Electromagnetic corrections to HVP: lattice QCD vs. data-driven evaluations

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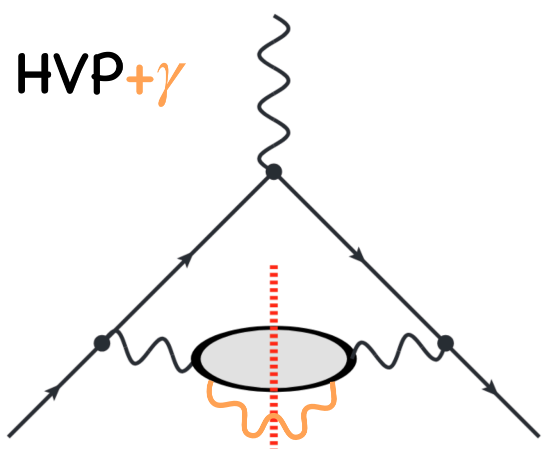
8th Plenary workshop of the Muon g-2 Theory Initiative 2025, Orsay, France

Hadronic vacuum polarization contribution to $(g-2)_\mu$

Data-driven approach
(timelike photon momentum)

Lattice QCD approach
(spacelike photon momentum)



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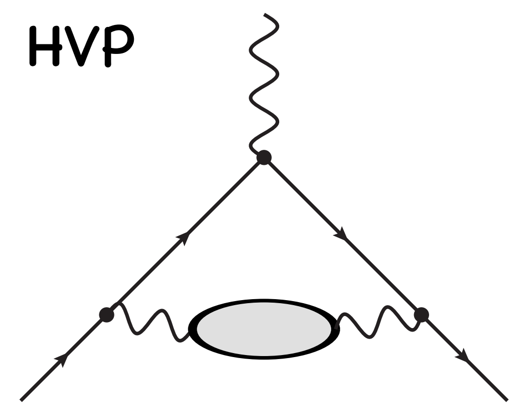

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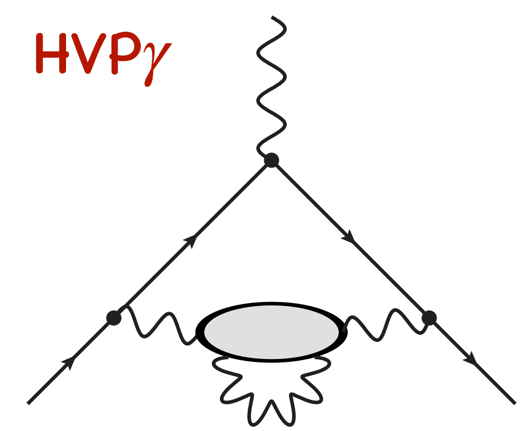


- sum over all $e^+e^- \to \gamma^* \to \text{hadrons}(+\gamma)$ channels

- sum of series expansion terms around isospin-symmetric point

$$a_\mu^{\text{HVP}}(\text{lattice}) = a_\mu^{\text{HVP}}(\text{isosym}) + \delta_m \frac{\partial a_\mu^{\text{HVP}}}{\partial \delta_m} + \alpha_{\text{em}} \frac{\partial a_\mu^{\text{HVP}}}{\partial \alpha_{\text{em}}}$$

challenging, uncertain

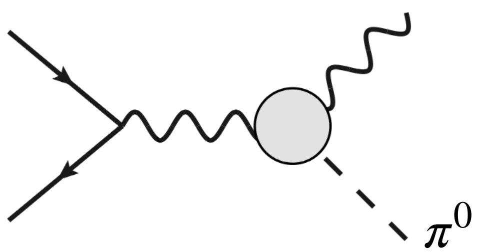
$\sim \text{few} \times 10^{-10}$

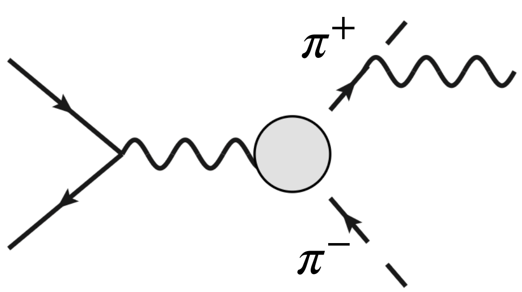
Phenomenological estimate of HVPγ

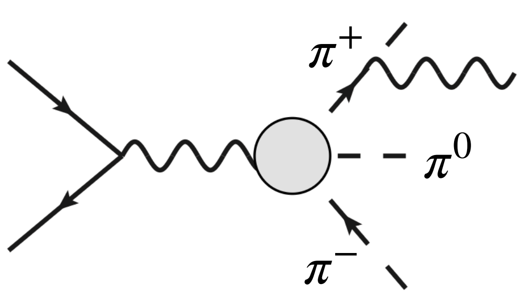
Data-driven-based estimate

- sum all channels with a photon in final state
  - most recent estimate - enters WP2025

