Axial-vector and tensor meson transition form factors

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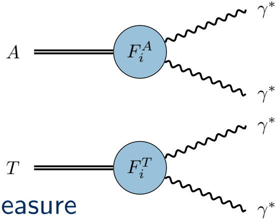
Significance of axial- and tensor-meson TFFs for $g-2$

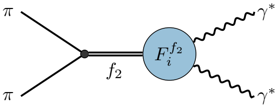
- axial/tensor mesons intermediate states contribute to HLbL
- interplay with short-distance constraints
- in dispersive framework:
  narrow-resonance approx.
  $\rightarrow$ need doubly-virtual TFFs



- significant contribution to uncertainty
- scarce experimental data, difficult to measure

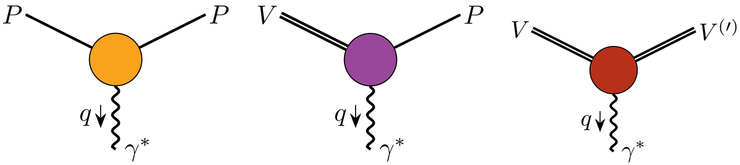
Currently used in disp. framework [Hoferichter et al., 2024] :

- axials (3 TFFs): exp. data and VMD model for $f_1(1285)$ [Zanke et al. 2021], $a_1(1260)$ and $f_1' \equiv f_1(1420)$ via $U(3)$ symm.
- tensors (5 TFFs): restrict to 1 TFF (hierarchy), simple dipole structure; plan [M.Zillinger's talk]:
  $f_2(1270)$ as $\pi\pi$ resonance

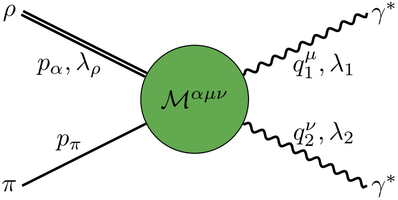


Situation for a_1 and a_2 (and potentially $f_1(1420) \equiv f_1'$)

- dominant decay channels: $a_1 \to 3\pi/\rho\pi$, $a_2 \to 3\pi/\rho\pi$, $(f_1' \to K\bar{K}\pi/K\bar{K}^*)$
- in isobar model, this is $A/T \to \rho\pi$
  $\to$ a_1 and a_2 similar to f_2, but resonances in $\rho\pi$ instead of $\pi\pi$
  $\to$ situation less symmetric and more complicated
- can relate a_1/a_2 TFFs to known form factors F_π^V, $F_{\rho\pi}$, G_i



$\rho\pi \to \gamma^*\gamma^*$ system



$$\langle \rho(p, \lambda_\rho)\pi(p_\pi)|\gamma^*(q_1, \lambda_1)\gamma^*(q_2, \lambda_2)\rangle \sim \epsilon_\mu^{\lambda_1 *}\epsilon_\nu^{\lambda_2 *}\epsilon_\alpha^{\lambda_\rho}\mathcal{M}^{\mu\nu\alpha}(q_1, q_2, p)$$

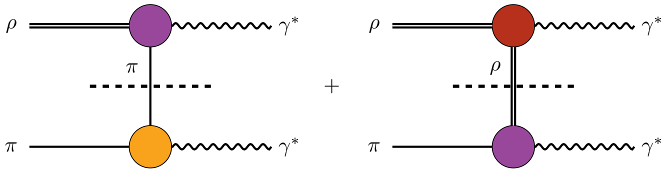
$$\mathcal{M}^{\mu\nu\alpha}(q_1, q_2, p) = \sum_i \mathcal{F}_i(q_1^2, q_2^2; s, t, u)\, T_i^{\mu\nu\alpha}(q_1, q_2, p)$$

- gauge-invariant description of $\rho\pi \to \gamma^*\gamma^*$
- need scalar functions $\mathcal{F}_i$ free of kinematic zeros and singularities $\to$ Bardeen-Tung-Tarrach (BTT) procedure
- capture effect of different form factors $F_\pi^V(q^2)$ and $G_i(q^2)$ for π and ρ, which is lost in scalar-QED $\times$ form factor calculation

