

# COMBINATION OF $2\pi$ AND $4\pi$ SPECTRA FROM TAU DECAYS AND APPLICATION TO MUON $g-2$

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## 8th Plenary Workshop Muon $g-2$ Theory Initiative

**ABSTRACT :** Due to the lack of agreement between different  $e^+e^-$  data sets for the  $\pi^+\pi^-$  channel, evaluating the HVP contribution to  $(g-2)_\mu$  from  $\tau$ -data input was again considered as an alternative. In this work, we discuss the HVP contribution from a new combination of the  $2\pi$ -channel from ALEPH, OPAL, and Belle  $\tau$ -data, without applying isospin breaking corrections. We also present a combination of ALEPH and OPAL  $\tau$ -data for the  $4\pi$ -channel, comparing the result with KNT19  $e^+e^-$  data using exact isospin symmetry.

## COMBINATION ALGORITHM

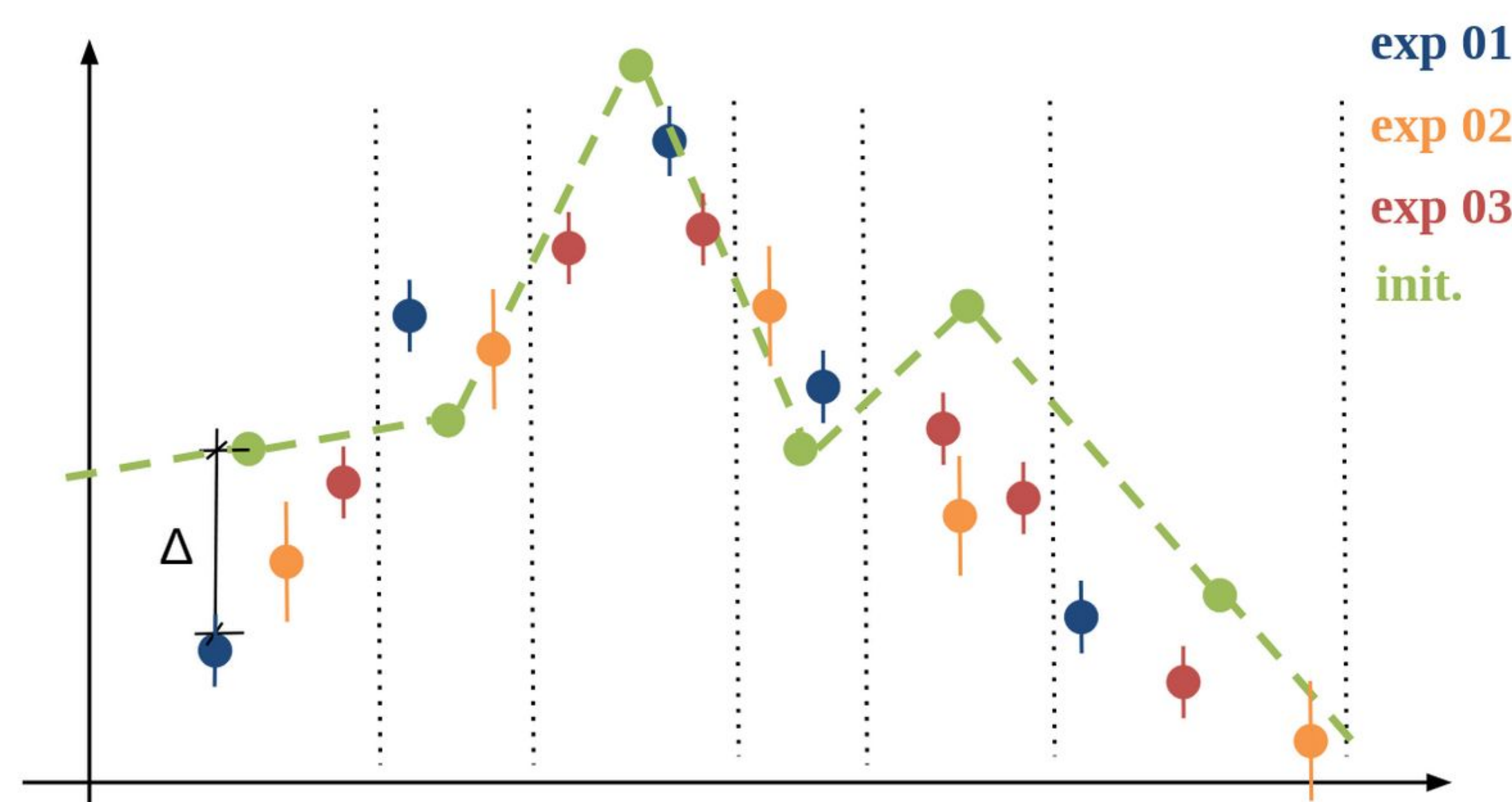
Boito, Eiben, Golterman, Maltman, Mansur, and Peris, **2502.08147**

Channel-by-channel combination adapted from the *KNT Algorithm*.

[Keshavarzi, Nomura, Teubner, '18]

