



# Experiment update - MUonE -

Riccardo Nunzio Pilato
University of Liverpool
on behalf of the MUonE Collaboration



rpilato@liverpool.ac.uk

8<sup>th</sup> Plenary Workshop of the Muon g-2 Theory Initiative IJCLab, 12<sup>th</sup> September 2025

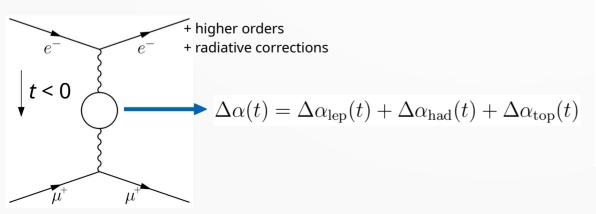


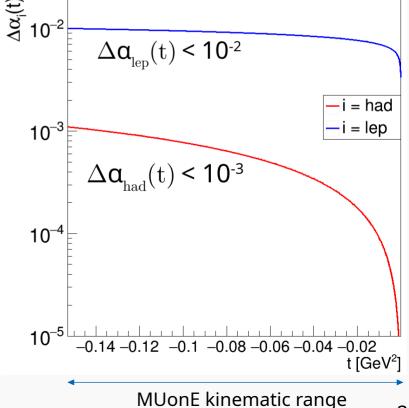
New independent evaluation of  $a_{\mu}^{\text{HVP,LO}}$ , based on the measurement of  $\Delta a_{\text{had}}(t)$  in the space-like region

Phys. Lett. B 746 (2015), 325 Eur. Phys. J. C 77.3 (2017), 139 Letter of Intent CERN-SPSC-2019-026 Proposal for Phase 1 of the MUonE experiment

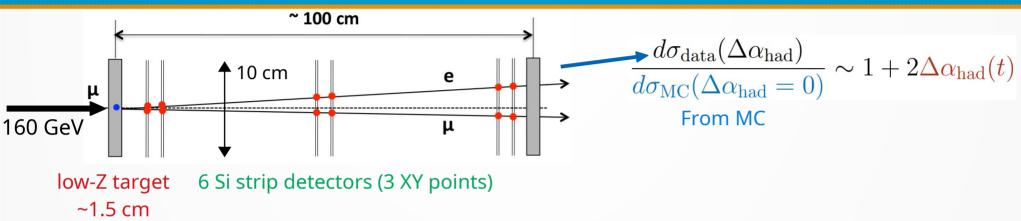
$$a_{\mu}^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 \! dx (1-x) \Delta \alpha_{had}[t(x)]^{-t(x)} = \frac{x^2 m_{\mu}^2}{x-1} < 0 \qquad \stackrel{\text{\tiny $\frac{2}{5}$}}{\text{\tiny $\frac{2}{5}$}} \mathbf{10}^{-2}$$

Extract  $\Delta \alpha_{had}(t)$  from the *shape* of  $\mu e \rightarrow \mu e$  differential cross section

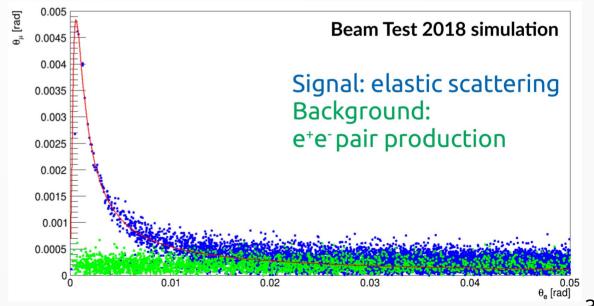




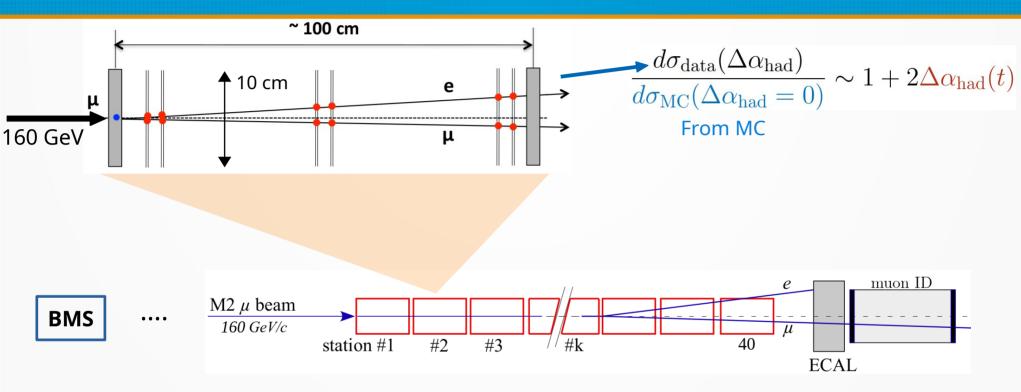




- Observables:  $(\theta_e, \theta_u)$
- Exploit (θ<sub>e</sub>, θ<sub>μ</sub>) correlation
  to reject background
  (main source: μ N → μ N e<sup>+</sup>e<sup>-</sup>)







Modular layout:
 each station measures
 the incident muon direction
 for the following one

- ECAL: PID + e energy
- Muon ID: PID
- BMS: beam momentum spectrometer



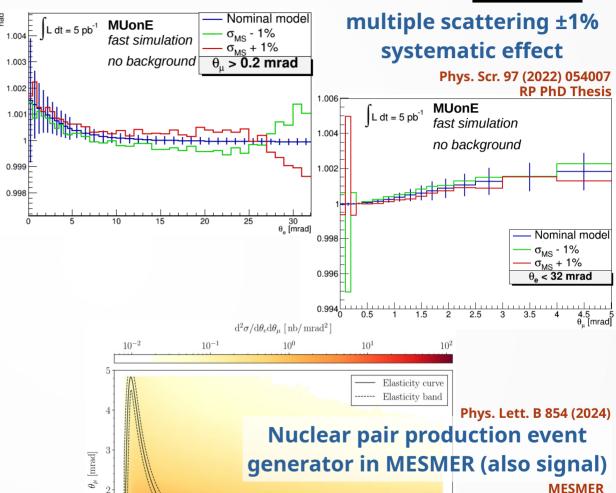
McMule

#### MUonE final goal:

- ~3 years post LS3 (>2030)
- 40 stations
- $a_{\mu}^{\text{HVP,LO}} < 0.5\%$

#### Systematic error goal: 10 ppm

- 10 µm longitudinal alignment
- Beam energy measured to few MeV
- Multiple scattering 1%
- Angular intrinsic resolution
- Uniform detector response over full angular range
- Need of dedicated MC generators: signal (>NNLO), main backgrounds



 $\theta_e$  [mrad]

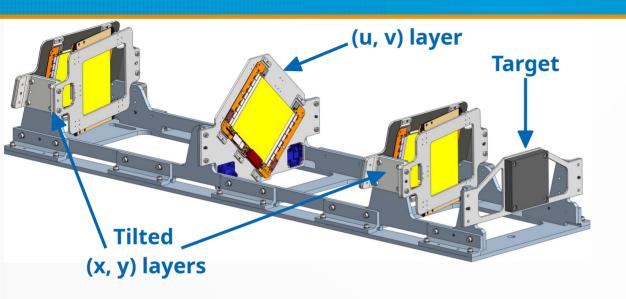
# Staged approach towards the full experiment



- 2017: test beam, multiple scattering studies JINST 15 (2020) P01017
- 2018: test beam, elastic scattering properties and event selection studies
- 2021: first joint test CMS-MUonE with 4 2S modules prototypes (parasitic)
- 2022:
  - test 1 tracking station
  - test the calorimeter
- **2023**: test with 2 tracking stations + calorimeter
- 2024: 2 tracking stations (DAQ tests) + calorimeter (characterization)
- **2025**: run with a scaled version of the complete apparatus

## **Tracking system**





INVAR (Fe/Ni alloy)

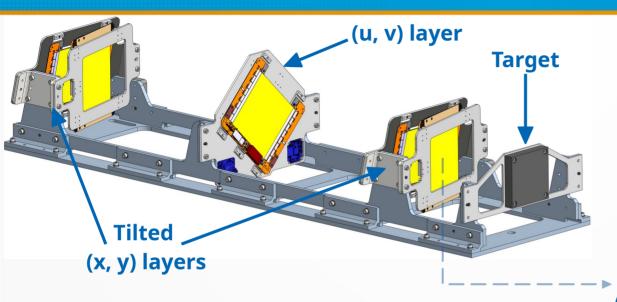
CTE ~ 1.2 ppm/K

Laser holographic system to monitor stability

- (x, y) layers: tilted by 233 mrad → ~2× hit resolution improvement
- (u, v) layers: solve reconstruction ambiguities

## **Tracking system**





INVAR (Fe/Ni alloy) CTE ~ 1.2 ppm/K

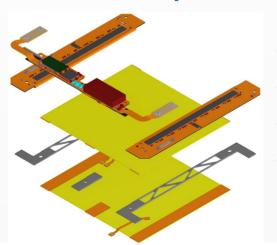
Laser holographic system to monitor stability