[Hoferichter et al., PRL (2023)]



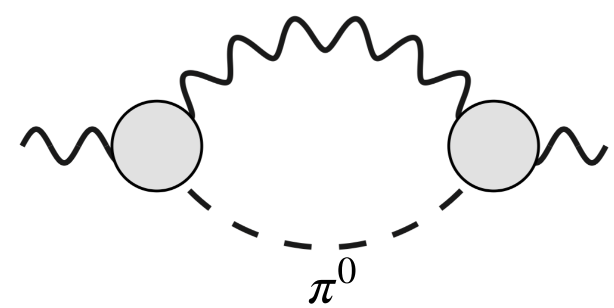




. . .

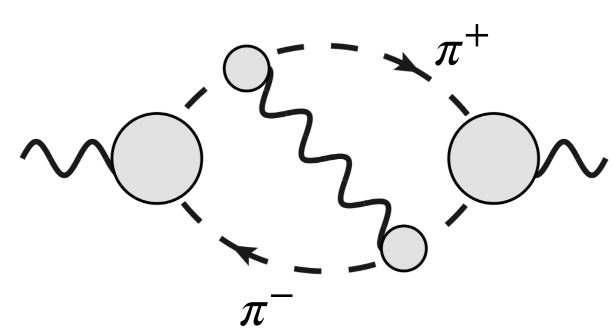
Spacelike models

- model the whole HVPγ blob
  - $\pi^0\gamma$-model [Blokland et al., PRL (2002)]

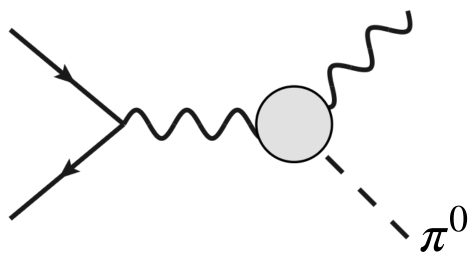


  - $\pi^+\pi^-\gamma$, $K^+K^-\gamma$ models + application in lattice analysis

[VB et al., JHEP (2023)] [Erb et al., arXiv (2025)]
[Parrino et al., JHEP (2025)]



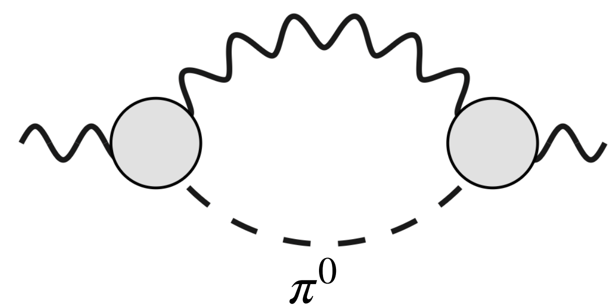
Timelike vs. Spacelike?



- data-driven evaluation:

[Hoid et al., EPJC (2020)]

$$a_\mu^{(\pi^0\gamma)}(\text{timelike}) = 4.38(6) \times 10^{-10}$$



- simple VMD-based model

[Blokland et al., PRL (2002)]

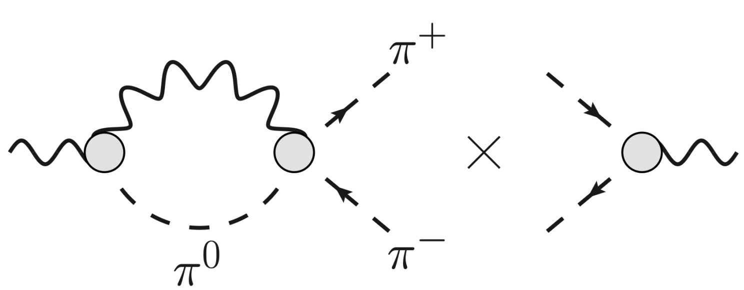
$$a_\mu^{(\pi^0\gamma)}(\text{spacelike}) \simeq 0.37 \times 10^{-10}$$

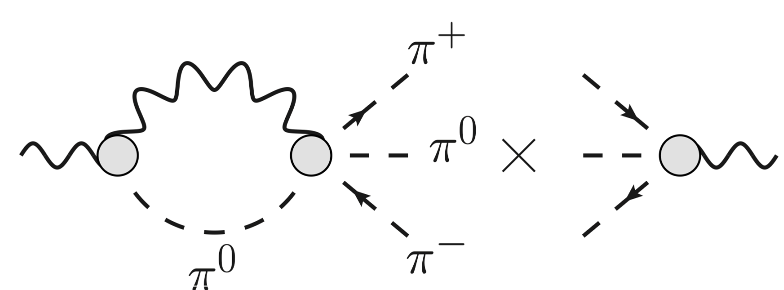
one order discrepancy!

cause large cancellations!