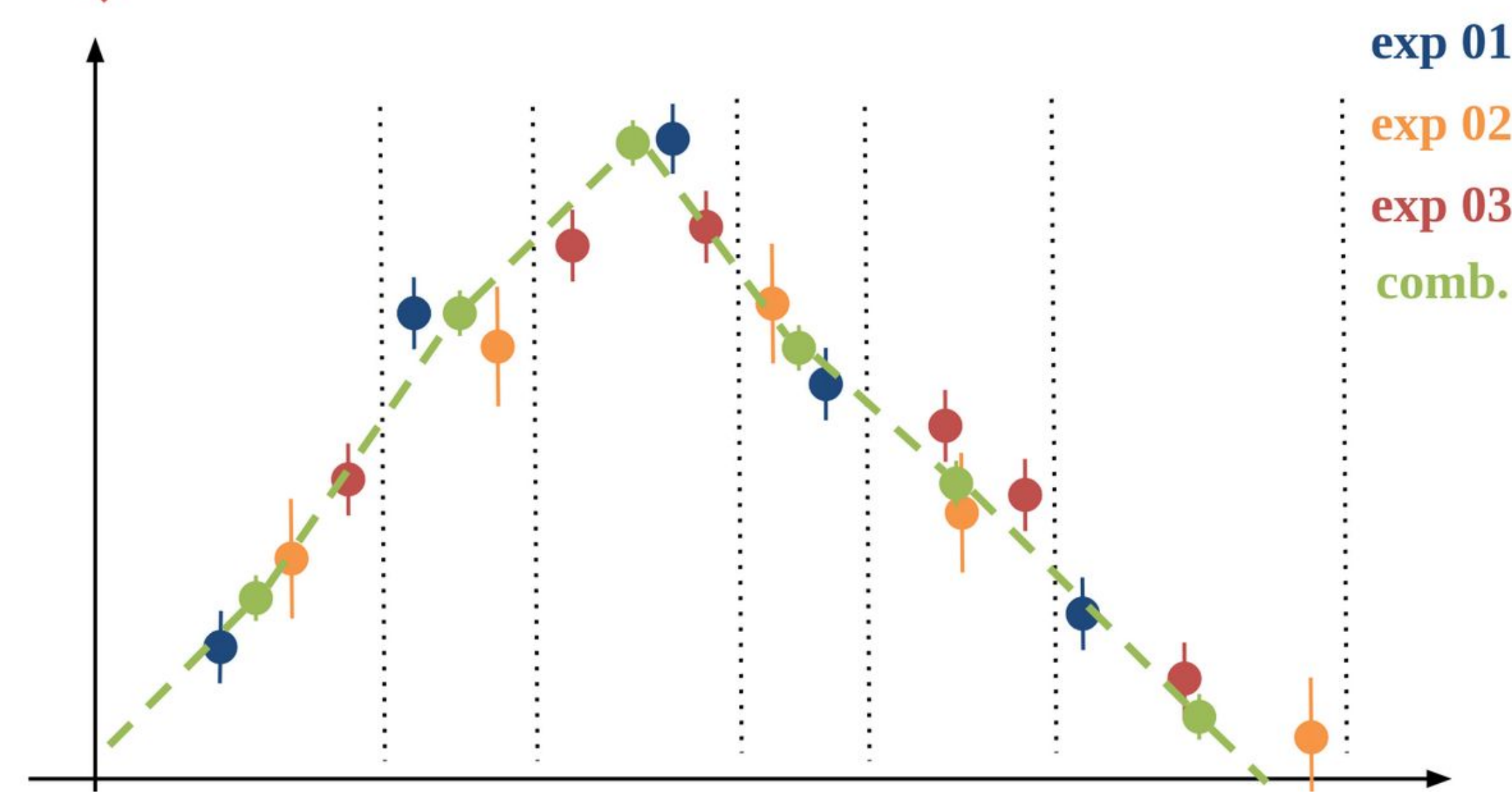
[Bruno & Sommer, '23]

