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INFN
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Antonio Evangelista on behalf of ETMC

Progress on the computation of the leading-order HVP contribution to the muon $g-2$ from ETMC.

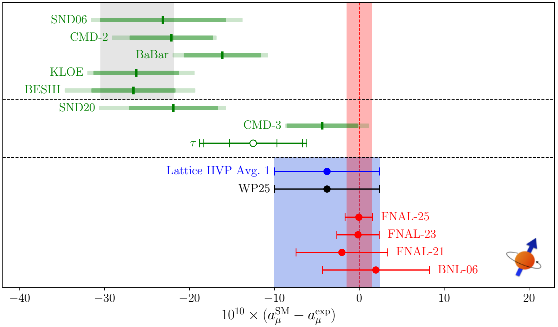
Extended Twisted Mass Collaboration

8th Plenary Workshop of the Muon $g-2$ Theory Initiative IJCLab Orsay, France

$$a_\mu = \frac{g_\mu - 2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$

$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 m_\mu \int_0^\infty \mathrm{d}t\ C(t)\, t^3 K(t) \qquad a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2}{3\pi^2} \int_0^\infty \frac{\mathrm{d}s}{s} R_{\mathrm{had}}(\sqrt{s})\, \tilde{K}(s)$$

[*R. Aliberti et al.* – WP25, arXiv:2505.21476]
accepted on Physics Reports

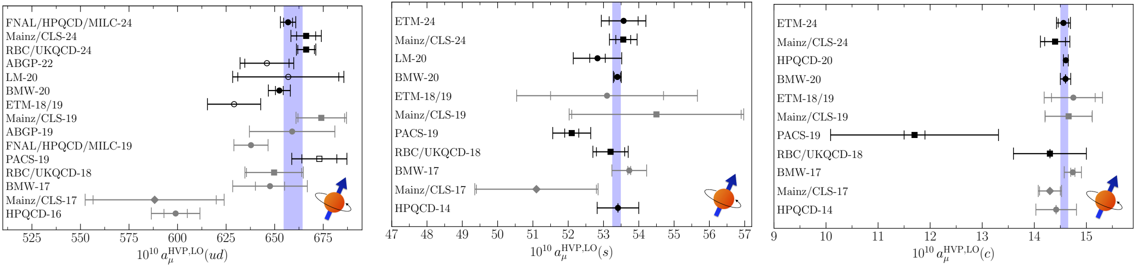


- Lattice QCD a_μ^{HVP} $\longrightarrow$ no tension
- Data-driven disp. a_μ^{HVP} $\longrightarrow$ unsolved puzzle

$a_\mu^{\mathrm{HVP,LO}}$ flavour decomposition status in the WP25 isoQCD scheme

- nice consistency among several collaborations on strange and charm quark-connected contributions
- light quark-connected contribution challenging

  interplay of lattice artifacts, statistical errors and mass dependence
- quark-disconnected contributions: agreement among a few results, error control important too



[*R. Aliberti et al.* – WP25, Fig. 34]

ETMC is finalising the $a_\mu^{\mathrm{HVP,LO}}$ in FLAG and (likely) WP25 isoQCD scheme

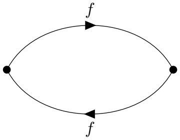
$$C(t) = -\frac{1}{3}\sum_{k=1,2,3}\int \mathrm{d}^3\boldsymbol{x}\ \langle 0|\hat{J}_k(\boldsymbol{x},t)\,\hat{J}_k(\boldsymbol{0},0)|0\rangle \qquad \hat{J}_\mu(x) \sum_{f=u,d,s,c,\dots} Z_V^f\, q_f\, \bar{\psi}_f(x)\gamma_\mu\psi_f(x)$$

mixed action setup

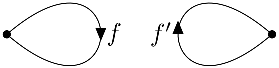
↓

two possible definitions of vector current

❖ connected contributions
subpercent relative error
two ways to approach the continuum



❖ disconnected contributions
larger relative error
one way to approach the continuum



ETMC strategy is using the isospin decomposition

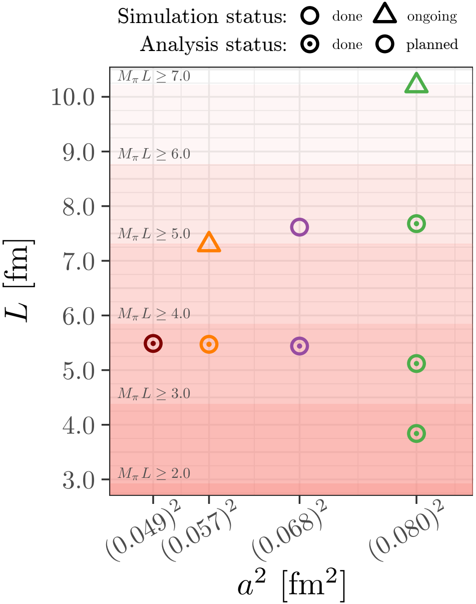
(as proposed by Mainz/CLS)

[*D. Djukanovic et al.* – JHEP 04 (2025) 098]

$$a_\mu^{\mathrm{HVP}}(I=1) = \frac{9}{10} a_\mu^{\mathrm{HVP}}(\ell)$$

$$a_\mu^{\mathrm{HVP}}(I=0) = \frac{1}{10} a_\mu^{\mathrm{HVP}}(\ell) + a_\mu^{\mathrm{HVP}}(s) + a_\mu^{\mathrm{HVP}}(c) + a_\mu^{\mathrm{HVP}}(\mathrm{disc.})$$

ensembles very close to the FLAG/Edinburgh and WP25 isoQCD definition



Used in this analysis:

- 4 different β's $\rightarrow$ $a \in 0.05 - 0.08$ fm
- Different linear size $\rightarrow$ $L \in 3.8 - 7.6$ fm
- Careful analysis of mistuning on μ_ℓ, μ_s, μ_c and $m_{\rm cr}$ to match exactly the FLAG isoQCD point

- Bounding method

$$0 \le C(t_c)\, e^{-m_{\rm eff}(t_c)(t-t_c)} \le C(t) \le C(t_c)\, e^{-E_0(t-t_c)} \quad \forall \; t > t_c$$

