Lattice HVP Discussion

Steven Gottlieb, Marina Marinkovic

- Overview of the strategies and plans by different collaborations [10‘]

- RBC: LD QED effects [10‘+5‘] J. Parrino, C. Lehner

- IB with CSS [10’+5‘] D. Erb

- EM corrections to HVP [15‘+5‘] V. Biloshytskyi

- Discussion [30‘]

Input from lattice HVP efforts:

- specific **goals for the immediate future**: what sources of error are you focusing on improving? Their strategy for the **long-distance contributions and QED** at long distance? [Talk by C. Lehner and J. Parrino]

- strategy for **scale setting** and **scheme separation**: which setup you intend to use, what is the scale setting **precision** you expect to achieve?

- projections for reducing overall uncertainties and the precision expected over the **next 2-3 years**: do you have an ultimate **precision goal**, and, if so, what is it?

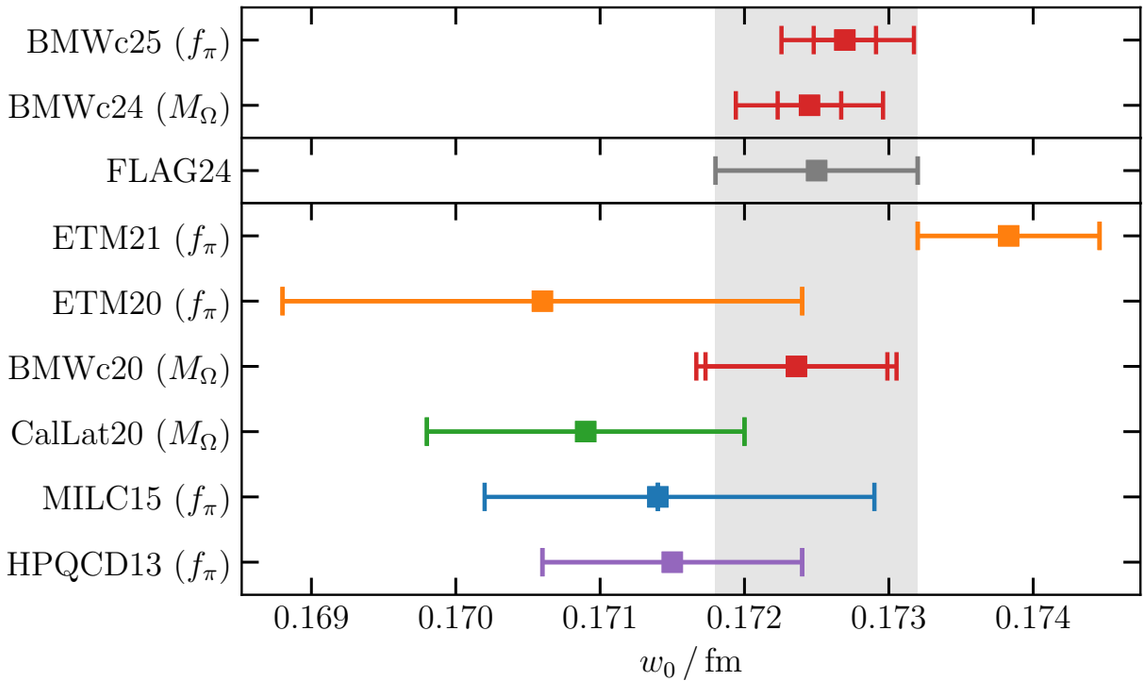
BMW/DMZ summary

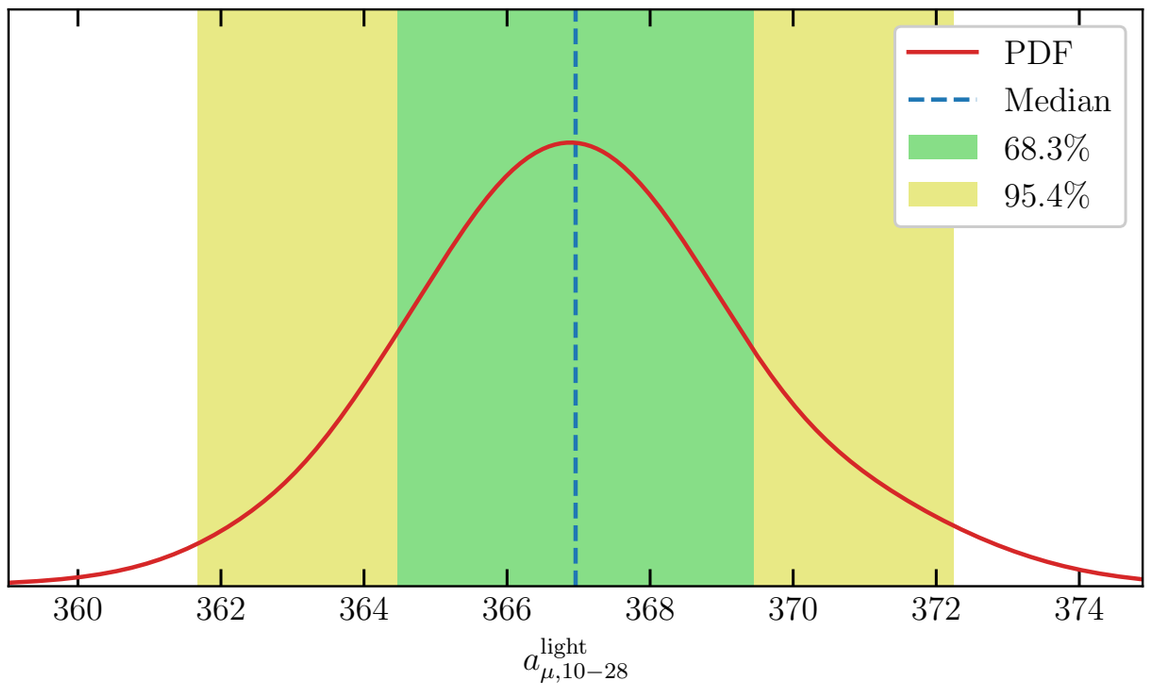
Scale setting and scheme

- Use BMW scheme (w_0, M_{uu}, M_{dd}, M_{ss})
- w_0 scale set via f_π
- f_π result consistent with M_Ω
- This was blinded

Long distance window

- Hybrid approach
- All contributions to 1.0–2.8fm window from lattice
- Remainder from data-driven
- $a_{\mu,10-\infty} = 410.0(3.2)$





ETM Collaboration plans for future on a_μ^{HVP}

1. immediate future goals