1. Division of the spectrum into **Clusters**.



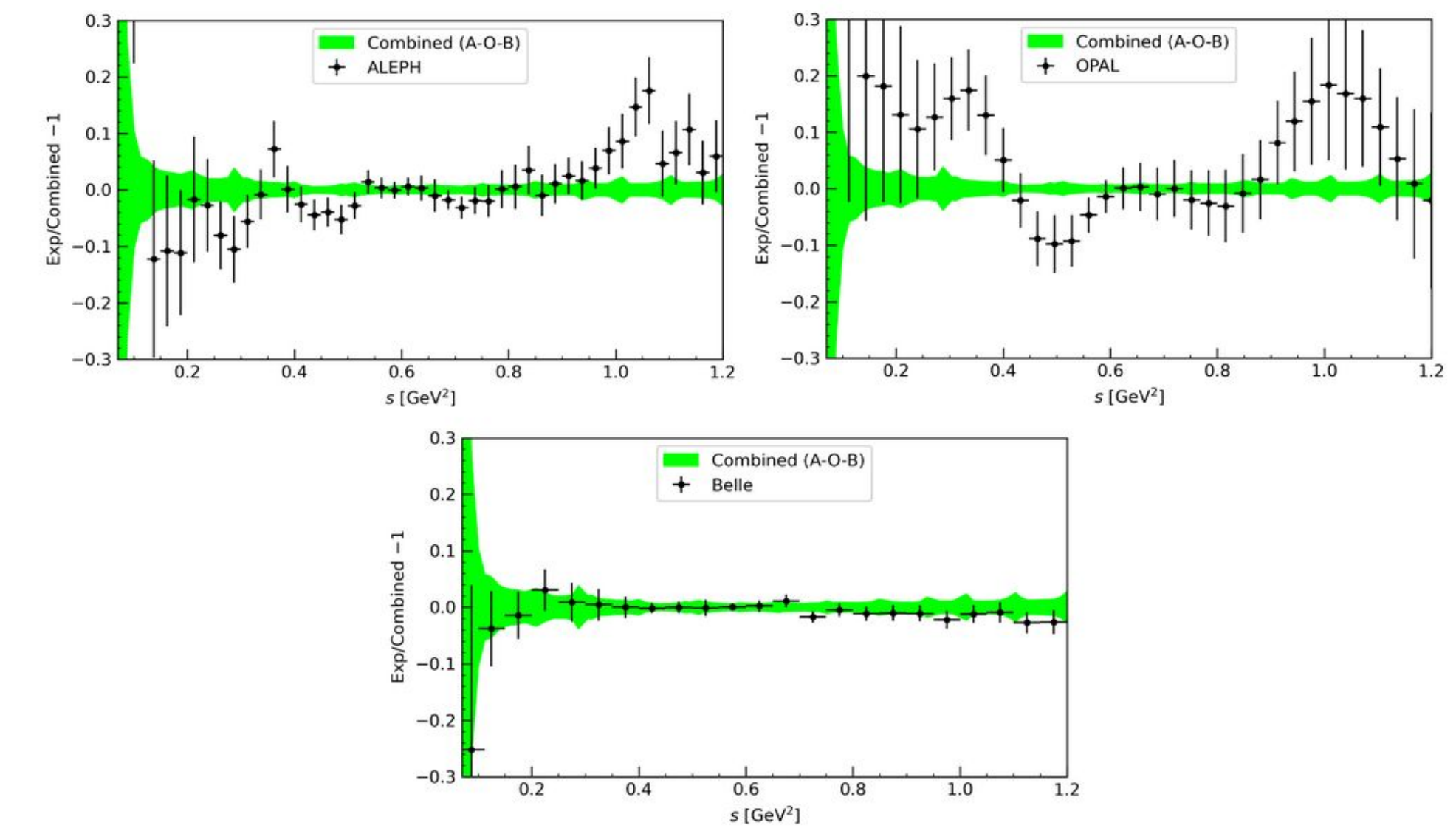
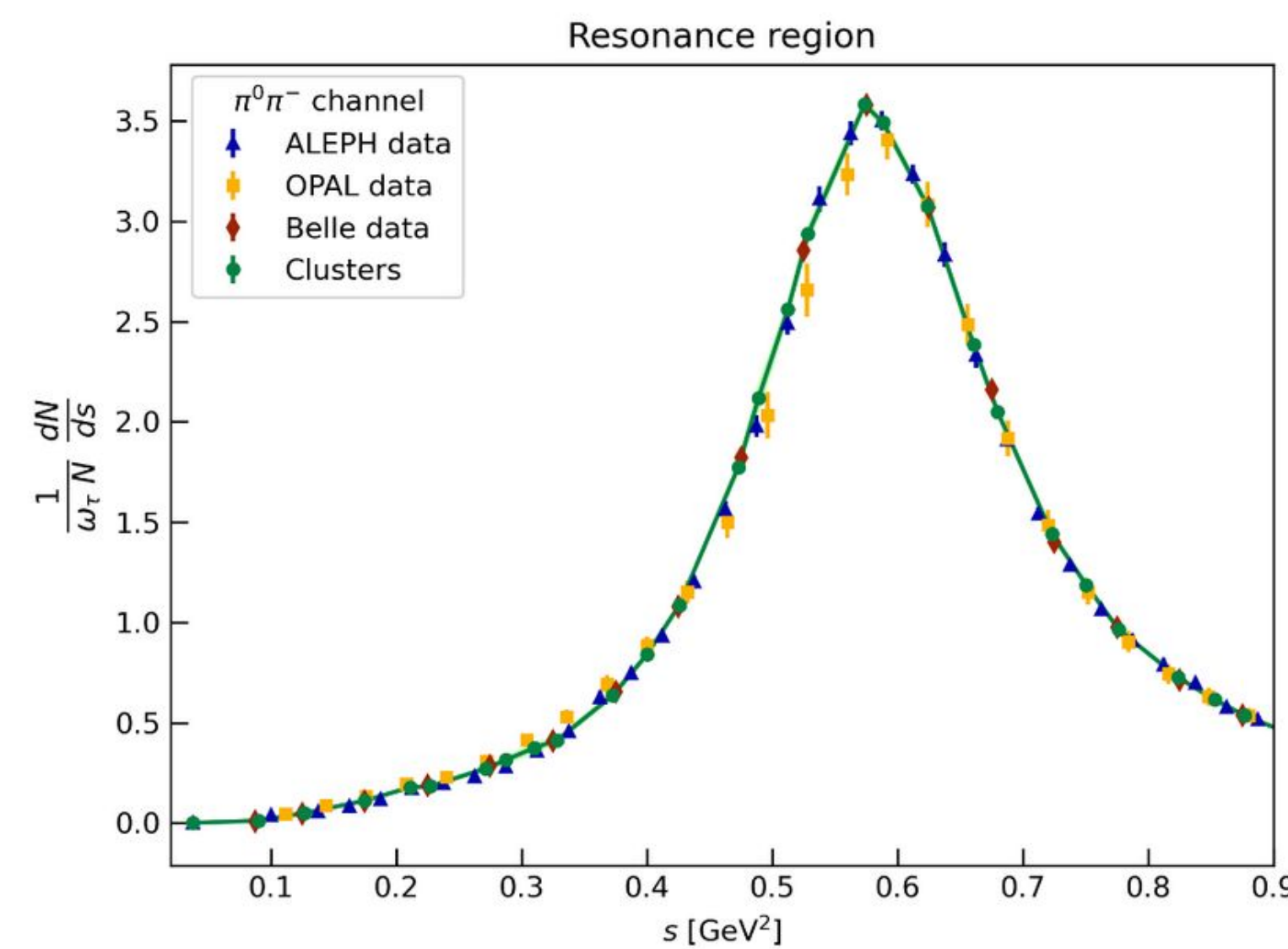
Minimization using a generalized  $Q^2$  function, and standard error propagation.

$$Q^2(\rho) = \sum_{i=1}^N \sum_{j=1}^N (d_i - R(s_i, \rho)) W_{ij} (d_j - R(s_j, \rho))$$