- corresponds to the lattice QCD computation

- $\mathcal{O}(\alpha_{\mathrm{em}})$ interference terms from hadronic channels reconcile the discrepancy





QFT calculation within VMD model

- Consider the VMD Lagrangian for pions [Kroll, Lee, Zumino, PR(1967)] and many others

conserved lepton current

$$\mathscr{L}_{\rm VMD} = \mathscr{D}^*_\mu \pi^+ \mathscr{D}^\mu \pi^- - m_\pi^2 \pi^+\pi^- - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{e}{2g_\gamma}V_{\mu\nu}F^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{m_\rho^2}{2}\rho_\mu\rho^\mu + A^\mu j^\ell_\mu + \Delta\mathscr{L}_{\pi^0}$$

$\rho - \gamma$ mixing

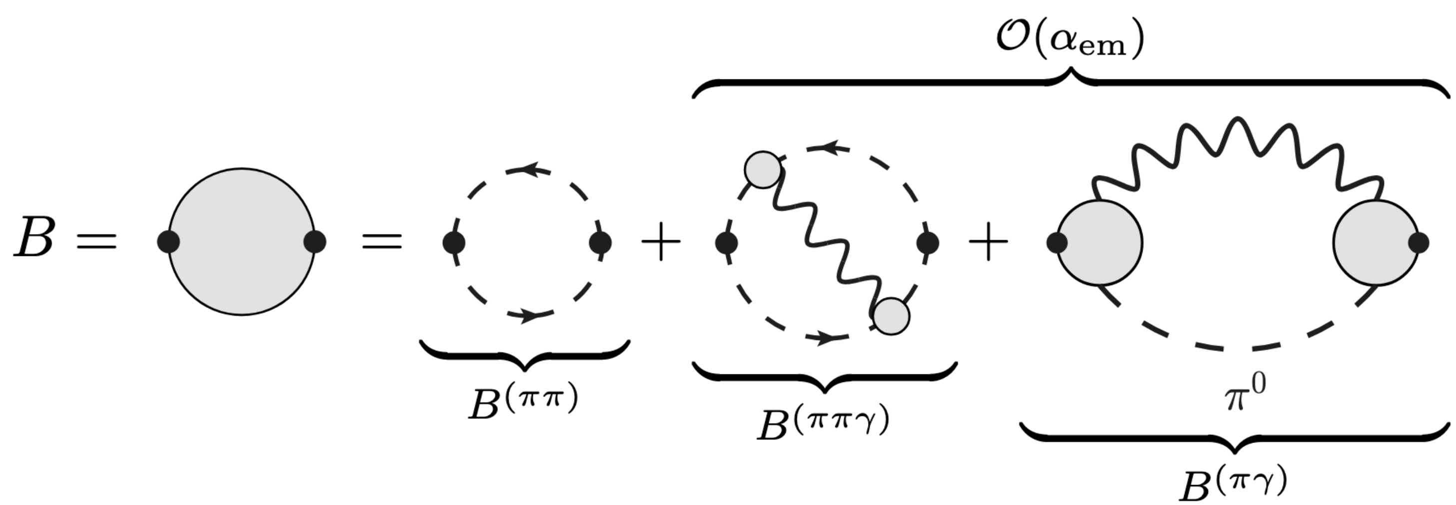
inclusion of π^0

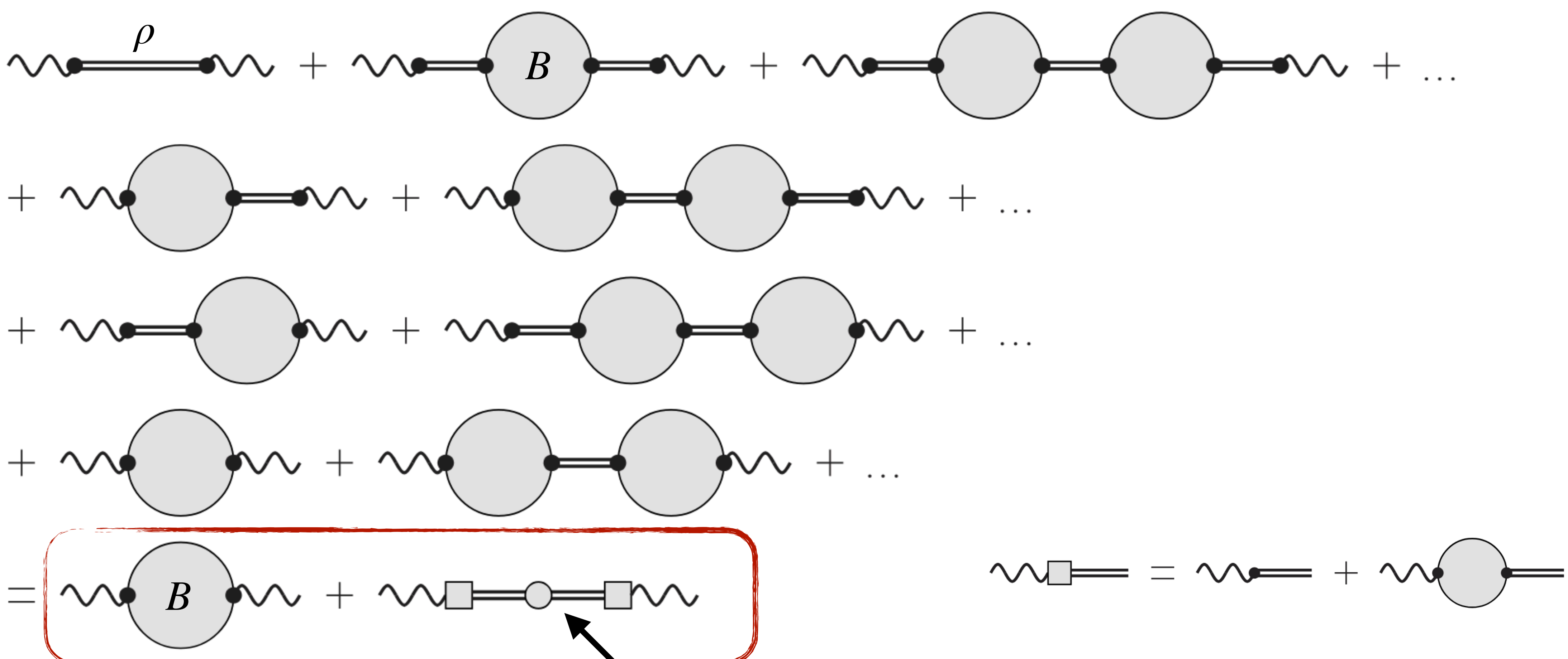
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad V_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu, \quad \mathscr{D}_\mu = \partial_\mu - ieA_\mu - ig_\rho \rho_\mu$$

- Determine g_ρ and g_γ from $\rho \to e^+e^-$ and $\rho \to \pi^+\pi^-$ decays

