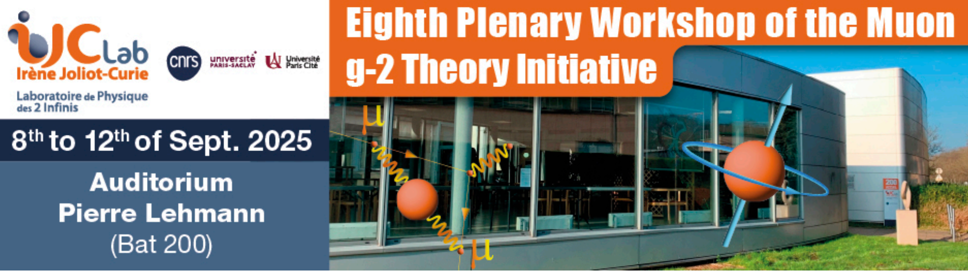

Tensor Meson pole contributions within RχT.

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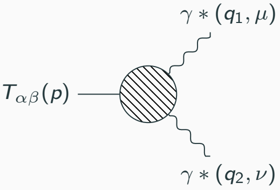


$$\mathcal{M}^{\mu\nu\alpha\beta} = \sum_{i=5}^{5} T_i^{\mu\nu\alpha\beta} \frac{1}{m_T^{n_i}} \mathcal{F}_i^T(q_1^2, q_2^2),$$

Where the tensor structures are:

$$T_1^{\mu\nu\alpha\beta} = g^{\mu\alpha} P_{21}^{\nu\beta} + g^{\nu\alpha} P_{12}^{\mu\beta} + g^{\mu\beta} P_{21}^{\nu\alpha} + g^{\nu\beta} P_{12}^{\mu\alpha} + g^{\mu\nu}(q_1^\alpha q_2^\beta + q_2^\alpha q_1^\beta) - q_1 \cdot q_2 (g^{\mu\alpha} g^{\nu\beta} + g^{\nu\alpha} g^{\mu\beta}),$$

$$T_2^{\mu\nu\alpha\beta} = (q_1^\alpha q_1^\beta + q_2^\alpha q_2^\beta) P_{12}^{\mu\nu}, \quad T_3^{\mu\nu\alpha\beta} = P_{11}^{\mu\alpha} P_{22}^{\nu\beta} + P_{11}^{\mu\beta} P_{22}^{\nu\alpha},$$

$$T_4^{\mu\nu\alpha\beta} = P_{12}^{\mu\alpha} P_{22}^{\nu\beta} + P_{12}^{\mu\beta} P_{22}^{\nu\alpha}, \quad T_5^{\mu\nu\alpha\beta} = P_{21}^{\mu\alpha} P_{11}^{\nu\beta} + P_{21}^{\mu\beta} P_{11}^{\nu\alpha}$$

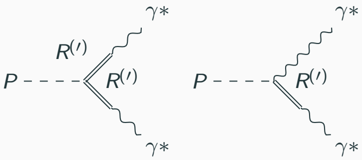
For WP25, the two determinations in the purely hadronic region (Hoferichter et al (2025) and Cappiello et al (2025)) ($Q_i < 1.5\mathrm{GeV}$) were in tension:

Region		Dispersive	hQCD	Regge	DSE/BSE
$Q_i > Q_0$		$6.2^{+0.2}_{-0.3}$	6.3(7)	4.8(1)	2.3(1.5)
Mixed	A, S, T	3.8(1.5)			
	OPE	10.9(0.8)			
	Effective pole	1.2			
	Sum	15.9(1.7)	13.5(2.4)	12.8(5)	10.1(3.0)
$Q_i < Q_0$	$A = f_1, f_1', a_1$	12.2(4.3)	13.1(1.5)	10.9(1.0)	8.6(2.6)
	$S = f_0(1370), a_0(1450)$	−0.7(4)			−0.8(3)
	$T = f_2, a_2$	−2.5(8)	2.9(4)		
	Other	2.0	8.0(9)	3.2(6)	2.8(6)
	Sum	11.0(4.4)	24.0(2.8)	14.1(1.2)	10.6(2.7)
Sum		33.2(4.7)	43.8(5.9)	31.7(1.6)	23.0(7.4)

An exploration within RχT could be done. Previous study obtained the optimal parameters for describing the radiative decay widths ($T \to \gamma\gamma$)(Chen, et al. (2023)). Adding resonances to this procedure would allow us to reproduce SDCs ($V + V'$)!