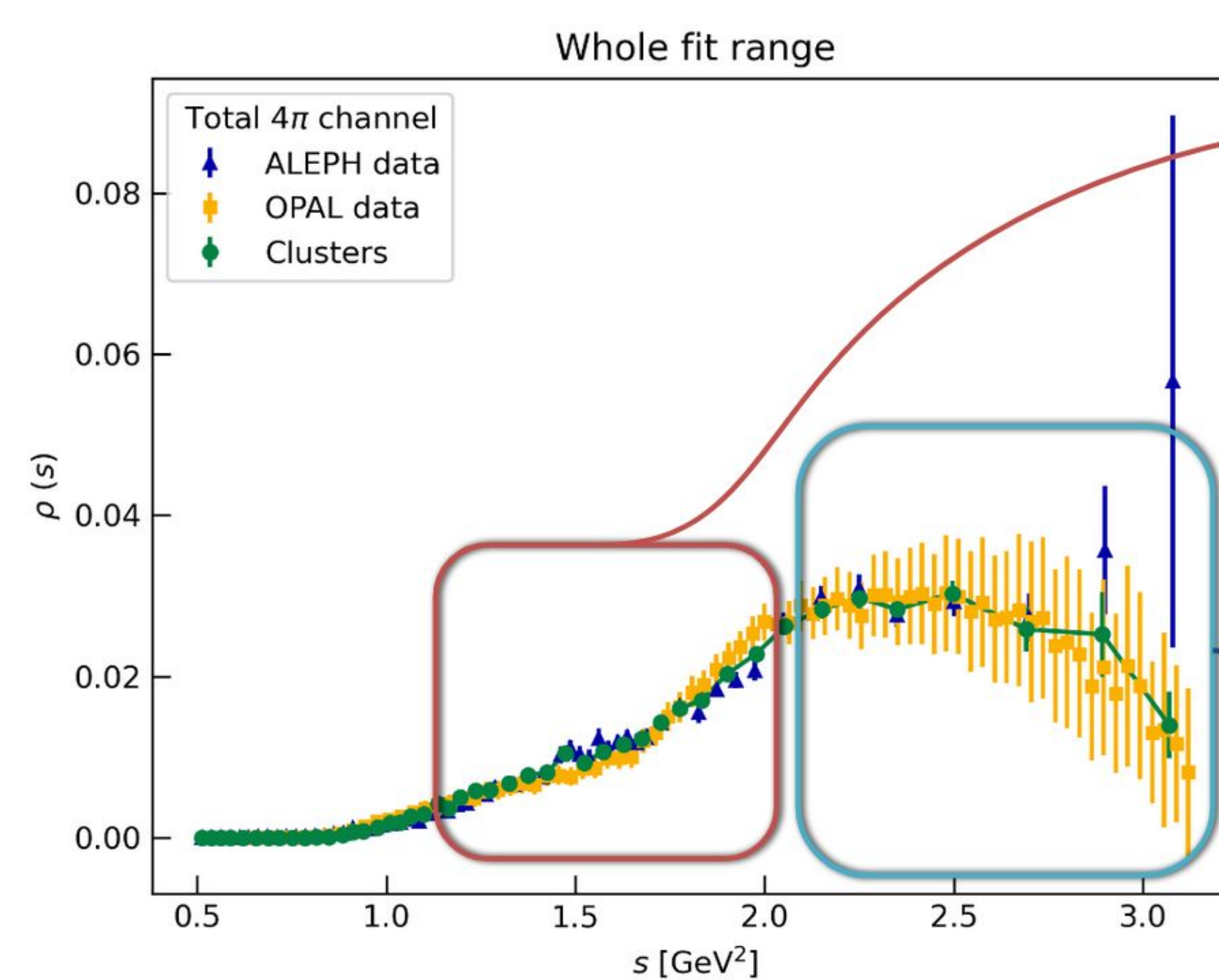
## COMBINATION OF EXCLUSIVE SPECTRA

### $2\pi$ channel results

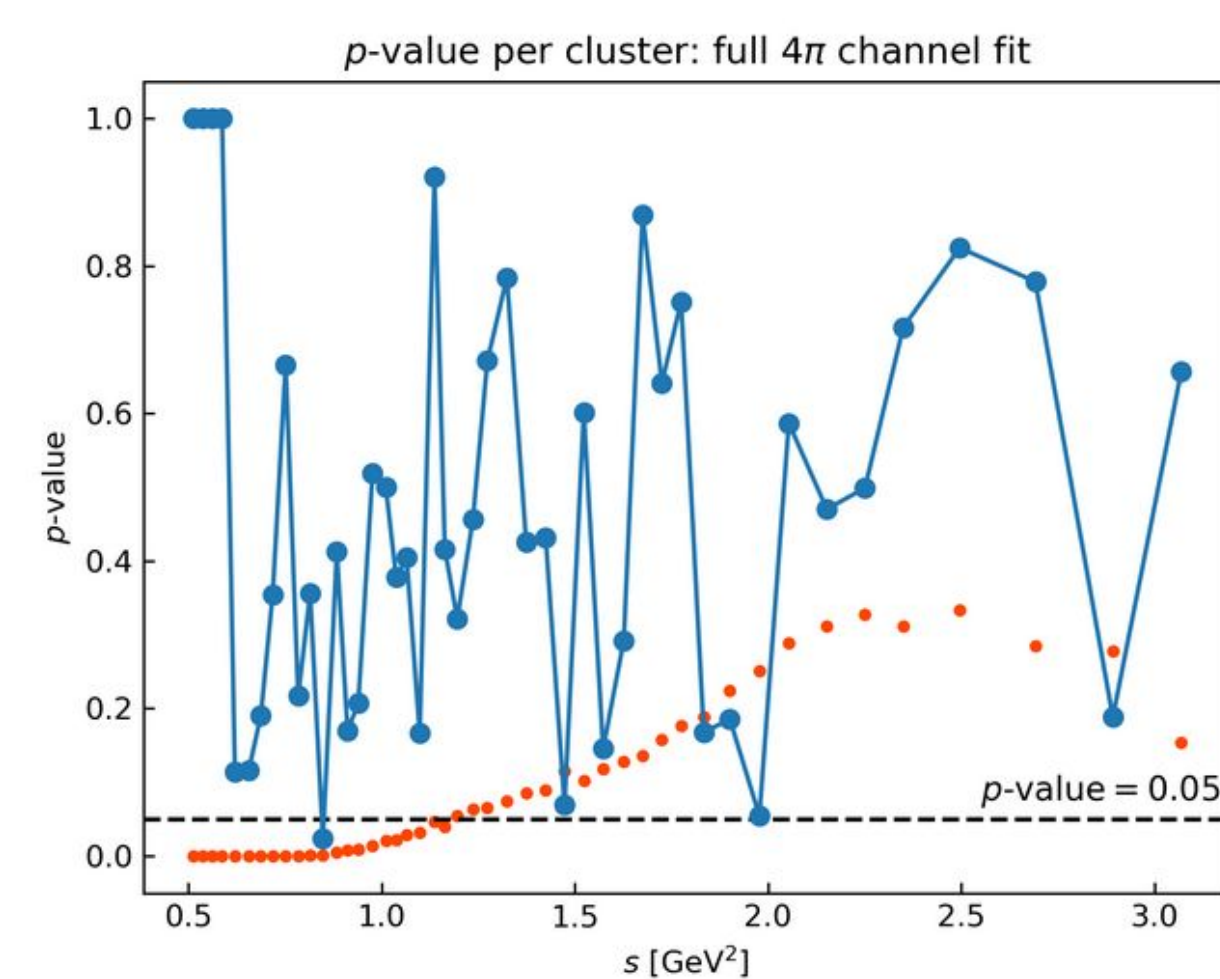
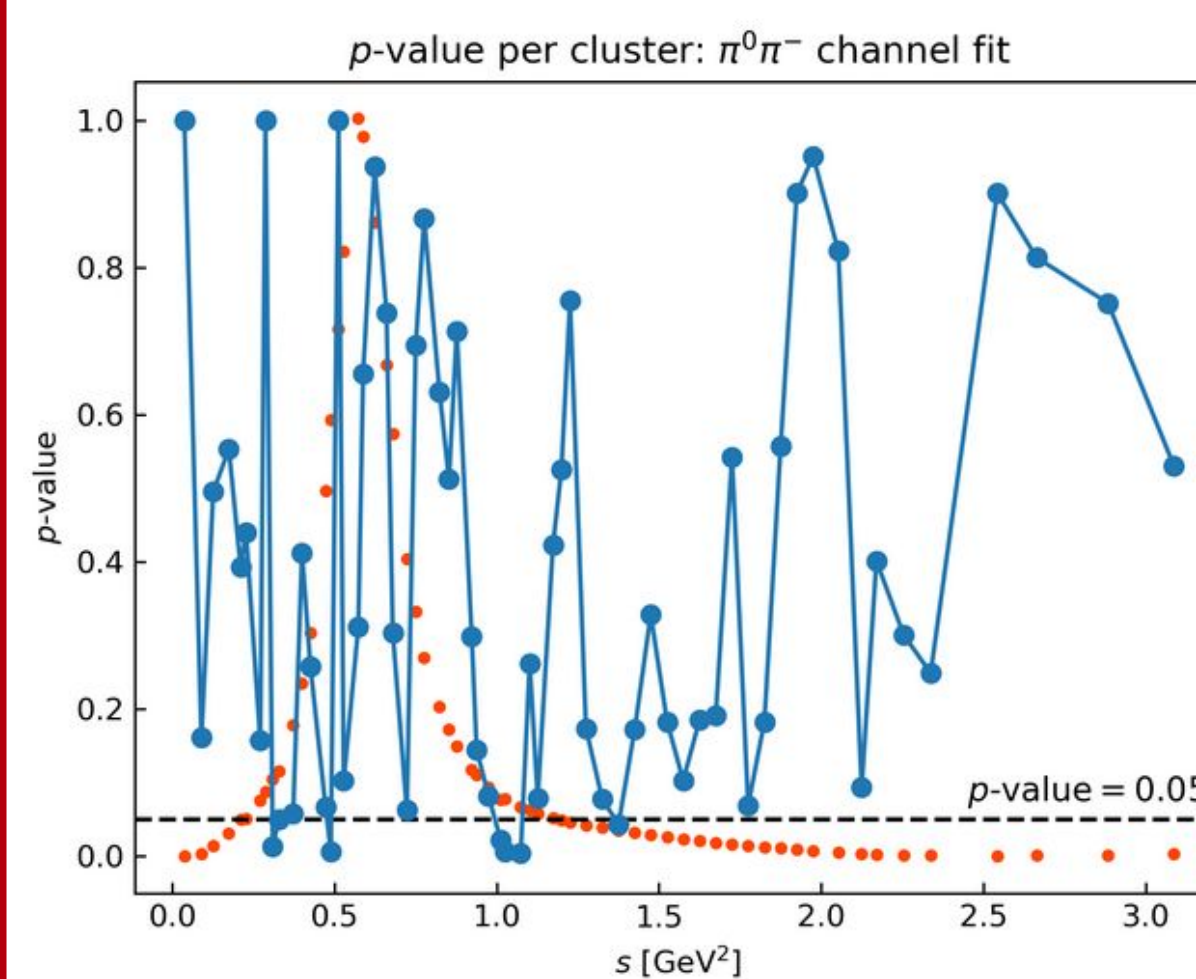