Input for scalar functions

- unitarity relation for $\rho\pi \to \gamma^*\gamma^*$, cut in t/u, dispersive reconstruction from 1-particle intermediate states
- $\epsilon_\mu^{\lambda_1 *}\epsilon_\nu^{\lambda_2 *}\epsilon_\alpha^{\lambda_\rho}\mathcal{F}_i T_i^{\mu\nu\alpha} = \frac{1}{\pi}\sum_{j=\pi,\rho} \frac{\hat{\rho}_j^t}{t'-t} + \frac{\hat{\rho}_j^u}{u'-u}$,

  $\hat{\rho}_{\pi/\rho}$ related to $F_\pi^V(q^2)$, $G_i(q^2)$, $F_{\rho\pi}(q^2)$



- this yields amplitude $\rho\pi \to \gamma^*\gamma^*$ for ρ, π **on-shell**
- project scalar functions accordingly to BTT projection of tensor structures (dual mapping)

Challenges in the $\rho\pi \to \gamma^*\gamma^*$ system

- problem: system not symmetric enough (compare $\pi\pi \to \gamma^*\gamma^*$)
  $\to$ cancellation of kinematic singularities in $\mathcal{F}_i$ would require

$$F_{\rho\pi}(q_2^2)F_\pi^V(q_1^2) = F_{\rho\pi}(q_2^2)G_1(q_1^2);$$

$$\text{but} \quad F_\pi^V(-Q^2) \sim Q^{-2}, \quad G_1(-Q^2) \sim Q^{-4}$$

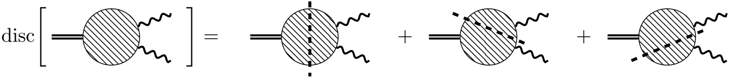
- solution: introduce additional heavier vector intermediate state R in LHC with FFs $H_i(q^2)$

$$\Rightarrow F_{\rho\pi}(q_2^2)\left[G_1(q_1^2) - F_\pi^V(q_1^2)\right] + F_{R\pi}(q_2^2)H_1(q_1^2)q_1^2 = 0$$

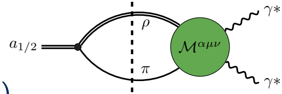
- additionally, need on-shell relations ($p^2 = M_\rho^2$) to cancel kinematic singularities, but ρ, π off-shell in loop
- projected scalar functions ambiguous under on-shell relations
- BTT basis $\{T_i^{\mu\nu\alpha}\}_{i=1}^{13}$, but need to include two more structures for generating set in all kinematic limits (Tarrach)

