Status of HVP from the BMW/DMZ collaboration

[arXiv:2407.10913v2]

Finn M. Stokes

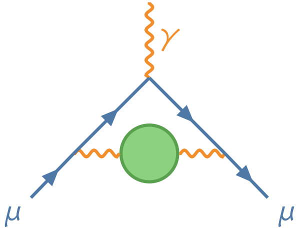
Bocalletti, Borsanyi, Davier, Fodor, Frech, Géradin, Giusti, Kotov, Lellouch, Lippert, Lupo, Malaescu, Mutzel, Portelli, Risch, Sjö, Stokes, Szabo, Toth, Wang, Zhang

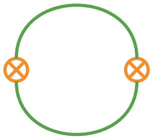
Lattice HVP

- Hadronic Vacuum Polarisation computed from vector-vector correlator
- Time-momentum representation
  [Bernecker & Meyer '11]

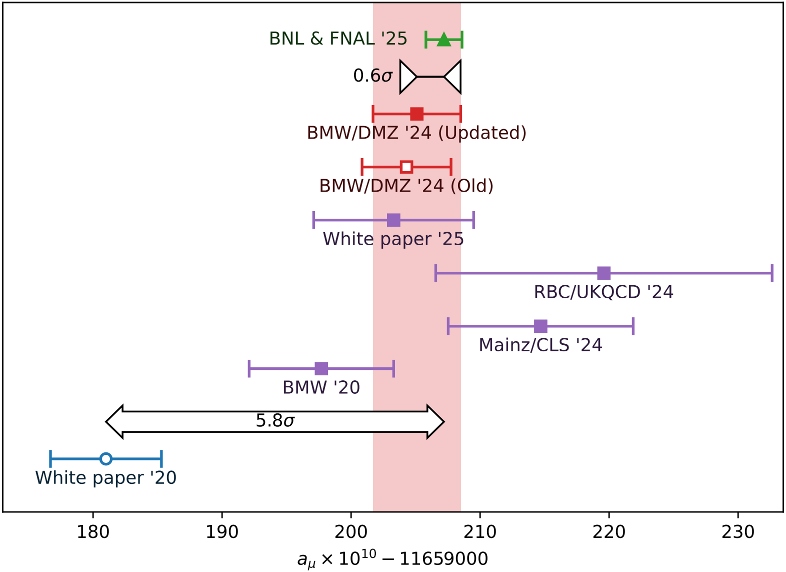
$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty K(t)C(t)dt$$

- $K(t)$ depends on Muon mass
  - Need to match dimensionless simulations onto physical units
- Result goes like square of scale
- To get $a_\mu^{\text{LO-HVP}}$ to 0.5%, need scale to around 0.25%