## 2S modules (CMS Phase2 upgrade)

**TDR CMS Tracker Phase2 Upgrade** 

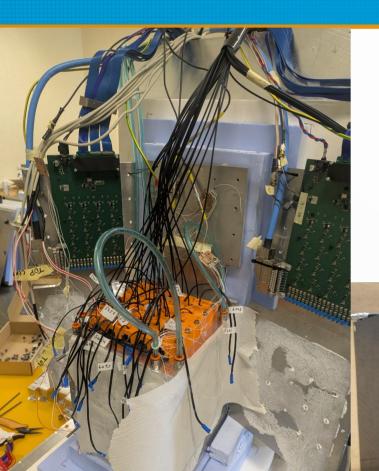
- ~90 cm<sup>2</sup> active area
- 2 × 320 µm thickness
- 40 MHz, binary readout
- 90 μm pitch
   (~26 μm hit resolution)

- (x, y) layers: tilted by 233 mrad → ~2× hit resolution improvement
- (u, v) layers: solve reconstruction ambiguities



## Calorimeter





• 5x5 PbWO<sub>4</sub> crystals, used in the CMS ECAL:

• area: 2.85×2.85 cm<sup>2</sup>

• length: 23 cm (~25 X<sub>0</sub>)

Total ECAL area: ~14×14 cm<sup>2</sup>

Readout: 10x10 mm<sup>2</sup> APD

- End of TR 2023: ECAL data integrated in the main DAQ
- TR 2025: tracker-ECAL time sync achieved

## **DAQ** system



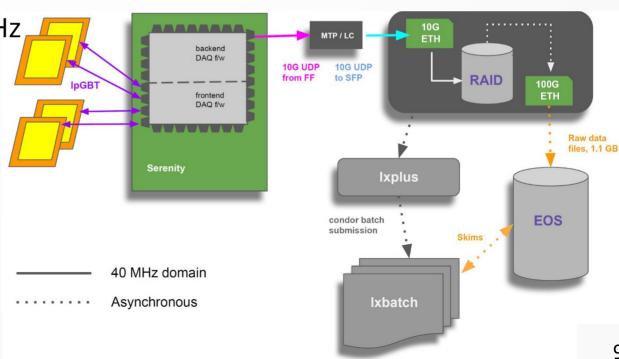
## M2 beam line at CERN:

unique environment

- High intensity:  $\sim 2 \times 10^8 \,\mu^+ / 5s$  spill  $\rightarrow \sim 40$  MHz
- Beam asynchronous to DAQ clock
- Serenity board (developed for CMS Phase2 Upgrade)

Triggerless readout @ 40 MHz

- Event aggregator on FPGA
- Data aggregation on 4 PCs
- Transmission to EOS into 1GB files

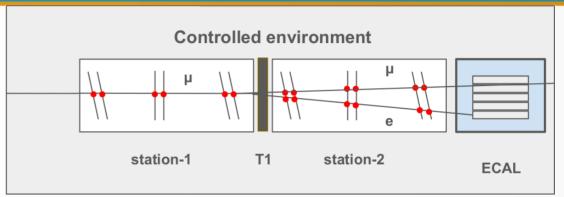


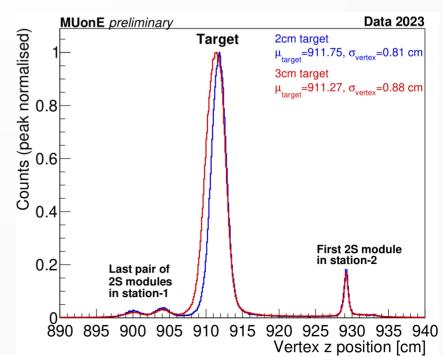
#### Test Run 2023



- 2 tracking stations
- C target
   (2 or 3 cm thickness used)
- ECAL

- Demonstrated continuous readout @40 MHz.
- Study detector performance, reconstruction algorithms, event selection.

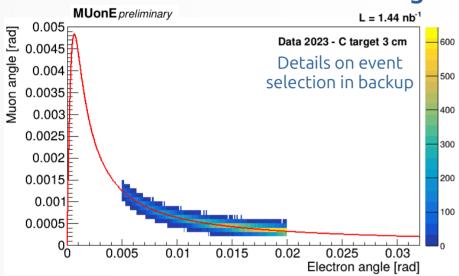




## Test Run 2023 - Data/MC comparison



#### Select elastic events in a clear region

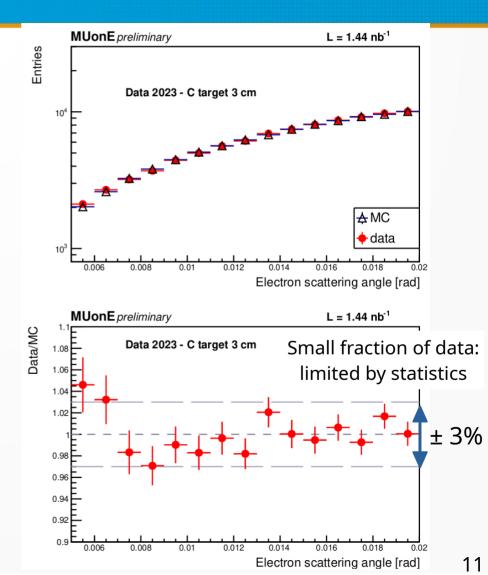


Count  $N_{\mu}$  on target  $\rightarrow$  <u>luminosity estimate</u>

Data/MC comparison of the cross section within event selection:

$$\sigma_{\text{data}} = (75.1 \pm 3.1) \, \mu \text{b}$$

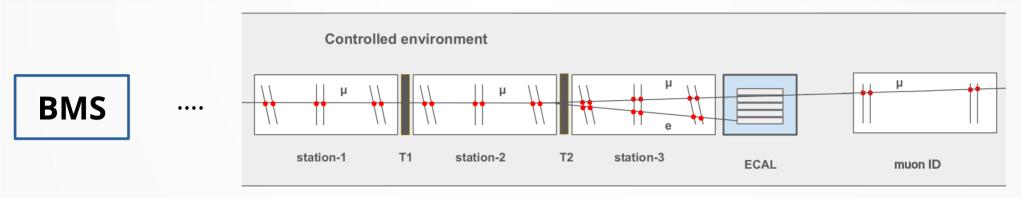
$$\sigma_{\rm MC} = (77.75 \pm 0.14) \,\mu b$$



#### **MUone Phase-1: Test Run 2025**



**Proposal for Phase 1 of the MUonE experiment** 

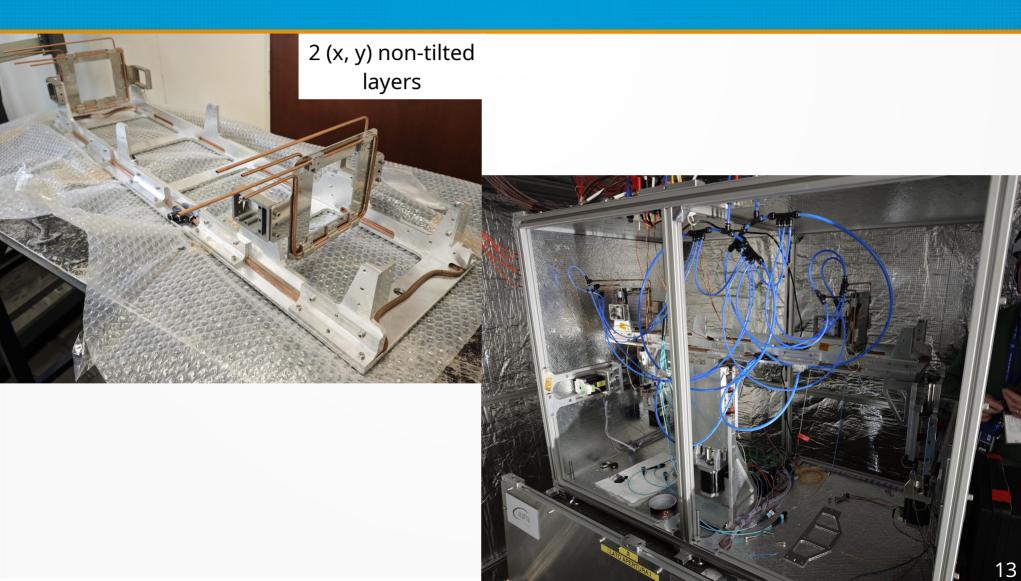


- 3 × tracking stations, each equipped with 6 pre-production 2S modules
- 2 × C targets (each 2 cm thick)
- ECAL: e- PID + E<sub>e</sub> measurement

- Timing detector: time of arrival of muons.
   2 plastic scintillators before and after the tracking stations
- Muon ID:  $\mu$  PID. Equipped with 4 prototype 2S modules
- BMS: measure pµ event by event.
   2 × tracking stations, each equipped with
   4 prototype 2S modules

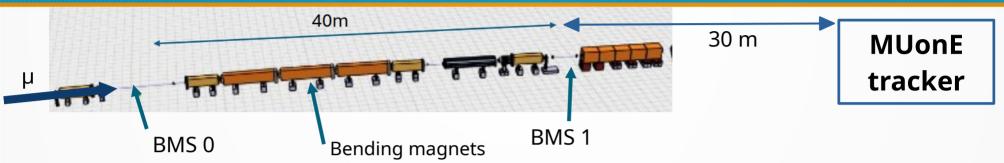
## **Muon ID**





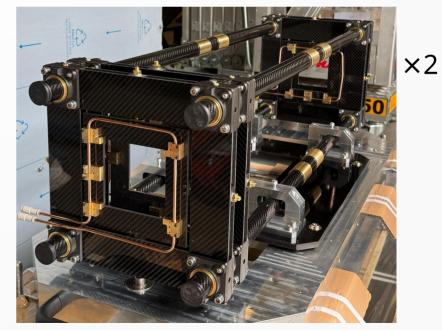
## **BMS (Beam Momentum Spectrometer)**





- Bending power: 16 T\*m
   (30 mrad @160 GeV)
- Proof of concept in 2025.
   Challenges:
  - Time synchronisation with the rest of the system
  - Alignment
  - B-field monitoring