Coupling constant	Operator
$C_{T\gamma\gamma}$	$\langle T^{\mu\nu}\{f^{\alpha}_{+\mu}, f_{+\alpha\nu}\}\rangle$
$C_{T\gamma V}$	$i\langle T^{\mu\nu}\{f^{\alpha}_{+\mu}, V_{\alpha\nu}\}\rangle$
$C_{T\gamma V'}$	$i\langle T^{\mu\nu}\{f^{\alpha}_{+\mu}, V'_{\alpha\nu}\}\rangle$
C_{TVV}	$\langle T^{\mu\nu}\{V^{\alpha}_{\mu}, V_{\alpha\nu}\}\rangle$
$C_{TV'V'}$	$\langle T^{\mu\nu}\{V'^{\alpha}_{\mu}, V'_{\alpha\nu}\}\rangle$
$C_{TVV'}$	$\langle T^{\mu\nu}\{V^{\alpha}_{\mu}, V'_{\alpha\nu}\}\rangle$

Leading Order in $1/N_C$ and in the chiral expansion. V+V$'$



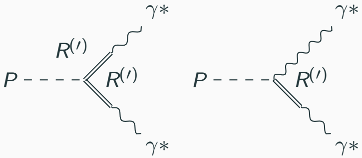
This set of operators generates $\mathcal{F}_1^T$.

Tensor Meson Poles: A Study within RχT

A computation within RχT could be done. Previous study obtained the optimal parameters for describing the radiative decay widths ($T \to \gamma\gamma$)(Chen et al (2023)). Adding resonances to this procedure would allow us to reproduce SDCs!

Coupling constant	Operator
$C_{T\gamma\gamma}$	$\langle T^{\mu\nu}\{f_{+\mu}^{\alpha}, f_{+\alpha\nu}\}\rangle$
$C_{T\gamma V}$	$i\langle T^{\mu\nu}\{f_{+\mu}^{\alpha}, V_{\alpha\nu}\}\rangle$
$C_{T\gamma V'}$	$i\langle T^{\mu\nu}\{f_{+\mu}^{\alpha}, V'_{\alpha\nu}\}\rangle$
C_{TVV}	$\langle T^{\mu\nu}\{V_{\mu}^{\alpha}, V_{\alpha\nu}\}\rangle$
$C_{TV'V'}$	$\langle T^{\mu\nu}\{V_{\mu}^{\prime\alpha}, V'_{\alpha\nu}\}\rangle$
$C_{TVV'}$	$\langle T^{\mu\nu}\{V_{\mu}^{\alpha}, V'_{\alpha\nu}\}\rangle$

Leading Order in $1/N_C$ and in the chiral expansion. V+V$'$



This set of operators generates $\mathcal{F}_1^T$.

If we minimally extend our Lagrangian ($\langle T^{\mu\nu}\{\nabla_\alpha R_{\mu}^{\alpha}, \nabla_\lambda R_{\nu}^{(\prime)\lambda}\}\rangle$), we get $\mathcal{F}_3^T$.

The following SDCs were imposed on $\mathcal{F}_1^T$ and $\mathcal{F}_3^T$ [1]:

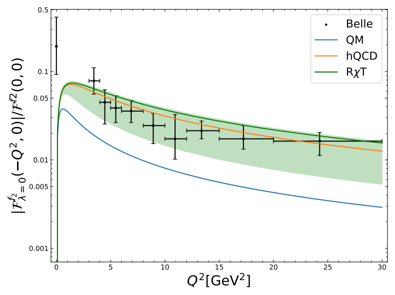
$$\lim_{Q^2\to\infty} \mathcal{F}_1^T(-Q^2,-Q^2) = -\frac{3F_T^{\rm eff} M_T^3}{14Q^4},$$

$$\lim_{Q^2\to\infty} \mathcal{F}_1^T(-Q^2,0) \sim \frac{1}{Q^4},$$

$$\mathcal{F}_3^T(-Q^2,-\lambda Q^2) \sim \frac{1}{Q^6}, \quad \lambda \in (0,1].$$

For $\mathcal{F}_3^T$, the DV SDC could be imposed in principle; however, it dominates the low-energy region because the model is too limited. It could be improved (chiral corrections, more resonances, but it requires DV and SV data).

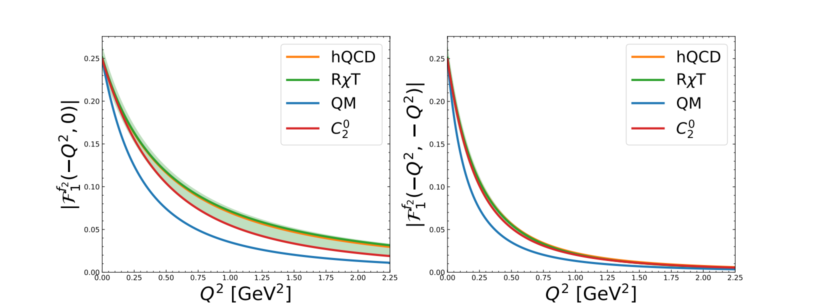
In this case, the $\Gamma(T \to \gamma\gamma)$ and the SDCs were fundamental as there is scarce data, which was successfully reproduced by our model (Only $\mathcal{F}_1^{f_2}$ SV is relevant for this data).



