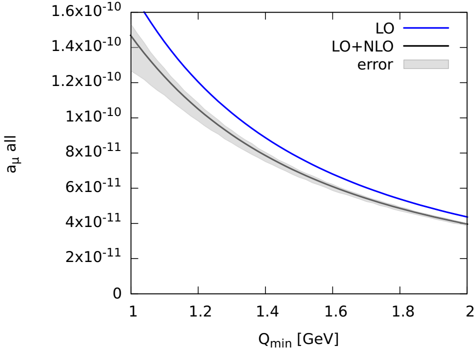
- $\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3) =$
  $-i\int d^4x\, d^4y\, d^4z\, e^{-i(q_1\cdot x+q_2\cdot y+q_3\cdot z)}\langle 0|\,T\{j^\mu_{\rm em}(x)j^\nu_{\rm em}(y)j^\lambda_{\rm em}(z)j^\sigma_{\rm em}(0)\}|0\rangle\,,$
- $\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma}\,\Pi_i$ Some ambiguity here
- $a_\mu^{\rm HLbL} =$
  $\frac{\alpha^3}{432\pi^2}\int_0^\infty d\Sigma\,\Sigma^3\int_0^1 dr\, r\sqrt{1-r^2}\int_0^{2\pi}d\phi\sum_{i=1}^{12} T_i(\Sigma,r,\phi)\,\bar\Pi_i(Q_1^2,Q_2^2,Q_3^2)$
- The $\bar\Pi_i$ are unique and have no ambiguities: useful to compare
- The T_i depend on $Q_i^2=-q_i^2$ at $q_4=0$ and m_μ
- But three scales and one at zero momentum

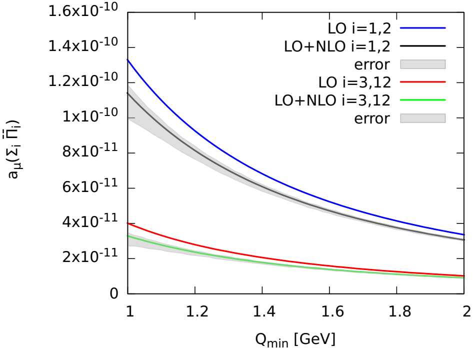
Short-distance 1

- Q_1^2, Q_2^2, Q_3^2 all large
- Can be done via OPE in a background field
- Paper I: J. Bijnens, N. Hermansson-Truedsson and A. Rodríguez-Sánchez, "Short-distance constraints for the HLbL contribution to the muon anomalous magnetic moment," Phys. Lett. B **798** (2019), 134994 [arXiv:1908.03331 [hep-ph]].
- Paper II: J. Bijnens, N. Hermansson-Truedsson, L. Laub and A. Rodríguez-Sánchez, "Short-distance HLbL contributions to the muon anomalous magnetic moment beyond perturbation theory," JHEP **10** (2020), 203 [arXiv:2008.13487 [hep-ph]].
- Paper III: J. Bijnens, N. Hermansson-Truedsson, L. Laub and A. Rodríguez-Sánchez, "The two-loop perturbative correction to the $(g2)_\mu$ HLbL at short distances," JHEP **04** (2021), 240 [arXiv:2101.09169 [hep-ph]].

Short-distance 1: Q_1^2, Q_2^2, Q_3^2 large

- Results:
  - Lowest order is massless quark-loop
  - Contributions from higher order operators small
  - α_S corrections about -10% (NLO in plot)