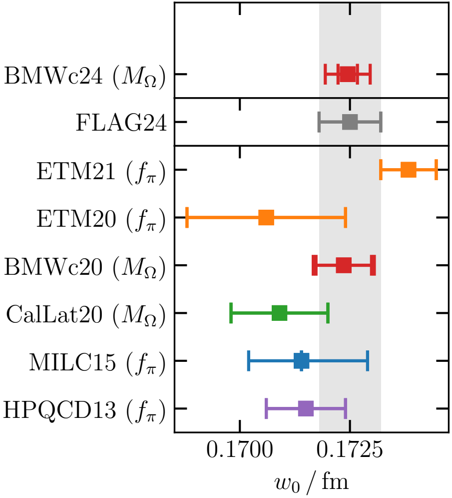
Summary



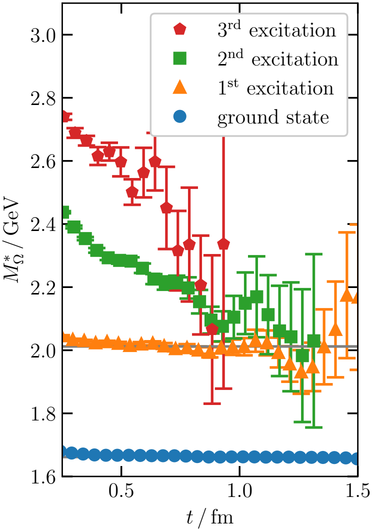
BNL & FNAL '25
0.6σ
BMW/DMZ '24 (Updated)
BMW/DMZ '24 (Old)
White paper '25
RBC/UKQCD '24
Mainz/CLS '24
BMW '20
5.8σ
White paper '20
180
190
200
210
220
230
$a_\mu \times 10^{10} - 11659000$

Scale setting

- We use the Wilson-flow based w_0 to set the scale [Lüscher '10, BMWc '12]
- Enables BMW isospin scheme, where w_0, M_{uu}, M_{dd}, M_{dd} are same in QCD and QCD + QED
- w_0 can't be measured experimentally
- Need to use another quantity to get physical value
- Previously, we have used the Ω baryon

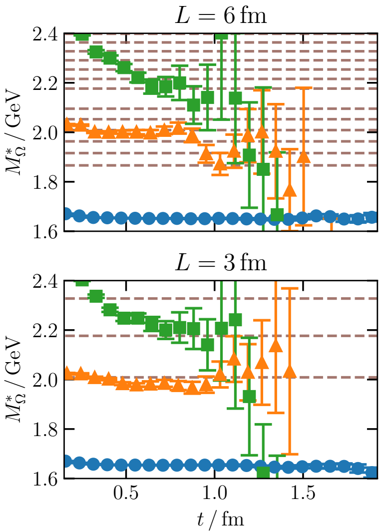


Omega baryon



- GEVP to isolate ground state
  - Wuppertal smearing and GPoF
- Localised states consistent with resonances & quark model
- However, nonlocal “scattering states” not extracted
- Search for resulting contamination
  - Change smearing
  - Change volume
  - Multi-state fit
- No sign of these contaminations
- However, no concrete bound on resulting uncertainty

Omega baryon



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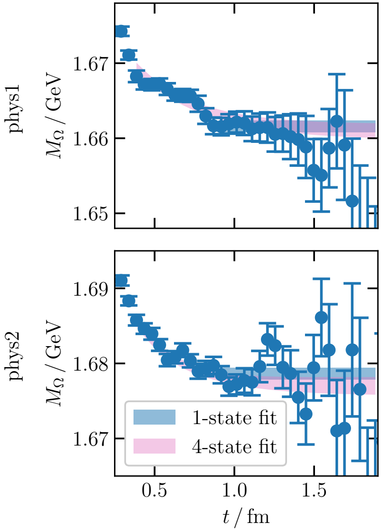

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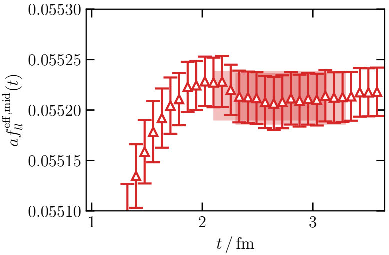
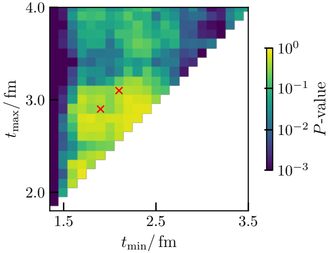
Pion decay constant

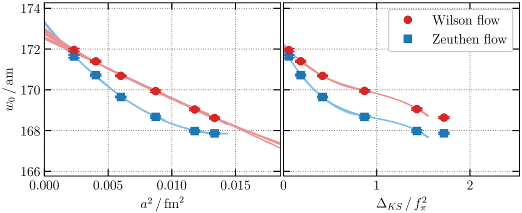
- Alternative scale setting quantity: f_π
- Effective decay constant from pseudoscalar-pseudoscalar correlator shows clear plateau
- Select plateau region using uncorrelated χ^2
  - Goodness of fit from Q-value [Bruno & Sommer '23]
  - Combine Q-values using one-sided Kolmogorov-Smirnov test
- Use heatmap to identify acceptable plateaus