[1] The mixed asymptotic behavior could not be reproduced for F_1 and the single virtual one could not be reproduced for $\mathcal{F}_3^T$. The systematic errors were addressed using a rational approximants approach that fulfilled the SDCs.

$\mathcal{F}_1^T$ in RχT

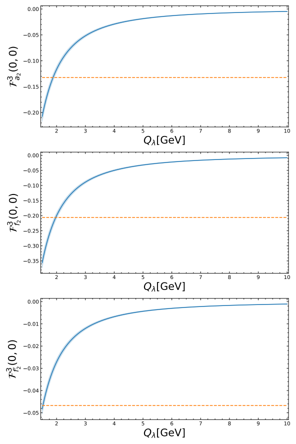
We show the plots for $\mathcal{F}_1^T$, as a representative example:



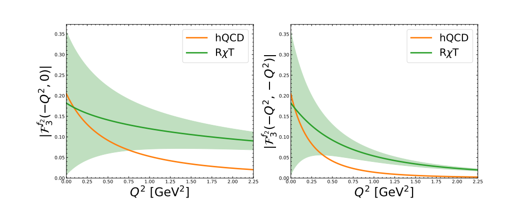
The $\mathcal{F}_3^T$ normalization requires $T \to \gamma^*\gamma^*$ data. In our attempt to solve this issue, we inferred the normalization (given our knowledge from $\mathcal{F}_1^T(0,0)$) by matching the ratios (Hoferichter, Stoffer (2020))

$$\frac{\mathcal{F}_1^{T-\,\mathrm{R}\chi\mathrm{T}}(-Q_\lambda^2,-Q_\lambda^2)}{\mathcal{F}_1^{T-\,\mathrm{LCE}}(-Q_\lambda^2,-Q_\lambda^2)} = \frac{\mathcal{F}_3^{T-\,\mathrm{R}\chi\mathrm{T}}(-Q_\lambda^2,-Q_\lambda^2)}{\mathcal{F}_3^{T-\,\mathrm{LCE}}(-Q_\lambda^2,-Q_\lambda^2)}$$

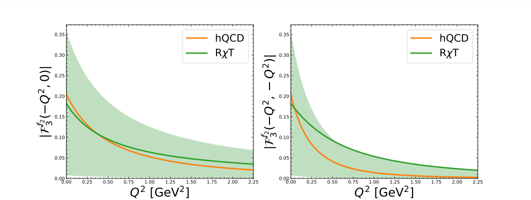
for different scales Q_λ, starting from $Q_0 = 1.5\mathrm{GeV}$.



In the first case, the high-energy behavior dominates the FF at low energies, being worse for smaller normalizations.



In the second case, the $\mathcal{F}_3^T$ is significantly lower, but without data, we cannot tell which one is more appropriate.



We compare our results to the ones from the different approaches:

$$a_\mu^{\mathrm{T-Poles:\,QM}} = -2.5(8), \qquad a_\mu^{\mathrm{T-Poles:\,hQCD}} = +3.2(4),$$

$$a_\mu^{\mathrm{T-Poles:\,R\chi T\,minimal}} = -4.5^{+0.5}_{-0.3}, \qquad a_\mu^{\mathrm{T-Poles:\,R\chi T\,extended:\,no\,SDC}} = +4(4),$$

$$a_\mu^{\mathrm{T-Poles:\,R\chi T\,extended}} = +10(5),$$

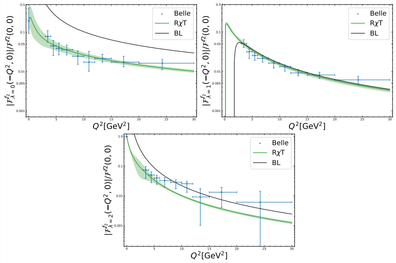
In spite of the absence of data for $\mathcal{F}_3^T(0,0)$, the presence of this Form Factor produces a positive sign for the T-pole contributions (as in hQCD) and also makes the data-driven determination of a_μ^{HLbL} compatible with the LQCD result within 1σ.