#### **New Carbon Fibre structure**

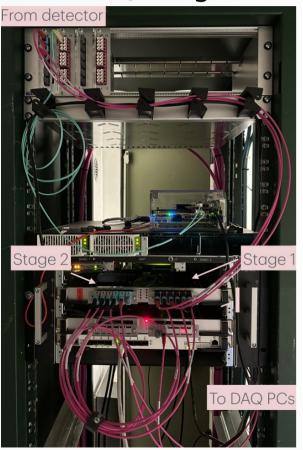


1m long, 2 (x, y) non-tilted layers

## System increasing in complexity...



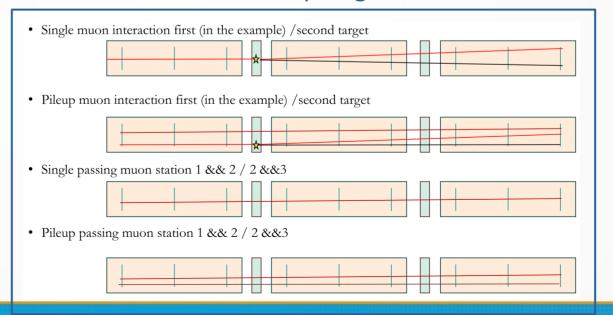
Move to a 2 stages DAQ design



- **Stage 1:** 36 communication links with subdetectors
  - 30×2S modules
- 2×Timing Detector

- 4×ECAL
- Online selection based on tracker modules occupancy.
   ~×100 reduction of recorded events compared to 2023.

#### Event topologies



## System increasing in complexity...



Move to a 2 stages DAQ design



- **Stage 1:** 36 communication links with subdetectors
  - 30×2S modules
    - 2×Timing Detector

- 4×ECAL
- Online selection based on tracker modules occupancy.
   ~×100 reduction of recorded events compared to 2023.

- Stage 2: event building.
  - Group information from all subdetectors in a time-coherent packet of data.
  - Online decoding of data provides ready-to-use ntuples for DQM and prompt analysis.



2S modules characterisation in lab

April

May

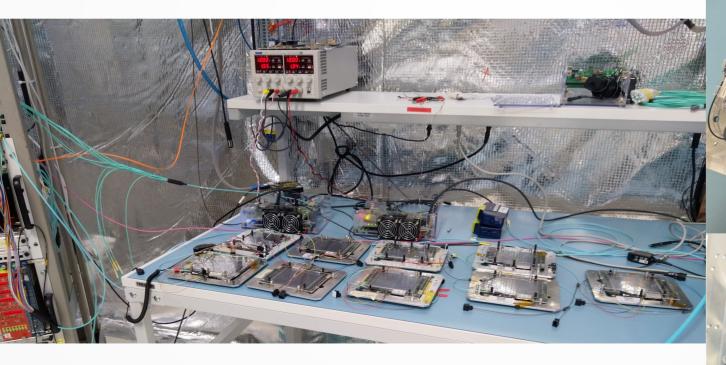
June

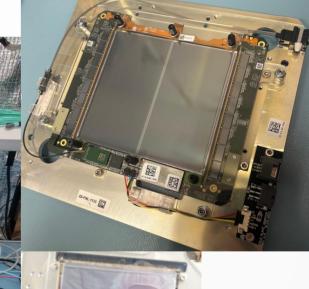
July

August

16

Noise tests + modules grading





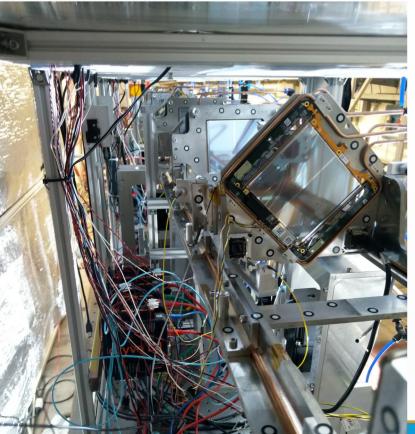


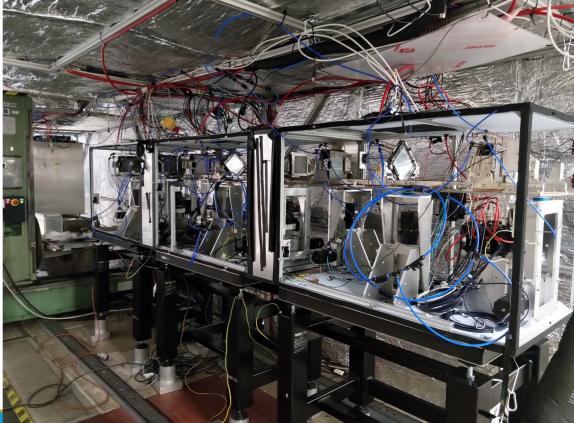
March April May June July August

2S modules characterisation

Tracker, ECAL, MuonID installation Inc.

Including infrastructure and services





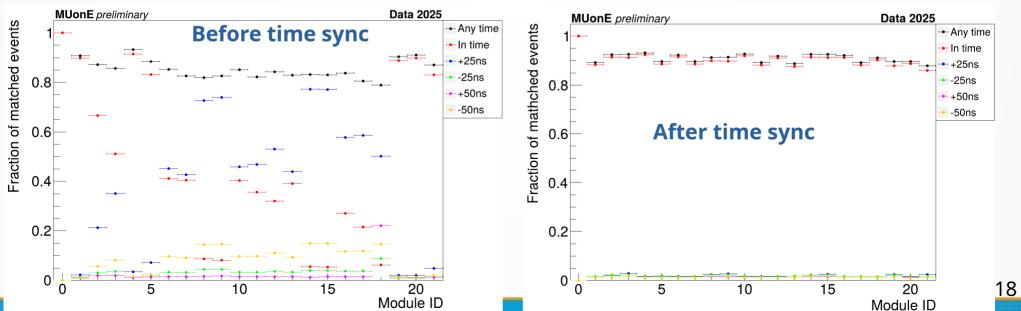




Numerous tests accomplished despite technical challenges (...and SPS inefficiencies)

#### **Tracker time synchronisation**

Fix the internal clock of a reference module, then scan the possible delays of the other modules to maximise their coincidences with reference



## DAQ and detector commissioning

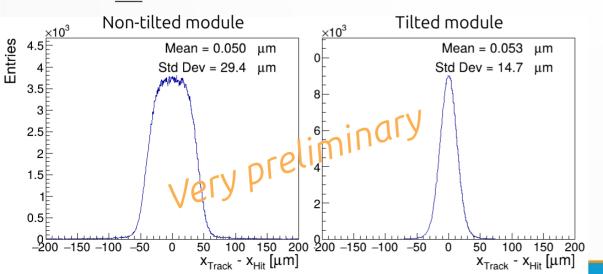


#### **Alignment**

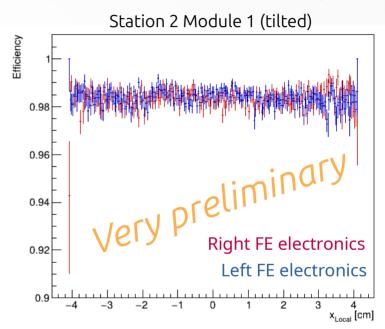
- Hardware (stepper motors): center the beam profile on each module, then align the 3 stations one relative to the other.
- Software: local  $\chi^2$  minimization on a sample of single passing muons.

#### Examples of residuals

\*Std Dev is not the hit resolution: track fit error to be subtracted



#### **Module efficiency**



Uniform efficiency over the entire modules surface

Work in progress: efficiency time uniformity over the entire data taking

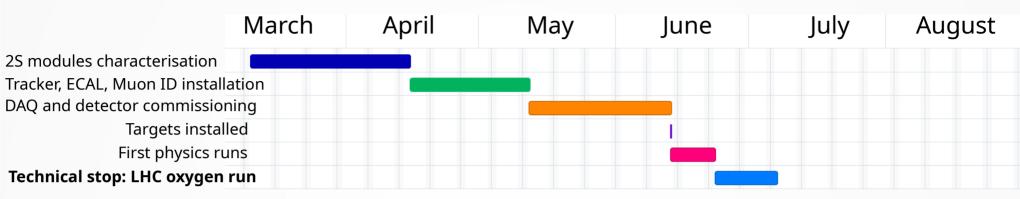


	March	April	May	June	July	August
2S modules characterisation						
Tracker, ECAL, Muon ID installat	tion					
DAQ and detector commissioni	ng					
Targets install	ed					
First physics ru	ns					