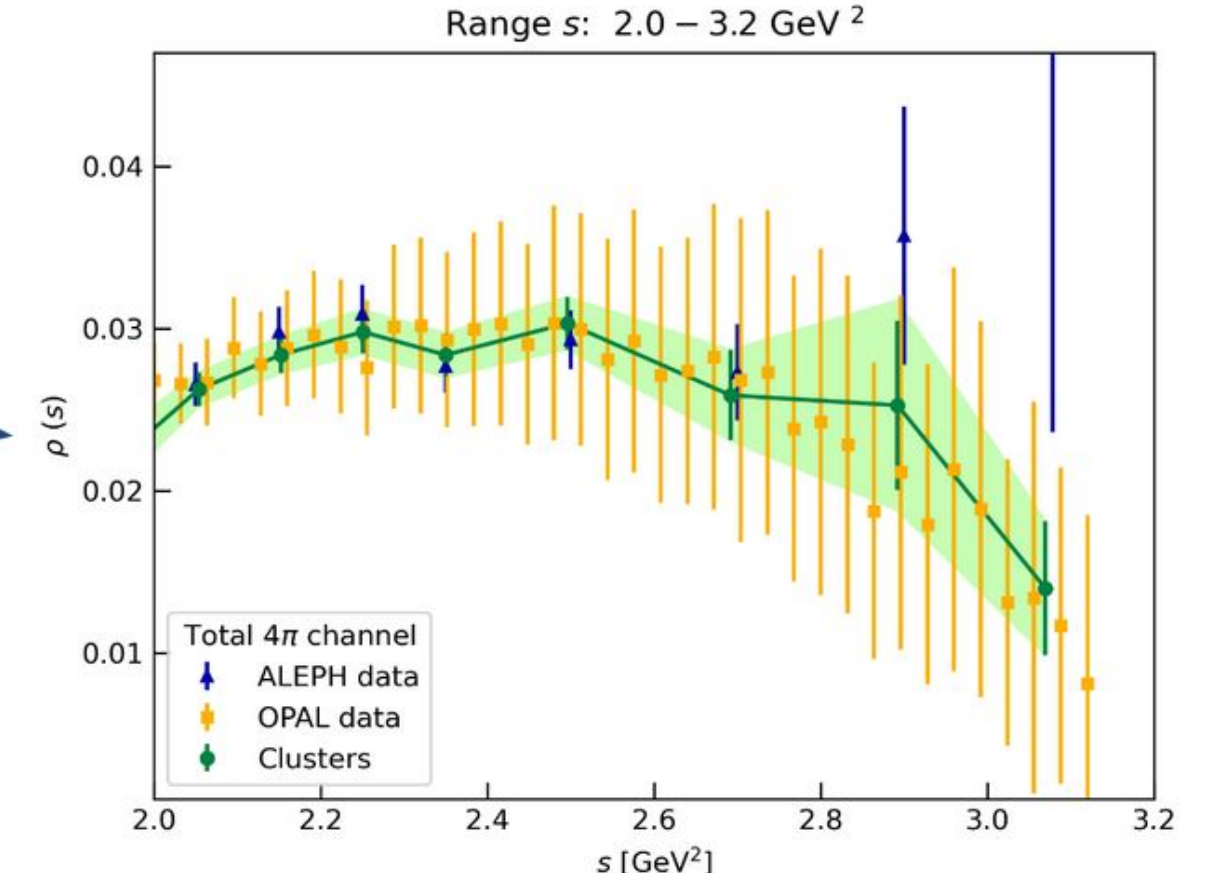
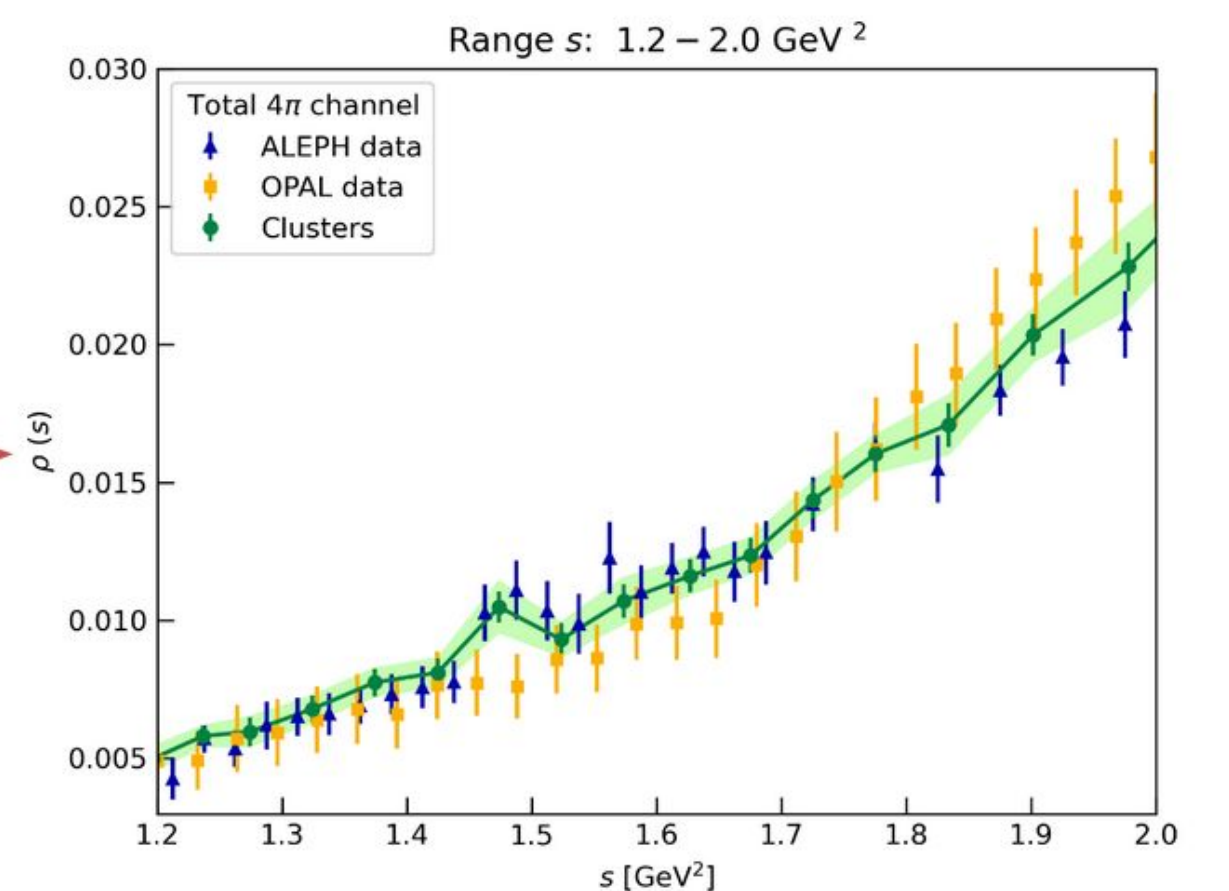