Dispersive reconstruction of $F_i^{a_1}$

- Cutcosky rules for the a_1 TFFs

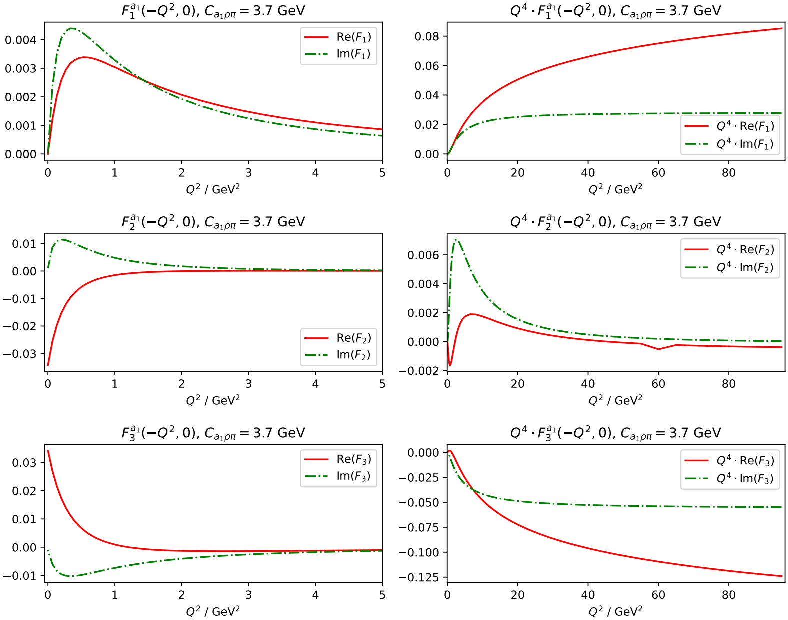


- no information about t/u-channel cuts $\to$ neglect for now
- s-channel cut: $\mathrm{disc}\, F_i^{a_1} = 2\mathrm{i}\, \mathrm{Im}\, F_i^{a_1}$ for space-like virtualities; $\mathrm{Im}\, F_i^{a_1}$ unambiguous
- neglecting rescattering effects in $\rho\pi$, reconstruct as $F_i^{a_1}(s, q_1^2, q_2^2) = \frac{1}{\pi} \int_{s_{\mathrm{thr}}}^{\infty} \mathrm{d}x \frac{\mathrm{Im}\, F_i^{a_1}(x, q_1^2, q_2^2)}{x-s}$
- take $\mathrm{Im}\, F_i^{a_1}(s, q_1^2, q_2^2)$ from loop calc., solve via PaVe decomp. (tensor coeffs.)

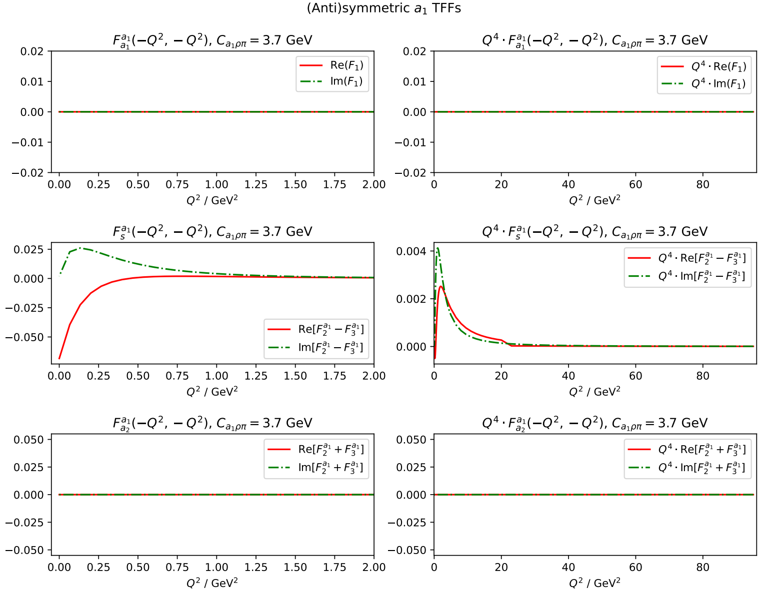


- neglect quadrupole FFs and full transversality of a_1, ρ (would require regularisation)
- can do this analogously for $F_i^{a_2}$, but need to regularise loop integrals

Preliminary results: singly-virtual $F_i^{a_1}$



Preliminary results: symmetrised $F_i^{a_1}$ with symm. virtualities



Discussion

Axials:

- $|F_2^{a_1}(0,0)| = |F_2^{a_1}(0,0)| \approx 0.035$ with $C_{a_1\rho\pi} = 3.7\,\text{GeV}$
- [Lüdtke et al. 2025]: $|F_2^{a_1}(0,0)| = |F_2^{a_1}(0,0)| \approx 0.35$
- mostly fall as expected from light-cone expansion [Zanke et al. 2021], but $F_1^{a_1}$ should fall as $\mathcal{O}\big(Q^{-6}\big)$

Tensors:

- loop integrals diverge $\rightarrow$ regularisation necessary
- if that is done, can apply this framework to a_2
- so far: projection to $F_i^{a_2}$ without solving integrals or dispersive reconstruction does not reveal any obvious hierarchy between different TFFs
  $\rightarrow$ maybe arises from further steps

Summary

What the current reconstruction from $\text{Im}\, F_i^{a_1}(q_1^2, q_2^2)$ **can** do:

- symmetries, Landau–Yang theorem (by construction)
- partly expected high-energy behaviour
- for a_1: complementary ansatz to Ref. [Lüdtke et al. 2025] and to $f_1 + U(3)$ symmetry
- test some variations of π/ρ FFs (see app.)