$$\begin{aligned} \Gamma(\rho \to \pi^+\pi^-) &\simeq 149.5\,\text{MeV} \\ \Gamma(\rho \to e^+e^-) &\simeq 7.04\,\text{keV} \end{aligned} \longrightarrow \begin{aligned} g_\rho &\simeq 5.98 \\ g_\gamma &\simeq 4.95 \end{aligned}$$

Bubble-chain resummation





resummed ρ propagator

$\pi^+\pi^-$ contribution

$$\boxed{\frac{1}{e^2}\Pi(q^2) = B_\gamma^{(\pi\pi)}(q^2) + q^2\left[g_\rho B_{\gamma\rho}^{(\pi\pi)}(q^2) - \frac{1}{g_\gamma}\right]^2 D(q^2)}$$

$$D(q^2) = \left[q^2 - m_\rho^2 - g_\rho^2 q^2 B_\rho(q^2)\right]^{-1}$$

- Different renormalization conditions: photon pole (γ), ρ-pole (ρ), $\gamma - \rho$ mixing ($\gamma\rho$)

- satisfies dispersion relation for $\Pi(q^2)$ with the data-driven Im part

$$\mathrm{Im}\Pi^{(\pi^+\pi^-)}(s) = e^2\left|F_\pi(s)\right|^2 \mathrm{Im}B^{(\pi\pi)}(s)$$

pion vector form factor

$$\overline{\Pi}^{(\pi^+\pi^-)}(q^2) = \frac{q^2}{\pi}\int \frac{\mathrm{d}s\ \mathrm{Im}\,\Pi^{(\pi^+\pi^-)}(s)}{s(s-q^2-i0^+)}$$

- fairly accurate prediction for $\pi^+\pi^-$ contribution:

$$\boxed{a_\mu^{(\pi^+\pi^-)} \simeq 509 \times 10^{-10}}$$

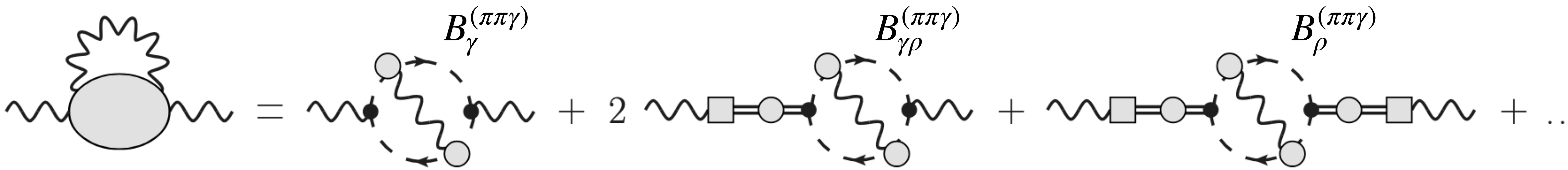
cf. $a_\mu^{(\pi^+\pi^-)}(\mathrm{WP2020}) = 495.0(2.36) \times 10^{-10}$