   - Leading isospin breaking (LIB) effects via RM123
     - valence contributions in progress
     - electro-unquenched corrections just started
   - control of Finite Volume effects and long-distance contributions
     - simulation of a new ensemble at larger linear extent $L \simeq 10.2\ \mathrm{fm}$
     - measurements on all the ensembles at $L \simeq 7.6\ \mathrm{fm}$
   - taming long-distance statistical errors via deflation methods

2. isosymmetric QCD scale setting setup
   - FLAG scheme $\longrightarrow$ lattice spacing accuracy at 0.1% level
   - WP25 scheme $\longrightarrow$ accuracy level under study (likely a bit worse)

3. final precision goal not set at the moment, it depends on
   - **accuracy for LIB effects**
   - feasibility MC simulations at very small a ($< 0.5\ \mathrm{fm}$)
   - efficiency of multi-level sampling for large t contributions

Aubin, Blum, Golterman, Jin, Moningi, Peris

Long distance contribution

- Continue calculation on MILC 2+1+1 HISQ 144^3 x 288, 0.042 fm lattice using AMA, LMA strategy (HH+HL+LL decomposition of connected light quark correlation function. H=high, L=Low)

- Implement HL (“rest-eigen”) machine learning strategy developed with Mainz group [arXiv: 2502.10237] (possibly on other ensembles too)

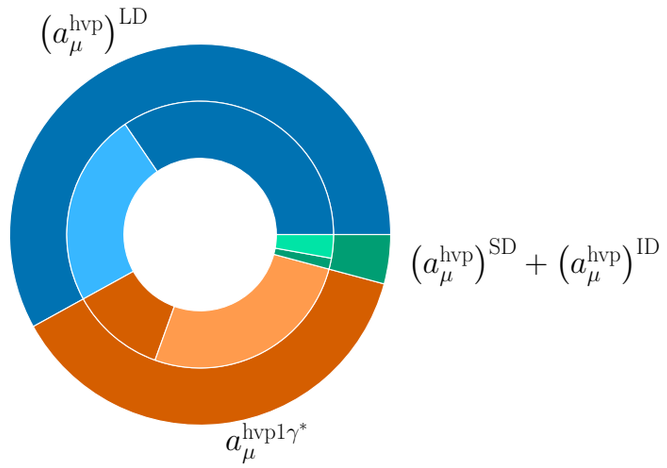
- Precision goal: sub-percent on connected light quark contribution

