A model for light tensor meson transition form factors

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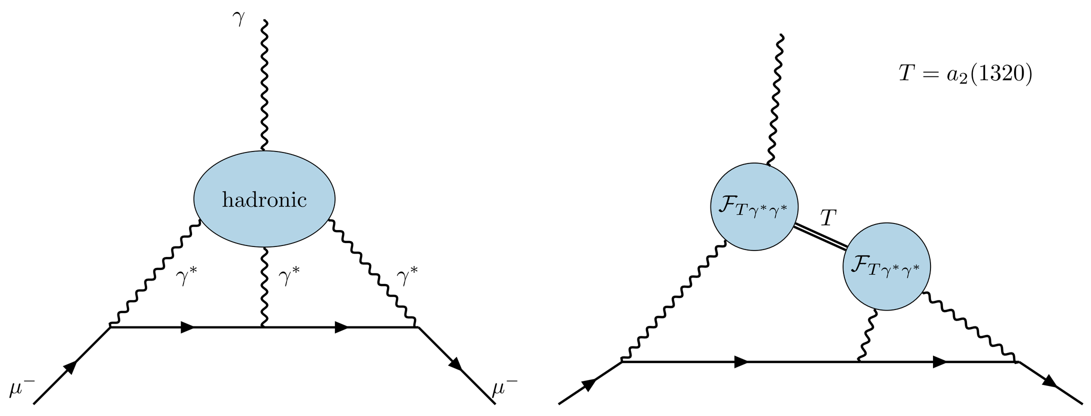
Motivation : $(g-2)_\mu$

Anomalous magnetic moment of the muon: hadronic contributions a_μ^{had}

- data-driven dispersive approach for hadronic contributions, hadronic vacuum polarization (HVP) and **hadronic light-by-light** (HLbL)

Focus on **tensor meson** resonance contribution to the HLbL



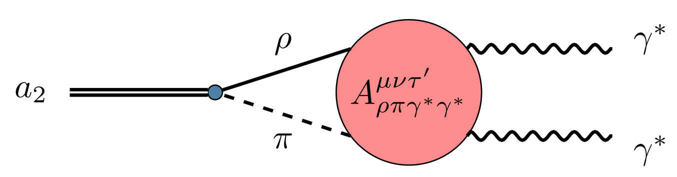
- description for the transition form factors (TFFs) $\mathcal{F}_{T\gamma^*\gamma^*}$, involving vector-meson dominance and dispersive treatment

Tensor mesons of $J^{PC}=2^{++}$

- multiplet of $\{a_2(1320), K_2^*(1430), f_2'(1525), f_2(1270)\}$
- specific case of $T=a_2(1320)$
  - ↪ utilize the main decay channel of 3π (or $\rho\pi$ via isobar model)
  - ↪ difficulty for phenomenological parameterization or modeling of $\mathcal{F}_{T\gamma^*\gamma^*}$ due to scarcity of data
- need 5 TFFs to describe the $T\to\gamma^*\gamma^*$ process

Framework

Model the dynamical behavior of $\mathcal{F}_{a_2\gamma^*\gamma^*}$ via $\rho\pi$ intermediate state

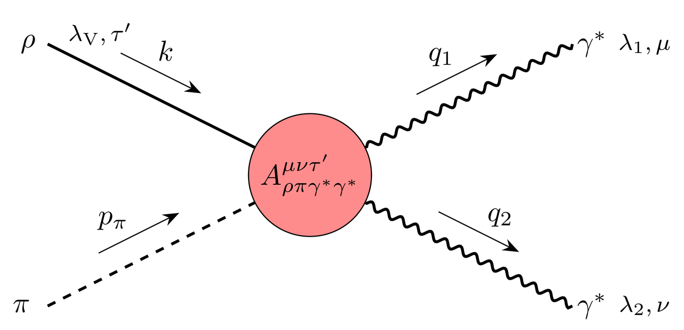


- framework based on ChPT including field representations for the pseudoscalar (P), vector (V_μ) and tensor ($T_{\mu\nu}$) multiplets [1] (containing only the relevant d.o.f. considering $a_2^0\to\rho^\pm\pi^\mp$)

$$P=\sqrt{2}\begin{pmatrix}0 & \pi^+ & 0\\ \pi^- & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\ ,\quad V_\mu=\sqrt{2}\begin{pmatrix}0 & \rho_\mu^+ & 0\\ \rho_\mu^- & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\ ,\quad T_{\mu\nu}=\begin{pmatrix}a^0_{2\mu\nu} & 0 & 0\\ 0 & -a^0_{2\mu\nu} & 0\\ 0 & 0 & 0\end{pmatrix}$$