- isospin-symmetric $\to$ physical point conversion

$$\boxed{\Delta a_\mu^{(\pi^+\pi^-)} \approx a_\mu^{(\pi^+\pi^-)}\Big|_{m_\pi = m_{\pi^\pm}} - a_\mu^{(\pi^+\pi^-)}\Big|_{m_\pi = m_{\pi^0}} \simeq -7.95 \times 10^{-10}}$$

cf. $\Delta a_\mu^{(\pi^+\pi^-)}(\mathrm{WP2025}) = -7.67(94) \times 10^{-10}$

$\pi^+\pi^-\gamma$ contribution



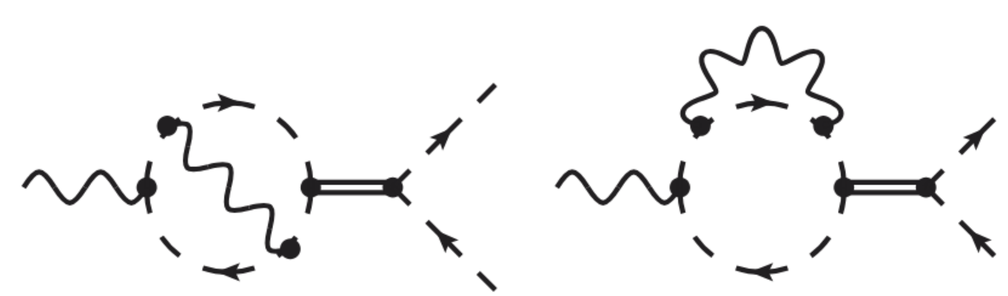
$+$ counterterm

- renormalization: $B^{(\pi\pi\gamma)} \to B^{(\pi\pi\gamma)} - \Sigma(m_\pi^2) \times \frac{\partial}{\partial m_\pi^2} B^{(\pi\pi)}$
- satisfies dispersion relation for $\Pi(q^2)$; the Im part has NOT ONLY the data-driven term!

- consider the simplified version of the model: $g_\gamma := g_\rho, \quad B_{\gamma\rho}^{(\pi\pi\gamma)} = B_\rho^{(\pi\pi\gamma)} := B_\gamma^{(\pi\pi\gamma)}$

$$\frac{1}{e^2}\overline{\Pi}^{(\pi^+\pi^-\gamma)}(q^2) \to F_\pi^2(q^2)\, B_\gamma^{(\pi\pi\gamma)}(q^2) \qquad \mathrm{Im}\left[F_\pi^2 B_\gamma^{(\pi\pi\gamma)}\right] = \underbrace{|F_\pi|^2\, \mathrm{Im} B_\gamma^{(\pi\pi\gamma)}}_{\equiv\text{“timelike” appr.}} + \boxed{\underbrace{2\mathrm{Re}\left[F_\pi B_\gamma^{(\pi\pi\gamma)}\right] \mathrm{Im} F_\pi}_{\text{cut through } F_\pi}}$$

a cut through the pion form factor is attributed to an $\mathcal{O}(\alpha_{\mathrm{em}})$ non-final-state-radiation contribution



$\pi^+\pi^-\gamma$ contribution: estimate

- We use exact VMD result for $\overline{\Pi}^{(\pi^+\pi^-\gamma)}$ with only simplification: $B^{(\pi^+\pi^-\gamma)} = B^{(\pi^+\pi^-\gamma)}_{\mathrm{sQED}}$

- Full result at isosymmteric point:

$$\boxed{a_\mu^{(\pi^+\pi^-\gamma)}(\text{iso-sym}) \simeq 0.74 \times 10^{-10}}$$

- (Data-driven) $\pi^+\pi^-\gamma$ - production channel:

$$a_\mu^{(\pi^+\pi^-\gamma)}[\pi^+\pi^-\gamma](\text{iso-sym}) \simeq 4.45 \times 10^{-10}$$

  at physical point:

$$a_\mu^{(\pi^+\pi^-\gamma)}[\pi^+\pi^-\gamma](\text{phys}) \simeq 4.41 \times 10^{-10} \qquad \textit{cf.}\ \ a_\mu^{(\pi^+\pi^-\gamma)}[\pi^+\pi^-\gamma](\text{WP2025}) = 4.42(4) \times 10^{-10}$$