  - E_0 is the energy of the lowest two-pion state ($I = 1$) and three-pion state ($I = 0$)
  - $m_{\rm eff}(t_c)$ can be safely replaced by $m_{\rm eff}(t^\star)$ choosing a suitable $t^\star < t_c$

    [*T. Blum et al.* – Phys.Rev.Lett. 121 (2018) 2]

- Additive blinding of the $I = 1$ data

$$C^{\rm reg}(an) \to C^{\rm reg}(an) + \frac{a^3}{Z_V^{\rm reg}} \sum_{k=1}^{K} A_k e^{-m_k \frac{T}{2}} \cosh\left(m_k\left(\frac{T}{2} - an\right)\right)$$

  - large enough K (we chose $K > 12$) does not allow for unblinding data
  - does not alter the cut-off effects $\longrightarrow$ removal of free-theory lattice artifacts possible;
  - Finite Volume Effects (FVE) not altered;

- Additive $a-$ and $L-$ independent blinding of the $I = 0$ data

$l = 1$ contribution

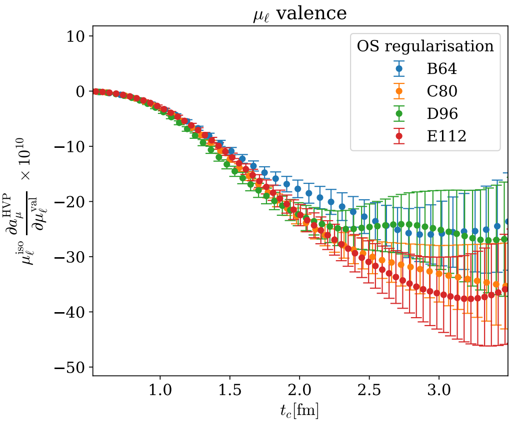
mistuning corrections

$$\mathcal{O}(\mu_f, m_{cr}|\mu_f^{val}, m_{cr}) = \mathcal{O}^{sim}(\mu_f^{val}, m_{cr}^{sim})$$

$$+ (m_{cr} - m_{cr}^{sim}) \left[\partial_{cr}^{sea} + \partial_{cr}^{val}\right] \mathcal{O}$$

$$+ \sum_{f=\ell,s,c} (\mu_f - \mu_f^{sim})\, \partial_f^{sea} \mathcal{O}$$

$$+ (\mu_\ell - \mu_\ell^{sim})\, \partial_\ell^{val}\, \mathcal{O}$$

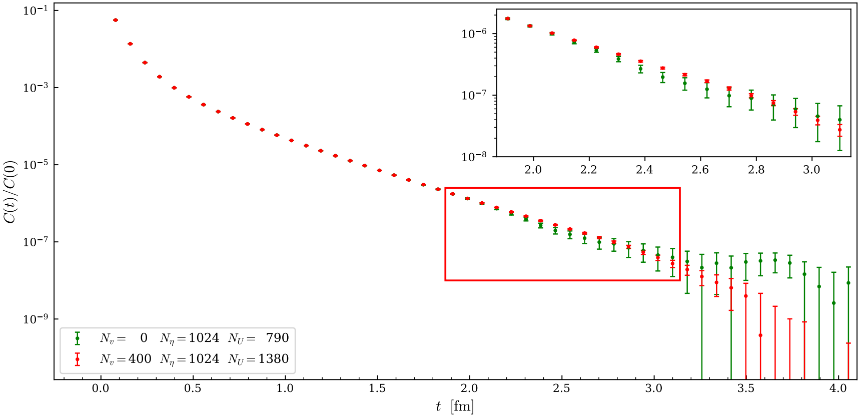


- m_{cr} both sea and valence $\longrightarrow$ small due to $\frac{m_{PCAC}}{\mu_\ell} \ll 1$ tuning, correction size is β dependent
- μ_f sea with re-weighting $\longrightarrow$ tipically small or similar w.r.t. statistical uncertainty
- μ_ℓ valence $\longrightarrow$ dedicated measurements at μ_ℓ close to the isoQCD FLAG point

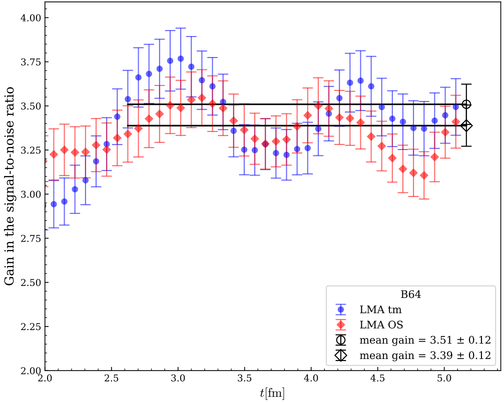
Low Mode Averaging setup

$$S^{\text{defl}}_{r,\eta}(x,y) - S_{r,\eta}(x,y) = \sum_{j=1}^{N_v} \frac{1}{\lambda_j + ir\mu} \left|v_j(x)\right\rangle \left\langle v_j(y)\right| \left(\mathbb{1} - \left|\eta(x)\right\rangle \left\langle \eta(y)\right| \right)\gamma_5 = S^{\text{IR}}_r(x,y) - S^{\text{IR}}_{r,\eta}(x,y)\,.$$

$$C^{\text{defl}}_{rr',\eta}(t) = C_{rr',\eta}(t) - C^{\text{IR}}_{rr',\eta}(t) + C^{\text{IR}}_{rr'}(t)$$



