HELICITY AMPLITUDES OF QED PROCESSES AT N^{3}LO

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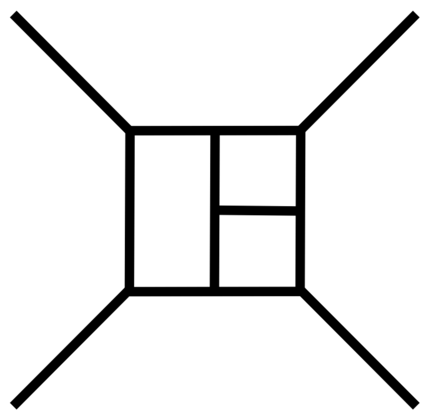
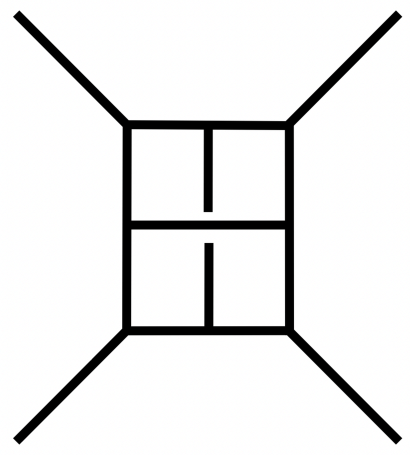




OUR MOTIVATION

In this research we aim to obtain analytical expressions for the helicity amplitudes for four QED processes within a massless model at N^{3}LO.

This will provide us with a first look at the complexity of calculations at this perturbative order, as well as providing an approximation for the helicity amplitudes.

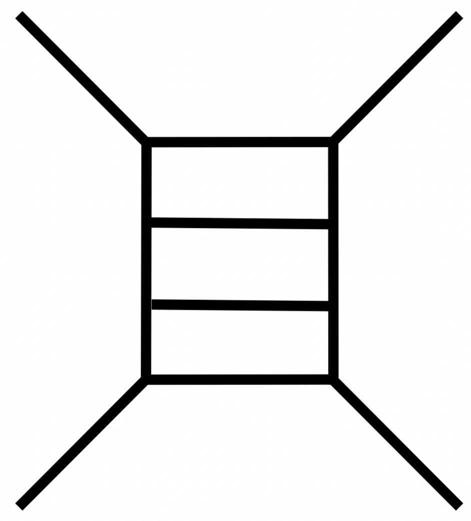
$$e^+e^- \to \mu^+\mu^- ,$$
$$e^+\mu^- \to e^+\mu^- ,$$
$$e^+e^- \to e^+e^- ,$$
$$e^+e^- \to \gamma\gamma .$$

WHY HELICITY AMPLITUDES?

Using tensor decomposition, we are able to obtain form factors that correspond to the coefficients of tensor structures within a helicity amplitude.

These form factors are comprised of Feynman integrals and form gauge invariant groups themselves. Therefore, we are able to perform all intermediate steps in scattering amplitude calculations on these expressions before reconstructing the helicity amplitudes in the final stages.

As we are able to compute numerical evaluations of these form factors we can use them obtain numerical evaluations of the interference for processes at higher orders, for example **|1-Loop|^2**.



HOW DOES TENSOR DECOMPOSITION WORK?

Tensor decomposition proceeds by identifying all possible tensor structures that can appear in an amplitude. From these structures, we can construct a projector that, when applied to a Feynman amplitude, isolates the corresponding form factors.

1) BUILD PROJECTORS

$$M_{ij} = \mathcal{T}_i^\dagger \mathcal{T}_j \qquad \mathcal{P}_i = \sum_j \left(M_{ij}\right)^{-1} \mathcal{T}_j$$