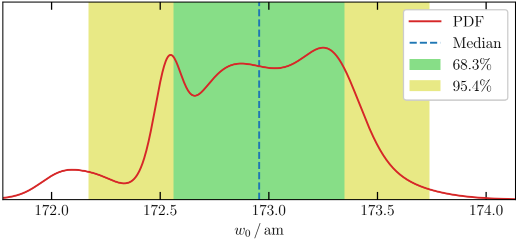




Continuum limit of $w_0 f_\pi$

- Continuum limit with our 6 finest lattice spacings
- Isosymmetric point in FLAG scheme
- Use Zeuthen flow as extra systematic [Ramos & Sint '16]
- As usual, compare 1σ and 2σ quantiles, and take the larger
  - Here that is 1σ

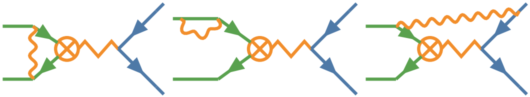




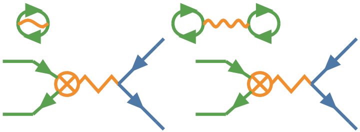
Electromagnetic effects

- Physical value from $\pi^+ \to \mu^+ \nu_\mu$
- Need to account for radiative corrections
- Valence-valence taken from electro-quenched RM123 [Di Carlo et. al. '19]
- Sea-sea computed using our fermion determinant derivatives
- From $SU(3)$ flavour suppression, sea-valence negligible

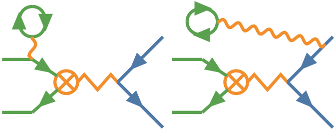
Valence-valence (and valence-lepton)



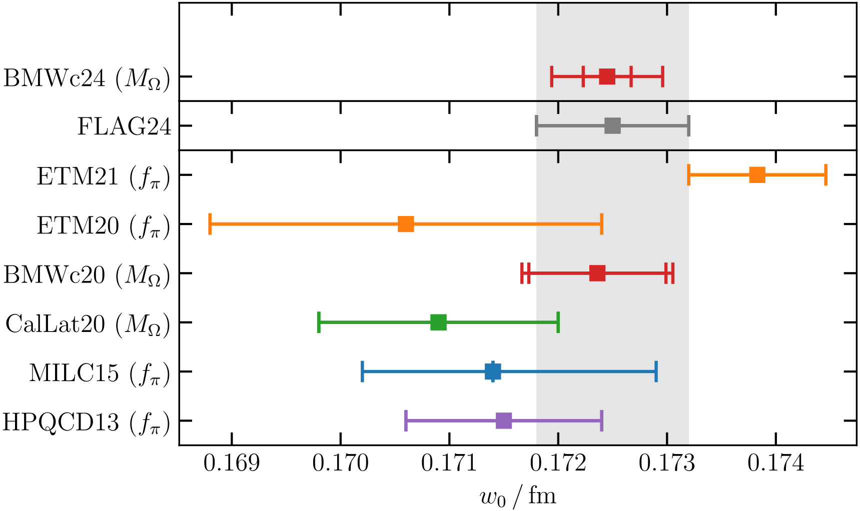
Sea-sea



Sea-valence (and sea-lepton)