S/N gain on vector-vector correlators

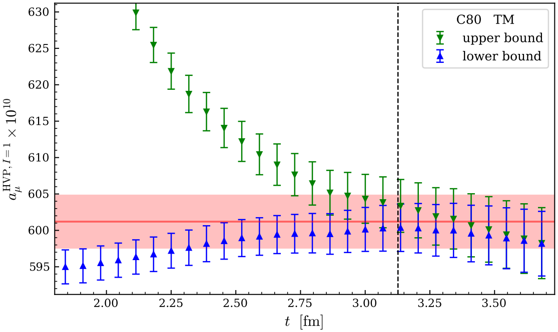


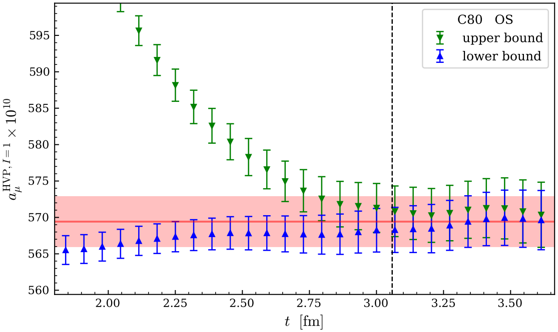
- N_v chosen to obtain a gain in S/N $\gtrsim 3.5$
- for a fixed gain the N_v values depend on physical volume

Ensemble	L [fm]	N_U	N_v	N_η	$a\mu_\ell^{\mathrm{sea}}$
B64	5.09	1380	400	1024	0.00072
C80	5.46	870	530	1600	0.00060
D96	5.46	450	530	960	0.00054
E112	5.46	420	530	1344	0.00044

C80 ensemble ($L \simeq 5.5$ fm) bounding

- upper bound $\longrightarrow$ two-pion (lattice) state with lowest relative momentum
- lower bound $\longrightarrow$ $m_{\text{eff}}(t^\star)$ with $t^\star = 1.8$ fm for all ensembles and both regularisation
- average over $\delta t \simeq 0.25$ fm after optimal t_c





reference volume interpolation L_{ref}=5.46 fm

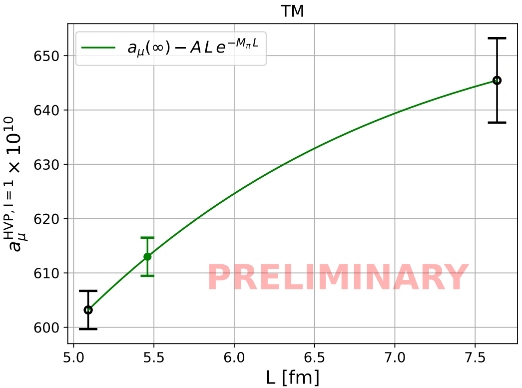
- B64 ensemble $L \simeq 5.1$ fm
- B96 ensemble $L \simeq 7.6$ fm

$\downarrow$

preliminary exponential interpolation
GS and HP inspired

[G.J. Gounaris and J.J. Sakurai – Phys.Rev.Lett. 21 (1968)]
[M.T. Hansen and A. Patella – Phys.Rev.Lett. 123 (2019) 172001]

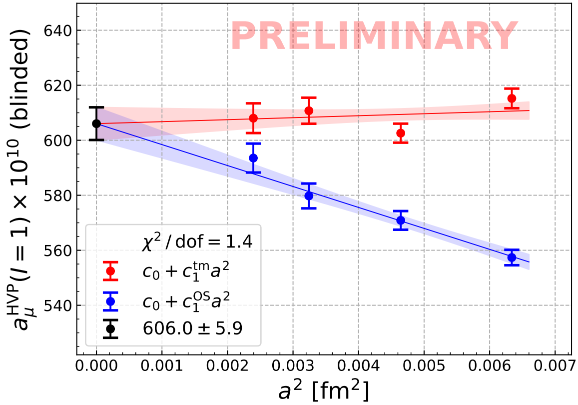
$$a_\mu(L) = a_\mu(\infty) - ALe^{-M_\pi L}$$



final interpolation via data-driven lattice-TM GS model

continuum limit extrapolation

$$a_\mu^{\rm HVP,reg}(I=1) = P_0 + P_1^{\rm reg} a^2$$



- $\sim 4 \cdot 10^{-10}$ error at fixed β
- increase of statistics on D96 and E112
- TM regularisation data almost flat in a^2

  compatible with constant within errors
- C.L. systematics to be estimated via BAIC

  $a^2 + a^4$ **terms on OS data**

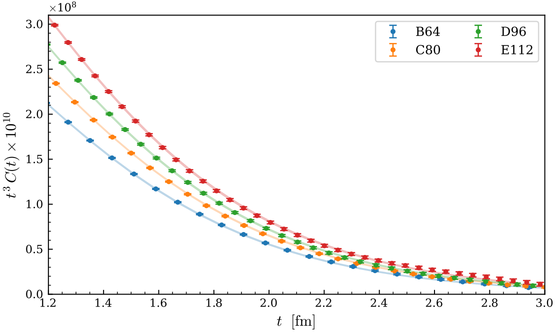
  $a^2 + a^2/\log(a^2\Lambda^2)$, ... **terms**

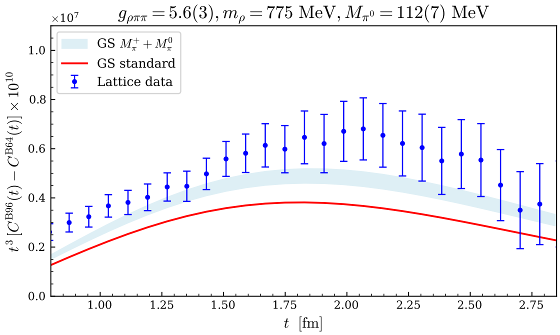
infinite volume limit

- lattice-TM GS model using $M_{\pi^\pm}$ and M_{π^0} for 2π energy levels
- standard GS parameters evaluated from global fit at 4β on TM regularisation
- validation for FVE between $L \simeq 5.1$ fm (B64) and $L \simeq 7.6$ fm (B96) with a target error $\sim 10\%$ in progress

$\downarrow$

increase of statistic on B96 in progress





$I = 0$ contribution

- strange and charm quark-connected contribution already published
  [*C. Alexandrou et al.* – Phys.Rev.D 111 (2025) 5, 054502]

no S/N problem at large Euclidean time

- no bounding
- no LMA

- mistuning for strange and charm quark-connected contributions taken into account
- mistuning on all quark-disconnected contributions negligible within errors
- charm quark-disconnected contribution expected to be negligible $\longrightarrow$ check in progress at a single beta