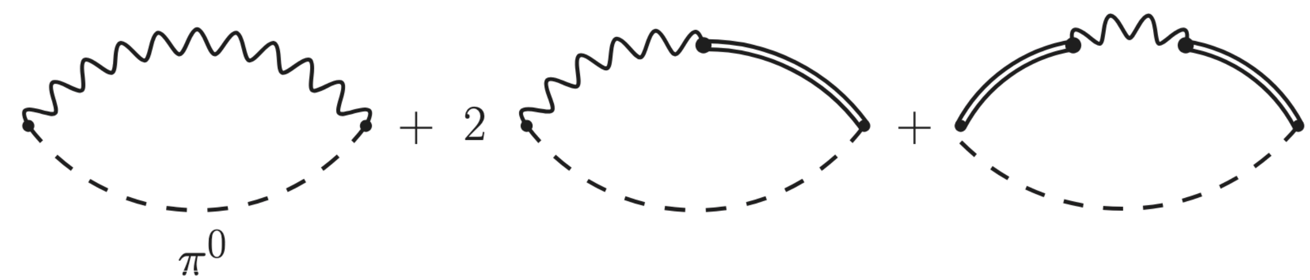
- $\mathcal{O}(\alpha_{\mathrm{em}})$ interference of electromagnetic non-FSR components:

$$a_\mu^{(\pi^+\pi^-\gamma)}[\pi^+\pi^-](\text{iso-sym}) \simeq -3.71 \times 10^{-10}$$

$$\longrightarrow \quad a_\mu^{(\pi^+\pi^-\gamma)} = a_\mu^{(\pi^+\pi^-\gamma)}[\pi^+\pi^-\gamma] + a_\mu^{(\pi^+\pi^-\gamma)}[\pi^+\pi^-]$$

$\pi^0\gamma$ contribution

$$\Delta\mathscr{L}_{\pi^0} = -\frac{1}{16\pi^2 f_\pi}(eF_{\mu\nu} + g_\rho V_{\mu\nu})(e\tilde{F}_{\mu\nu} + g_\rho \tilde{V}_{\mu\nu}) + \dots$$

- Simplifications:
  - stay within one-loop : $B^{(\pi\gamma)} =$ 

  - equal couplings and photon-pole renormalization: $g_\gamma := g_\rho, \quad B^{(\pi\gamma)}_{\gamma\rho} = B^{(\pi\gamma)}_{\rho} := B^{(\pi\gamma)}_{\gamma}$

- Model: $\overline{\Pi}^{(\pi^0\gamma)}(q^2) = e^2 F^2(q^2) B^{(\pi\gamma)}_{\gamma}(q^2)$ — satisfies dispersion relation for $\Pi(q^2)$

- Pion transition form factor: $F_{\pi^0\gamma\gamma}(q^2, k^2) = \dfrac{1}{4\pi^2 f_\pi} F(q^2)\, F(k^2)$

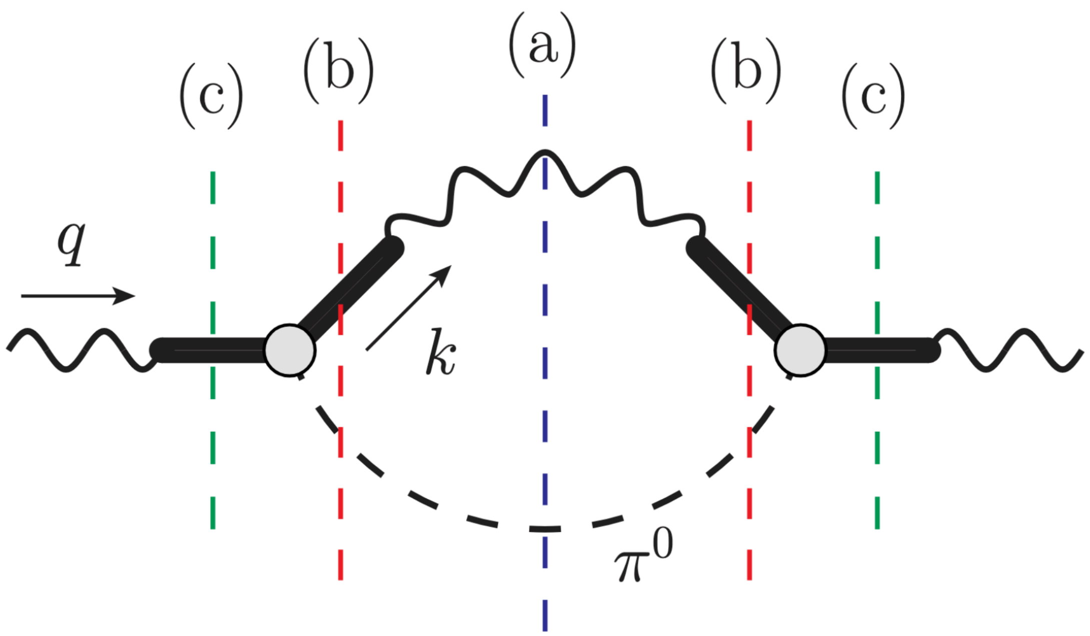
$$F(q^2) = \left[1 - \frac{q^2}{m_\rho^2}\frac{1 - g_\rho^2\, B^{(\pi\pi)}_{\gamma\rho}(q^2)}{1 - (q^2 - m_\rho^2)\, g_\rho^2\, B^{(\pi\pi)\prime}(q^2 = m_\rho^2)}\right]^{-1} \qquad F(k^2) = \frac{1}{1 - k^2/m_\rho^2 - i0^+}$$