Summary

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What it **can't** do:

- expected normalisation
- expected asymptotic behaviour for $F_1^{a_1}$
- real form factors
- hierarchy of TFFs (so far)

Therefore, want to implement proper unitarisation

Appendix

Dispersive reconstruction of $\rho^-\pi^+ \to \gamma^*\gamma^*$

$$\epsilon_\mu^{\lambda_1 *}\epsilon_\nu^{\lambda_2}\epsilon_\alpha^{\lambda_\rho *}\mathrm{Im}\,{}^{t}_{\pi}\mathcal{M}^{\mu\nu\alpha} \quad \text{(crossed channel)}$$
$$=\pi\delta(t-M_\pi^2)\langle\rho^+(-p,\lambda_\rho)\gamma^*(q_1,\lambda_1)|\pi^+(k)\rangle\langle\pi^+(p_\pi)\gamma^*(q_2,\lambda_2)|\pi^+(k)\rangle^*$$
$$=\epsilon_\mu^{\lambda_1 *}\epsilon_\nu^{\lambda_2}\epsilon_\alpha^{\lambda_\rho *}\pi\delta(t-M_\pi^2)F_{\rho\pi}(q_1^2)F_\pi^V(q_2^2)(2p^\nu-2q_1^\nu-q_2^\nu)\epsilon^{\alpha\mu p q_1}$$

$\cdots$

$$\epsilon_\mu^{\lambda_1}\epsilon_\nu^{\lambda_2 *}\epsilon_\alpha^{\lambda_\rho}\mathrm{Im}\,{}^{u}_{\rho}\mathcal{M}^{\mu\nu\alpha}$$
$$=\pi\delta(u-M_\rho^2)\langle\rho^+(-p,\lambda_\rho)\gamma^*(q_2,\lambda_2)|\rho^+(k)\rangle\langle\pi^+(p_\pi)\gamma^*(q_2,\lambda_2)|\rho^+(k)\rangle^*$$
$$=\epsilon_\mu^{\lambda_1}\epsilon_\nu^{\lambda_2 *}\epsilon_\alpha^{\lambda_\rho}\pi\delta(u-M_\rho^2)F_{\rho\pi}(q_2^2)G_1(q_1^2)\times$$
$$\times\left[(2p^\nu-q_2^\nu)\epsilon^{\alpha\mu p q_1}+(2p^\nu-q_2^\nu)\epsilon^{\alpha\mu q_1 q_2}\right]$$

$$\epsilon_\mu^{\lambda_1 *}\epsilon_\nu^{\lambda_2}\epsilon_\alpha^{\lambda_\rho *}\mathcal{F}_i T_i^{\mu\nu\alpha}=\frac{1}{\pi}\sum_{j=\pi,\rho}\frac{\hat\rho_j^t}{t'-t}+\frac{\hat\rho_j^u}{u'-u} \quad \text{(read } \hat\rho \text{ from above)}$$

Implementation BTT basis

- find 13 independent gauge-invariant structures (matches # helicity amplitudes), "basis" $\{T_i^{\mu\nu\alpha}\}_{i=1}^{13}$
- problem: Tarrach structures T_{14}, T_{15}, not a basis in all kinematic limits

$$(p \cdot q_1)\big(T_9^{\mu\nu\alpha} - T_{11}^{\mu\nu\alpha}\big) - q_1^2\big(T_{10}^{\mu\nu\alpha} - T_{12}^{\mu\nu\alpha}\big) = 2(q_1 \cdot q_2)\,T_{14}^{\mu\nu\alpha},$$
$$(p \cdot q_2)\big(T_9^{\mu\nu\alpha} + T_{11}^{\mu\nu\alpha}\big) - q_2^2\big(T_{10}^{\mu\nu\alpha} + T_{12}^{\mu\nu\alpha}\big) = 2(q_1 \cdot q_2)\,T_{15}^{\mu\nu\alpha}$$