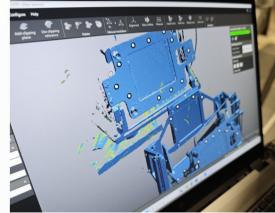




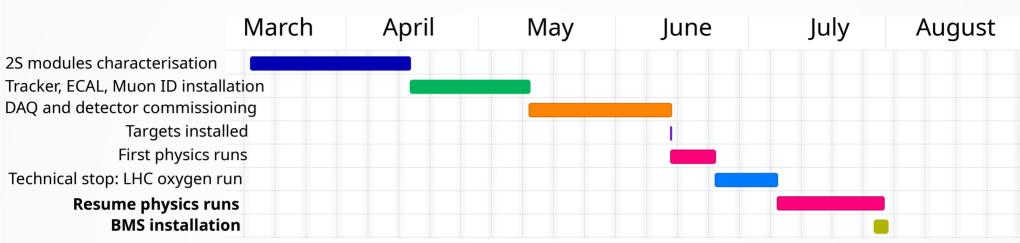
#### Metrology measurements of the detector (100 µm precision)

- 3D scanner photogrammetry: 
   position and orientation
   of each module within a station
- <u>Laser survey</u>:
   relative position of the different
   subdetectors; absolute position
   with respect to beam elements
- To be used as starting point of software alignment



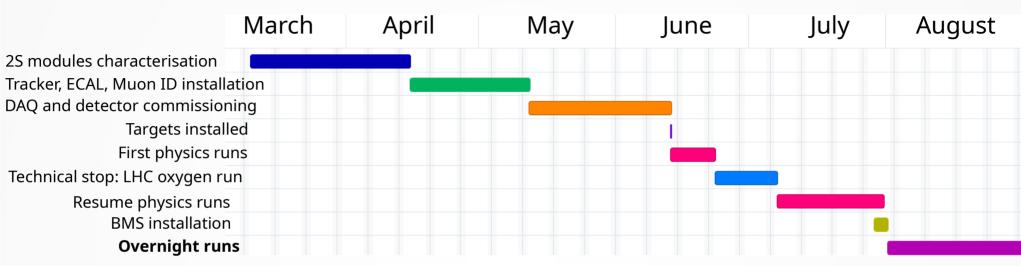








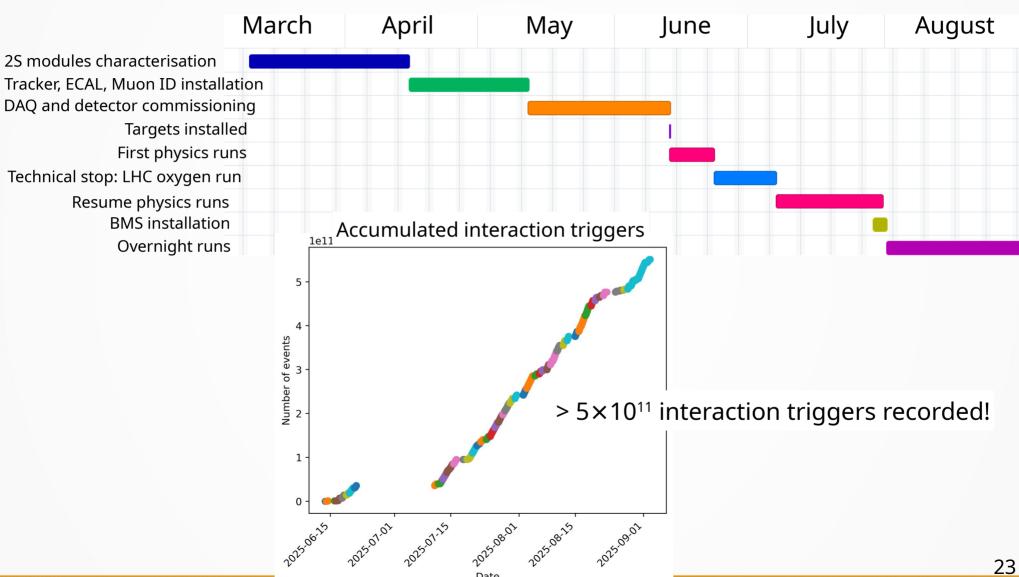




AMBER takes over as main user.

They kindly agreed that MUonE could continue
to take data during nights, while they exploit daytime
to install new hardware

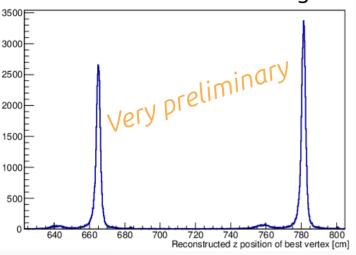




## **Elastic scattering events**



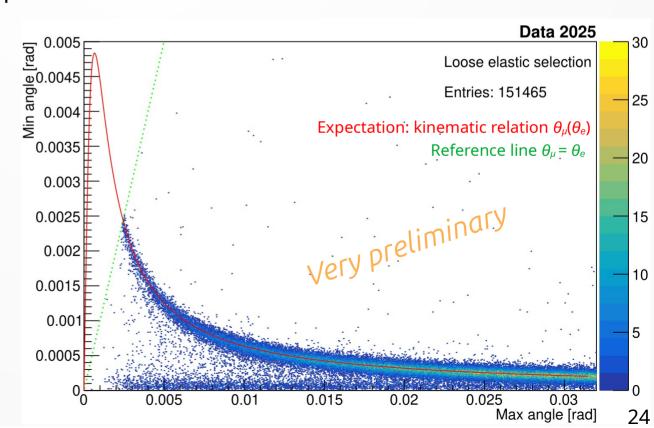




## Can use ECAL or MuonID to resolve the ambiguity

#### **Tracker-only analysis of elastic events**

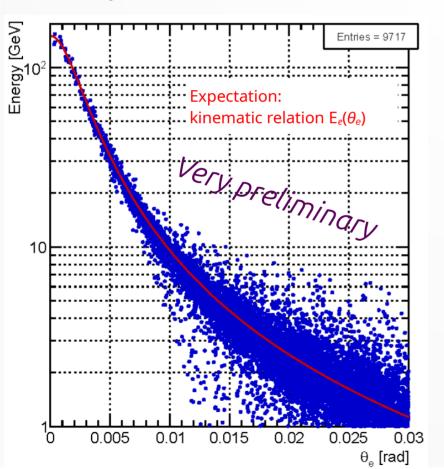
Not able to discriminate between  $\mu$  and e: plot  $\theta_{\text{max}}$  vs  $\theta_{\text{min}}$  to avoid misidentification when  $\theta_{\text{e}}$  < 5mrad

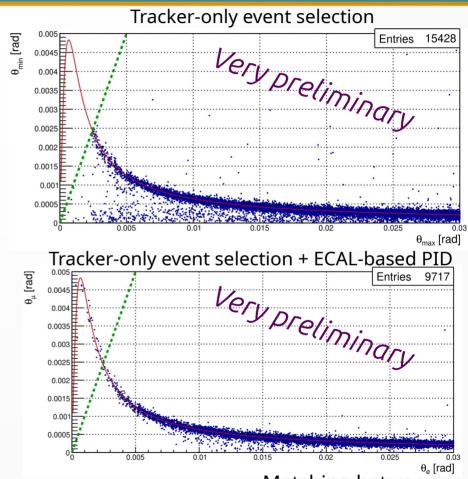


## **ECAL-based PID**



Correlation between ECAL energy deposit and  $\theta_e$  reconstructed in the tracker



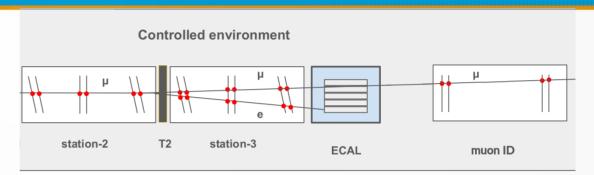


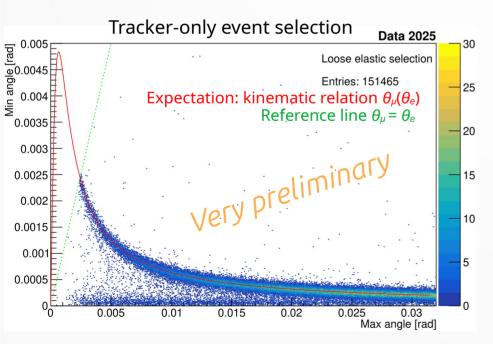
- ECAL energy > 1 GeV
  - Loose elastic selection
- Matching between candidate *e* track and ECAL cluster centroid

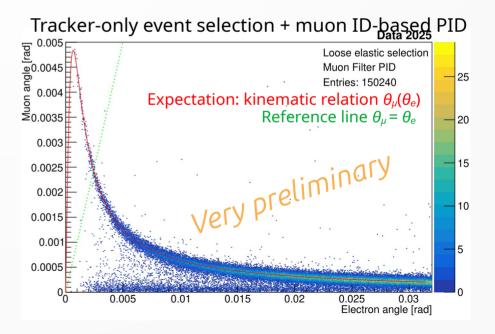
## **Muon ID-based PID**



- Propagate tracks to the Muon ID
- Look for matching between a track and muon ID hits: select the muon track





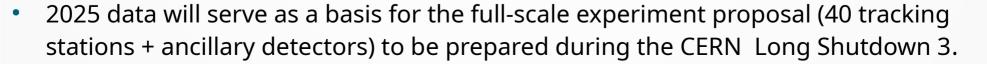


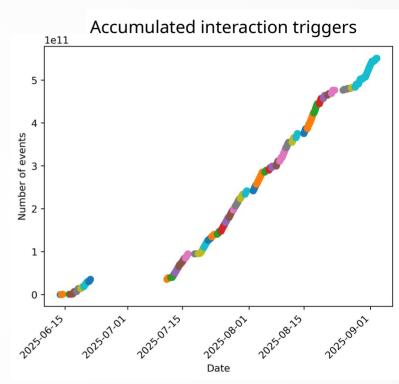
Work in progress: ECAL + muon ID combined PID

## **Conclusions**



- MUonE 2025 Test Run:
  - Successful ~3 months data taking with 3 stations (2 targets), ECAL and muon ID.
  - Integration of the BMS in the main DAQ in the last few days of run.
  - Opportunity to run further tests parasitically in September (no ECAL).
- Analysis campaign underway. Goals:
  - Proof of principle measurement of  $\Delta \alpha_{lep}(t)$  (and comparison with 2023 data).
  - Preliminary measurement of  $\Delta \alpha_{had}(t)$  (~20% statistical error + similar systematic).
  - Study systematic effects.