$\pi^0\gamma$ contribution: estimate

- Add ω for better description of $\pi^0\gamma$-production channel (similar to [Crivellin and Hoferichter, PRD (2023)])

$$F(q^2) = \frac{1}{2}\left\{\left[1 - \frac{q^2}{m_\rho^2}\frac{1 - g_\rho^2 B_{\gamma\rho}^{(\pi\pi)}(q^2)}{1-(q^2-m_\rho^2)\, g_\rho^2\, B^{(\pi\pi)\prime}(q^2=m_\rho^2)}\right]^{-1} + \frac{m_\omega^2}{m_\omega^2 - q^2 - i m_\omega \Gamma_\omega\, \theta\left(q^2 - 9m_\pi^2\right)}\right\}$$

$$\boxed{a_\mu^{(\pi^0\gamma)} \simeq 0.10 \times 10^{-10}}$$



(a) $a_\mu^{(\pi^0\gamma)}[\pi^0\gamma] \simeq 4.15 \times 10^{-10}$

cf. $a_\mu^{(\pi^0\gamma)}(\mathrm{WP2025}) = 4.38(6) \times 10^{-10}$

(b) $a_\mu^{(\pi^0\rho)} \simeq -0.03 \times 10^{-10}$

(c) $a_\mu^{(\pi^0\gamma)}[\pi^+\pi^-, \pi^+\pi^-\pi^0] \simeq -4.02 \times 10^{-10}$

$$\longrightarrow\ a_\mu(\pi^0\gamma) = \underbrace{a_\mu[\pi^0\gamma]}_{\text{(a)}} + \underbrace{a_\mu[\pi^0\gamma \in \pi^+\pi^-, \pi^+\pi^-\pi^0]}_{\text{(c)}} + \underbrace{a_\mu[\pi^0\rho]}_{\text{(b)}} + \mathcal{O}(10^{-12})$$

due to the constant-width ω

Conclusions

- Data-driven contributions containing the photon explicitly ($\pi^0\gamma$, $\pi^+\pi^-\gamma$, …) are substantially reduced by destructive interference of $\mathcal{O}(\alpha_{\mathrm{em}})$ terms in purely hadronic channels ($\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, etc.)

- Interference terms are model-dependent, but necessary to include for obtaining the correspondence with lattice QCD

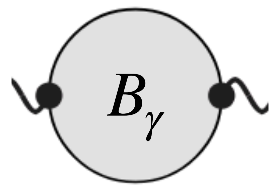
- State-of-the-art lattice-QCD-tailored phenomenological estimate of electromagnetic IB corrections to HVP should be completed by accounting for these terms

Thank you for attention!

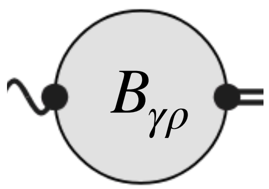
Backup

Renormalization conditions

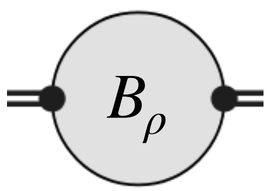
On-shell renormalization scheme similar to [Jegerlehner, EPJC (2011)]



$$B_\gamma(q^2) = B(q^2) - B(0)$$



$$B_{\gamma\rho}(q^2) = B_\gamma(q^2) - \mathrm{Re}\, B_\gamma(m_\rho^2)$$



$$B_\rho(q^2) = B_\gamma(q^2) - \mathrm{Re}\, B_\gamma(m_\rho^2) - (q^2 - m_\rho^2)\frac{m_\rho^2}{q^2}\left[\frac{\mathrm{d}}{\mathrm{d}k^2}\mathrm{Re}\, B_\gamma(k^2)\right]_{k^2=m_\rho^2}$$

