Discussion session: data-driven / analytic HLbL

Peter Stoffer

University of Zurich and Paul Scherrer Institut

September 12, 2025

8th Plenary workshop of the Muon $g - 2$ Theory Initiative 2025 – IJCLab, Orsay

HLbL: current situation and open issues
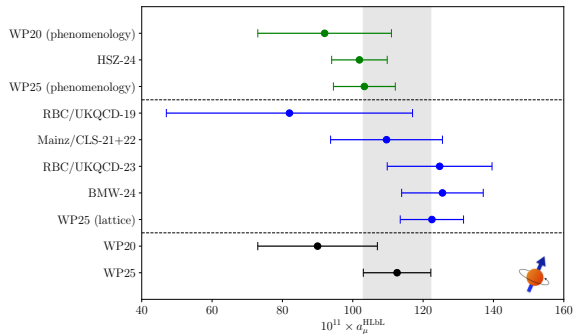
- HLbL pheno and lattice **agree at the 1.5σ level**

- lattice systematically higher

- hQCD suggests explanation: large **tensors** and excited states?

  → talk by Jonas Mager

- SDCs understood and under good control (both pert./non-pert. corrections) → talk by Hans Bijnens

→ Image: WP25

2

HLbL: dispersive approach in four-point kinematics

- complete evaluation available
- **tensor-meson** contributions based on **quark model** and only $\mathcal{F}_1^T$ TFF, $\mathcal{F}_{2,3,4,5}^T = 0$
- low-energy contributions of matching to SDCs: effective P and A poles in triangle kinematics
- tensor contribution with 100% uncertainty
- uncertainties conservative enough?

HLbL: dispersive approach in triangle kinematics

→ talks by Emilis Kaziukėnas, Jan-Niklas Toelstede

- offers solution to roadblock for tensor/spin-2 contributions: **no kinematic singularities**

- more complicated unitarity relations and sub-processes

- dispersive reconstruction in progress (f_2 in NWA and $\pi\pi$ rescattering)

- reshuffling and different matching to SDCs $\Rightarrow$ get handle on uncertainties

→ Lüdtke, Procura, Stoffer (2023)

4

HLbL: discussion items 1/2

- input for axial-vector meson TFFs? → talk by Hannah Schäfer
- input for tensor-meson TFFs?
  - diverse dispersive approaches → talks by Maximilian Zillinger, Emilis Kaziukėnas, Hannah Schäfer
  - model approaches; limitations of $R\chi T$ → talks by Jonas Mager, Emilio Estrada
  - how to validate results for TFFs? (BL, singly-virtual data, $\gamma\gamma \to \pi\pi$)
  - hierarchy $\mathcal{F}_1^T$, $\mathcal{F}_3^T$ vs rest?
- how to get tensor contribution to a_μ?
  - narrow resonances vs rescattering
  - imaginary parts of TFFs for heavy resonances
  - 4pt kinematics: sum-rule ambiguities
  - 3pt kinematics: missing sub-processes
  - reshuffling, assessment of matching uncertainties

HLbL: discussion items 2/2

- comparison of lattice and data-driven evaluations
  - lattice definition of pion pole $\leftrightarrow$ dispersive pole in 4pt kinematics: obvious?
  - HLbL window quantities?
  - position-space 12-term master formula

Reshuffling and matching uncertainties: lessons from VVA?

→ Lüdtke, Procura, Stoffer (2025)