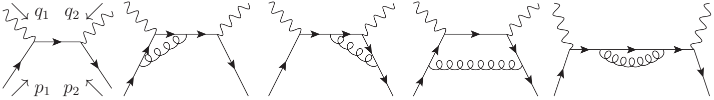




Short-distance 2: Two Q_i^2 large

- Euclidean thus $Q_1^2 \approx Q_2^2 \gg Q_3^2$ and cyclic
- Idea: two of the currents are close to each other
- Started in 2003
  K. Melnikov and A. Vainshtein, "Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment revisited," Phys. Rev. D **70** (2004), 113006 [arXiv:hep-ph/0312226 [hep-ph]].
- Leading term is axial current: anomaly: many papers discussing it
- We went to higher order in OPE and gluonic corrections
- Paper IV: J. Bijnens, N. Hermansson-Truedsson and A. Rodríguez-Sánchez, "Constraints on the hadronic light-by-light in the Melnikov-Vainshtein regime," JHEP **02** (2023), 167 [arXiv:2211.17183 [hep-ph]].
- Paper V: J. Bijnens, N. Hermansson-Truedsson and A. Rodríguez-Sánchez, "Constraints on the hadronic light-by-light tensor in corner kinematics for the muon $g-2$," JHEP **03** (2025), 094 [arXiv:2411.09578 [hep-ph]].

Short-distance 2: Two Q_i^2 large



- Do the OPE for two of the four currents

$$D=3: \quad \mathcal{O}_{q3}^{\alpha\beta\gamma} = \overline{q}\left[\gamma^\alpha\gamma^\beta\gamma^\gamma - \gamma^\gamma\gamma^\beta\gamma^\alpha\right]q\,,$$

$$D=4: \quad \mathcal{O}_{q4}^{\alpha\beta} = \overline{q}\gamma^\beta\left[\overrightarrow{D}^\alpha - \overleftarrow{D}^\alpha\right]q\,,$$

$$\mathcal{O}_{FF,1}^{\alpha\beta} = F^{\alpha\gamma}F_\gamma^{\ \beta}\,, \qquad \mathcal{O}_{FF,2}^{\alpha\beta} = F^{\gamma\delta}F_{\gamma\delta}\,g^{\alpha\beta}\,, \qquad \mathcal{O}_F = F\times F\,,$$

$$\mathcal{O}_{GG,1}^{\alpha\beta} = G^{\alpha\gamma}G_\gamma^{\ \beta}\,, \qquad \mathcal{O}_{GG,2}^{\alpha\beta} = G^{\gamma\delta}G_{\gamma\delta}\,g^{\alpha\beta}\,, \qquad \mathcal{O}_G = G\times G\,.$$

- Now take matrix element with two softer photons
- $D=3$ two formfactors: ω_L and ω_T
- $D=4$ many more formfactors: consistency conditions to have well defined $\bar{\Pi}_i$

New variables

- Define new variables

$$\hat{q} = \frac{1}{2}\left(q_1 - q_2\right)\,, \qquad q_{1,2} = \pm\hat{q} - \frac{1}{2}\left(q_3 + q_4\right)$$

$$\underbrace{\overline{Q}_3}_{\text{large}} = Q_1 + Q_2\,, \qquad \delta_{12} = Q_1 - Q_2$$

- OPE done in terms of $\hat{Q}$ for tensor, then $\overline{Q}_3$ for $\hat{\Pi}_i$ and a_μ^{HLbL}
- Related through

$$\underbrace{\hat{Q}^2}_{\text{large}} = \frac{1}{4}\left(\overline{Q}_3^2 + \delta_{12}^2 - Q_3^2\right)$$

- For OPE, gauge invariance in $\hat{q}$ only perturbative

OPE and matrix elements

- OPE is done in terms of $\hat{q}^2$ large, $q_3 = -q_1 - q_2$ small
- OPE only depends on $\hat{q}$
- Matrixelements of OPE given in terms of their Lorentz structure and q_3.
- Together with Wilson coefficients fixes $q_3 \cdot \hat{q}$ dependence.
- Many formfactors in the matrix-elements but to remove kinematic singularities there are relations
- These are satisfied by the perturbative matrix-elements including gluonic corrections
- Must be satisfied in general