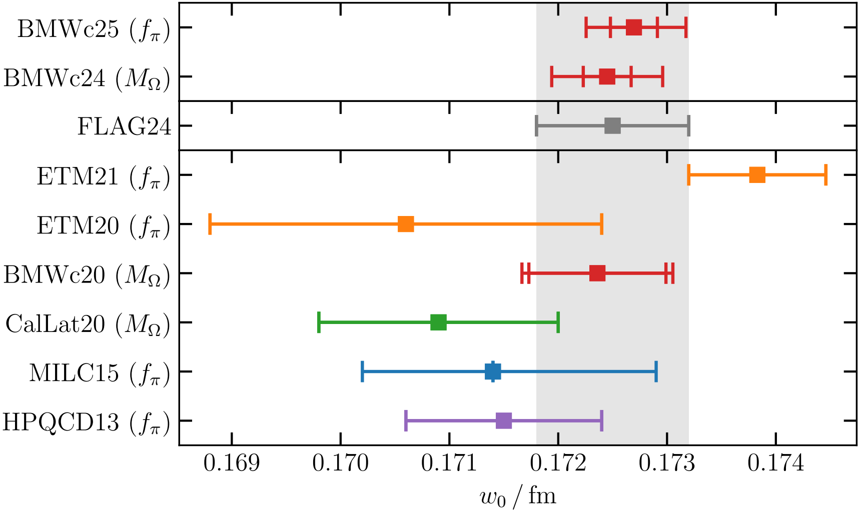


Unblinding



BMWc24 (M_Ω)
FLAG24
ETM21 (f_π)
ETM20 (f_π)
BMWc20 (M_Ω)
CalLat20 (M_Ω)
MILC15 (f_π)
HPQCD13 (f_π)
0.169
0.170
0.171
0.172
0.173
0.174
w_0 / fm

Unblinding



BMWc25 (f_π)
BMWc24 (M_Ω)
FLAG24
ETM21 (f_π)
ETM20 (f_π)
BMWc20 (M_Ω)
CalLat20 (M_Ω)
MILC15 (f_π)
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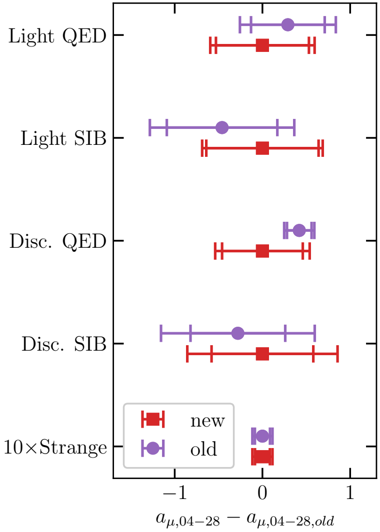
0.169
0.170
0.171
0.172
0.173
0.174
w_0 / fm

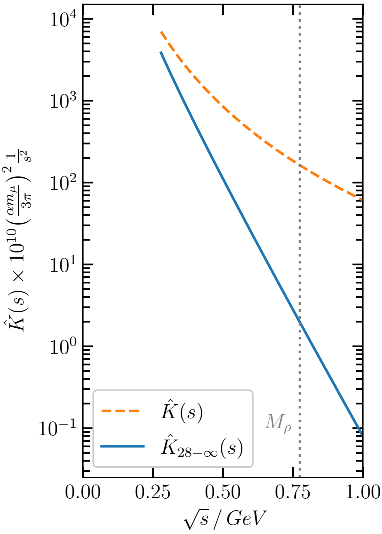
Isospin-breaking windows

- Recomputed with true window functions
- Upper bounds removed
- Otherwise analysis was identical and blinded
- Negligible impact on final result



Data-driven tail

- Long-distance region above 2.8 fm computed in data-driven approach
- Contributes less than 5% of the final value
- Strong suppression of rho peak $\rightarrow$ tensions between experiments significantly reduced
- Independently integrate each experiment that has over 95% coverage
- Average them together (inner error bars)



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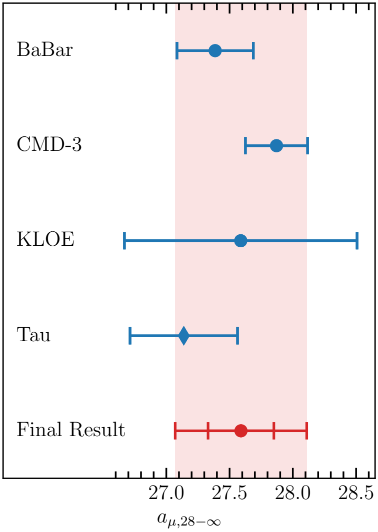