If we account for all possible operators with derivatives within RχT, we can generate all 5 transition form factors. Although not all SDCs are possible to fulfill with 2 V multiplets of resonances, if we use a rational approximant based on the low-energy constants of the RχT model, a toy fit can be done to the single virtual data:

$$\mathcal{F}^T_{\lambda=0} = \frac{Q^2}{\sqrt{6}M_T^2}\mathcal{F}_1^T(-Q^2,0) - \frac{(M_T^2+Q^2)^2}{2\sqrt{6}M_T^4}\mathcal{F}_2^T(-Q^2,0) + \frac{Q^2}{\sqrt{6}M_T^2}\mathcal{F}_5^T(-Q^2,0),$$

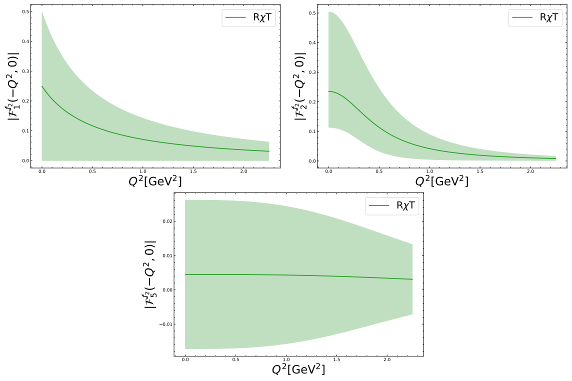
$$\mathcal{F}^T_{\lambda=1} = \frac{\sqrt{Q^2}}{\sqrt{2}M_T}\mathcal{F}_1^T(-Q^2,0) + \frac{\sqrt{Q^2}(M_T^2-Q^2)}{2\sqrt{2}M_T^3}\mathcal{F}_5^T(-Q^2,0),$$

$$\mathcal{F}^T_{\lambda=2} = -\mathcal{F}_1^T(-Q^2,0) + \frac{Q^2}{M_T^2}\mathcal{F}_5^T(-Q^2,0).$$



Non-minimal extension: Toy fit

We can infer information for the normalization of $\mathcal{F}_2^T$ and $\mathcal{F}_5^T$:



Conclusions

- Given the tension between the dispersive (with quark model form factor) and holographic determinations of $a_\mu^{T-poles}$, it is timely to explore them in other approaches. We used Resonance Chiral Theory.
- Taking into account the operators leading in $1/N_C$ and neglecting those with derivatives, we only get contributions to the $\mathcal{F}_1$ form factor, with results that agree closely with the holographic group computation when only this FF is kept, $\sim -4.5(5) \times 10^{-11}$ $(a_2 + f_2 + f_2')$.
- When we extend minimally our Lagrangian, with operators including two derivatives that only contribute –additionally– to $\mathcal{F}_3$, our result switches sign, to $+10(5) \times 10^{-11}$, which exceeds in magnitude the holographic result. This would bring the phenomenological a_μ^{HLbL} determination closer to the lattice results. We still have to complete consistently the Lagrangian to draw firm conclusions.
- Preliminary fits were shown, aiming to improve our understanding of these form factors. For $\mathcal{F}_2$ the results seem particularly promising.

Backup

Expresions for the TFFs

FF1

$$\mathcal{F}_1^T(q_1^2,q_2^2) = c_T \frac{9\,F_T^{\rm eff} M_T^3(M_V^4 - q_1^2 q_2^2) - 14\sqrt{2} C_{TVV} F_V^2 M_T (M_V^2 - M_{V'}^2)^2}{42\,D_{M_V}(q_1^2) D_{M_V}(q_2^2) D_{M_{V'}}(q_1^2) D_{M_{V'}}(q_2^2)},$$

FF3 with DV SDC at $1/Q^6$ unmatched

$$\mathcal{F}_3^T(q_1^2,q_2^2) = \left(\mathcal{F}_3^T(0,0)\frac{M_V^4 M_{V'}^4}{M_V^2 + M_{V'}^2}\right) \frac{M_V^2 + M_{V'}^2 - q_1^2 - q_2^2}{D_{M_V}(q_1^2) D_{M_V}(q_2^2) D_{M_{V'}}(q_1^2) D_{M_{V'}}(q_2^2)},$$

FF3 with DV SDC at $1/Q^6$ matched

$$\mathcal{F}_3^T(q_1^2,q_2^2) = \frac{\frac{4}{21} F_T^{\rm eff} M_T^5 \left(q_1^2 + q_2^2\right) + \mathcal{F}_3^T(0,0) M_V^4 M_{V'}^4}{D_{M_V}(q_1^2) D_{M_V}(q_2^2) D_{M_{V'}}(q_1^2) D_{M_{V'}}(q_2^2)},$$