- phenomenological description of $a_2\rho\pi$ interaction vertex $A^{\mu\nu\tau}_{a_2\rho\pi}$ with proper spin-2 tensor field representation → tracelessness, transversality

Connect to $\rho\,\pi\to\gamma^*\gamma^*$ amplitude



- Lorentz decomposition of amplitude as $A^{\mu\nu\tau'}_{\rho\pi\gamma^*\gamma^*}=\sum_i \mathcal{F}_i T_i^{\mu\nu\tau'}$
- employ BTT procedure [2–4] for the initial set of structures
  - ↪ ensure gauge invariance and freedom from kinematic singularities and zeros for the Lorentz structures $T_i^{\mu\nu\tau'}$
  - ↪ form linear combinations in order to remove poles
  - ↪ use Schouten identities to find a minimal basis
- dispersion relations for left-hand cuts $^{(*)}$
  - ↪ consider only one-particle ρ/π contributions
  - ↪ imaginary part of amplitude described in terms of $\{F_\pi^V, F_{\rho\pi}, G_1, G_2, G_3\}$
- derived a minimal basis of structures $\{\tilde{T}_i^{\mu\nu\tau'}\}_{i=1}^{13}$ and the corresponding form factors $\tilde{\mathcal{F}}_i$

Form factor derivation

Combine everything and calculate the amplitude

$$i\mathcal{M}^{\alpha\beta\mu\nu}_{a_2\gamma^*\gamma^*}=\sum_{i=1}^{13}\int\frac{d^4k}{(2\pi)^4}A^{s\ \ \alpha\beta\tau}_{a_2\rho\pi}\cdot\frac{(g_{\tau\tau'}-k_\tau k_{\tau'}/M_\rho^2)}{(k^2-M_\rho^2)}\frac{1}{[(k-p_{\mathrm{T}})^2-M_\pi^2]}\cdot\tilde{T}_i^{\mu\nu\tau'}\tilde{\mathcal{F}}_i$$

- solve integrals using Passarino–Veltman decomposition

Calculate helicity amplitudes

$$H_{\lambda_{\mathrm{T}};\lambda_1,\lambda_2}=\mathcal{M}^{\mu\nu\alpha\beta}_{a_2\gamma^*\gamma^*}\cdot\epsilon^{\lambda_{\mathrm{T}}}_{\alpha\beta}(p_{\mathrm{T}})\,\epsilon_\mu^{\lambda_1 *}(q_1)\,\epsilon_\nu^{\lambda_2 *}(q_2)$$

- non-vanishing helicity amplitudes satisfy : $\lambda_{\mathrm{T}}=\lambda_1-\lambda_2$

Results expressed $\propto\{f_i\}_{i=1}^4$, with

$$f_1=G_1(q_1^2)F_{\rho\pi}(q_2^2)\qquad f_2=G_1(q_2^2)F_{\rho\pi}(q_1^2)$$
$$f_3=F_\pi^V(q_1^2)F_{\rho\pi}(q_2^2)\qquad f_4=F_\pi^V(q_2^2)F_{\rho\pi}(q_1^2)$$