Data-driven tail

- We have updated our uncertainties to be over-conservative
- No change to central value
- Additional uncertainties associated with:
  - Ordering of averaging and integration
  - Other experiments with less coverage
  - Effect of CMD3 on the average
  - Difference between KLOE and BaBar on the rho peak
- Negligible impact on final result

Integral of avg.	0.12
Other exp.	0.16
CMD3	0.31
BaBar/KLOE	0.26

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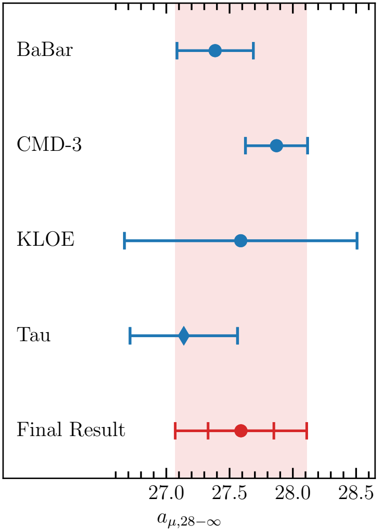
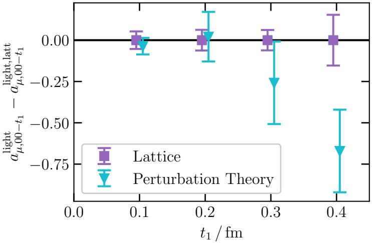
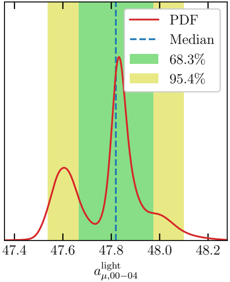
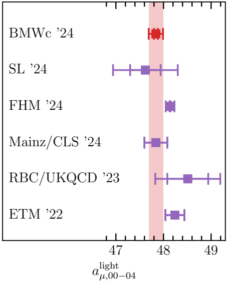


Short-distance window (below 0.4 fm)

- New test of agreement between lattice and perturbation theory
  - Good agreement below 0.3 fm
  - Perturbation-theory-motivated fit form justified
- Results updated with new scale setting: Very little change

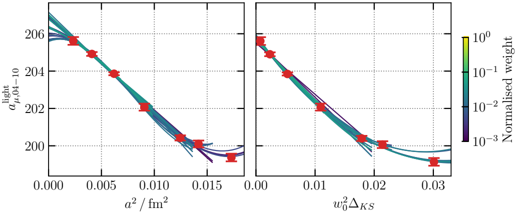


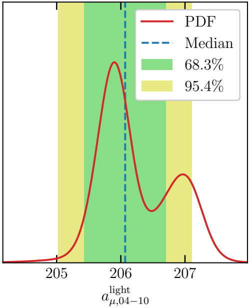


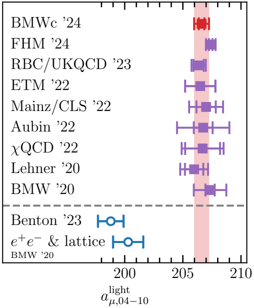


Intermediate window (0.4 fm to 1.0 fm)

- New check: how much did the new ensemble improve precision?
  - Error without new ensemble: 0.84
  - Error with new ensemble: 0.64
  - Improvement: 27%
- Results updated with new scale setting: Very little change