## **BACKUP**

#### Data-MC comparison of elastic events

Data sample: run  $6 \rightarrow 97 \times 10^6$  events after skimming to be reconstructed

 $\underline{\text{MC}}$  sample: MESMER generated <u>signal elastic events</u>  $\rightarrow 16.5 \times 10^6$  to be reconstructed with **realistic misalignment** scenario (simulated geometry from real metrology followed by track-based alignment as with real data)

#### Fiducial selection:

- $N_{hits_{S0}} = 6 \rightarrow 1 \text{ per module: } \underline{\text{golden muon}} \text{ (GM)};$
- GM impinges last 2 modules in S0 within  $\pm 1.5$  cm from centre in X and Y;
- Reconstructed GM with  $\theta < 4 mrad$ .

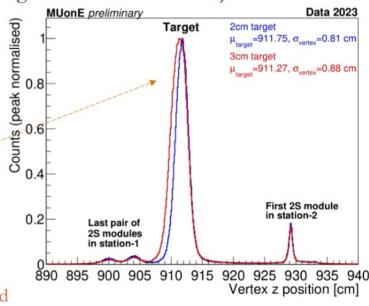
#### Elastic selection:

- $N_{hits_{s_1}} \le 15$ ;
- Reconstructed Z vertex > 906 *cm*;
- $\theta_{u} > 0.2 \, mrad$ ,  $5 < \theta_{e} < 20 \, mrad$ ;
- Acoplanarity  $|A_{\phi}| < 0.4 \ rad;$

- >0.2 mrad: main background removed
- >5 mrad: Avoid ambiguities in PID
- <20 mrad: region less affected by systematics

• Elasticity condition: 
$$|\theta_{\mu} - \theta_{\mu}^{exp}(\theta_{e})| < 0.2 \ mrad$$

$$\theta_{\mu}^{exp}(\theta_{e}) = \arcsin \left\{ \sin \theta_{e} \sqrt{\frac{E_{e}^{2}(\theta_{e}) - m_{e}^{2}}{[E_{\mu} + m_{e} - E_{e}(\theta_{e})]^{2} - m_{\mu}^{2}}} \right\}$$



## Absolute luminosity normalization

From the **knowledge of the number of golden muons** (passing the fiducial selection) that can potentially interact in the target, we can <u>estimate luminosity</u>:

#### Fiducial selection:

 $N_{hits_{S0}} = 6 \rightarrow 1 \text{ per module: } \underline{\text{golden muon}} \text{ (GM)};$ 

GM impinges last 2 modules in S0 within  $\pm 1.5$  cm from centre in X and Y;

Reconstructed GM with  $\theta$  < 4 mrad.

Luminosity real data:

$$L = N_{\mu \text{oT}} \cdot d_{target} \cdot \rho_{target}^{e} =$$

Run 6 =  $(1443.0 \pm 8.0) \mu b^{-1}$ 

Golden muons on target

Target thickness

Electron density target  $\rho_{target}^e = \rho \cdot \frac{Z}{A} \cdot N_A$ 

$$\sigma = \frac{N_{elastic}}{\epsilon_{hw}L}$$

$$\epsilon_{hw} = 0.850 \pm 0.035$$
:

2 tracks reconstruction efficiency which depends on modules efficiency  $(\epsilon_{mod} = 0.980 \pm 0.005)$  Main error on:

$$\rho = (1.83 \pm 0.01)g/cm^3$$

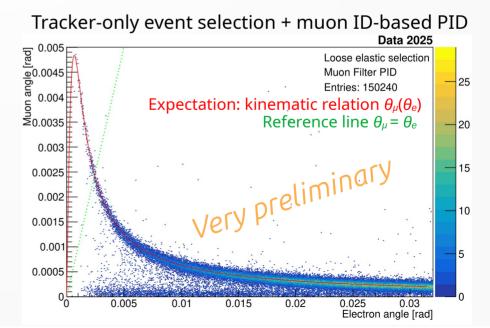
$$d_{target} = (3.000 \pm 0.001) cm$$

MC selection efficiency on  $\sigma_{MC}$  estimate: 76.5%

#### **Muon ID-based PID event selection**



- 1stub/module for each track
- 1 track in the station before target; 2 tracks in the station after target
- No stubs shared between different tracks
- $|z_{\text{vtx}} z_{\text{target}}| < 2 \text{ cm}$
- Reject acoplanar events (acoplanarity < 0.4 rad)</li>



## Summary of the main sources of systematic errors and corresponding uncertainties for the 2025 run



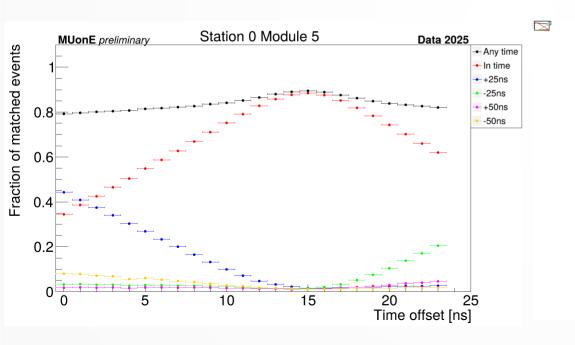
Source of systematic error	Uncertainty on the	Uncertainty	Uncertainty
	systematic source	on $\Delta lpha_{had}$	on $d\sigma$
Intrinsic angular resolution	0.5%	10%	200 ppm
Multiple scattering	1%	10%	200 ppm
Beam energy scale	25 MeV	10%	200 ppm
z coordinate	100 μm	10%	200 ppm
Beam energy spread (3.75%)			
current BMS	$4\% (\sigma_p/p \sim 1\%)$	1%	20 ppm
upgraded BMS	$1\% (\sigma_p/p < 0.5\%)$	< 0.5%	5ppm
Other contributions			
(i.e. tracking efficiency and		15%	300 ppm
reconstruction uniformity,			
residual background)			
Total		25%	500 ppm

## TR 2025 - tracker time synchronisation



Delay:

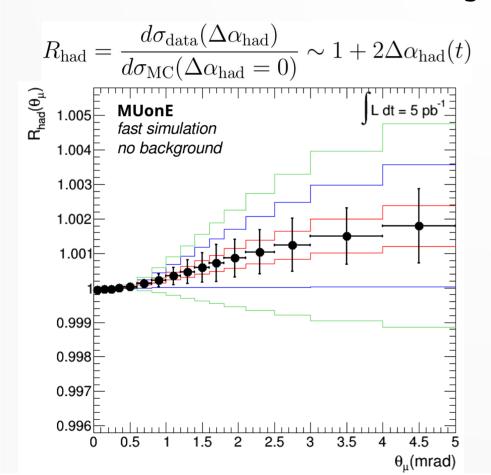
Delay scanned [0, 24] ns



### First measurement of $\Delta\alpha_{had}(t)$



Expected event yield: ~10° elastic events within acceptance (one order of magnitude larger than 2023)



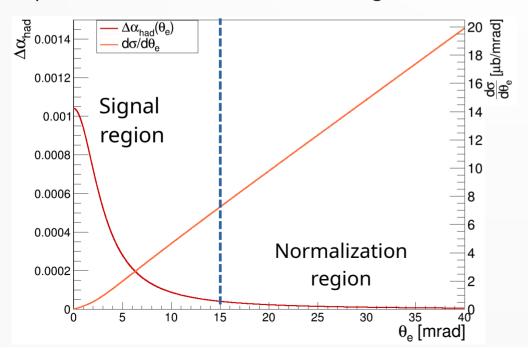
Template fit procedure to extract  $\Delta \alpha_{had}(t)$  $\Delta \alpha_{had}(t) \simeq -\frac{1}{15}Kt$ 50  $K = 0.136 \pm 0.026$ 40 (20% stat error) 30 20 10 **MUonE** fast simulation L dt =  $5 \text{ pb}^{-1}$ no background 0.2 0.1 0.3

### Strategy for the systematic effects

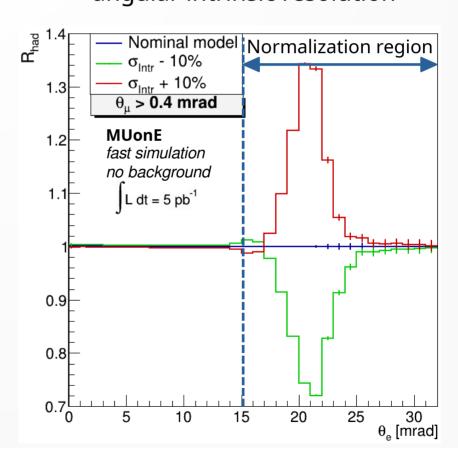


#### Promising strategy:

- Study the main systematics in the normalization region (large systematic effects but no sensitivity to  $\Delta\alpha_{\rm bad}$ ).
- Include residual systematics as nuisance parameters in a combined fit with signal.