OPE and matrix elements

- Lorentz decompose $D = 4$:

$$\lim_{q_4\to 0} \partial_{q_4}^{\nu_4} \left\langle \bar{q}(0) \left[\overrightarrow{D}^{\alpha} - \overleftarrow{D}^{\alpha}\right] \gamma^{\beta} q(0) \right\rangle^{j,\mu_3,\mu_4}_{\overline{\mathrm{MS}}(\mu),(8)} = \sum_{i=1}^{6} \omega_{(8)}^{D,i} L_i^{\alpha\beta\mu_3\mu_4\nu_4}$$

- Cancellation of collinear singularities: Nontrivial relations

$$\omega_{(8)}^{D,2} = -2\,\omega_{(8)}^{D,1} + \omega_{(8)}^{D,5} - \frac{\omega_{(8)}^{D,6}}{2} - \frac{\omega_{T,(8)}\,Q_i^2}{8\pi^2}$$

$$\omega_{(8)}^{D,3} = -2\,\omega_{(8)}^{D,1} + \omega_{(8)}^{D,5} - \frac{\omega_{(8)}^{D,6}}{2} + \frac{\omega_{T,(8)}\,Q_i^2}{8\pi^2}$$

$$\omega_{(8)}^{D,4} = \omega_{(8)}^{D,5}$$

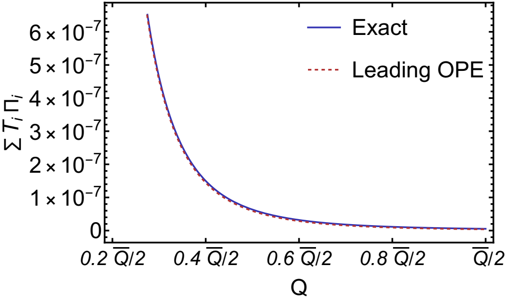
- Some follow from EOM (multiply by $g_{\alpha\beta}$)
- In the end only $\omega_{(8)}^{D,1}$, $\omega_{(8)}^{D,5}$ and $\omega_{(8)}^{D,6}$ contribute
- Expressions can be found in Paper IV
- Agree with corner expansion of short-distance 1
- Only three more formfactors needed

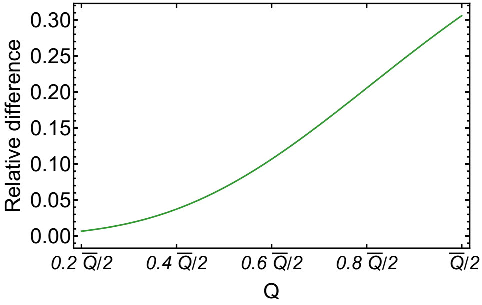
Surprise

- Sum over all three corners: $Q_1^2, Q_2^2 \gg Q_3^2$; $Q_2^2, Q_3^2 \gg Q_1^2$; $Q_3^2, Q_1^2 \gg Q_2^2$
- Expand the T_i appropriately
- Do the integration symmetrically
- Use the constraints: all the new form-factors cancel
- Very nontrivial. $D = 4$ contributes but can be written in terms of the two $D = 3$ form-factors
- Note that this is true for the third Q_i^2 small, not only in the perturbative regime for it.

Final comment

- When to use which short-distance?
- It turns out that the MV perturbative result is quite good even into the region where all three are similar
- Q_3 is the small one; figure for $Q_1 = Q_2 = 5\,GeV$ and summed over cyclic points.
- Check only valid in the fully perturbative regime





Conclusions and outlook

- We have solved the problem of the short-distance regime for HLbL
- Outlook 1: for the all scales large case next order becomes three-loops; possibly in the rather far future
- Outlook 2: In the MV case we do expect quark-mass corrections to be small (at short-distances) but some more work can be done here; quark-mass corrections in the formfactors at low-energies have already been studied
- Outlook 3: $D = 5$ in the MV limit has very many operators; not obvious if needed even for another factor of 2 precision