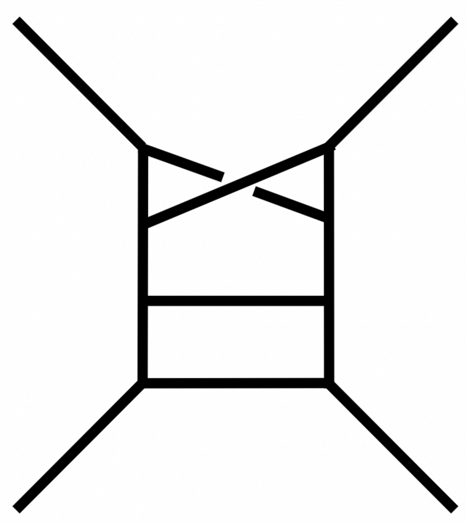
2) APPLY PROJECTORS

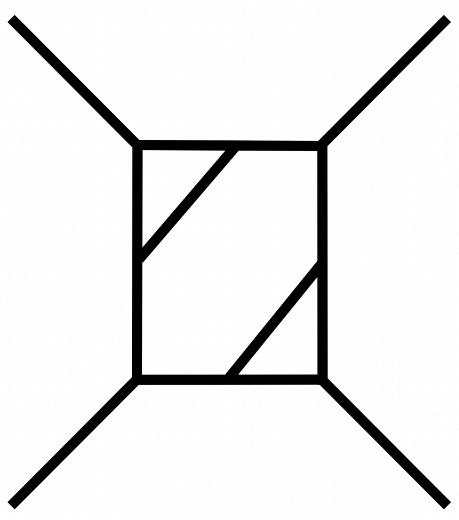
$$\mathcal{F}_i^{(l)} = \mathcal{P}_i \quad \mathcal{A}_B^{(l)}$$

Once we have our form factors, we can perform all intermediate steps. Then reconstruct the helicity amplitudes via:

3) RECONSTRUCT AMPLITUDE

$$\mathcal{A}^{(l)} = \sum_{i=1}^{n} \mathcal{F}_i^{(l)} \, \mathcal{T}_i^{(l)}$$





METHODOLOGY

Diagram generation + Tensor Decomposition

Here we generate all diagrams relevant to a process at a given loop order.

We then perform a tensor reduction to produce form factors that only contain scalar products of the momenta. Confining all of the complexity of spinors in tensor structures.

Topology mapping

In this stage we group Feynman Diagrams into "Families" that share internal propagators.

Some diagrams must undergo shifts of loop momenta to group them into a family.

Integration-by-parts Reduction

The form factors we have contain many Feynman integrals. This technique allows us to write these expressions in terms of a minimal basis of Feynman integrals known as Master Integrals.

Differential equations

We can now find analytical expressions for the master integrals that appear in our expressions, by using differential equations to form a power series in the dimensional regulator.

These expressions take the form of generalised polylogarithms.

Removal of Poles

At this stage we have a power series with negative powers of epsilon.

These are singularities that need to be removed. We achieve this through UV Renormalisation and IR Subtraction.

Construct the Helicity Amplitudes

Lastly, we can multiply our finite expressions for the form factors with the tensor structures they have been decomposed into. We consider each of the possible spin configurations for each process to obtain helicity amplitudes.

Now we are finished!

PROGRESS AND CONCLUSIONS

- So far we have managed to reduce all form factors for **$e^+e^- \to \mu^+\mu^-$** to a set of master integrals which we will be able to express in terms of generalised polylogarithms.
- From this process we will be able to find the helicity amplitudes for both **$e^+\mu^- \to e^+\mu^-$** as well as **$e^+e^- \to e^+e^-$**, using external momenta crossings.
- Good progress is also being made with **$e^+e^- \to \gamma\gamma$**. The tensor decomposition of all amplitudes into form factors has been completed, and integration-by-parts reduction is ready to take place on these form factors.

Once we have obtained helicity amplitudes for all processes, we will be able to use these expressions in Monte Carlo generators to approximate the contributions from QED processes at N^{3}LO to the anomalous magnetic moment of the muon.

We also aim to apply these same techniques to other processes relevant to the g-2 anomaly, such as **$e^+e^- \to \pi^+\pi^-\gamma$**. Early studies into the tensor decomposition have been very promising and will be very useful towards finding the NNLO contributions to the g-2 anomaly from this process.

RELEVANT WORKS

[1] [2] [3]

Check out our NNLO Paper!

THANK YOU FOR READING AND PLEASE FEEL FREE TO ASK ANY QUESTIONS!