- need to include $T_{14}^{\mu\nu\alpha}$, $T_{15}^{\mu\nu\alpha}$ into generating set
- project scalar functions to "basis" $\{\mathcal{F}_i\}_{i=1}^{13}$, manually shuffle parts of $\mathcal{F}_9, \mathcal{F}_{10}, \mathcal{F}_{11}, \mathcal{F}_{12}$ to $\mathcal{F}_{14}, \mathcal{F}_{15}$ (no double counting)
- additionally, need relations like $t - M_\rho^2 = q_1^2 - 2(p \cdot q_1)$ (implies $M_{\rho/\pi}^2$ in LHC $= p_{\rho/\pi}^2$ in intermediate state)

Definition of $\mathcal{F}_i^{a_1}$ [Hoferichter, Stoffer 2020]

$$\mathcal{M}^{\mu\nu\beta}_{a_1\gamma^*\gamma^*} = \sum_{i=1}^{3} F_i^{a_1}\, \tilde{T}_i^{\mu\nu\beta}$$

$$\tilde{T}_1^{\mu\nu\beta}(q_1,q_2) = \varepsilon^{\mu\nu q_1 q_2}(q_1-q_2)^\beta,$$
$$\tilde{T}_2^{\mu\nu\beta}(q_1,q_2) = \varepsilon^{\beta\nu q_1 q_2} q_1^\mu + \varepsilon^{\beta\mu\nu q_2} q_1^2,$$
$$\tilde{T}_3^{\mu\nu\beta}(q_1,q_2) = \varepsilon^{\beta\mu q_1 q_2} q_2^\nu + \varepsilon^{\beta\mu\nu q_1} q_2^2$$

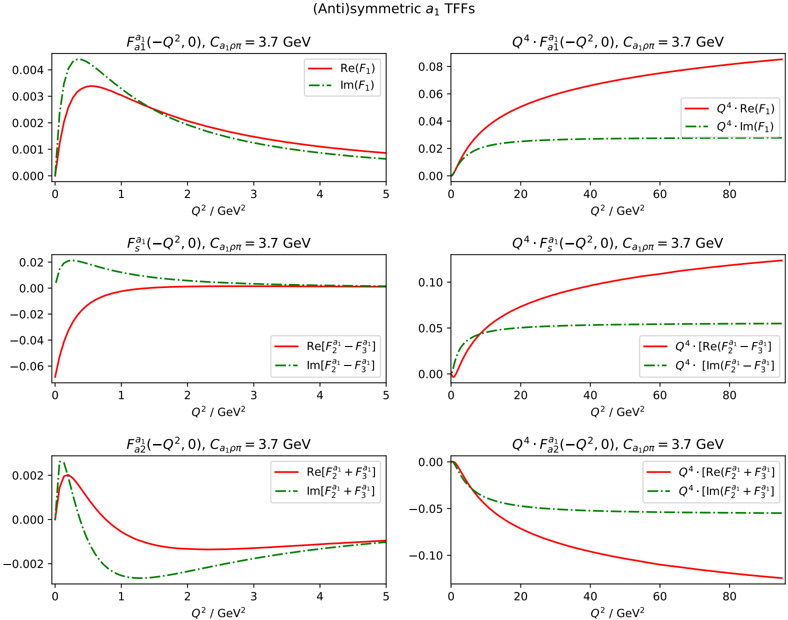
$$\tilde{T}_{a1}^{\mu\nu\beta}(q_1,q_2) = \tilde{T}_1^{\mu\nu\beta}(q_1,q_2),$$
$$\tilde{T}_{a2}^{\mu\nu\beta}(q_1,q_2) = \left[\tilde{T}_2^{\mu\nu\beta}(q_1,q_2) + \tilde{T}_3^{\mu\nu\beta}(q_1,q_2)\right],$$
$$\tilde{T}_{s}^{\mu\nu\beta}(q_1,q_2) = \left[\tilde{T}_2^{\mu\nu\beta}(q_1,q_2) - \tilde{T}_3^{\mu\nu\beta}(q_1,q_2)\right]$$

$$F_{a1}^{a_1} = F_1^{a_1}, \quad F_{a_2}^{a_1} = F_2^{a_1} + F_3^{a_1}, \quad F_s^{a_1} = F_2^{a_1} - F_3^{a_1}$$