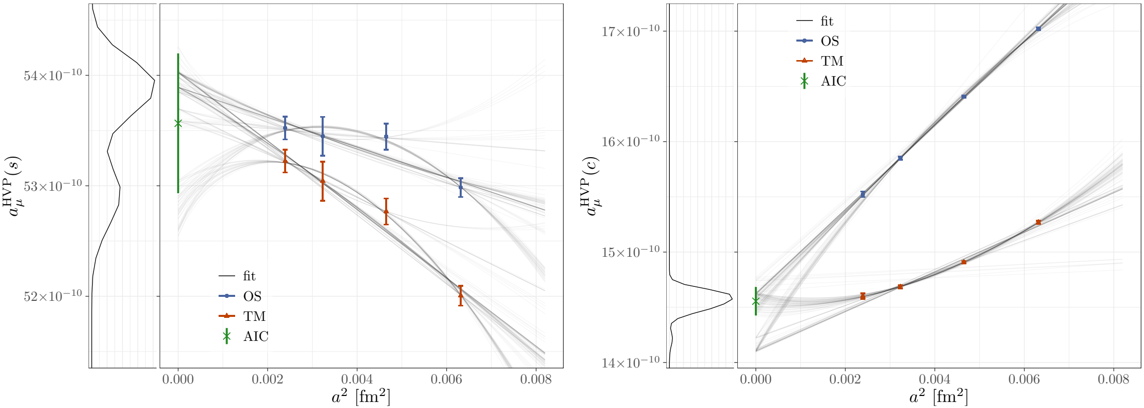
only light and strange contributions taken into account (for now)

$$a_\mu^{\mathrm{HVP}}(I=0) = \frac{1}{10} a_\mu^{\mathrm{HVP}}(\ell) + a_\mu^{\mathrm{HVP}}(s) + a_\mu^{\mathrm{HVP}}(\mathrm{disc.}; \ell - s)$$

strange and charm quark-connected contributions

- FVE negligible but taken into account as systematic effect
- continuum limit systematics via Bayesian AIC

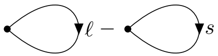
$$a_\mu^{\mathrm{HVP,reg}}(f) = P_0 + P_1^{\mathrm{reg}} a^2 + P_2^{\mathrm{reg}} a^4 \qquad f = s, c$$



light-strange disconnected correlators estimation

- $B(\ell) - B(s)$ computable via Axial WI

$$\sum_{f=u,d,s} q_f B_f^{\mu}(z) = Z_V(\mu_\ell + \mu_s) a^4 \sum_x \langle P_{\ell s}(x) A_{\ell s}^{\mu}(z) \rangle \ ,$$



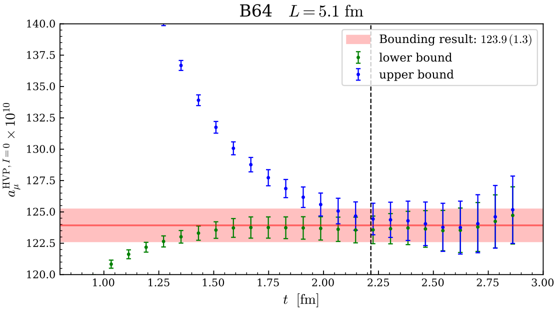
- frequency splitting improvement with 4 levels ($N = 3$)

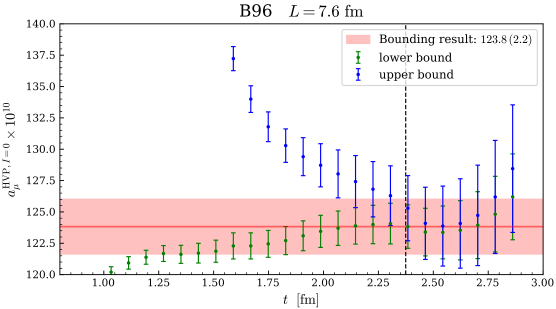
[*L. Giusti et al.* – Eur.Phys.J.C 79 (2019) 7, 586]

$$B^{\mu}(\mu_\ell) - B^{\mu}(\mu_s) = \left[B^{\mu}(\mu_\ell) - B^{\mu}(\mu_1)\right] + \left[B^{\mu}(\mu_1) - B^{\mu}(\mu_2)\right] + \ \ldots \ + \left[B^{\mu}(\mu_N) - B^{\mu}(\mu_s)\right] \ .$$

$I = 0$ bounding procedure

- lower bound $\longrightarrow$ correlator positivity
- upper bound $\longrightarrow$ $E_0 = 2E_{\pi^\pm}(\boldsymbol{p}) + m_{\pi^0}$ with $\boldsymbol{p} = \pm\frac{2\pi}{L}\boldsymbol{n}$ (state relevant only at $\mathcal{O}(a)$ in tm-LQCD)
- average over $\delta t \simeq 0.25$ fm after optimal t_c





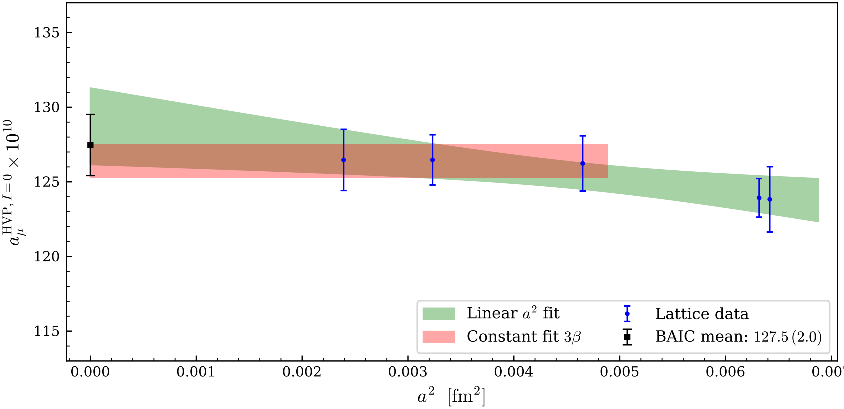
FVE negligible within statistical errors as expected

very small lattice artifacts

↓

- constant fit with 3β
- linear fit in a^2 with 4β

blinded preliminary result



Leading isospin breaking corrections

towards LIB effects from ETMC

$\downarrow$

computing $a_\mu^{\mathrm{HVP,LO}}$ corrections via RM123 method

$$\Delta a_\mu^{\mathrm{HVP}}(f) = 2\alpha^2 \lim_{t_c \mapsto +\infty} \int_0^{t_c} \mathrm{d}t \; t^2 K(m_\mu t) \; Z_V^2 \, \Delta C^f(t) + 2 \frac{\Delta Z_{V,f}}{Z_V} \left(a_\mu^{\mathrm{HVP}}(f) \right)^{\mathrm{iso}}$$