- For this particular channel, we combined the unit-normalized spectral function from **ALEPH, OPAL, and Belle** experiments (**CLEO not included in our main results** due to a lack of systematic uncertainties bin-by-bin).

### $4\pi$ channel results



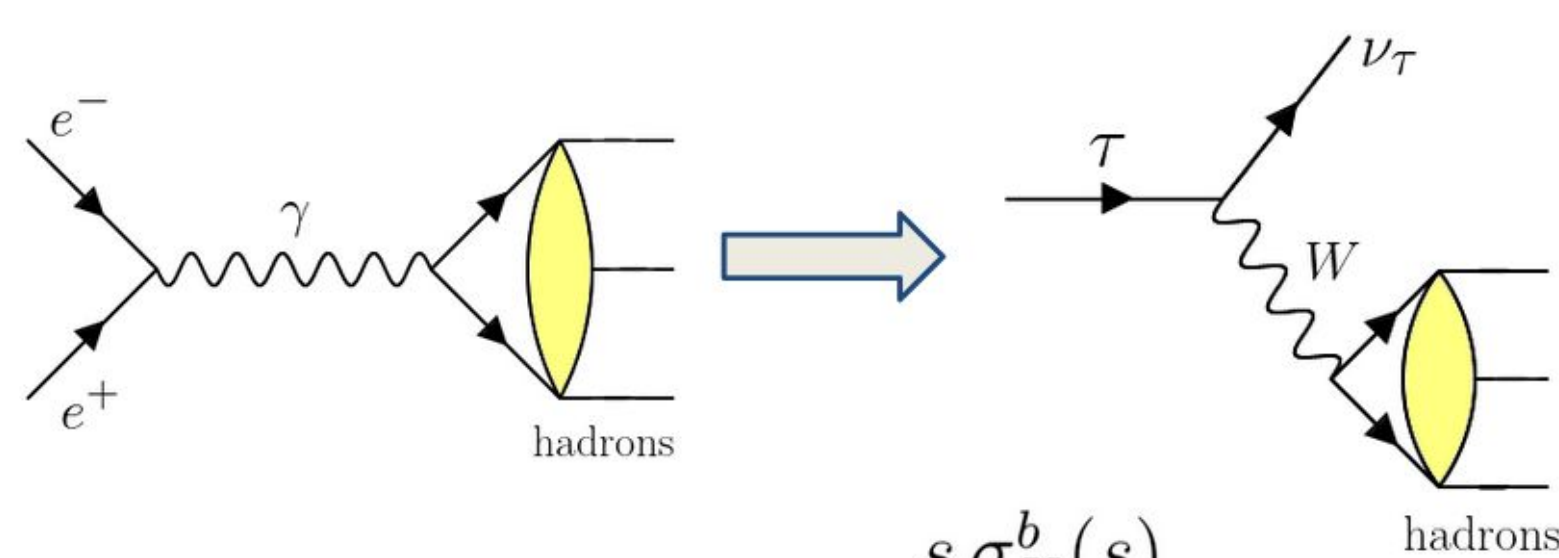
- Green bands** represent the inflated errors for the clusters with  $Q^2_{\text{cluster}} / \langle Q^2 \rangle > 1$



- Goodness-of-fit** computed using a Monte Carlo method for general  $Q^2$ -function minimizations.  
[Bruno & Sommer, '23]
- Adaptation of algorithm in the  $4\pi$  channel to deal with strong correlation in OPAL data, and avoid *d'Agostini bias* in multiplication by global factors.  
[Ball et al. (NNPDF), '10]

## HVP CONTRIBUTION IN THE $2\pi$ - CHANNEL

From the **Conserved Vector Current (CVC)** relation, we can associate the hadronic weak charged current and the isovector component of EM current by a factor



$$[\rho_{ud}; V(s)]_{X-} = \frac{s \sigma_X^b(s)}{8\pi^3 \alpha_{EM}^2}$$

without IB corrections  $\Rightarrow S_{EW} = 1.0$ ,  $R_{IB} = 0$

From our definition of spectral function

$$\rho_{\pi^\pm\pi^0}(s) = \frac{m_\tau^2}{12\pi^2 B_e |V_{ud}|^2 w_T^{av}(s; m_\tau^2)} \frac{B_{\pi^\pm\pi^0} dN(s)}{N ds}$$

we arrive at

$$a_\mu^{\text{HVP, LO}}[\pi\pi, \tau] = 2\alpha^2 \int_{4m_\pi^2}^{m_\tau^2} \frac{K(s)}{s} [\rho_{\pi^\pm\pi^0}(s)]$$

Using CVC, **without IB corrections** included, we find the results (considering only experimental uncertainties)

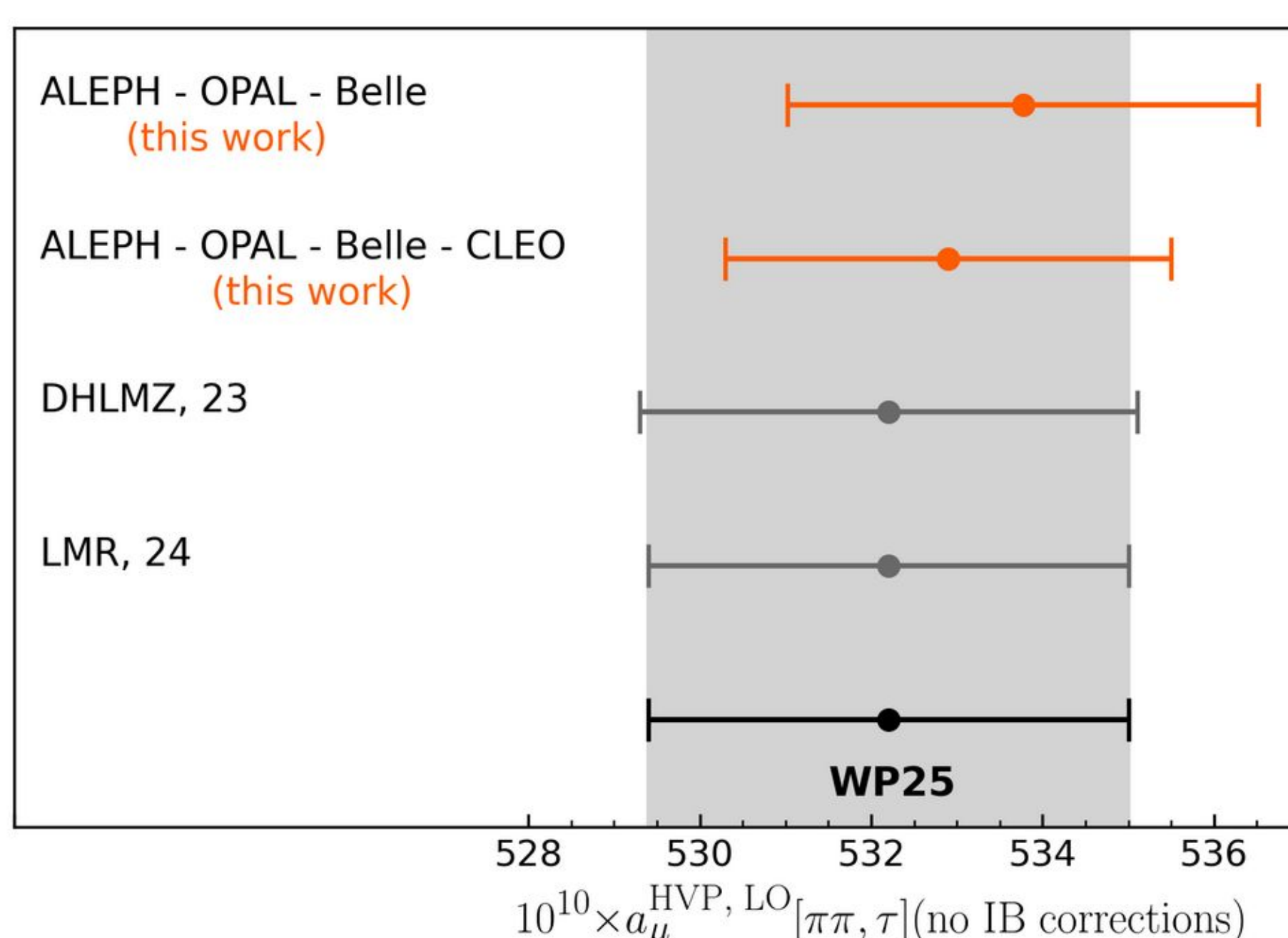
ALEPH (A), OPAL (O), Belle (B), and CLEO (C)