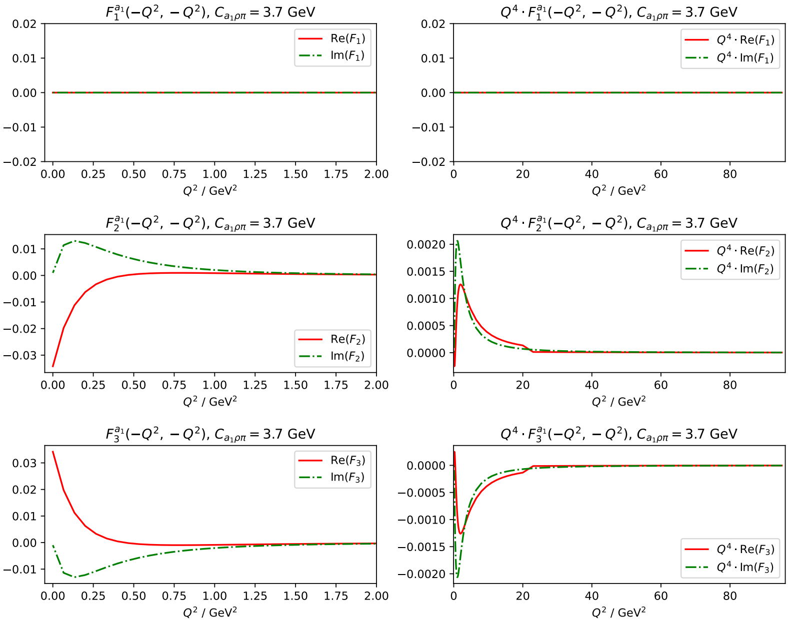
Projection to $\mathcal{F}_i^{A/T}$ and implementation

- reconstruct off-shell amplitude for loop calculation from unambiguous **imaginary part** (for space-like virtualities)
- loop integral calculation via PaVe decomposition (tensor coefficients), evaluation implemented in two versions for cross-check
- project integrals to $\{\mathcal{F}_i^A\}_{i=1}^3$ and $\{\mathcal{F}_i^T\}_{i=1}^5$
- for A, build projectors; take care as unphysical degree of freedom is present from $A \to VP$
- for T, more (physical and unphysical) structures $\to$ either use BTT projection of tensor TFFs or helicity amplitudes [E.Lymperiadou's poster]
- dispersively reconstruct projected objects (two implementations)

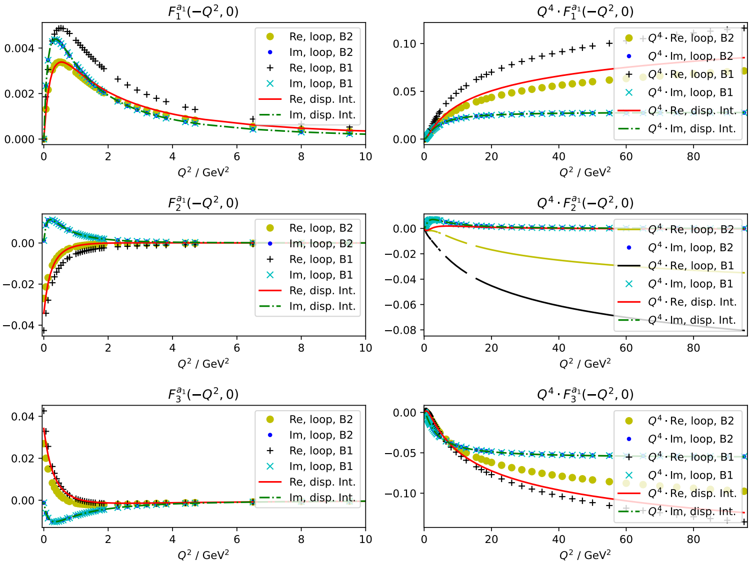
Preliminary results: symmetrised singly-virtual $F_i^{a_1}$



Preliminary results: $F_i^{a_1}$ with symmetric virtualities



Comparison of real parts — singly-virtual form factors (prelim.)