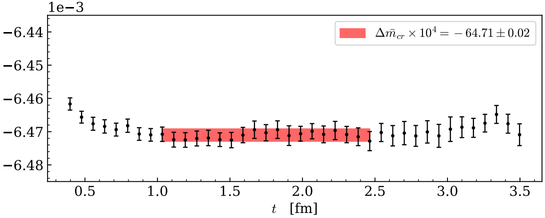
- valence contributions $\longrightarrow$ started and well advanced
- sea and sea-valence contribution $\longrightarrow$ starting now

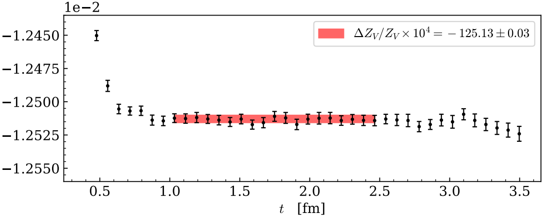
valence quark-connected leading isospin breaking corrections

$$\Delta C^f(t) = e_f^2 \frac{\partial C^f}{\partial e^2}(t) + \Delta m_{cr}^f \frac{\partial C^f}{\partial m_{cr}^f}(t) + \Delta \mu_f \frac{\partial C^f}{\partial \mu_f}(t)$$

- e^2 → discrete derivative
- μ_f, $m_{\rm cr}$ → operator insertion

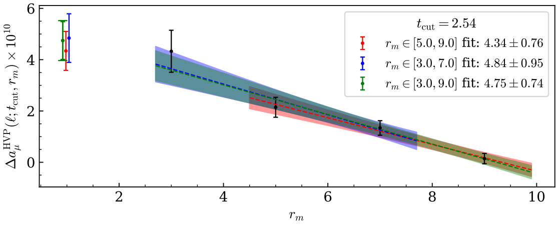
- $\Delta m_{\rm cr}$ from parity restoration in QCD+QED
- $\Delta \mu_f$ from matching physical $\pi^{\pm}$, $K^{\pm}$, K^0 and D_s meson masses
- $\Delta Z_V^f / Z_V$ from axial WI as in tmQCD by using mixed action setup

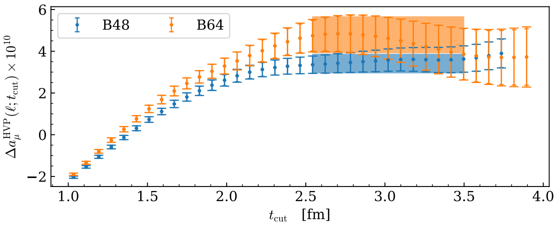




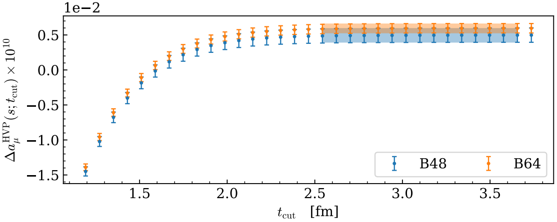
2 volumes: $L_1 \simeq 3.8$ fm and $L_2 \simeq 5.1$ fm at fixed lattice spacing $\simeq 0.08$ fm

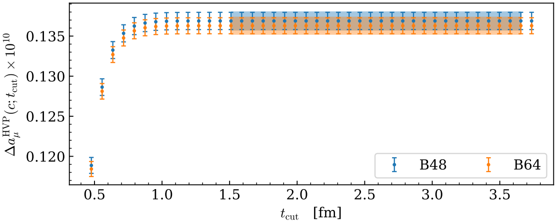
❖ $u + d$ contribution needs chiral extrapolation





❖ strange and charm show no S/N problem





isoQCD results

- $a_\mu^{\rm HVP}(s)$ and $a_\mu^{\rm HVP}(c)$ published with $\sim 1\%$ accuracy
- $a_\mu^{\rm HVP}(I=0)$ preliminary blinded result with competitive error ($\sim 2 \times 10^{-10}$)
- $a_\mu^{\rm HVP}(I=1)$ almost completed
  - increase of statistics on the two finest lattice spacings and on the large volume
  - careful continuum extrapolation study to establish systematics (target error $\sim 5 \times 10^{-10}$)
  - TM-lattice GS model for FVE estimation under validation (target error $\sim 3 \times 10^{-10}$)

$$\downarrow$$

final accuracy on $\boldsymbol{a_\mu^{\rm HVP}}$ **close or below** 1% **achievable**

leading valence isospin breaking effects

- valence quark-connected data ready to be analysed on 2 finer lattice spacings
- third volume ($L \simeq 7.6$ fm) at coarsest lattice spacing to check FVE

future directions

- **isoQCD** simulations at a bigger volume $L \simeq 10.2$ fm just started
- including **electro-unquenched** leading isospin breaking effects at several lattice spacings
- **smeared *R*-ratio** with energy resolution down to ~ 200 MeV just started

  interesting in view of experimental $e^+e^- \mapsto \mathrm{hadrons}$ updates

Thank you for the attention!