#### Example: ±10% systematic error on the angular intrinsic resolution

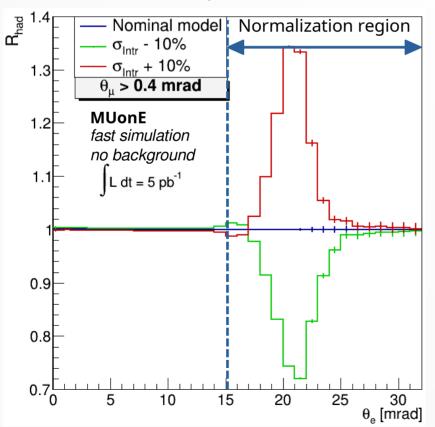


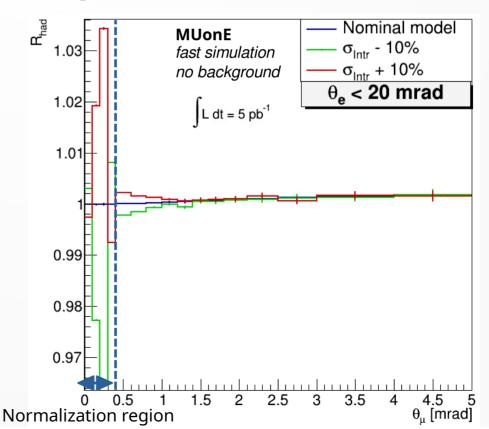
## The need of including systematic effects in the analysis



Some systematic effects can produce huge distortions in the shape of the elastic scattering cross section

Example: ±10% error on the angular intrinsic resolution



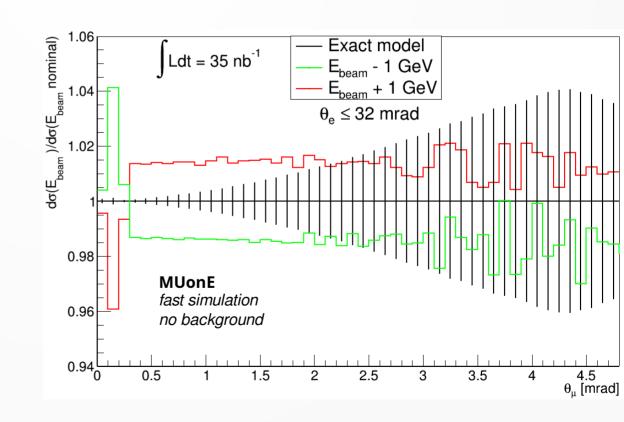


# Systematic error on the muon beam energy



Accelerator division provides E<sub>beam</sub> with O(1%) precision (~ 1 GeV)

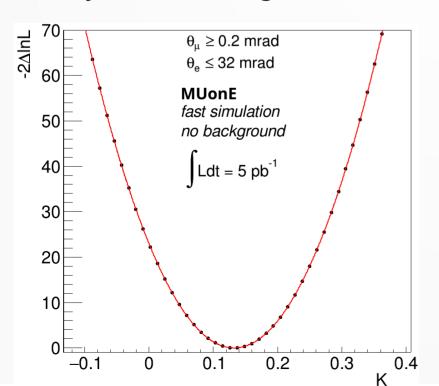
This effect can be seen from our data in 1h of data taking per station



## Combined fit signal + systematics



- Include residual systematics as nuisance parameters in the fit.
- Simultaneous likelihood fit to K and systematics using the Combine tool.



- $K_{ref} = 0.137$
- shift MS: +0.5%
- shift intr. res: +5%
- shift E<sub>beam</sub>: +6 MeV

Selection cuts	Fit results	
$\theta_e \le 32  \text{mrad}$ $\theta_\mu \ge 0.2  \text{mrad}$	$K = 0.133 \pm 0.028$	
	$\mu_{\rm MS} = (0.47 \pm 0.03)\%$	
	$\mu_{\text{Intr}} = (5.02 \pm 0.02)\%$	
	$\mu_{\rm E_{Beam}} = (6.5 \pm 0.5) \; {\rm MeV}$	
	$\nu = -0.001 \pm 0.003$	

Similar results also for different selection cuts

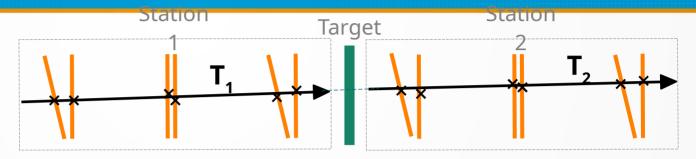
Input shifts identified correctly.

No degradation on the signal parameter

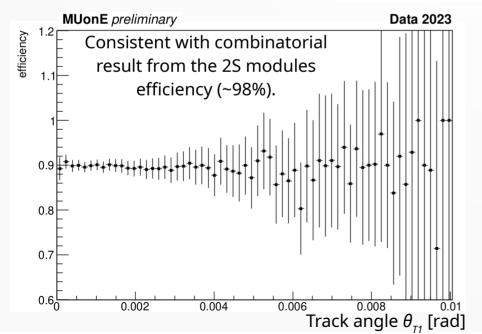


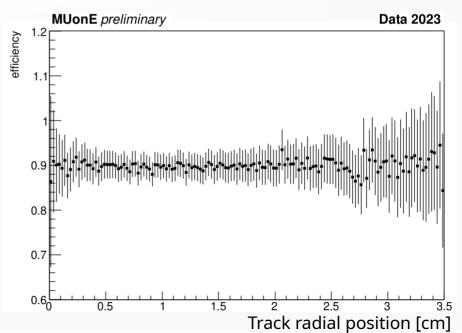
### TR 2023 – tracking efficiency

Select events with single passing muons.



Tracking efficiency = 
$$\frac{N(T_2 \cdot T_1)}{N(T_1)}$$

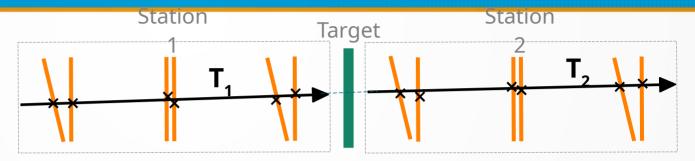


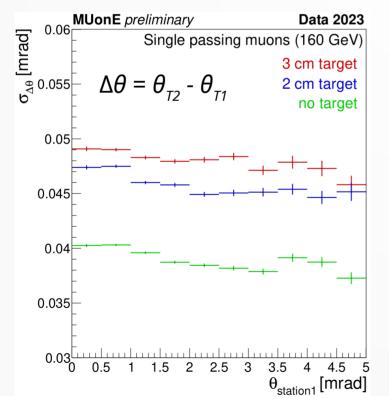


## TR 2023 – angular resolution and MS effects



Select events with single passing muons.



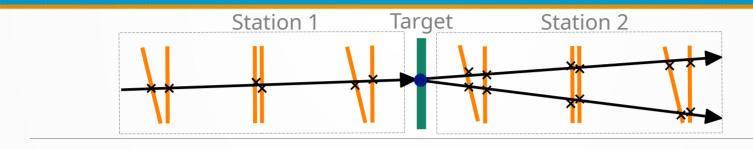


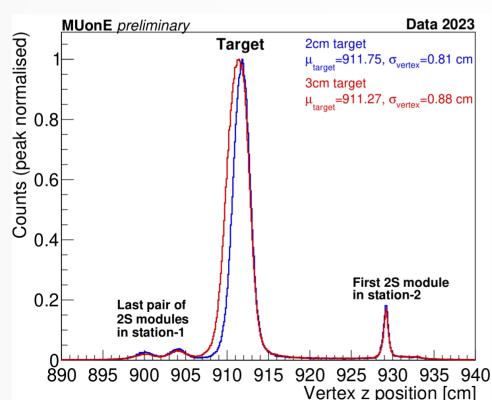
Target	$\sigma_{\Delta\theta}$ [ $\mu$ rad]	$\sigma_{MS}(\text{target}) [\mu \text{rad}]$	$\sigma_{MS}^{PDG}$ (target) [ $\mu$ rad]
3 cm C	$48.9 \pm 2.1$	$28.1 \pm 0.6$	28.2
2 cm C	$46.8 \pm 2.1$	$24.3 \pm 1.4$	22.6
No Target	$40.0 \pm 2.2$		

- Angular resolution of a station: ~28 µrad
- Target MS effects: good agreement with the expectations

### TR 2023 - vertexing







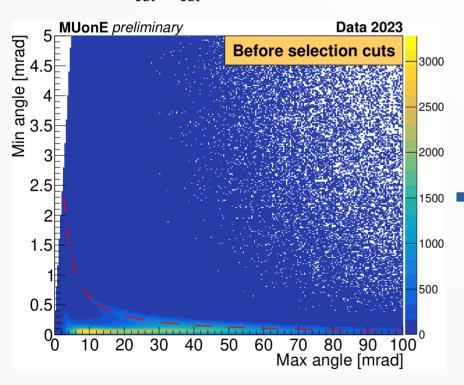
- Simple selection: events with 2 outgoing tracks within geometrical acceptance (0.2 – 32 mrad)
- The target center is shifted by 0.5 cm by changing between 3 cm and 2 cm target
- Interactions in the Si sensors are visible
- Vertex resolution: ~8 mm