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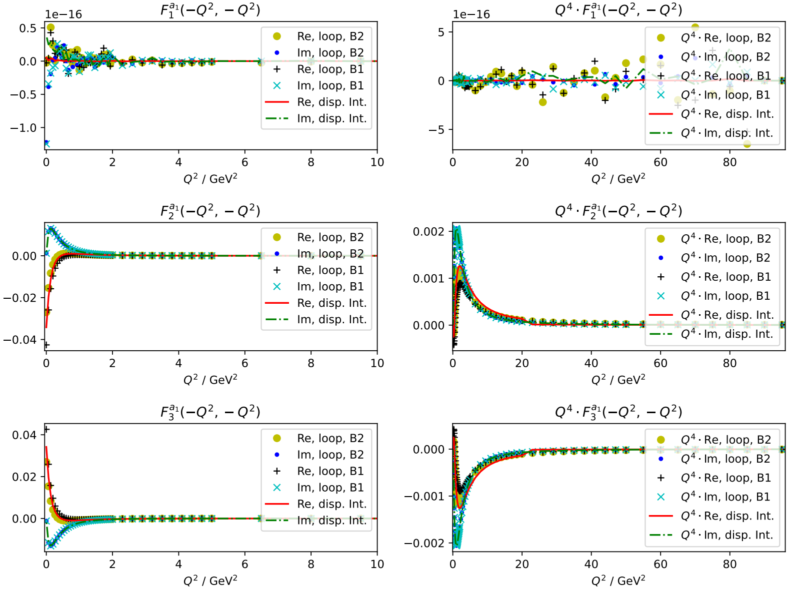
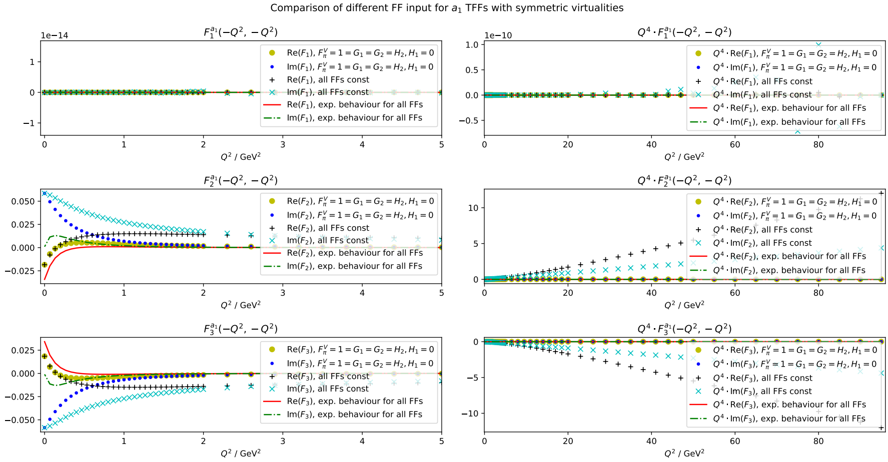
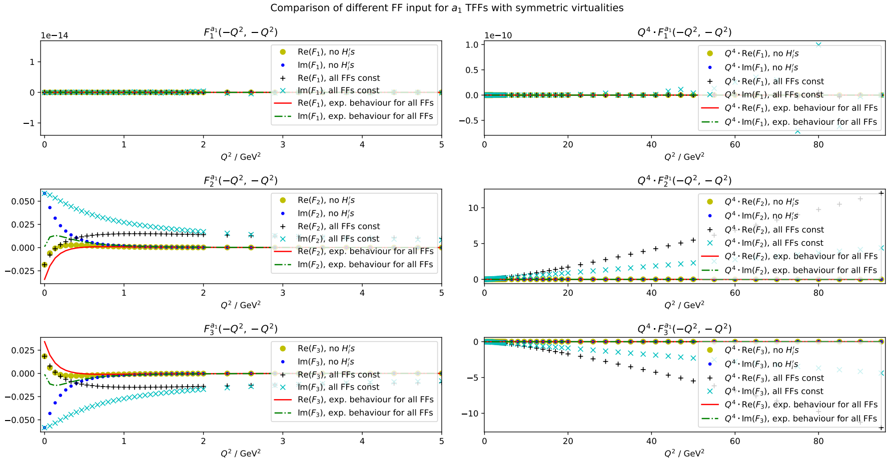