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RESEARCH & INNOVATION FOUNDATION

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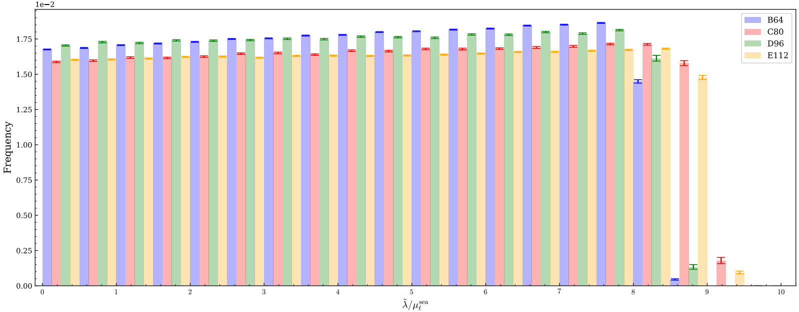
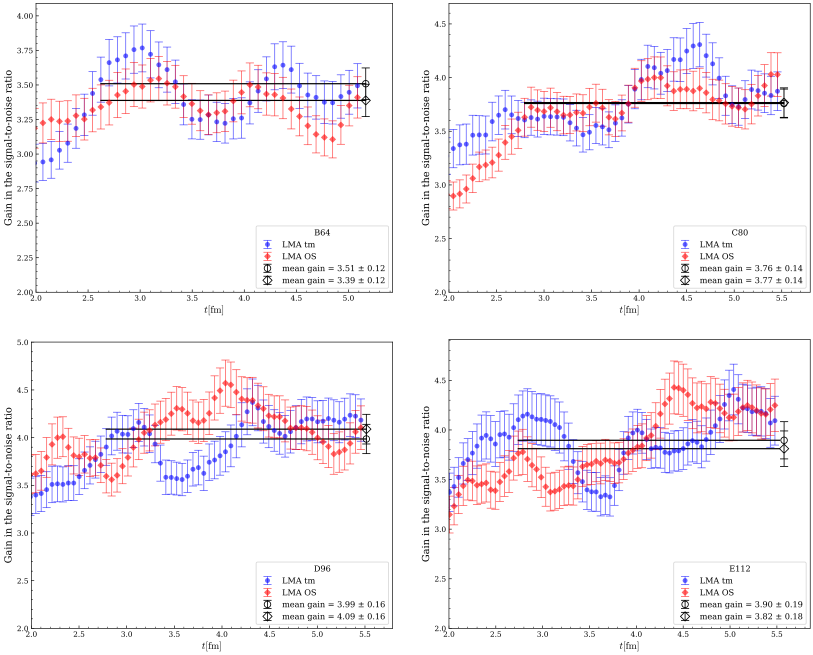
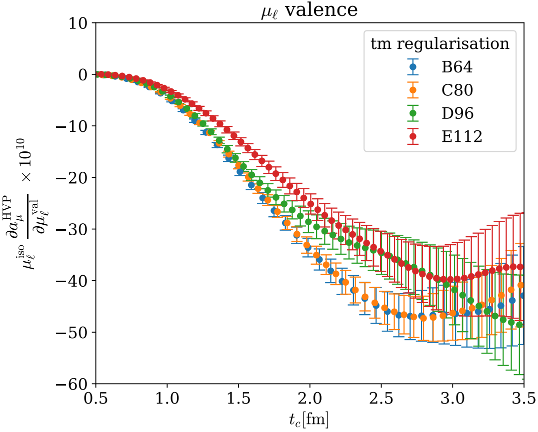
Backup

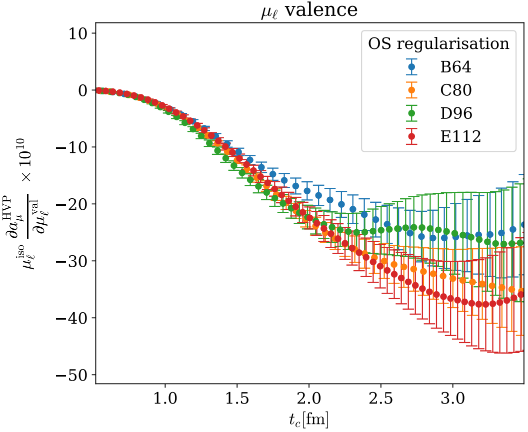
$$\frac{\Delta N}{\Delta\tilde{\lambda}}(0) \propto \Lambda^3_{\mathrm{QCD}} L^3 T\,.$$