Fermilab/HPQCD/MILC HVP Summary

- Determination of HVP LO to ~1% precision (upcoming paper)
  - LD contributions: light, strange, charm, disc., SIB (conn. and disc.), QED (conn valence)
  - Focused on improving uncertainty from statistical noise in LD tail (correlator recon. with cross-checks), continuum limit (new data), and disc. and iso-breaking contributions (new precision analyses)
  - Lattice determination of conn valence QED effects (SD, W, and LD): chiral continuum extrapolation of light contribution from (3/5/7)m_l, physical-mass extrapolation of strange contribution
- Scale setting and separation scheme
  - New determination of w_0 (fm) (4 per mil) and M_Ω ($\lesssim$2 per mil with aM_Ω [1]) scales
  - Separation scheme from FLAG24
  - Analysis variations with f_π scale and TI scheme performed for comparison purposes
- Ongoing data generation aimed at ~5 per mil precision
  - Large volume (L ~ 11 fm) at a ~ 0.09 fm
  - Light-quark conn. statistics at a ~ 0.042 fm
  - Disc. at a ~ 0.06 fm
  - Two-pion for a < 0.15 fm

[1] This precision for a~0.04 fm in progress

Mainz/CLS computation of a_μ^{hvp}



- Main sources of uncertainty for a_μ^{hvp}:
  - Statistical uncertainties in the long-distance regime.
  - Continuum extrapolation in the long-distance regime.
  - Isospin breaking effects: Including the full set of diagrams.
- Scales and schemes
  - Currently work with f_π and $f_K \rightarrow$ FLAG scheme might be an option for isoQCD.
  - Derivatives with respect to f_π, f_K, m_π, m_K are published.
  - We work on computing isospin-breaking effects in the baryon octet and decuplet.
  - We see large cutoff effects in $(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$ when using w_0 as intermediate scale.
- Desirable to arrive at 0.5% precision in the long-term future.

RBC/UKQCD plans

- Last complete published result for HVP QCD+QED had total uncertainty of 19×10^{-10} (PRL121(2018)022003)
- Target: 1.5×10^{-10} to match Fermilab E989
- Largest uncertainty was in light-quark connected long-distance contribution which we have reduced now to 5×10^{-10} in PRL134(2025)201901
- Light-quark short-distance and intermediate-distance windows in isospin symmetric limit are already at target precision (PRD108(2023)054507, PRD111(2025)114517)
- Current focus reducing uncertainties to target precision of: QED/SIB corrections, I=0 contributions (see talk by Julian Parrino); goal is to finish this by early 2026
- So far, used RBC/UKQCD14 hadronic scheme (pion, kaon, Omega baryon masses) and BMW20 scheme (pion, ss*, w0); in future will include also WP25 scheme
- QED corrections to Omega mass were already calculated in electroquenched approximation in PRL121(2018)022003, currently extending to all diagrams
- Particular challenge for QED long-distance: employ a exclusive finite-volume state reconstruction just put out as a preprint (arXiv:2508.21685), see talk by Christoph Lehner
- HVP from hadronic τ decay program: method paper to appear this month, see talk by Mattia Bruno
- Generate new $128^3 \times 288$ ensemble at $a^{-1} = 3.5\ \text{GeV}, m_\pi = 135\ \text{MeV}, m_\pi L = 4.9$ right now; will allow for reduction of long-distance light-quark connected contribution uncertainty to target uncertainty over the next years

Discussion session: RC* Collaboration Future Plans

1. **Immediate plans regarding HVP**
   - extend U-spin HVP study (Altherr et al., 2506.19770) with scaling in L (and a)
   - "isospin-violating" correlator $a_\mu^{\mathrm{HVP},38}$ à la Mainz (arXiv:2505.24344)
     $\rightsquigarrow$ cross-check at $m_\pi = 400\,\mathrm{MeV}$ with full QCD+QED
     $\rightsquigarrow$ computation of disconnected diagrams in dynamical QCD+QED
   - employ multigrid LMA for long-distance (R. Gruber et al., arXiv:2412.06347)
2. **Scheme and scale setting**
   - next-generation QCD+QED configurations $m_\pi \to 300\,\mathrm{MeV}$ retain CLS-type scheme
   - new baryon spectrum computations, focus on Ω_- (S. Rosso et al., arXiv:2502.03961)

Many theoretical developments still taking place for $C^\star$ bcs and $N_\mathrm{f} = 1+1+1+1$ in renormalization and FV effects which will have wider impact than HVP

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