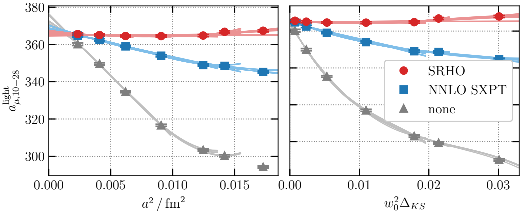


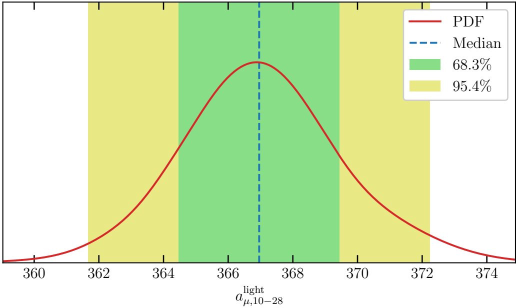




Long-distance window (above 1.0 fm)

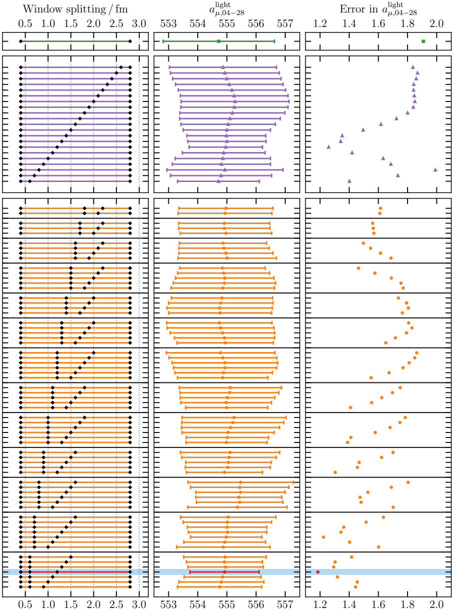
- New result, add together:
  - Lattice calculations for all contributions to 1.0 – 2.8fm
  - Data-driven tail for 2.8 fm to infinity
- Lattice results corrected with SRHO taste-breaking model with uncertainty from NNLO SXPT
- Uncertainty dominated by scale setting and statistics



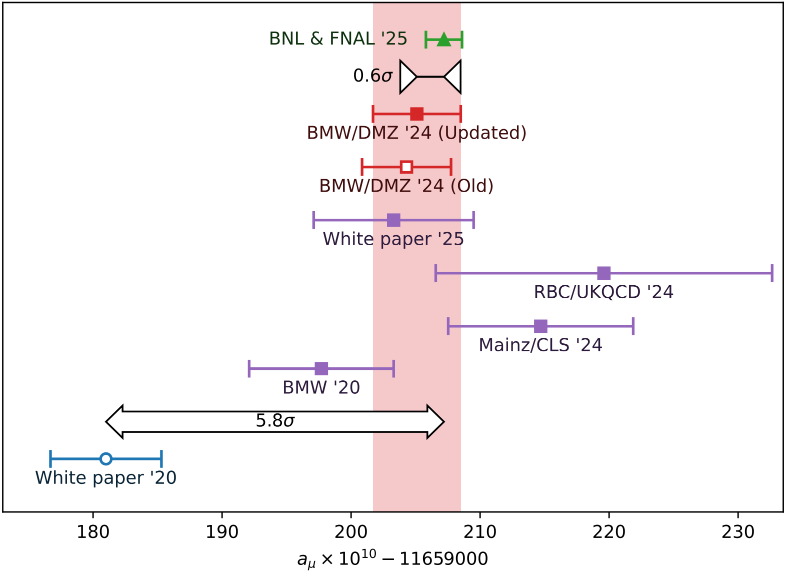


Optimised sub-windows

- For the full result, use different set of sub-windows
- Optimise for minimum final variance
- 0.0 – 0.4fm was best SD window found
- Try different splittings of 0.4 – 2.8fm into one, two, or three sub-windows
- Best was:
  1. 0.4 – 0.6fm,
  2. 0.6 – 1.2fm, and
  3. 1.2 – 2.8fm



Summary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