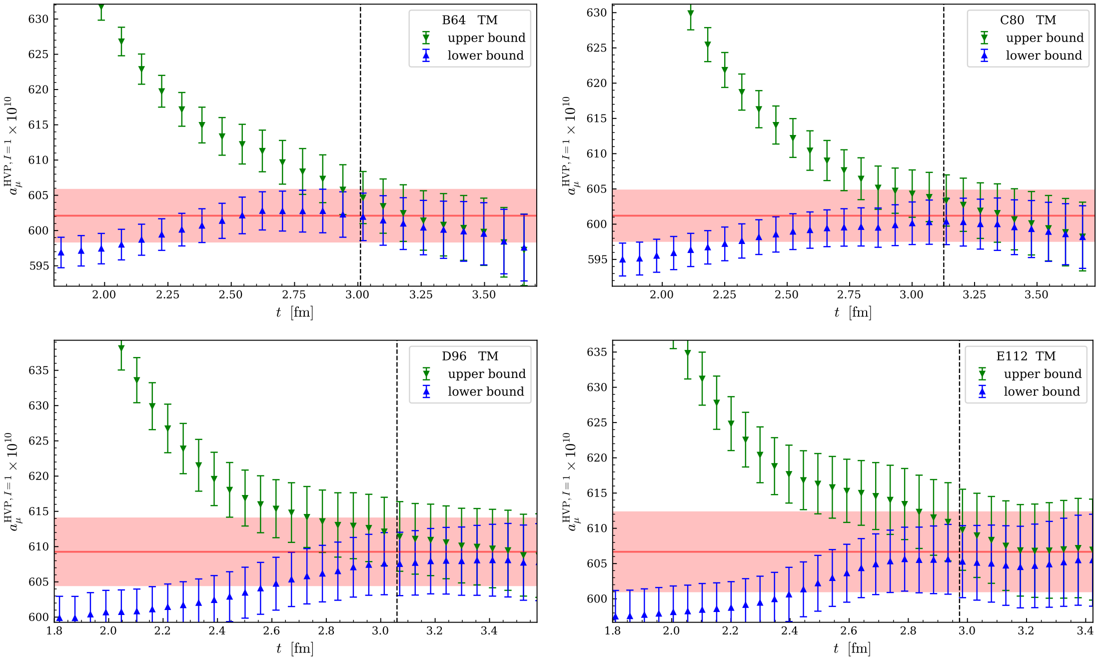


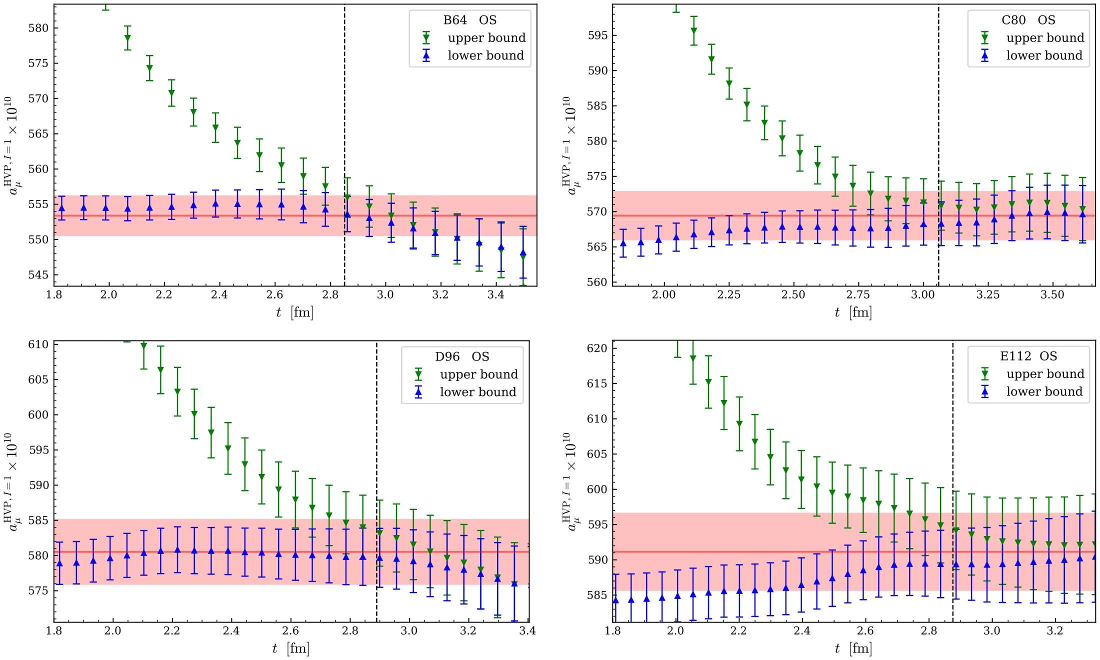

Gain in the signal-to-noise ratio
t[fm]
B64
LMA tm
LMA OS
mean gain = 3.51 ± 0.12
mean gain = 3.39 ± 0.12
C80
LMA tm
LMA OS
mean gain = 3.76 ± 0.14
mean gain = 3.77 ± 0.14
D96
LMA tm
LMA OS
mean gain = 3.99 ± 0.16
mean gain = 4.09 ± 0.16
E112
LMA tm
LMA OS
mean gain = 3.90 ± 0.19
mean gain = 3.82 ± 0.18

μ_ℓ valence mistuning

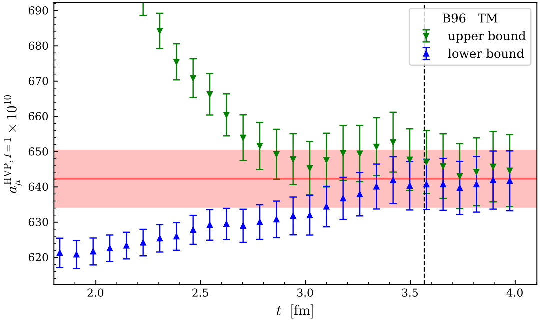


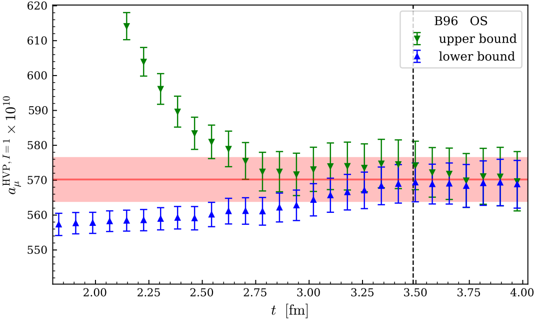


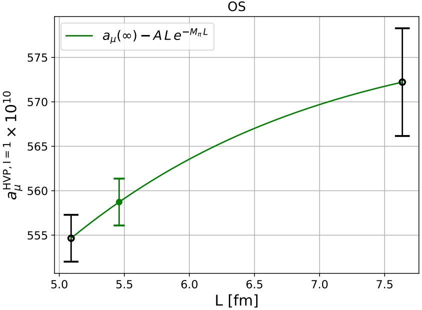

B64 TM
upper bound
lower bound
C80 TM
upper bound
lower bound
D96 TM
upper bound
lower bound
E112 TM
upper bound
lower bound
$a_\mu^{\mathrm{HVP},I=1} \times 10^{10}$
t [fm]

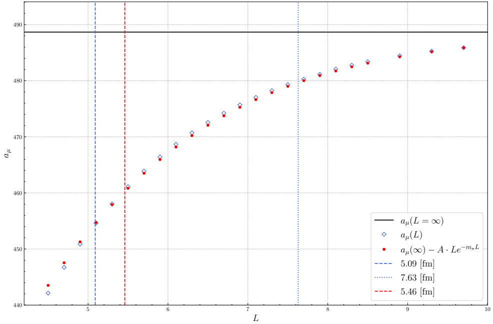


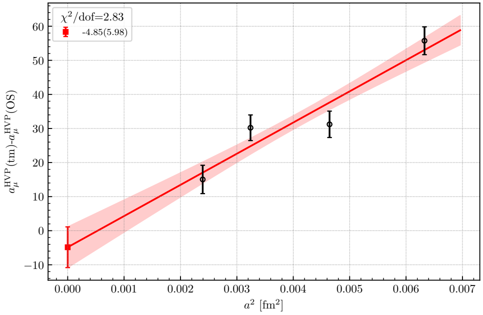
B64 OS
upper bound
lower bound
C80 OS
upper bound
lower bound
D96 OS
upper bound
lower bound
E112 OS
upper bound
lower bound
$a_\mu^{\mathrm{HVP},I=1} \times 10^{10}$
t [fm]