## TR 2023 $\mu$ -e elastic scattering event selection



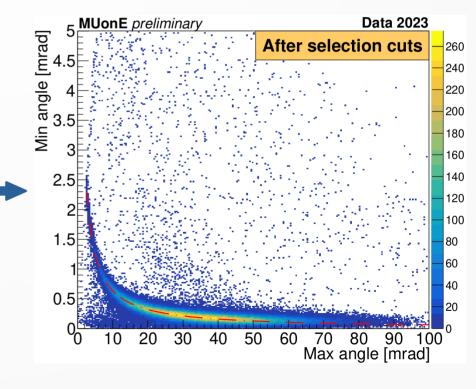
#### **Pre-selection**

- Single  $\mu_{in}$  candidate
- $\mu_{out}$ ,  $e_{out}$  pair candidate



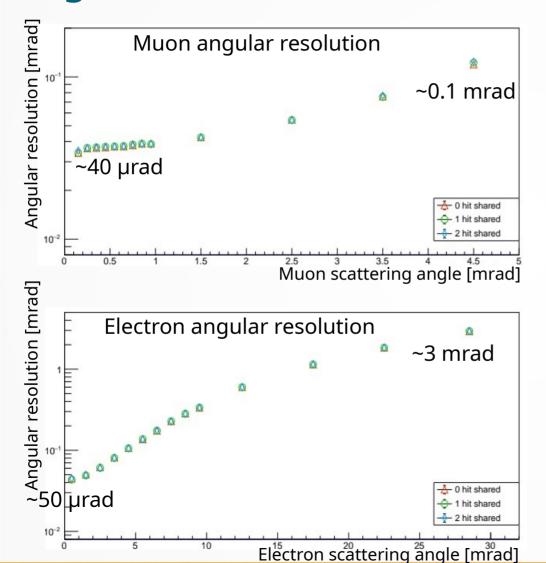
#### **Initial event selection**

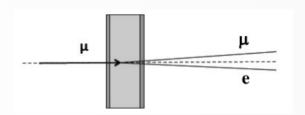
- ≥ 1 hit/module
- Cut on  $N_{hits}$ (station2)
- $|z_{vtx} z_{target}| < 3 \text{ cm}$
- Acoplanarity cut



## TR 2023 - MC performance: angular resolution of scattered particles



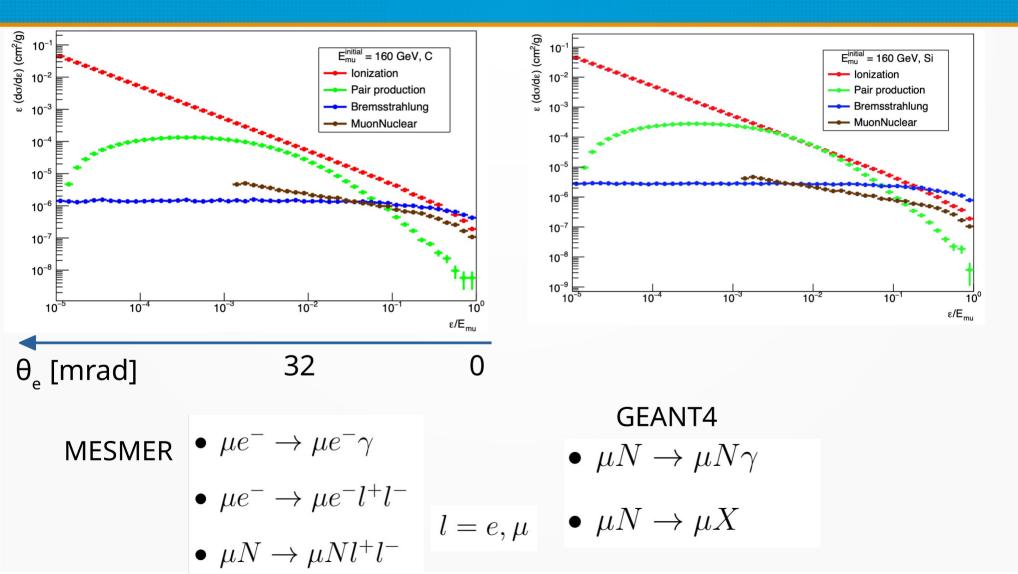




- Compare track reconstruction with MC truth.
- Muon angle: ~40 µrad resolution for small scattering angles.
- Electron angle: stronger impact of MS.
   Resolution is ~3 mrad for large scattering angles (E<sub>e</sub> ~1–2 GeV).

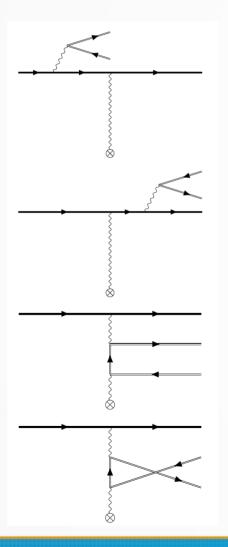
### Backgrounds



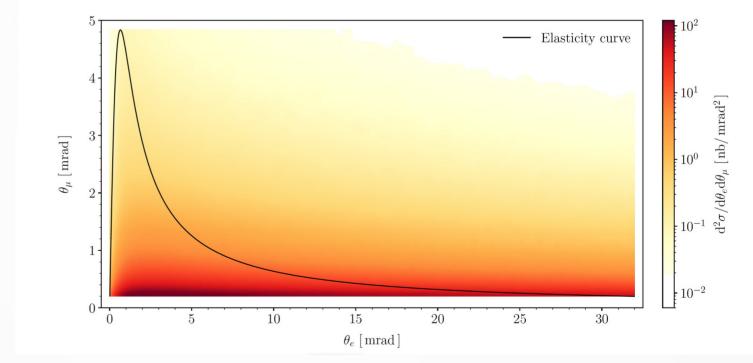


### **New Background MC generator**

Main background: e+e- pair production
Implemented in MESMER
and interfaced with the MUonE detector simulation

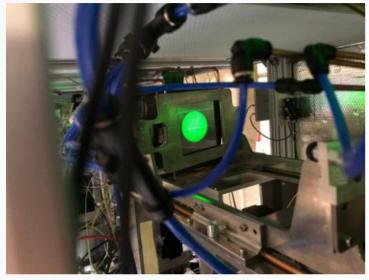


#### Numerical results for $\mu^+ C \rightarrow \mu^+ C e^+ e^-$ (3)

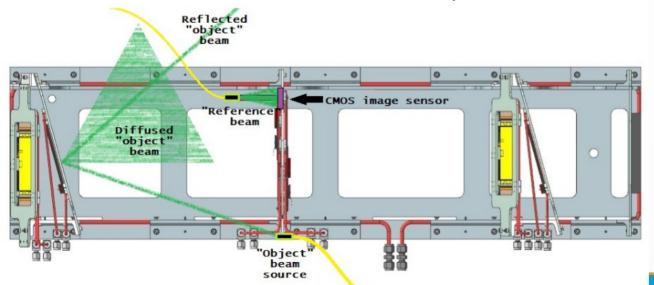


### Laser holographic system



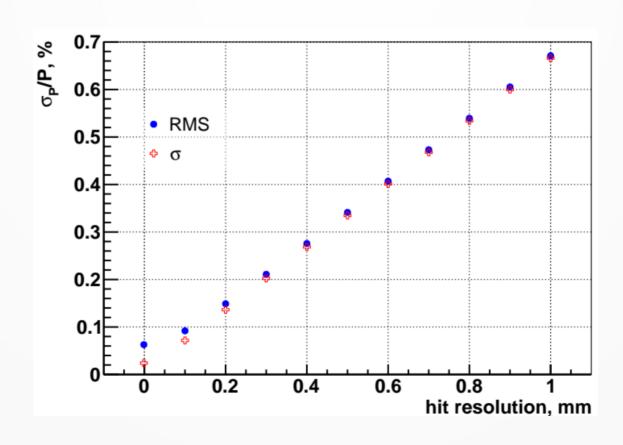


- Compare holographic images of the same object at different time
- Fringe pattern is related to deformations of the mechanical structure
- 532 nm fiber-coupled laser.
   Resolution: ~0.25 µm (half wavelength)
- Current limitation: Si sensors are sensitive to visible light → continuous monitoring is not possible
- Improvement: use >1500 nm laser (IR)



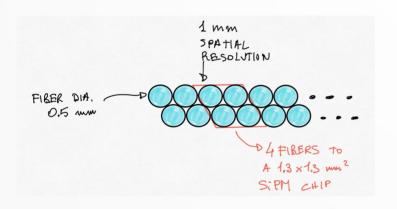
### **BMS (Beam Momentum Spectrometer)**