Use the BTT result for $\mathcal{M}^{\mu\nu\alpha\beta}_{T\gamma^*\gamma^*}$ [5] as a basis

$$\mathcal{M}^{\mu\nu\alpha\beta}_{T\gamma^*\gamma^*}=\sum_{i=1}^{5}T_i^{\mathrm{T}\mu\nu\alpha\beta}\frac{1}{m_T^{n_i}}\mathcal{F}_i^{\mathrm{T}}$$

and project our results onto the five $\mathcal{F}_i^{\mathrm{T}}$

- **Invariant amplitude approach**
  - ↪ Define projectors that isolate each TFF
  $$P_{i\mu\nu\alpha\beta}\,\mathcal{M}^{\mu\nu\alpha\beta}_{T\gamma^*\gamma^*}=\mathcal{F}_i^{\mathrm{T}}$$
  - ↪ Contract with our amplitude to find $\mathcal{F}_i^{\mathrm{T}}=\sum_{j=1}^4 c_{ij}\cdot f_j$
- **Helicity amplitude approach**
  - ↪ Calculation of helicity amplitudes in both cases
  $$\vec{H}=M_2\cdot\vec{f}\ ,\ \vec{H}^{\mathrm{T}}=M_1\cdot\vec{\mathcal{F}}^{\mathrm{T}}$$
  - ↪ Inverse matrix to find the relations
  $$\vec{\mathcal{F}}^{\mathrm{T}}=M_1^{-1}M_2\cdot\vec{f}\to\mathcal{F}_i^{\mathrm{T}}=\sum_{j=1}^4\tilde{c}_{ij}\cdot f_j$$

Further work

Further adaptations and ideas for future implementation:

- Consideration of structure basis that includes heavy ρ in LHCs
- Solve integrals and express results w.r.t. Feynman integrals
- Consider rescattering effects of the $\rho\pi$–state for the dispersive description

Final steps:

- Evaluate and finalize the transition form factor functions
- Treat potential UV–divergencies
- Study whether the form factors satisfy the required asymptotic behavior

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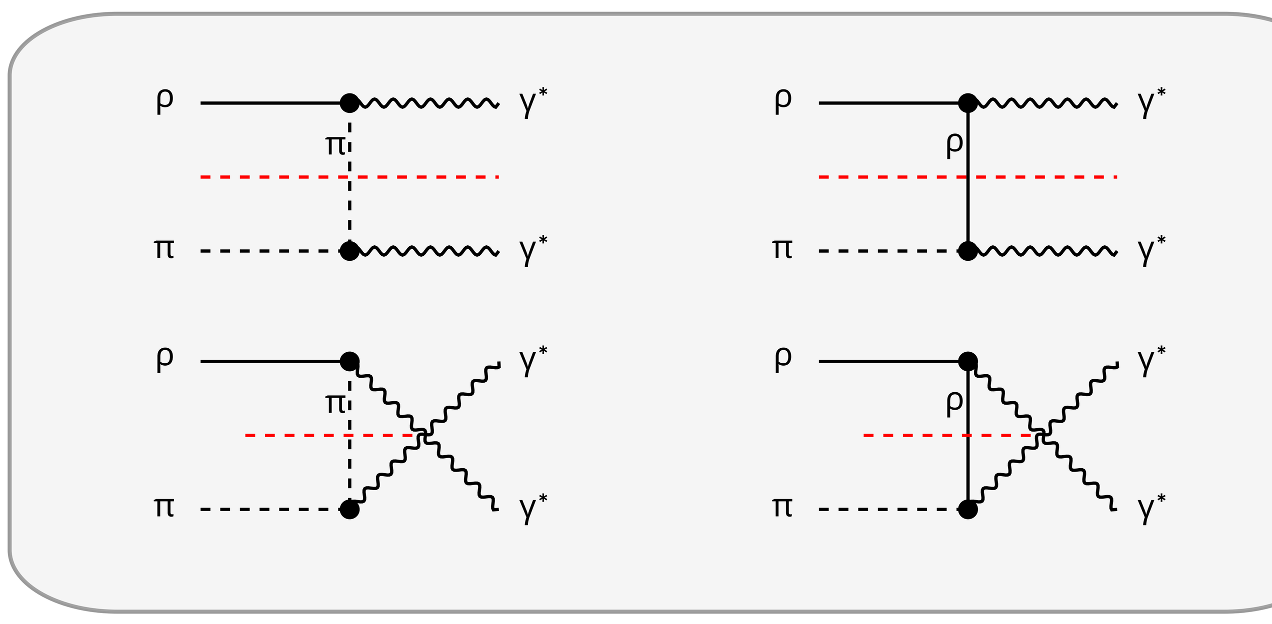

References

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