$$a_\mu^{\text{HVP, LO}}[\pi\pi, \tau](A-O-B) = 533.8(2.8) \times 10^{-10}$$

$$a_\mu^{\text{HVP, LO}}[\pi\pi, \tau](A-O-B-C) = 532.9(2.6) \times 10^{-10}$$

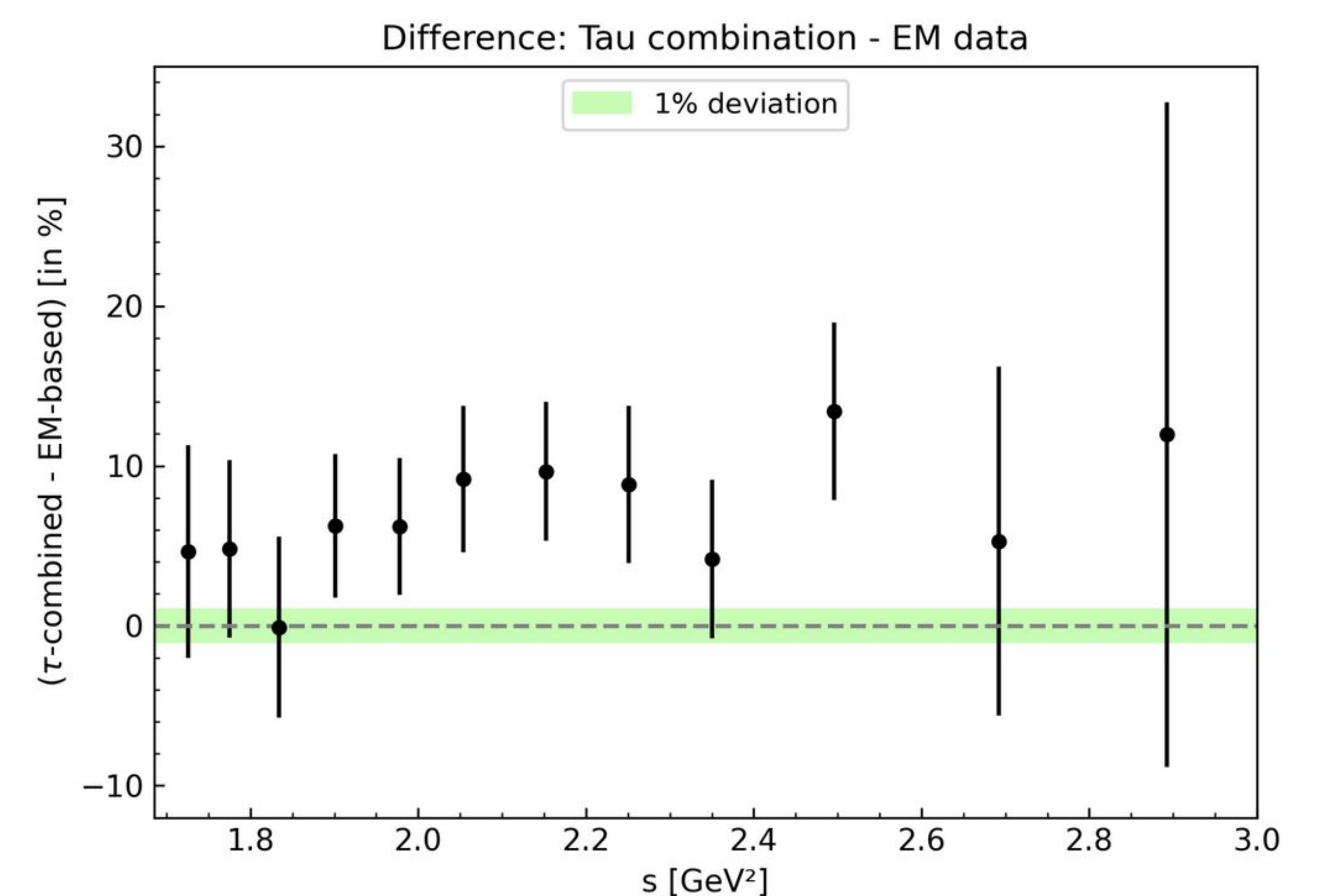
The reference value from the WP25 [Aliberti, et al., '25]

$$a_\mu^{\text{HVP, LO}}[\pi\pi, \tau](\text{WP25}) = 532.2(2.8) \times 10^{-10}$$



## $4\pi$ - CHANNEL: COMBINED vs. EM-BASED DATA

From the **Pais Relations**, we can obtain a EM-based spectral function using the data from KNT19 [Keshavarzi, Nomura, Teubner. 18'].



- The difference observed between EM-based data and the combined  $4\pi$ -channel from tau is **above the expected ~1% IB deviation** for these channels.
- The discrepancy appears to be **mostly** coming from the  $2\pi\pi^+\pi^0$  channel ("easier"  $\tau$ -decay mode).
- New experimental inputs** for the  $2\pi\pi^+\pi^0$  and  $\pi^3\pi^0$  can play an important role in the understanding of the above discrepancies.