B96 TM
upper bound
lower bound
$a_\mu^{\mathrm{HVP},\,I=1} \times 10^{10}$
t [fm]
690
680
670
660
650
640
630
620
2.0
2.5
3.0
3.5
4.0

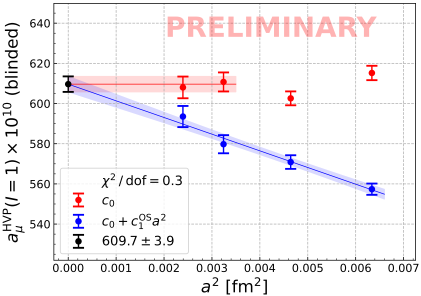

B96 OS
upper bound
lower bound
$a_\mu^{\mathrm{HVP},\,I=1} \times 10^{10}$
t [fm]
620
610
600
590
580
570
560
550
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
4.00


OS
$a_\mu(\infty) - A\,L\,e^{-M_\pi L}$
$a_\mu^{\mathrm{HVP,\,I=1}} \times 10^{10}$
L [fm]
575
570
565
560
555
5.0
5.5
6.0
6.5
7.0
7.5

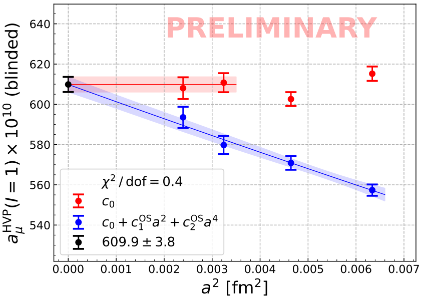

$a_\mu(L=\infty)$
$a_\mu(L)$
$a_\mu(\infty) - A \cdot Le^{-m_\pi L}$
5.09 [fm]
7.63 [fm]
5.46 [fm]
a_μ
L



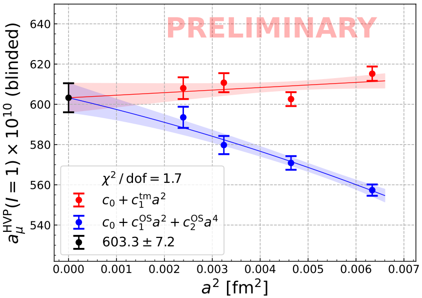
χ²/dof=2.83
-4.85(5.98)
a_μ^HVP(tm)-a_μ^HVP(OS)
a² [fm²]



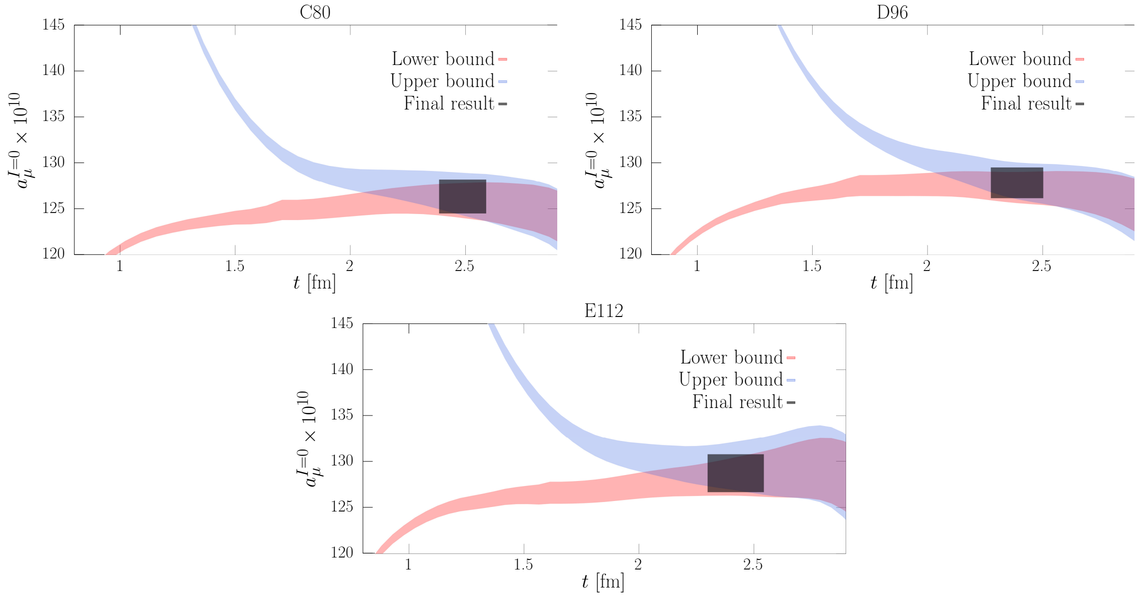
PRELIMINARY
χ² / dof = 0.3
c₀
c₀ + c₁^OS a²
609.7 ± 3.9
a_μ^HVP(I = 1) × 10¹⁰ (blinded)
a² [fm²]



PRELIMINARY
χ² / dof = 0.4
c₀
c₀ + c₁^OS a² + c₂^OS a⁴
609.9 ± 3.8
a_μ^HVP(I = 1) × 10¹⁰ (blinded)
a² [fm²]



PRELIMINARY
χ² / dof = 1.7
c₀ + c₁^tm a²
c₀ + c₁^OS a² + c₂^OS a⁴
603.3 ± 7.2
a_μ^HVP(I = 1) × 10¹⁰ (blinded)
a² [fm²]



C80
D96
E112
Lower bound
Upper bound
Final result
$a_\mu^{I=0} \times 10^{10}$
t [fm]
145
140
135
130
125
120
1
1.5
2
2.5