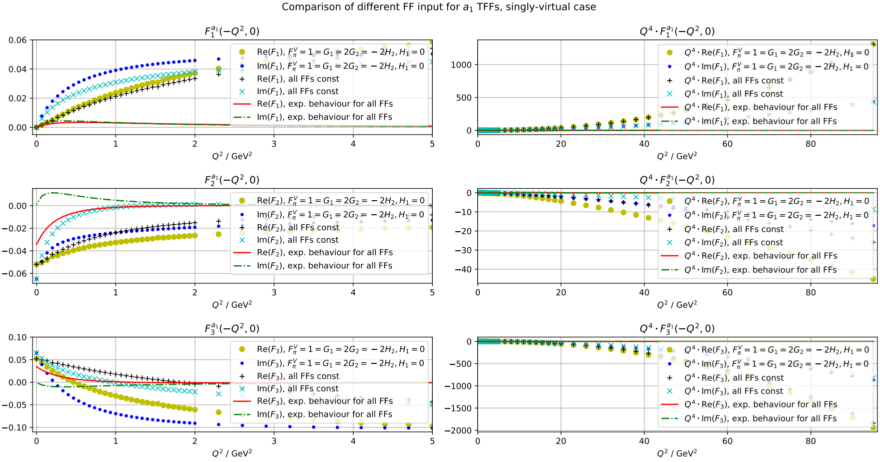
Influence of F_π^V, $F_{\rho\pi}$, and G_1 — symm. virtualities (prelim.) I



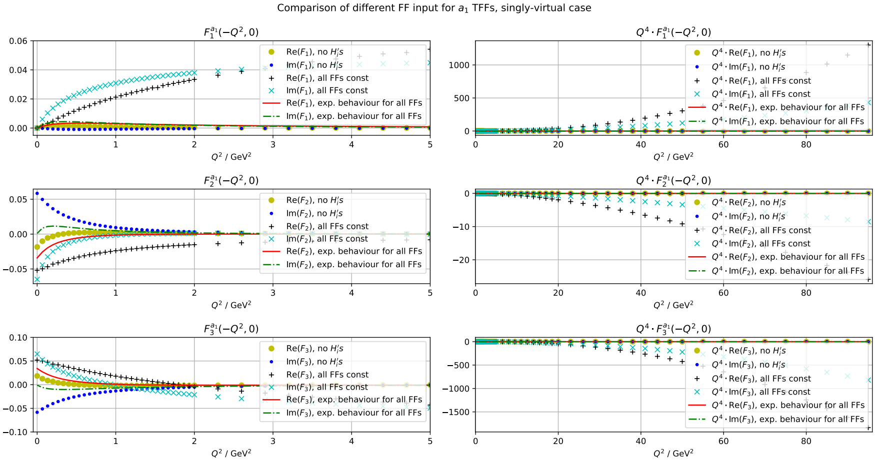
Influence of F_π^V, $F_{\rho\pi}$, and G_1 — symm. virtualities (prelim.) II



Influence of F_π^V, $F_{\rho\pi}$, and G_1 — singly-virtual (prelim.) I



Influence of F_π^V, $F_{\rho\pi}$, and G_1 — singly-virtual (prelim.) II



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