#### Muon ID - SciFi

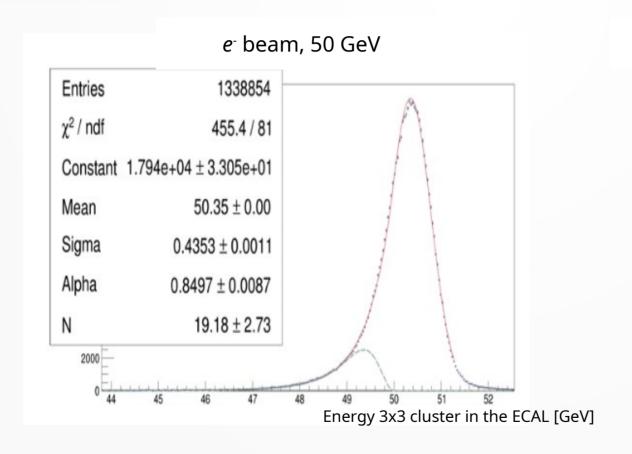




#### prototype

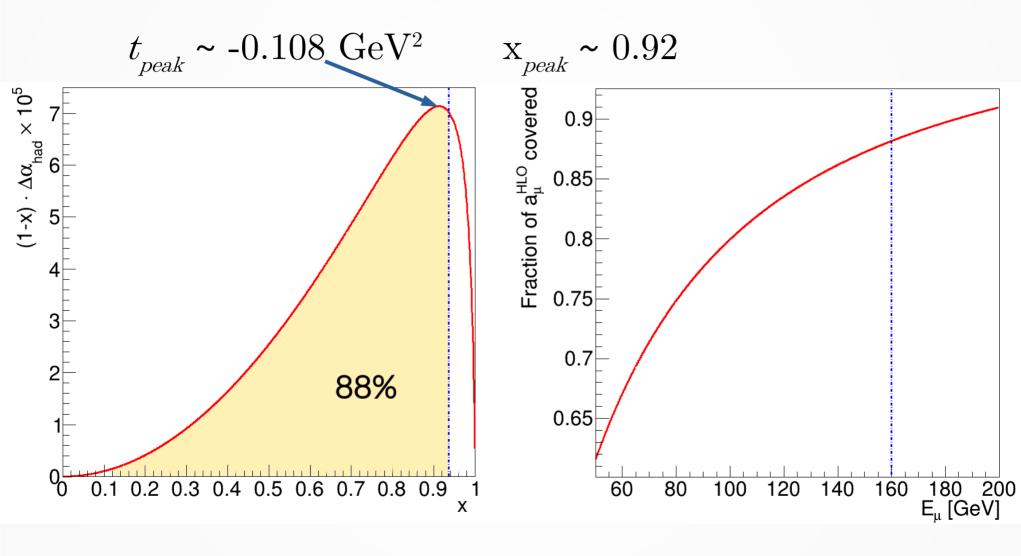


- Polistyrene round fibres. 4 fibres coupled to 1 SiPM.
- <0.5 ns timing resolution.</li>
- Pitch: 1.25mm. Expected resolution: ~360 μm
- Same technology could be used as timing detector between BMS and main tracker.



TB 2024, M2 beamline

x < 0.936

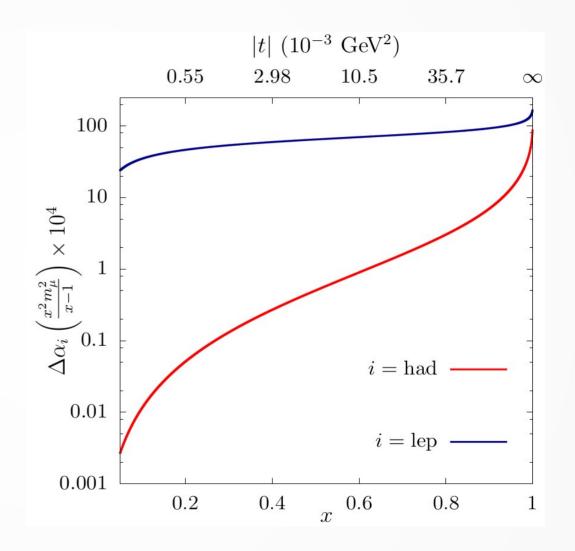


 160 GeV muon beam on atomic electrons.

$$\sqrt{s} \sim 420 \, \mathrm{MeV}$$

$$-0.153 \,\mathrm{GeV}^2 < t < 0 \,\mathrm{GeV}^2$$

$$\Delta \alpha_{had}(t) \lesssim 10^{-3}$$



### $\Delta\alpha_{had}$ parameterization



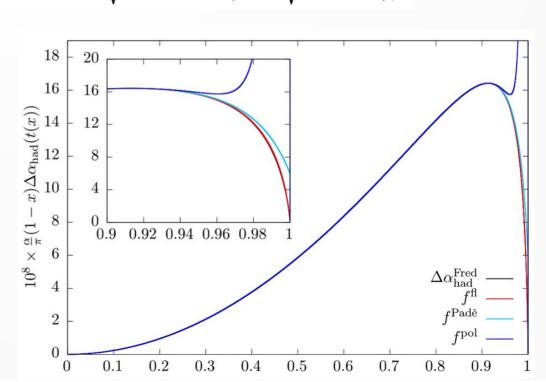
Inspired from the 1 loop QED contribution of lepton pairs and top quark at t < 0

$$\Delta\alpha_{had}(t) = KM \left\{ -\frac{5}{9} - \frac{4}{3}\frac{M}{t} + \left(\frac{4}{3}\frac{M^2}{t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1 - \frac{4M}{t}}} \ln \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\} \text{ 2 parameters: } \text{K, M}$$

Allows to calculate the full value of  $a_{\mu}^{\;\;\mathrm{HLO}}$ 

Dominant behaviour in the MUonE kinematic region:

$$\Delta \alpha_{had}(t) \simeq -\frac{1}{15}Kt$$

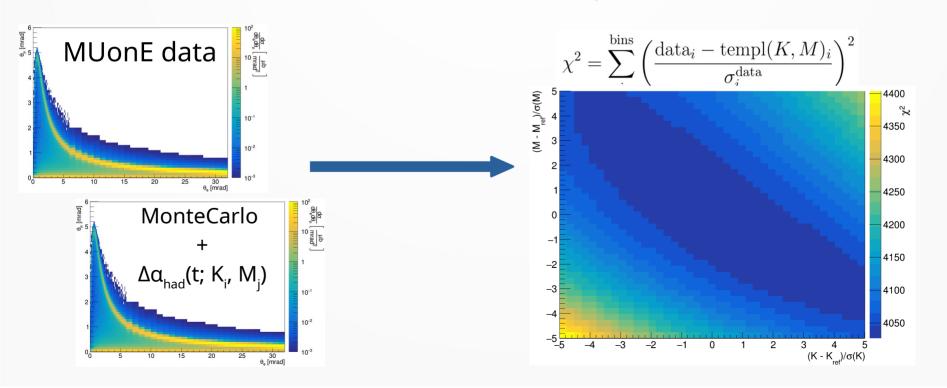


### **Extraction of** $\Delta\alpha_{had}(t)$



$$\Delta\alpha_{had}(t) = KM \left\{ -\frac{5}{9} - \frac{4}{3} \frac{M}{t} + \left( \frac{4}{3} \frac{M^2}{t^2} + \frac{M}{3t} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M}{t}}} \ln \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\}$$
 2 parameters: K, M

Template fit to the 2D ( $\theta_{\rm e}$ ,  $\theta_{\rm u}$ ) distribution:



## Extraction of $a_{..}$



Extraction of  $\Delta \alpha_{had}(t)$  through a template fit to the 2D ( $\theta_e$ ,  $\theta_u$ ) distribution

$$R_{had} = \frac{d\sigma(\Delta\alpha_{had})}{d\sigma(\Delta\alpha_{had} = 0)}$$

$$R_{had} = \frac{d\sigma(\Delta\alpha_{had})}{d\sigma(\Delta\alpha_{had} = 0)} \qquad a_{\mu}^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx (1 - x) \Delta\alpha_{had}[t(x)]$$

Results from fast simulation assuming the final statistics:

$$a_{\mu}^{\rm HLO}$$
 = (688.8 ± 2.4) 10<sup>-10</sup>

Input value:
 $a_{\mu}^{\rm HLO}$  = 688.6 10<sup>-10</sup>

## Alternative method to compute $a_{\mu}^{ m HLO}$ from MUonE data



$$a_{\mu}^{\mathrm{HLO}} = a_{\mu}^{\mathrm{HLO}\;\mathrm{(I)}} + a_{\mu}^{\mathrm{HLO}\;\mathrm{(II)}} + a_{\mu}^{\mathrm{HLO}\;\mathrm{(III)}} + a_{\mu}^{\mathrm{HLO}\;\mathrm{(IV)}}$$

Ignatov, RP, Venanzoni, Teubner, Phys. Lett. B 848 (2024) 138344

$$a_{\mu}^{\text{HLO (I)}} = -\frac{\alpha}{\pi} \sum_{n=1}^{3} \frac{c_{n}}{n!} \frac{d^{(n)}}{dt^{n}} \Delta \alpha_{had}(t) \Big|_{t=0}$$

$$a_{\mu}^{\text{HLO (II)}} = \frac{\alpha}{\pi} \frac{1}{2\pi i} \oint_{|s|=s_{0}} \frac{ds}{s} c_{0} s \, \Pi_{had}(s) \Big|_{\text{pQCD}}$$

$$a_{\mu}^{\text{HLO (III)}} = \frac{\alpha^{2}}{3\pi^{2}} \int_{s_{\text{th}}}^{s_{0}} \frac{ds}{s} [K(s) - K_{1}(s)] R(s)$$

$$a_{\mu}^{\text{HLO (IV)}} = \frac{\alpha^{2}}{3\pi^{2}} \int_{s_{0}}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_{1}(s)] R(s)$$

MUonE 99% Time-like data + pQCD

1%

Competitive results independently of the parameterization chosen to fit  $\Delta\alpha_{had}(t)$ 