General expression for $\pi^+\pi^-\gamma$ contribution

$$\frac{1}{e^2}\Pi^{(\pi^+\pi^-\gamma)}(q^2) = B_\gamma^{(\pi\pi\gamma)}(q^2) + 2q^2\left(g_\rho^2\, B_{\gamma\rho}^{(\pi\pi)}(q^2) - g_\rho/g_\gamma\right) D(q^2)\, B_{\gamma\rho}^{(\pi\pi\gamma)}(q^2)$$

$$+q^4\left(g_\rho^2\, B_{\gamma\rho}^{(\pi\pi)}(q^2) - g_\rho/g_\gamma\right)^2 D^2(q^2)\, B_\rho^{(\pi\pi\gamma)}$$

$$\text{counterterm} = -\,\Sigma(m_\pi^2)\,\frac{\partial}{\partial m_\pi^2}\overline{\Pi}^{(\pi^+\pi^-)}(q^2)$$

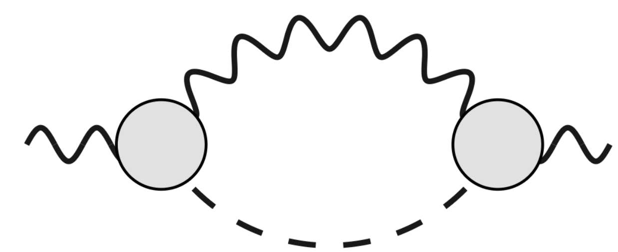
$$= -\,\Sigma(m_\pi^2)\times\Bigg[\frac{\partial}{\partial m_\pi^2}B_\gamma^{(\pi\pi)}(q^2)$$

$$+2q^2\left(g_\rho^2 B_{\gamma\rho}^{(\pi\pi)}(q^2) - g_\rho/g_\gamma\right)\frac{\partial}{\partial m_\pi^2}B_{\gamma\rho}^{(\pi\pi)}(q^2)$$

$$+q^4\left(g_\rho^2 B_{\gamma\rho}^{(\pi\pi)}(q^2) - g_\rho/g_\gamma\right)^2\frac{\partial}{\partial m_\pi^2}B_\rho^{(\pi\pi)}(q^2)\Bigg]$$

Stability test of spacelike model

	TFF model	$a_\mu^{(\pi^0\gamma)} \times 10^{-10}$
	VMD	0.104
[Gerardin et al., PRD (2019)]	LMD+V	0.098
	z-expansion	0.098
[Danilkin et al., Prog.Part.Nucl.Phys. (2019)]	pQCD-inspired	0.096
[Hoferichter et al., JHEP (2018)]	dispersive	0.105



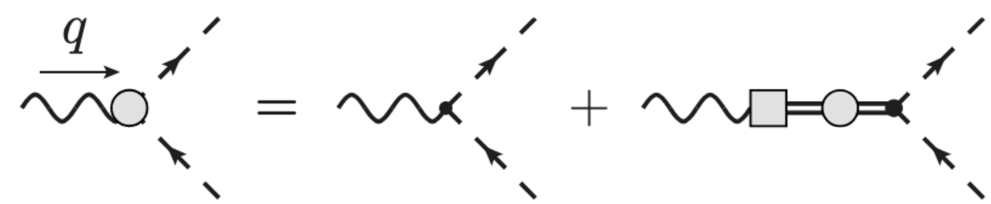
VMD: $$F_{\pi^0\gamma\gamma}(q^2,k^2) = \frac{1}{4\pi^2 f_\pi}\frac{1}{\left(1-q^2/m_\rho^2\right)\left(1-k^2/m_\rho^2\right)}$$

pQCD-inspired: $$F_{\pi^0\gamma\gamma}(q^2,k^2) = \frac{1}{4\pi^2 f_\pi}\frac{f(\omega_\Lambda)}{1-\frac{q^2+k^2}{\Lambda^2}}$$

$$f(\omega_\Lambda) \equiv \frac{1}{\omega_\Lambda^2}\left(1-\frac{1-\omega_\Lambda^2}{2\omega_\Lambda}\log\frac{1+\omega_\Lambda}{1-\omega_\Lambda}\right), \quad \omega_\Lambda = \sqrt{\frac{(q^2-k^2)^2+\Lambda^4}{(q^2+k^2)^2+\Lambda^4}}$$

Pion vector form factor in VMD

$$F_\pi(q^2) =$$



$$= \frac{q^2\left(1 - \frac{g_\rho}{g_\gamma}\right) - m_\rho^2 - g_\rho^2 q^2\left[B_\rho^{(\pi\pi)}(q^2) - B_{\gamma\rho}^{(\pi\pi)}(q^2)\right]}{q^2 - m_\rho^2 - g_\rho^2 q^2 B_\rho^{(\pi\pi)}(q^2)}$$