The pion form factor in a coupled-channel system

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Coupled-channel formalism

As discussed in [1]: Main tension between $\pi\pi$ data sets is absorbed in inelastic parameters

$\Rightarrow$ those parameters belong to exclusive channels – what happens if we build them in explicitly?

Idea: Do different $\pi\pi$ data sets agree better with the other exclusive channels?
We construct three different channel structures:

- pseudoscalar + pseudoscalar ($\pi\pi, K\bar{K}$) $[PP]$
- vector + pseudoscalar ($\rho\eta, \omega\pi, K\bar{K}^*$) $[VP]$
- axial vector + pseudoscalar ($a_1\pi$) $[AP]$

Components [2]

The $\mathcal{S}$-, $\mathcal{M}$-, and $\mathcal{T}$-matrices:

$$_{\text{out}}\langle q_1 q_2, b|\, \mathcal{S} - 1\, |p_1 p_2, a\rangle_{\text{in}} = i(2\pi)^4 \delta^4(p_1 + p_2 - q_1 - q_2)\, \mathcal{M}_{ba}\,,$$

$$\mathcal{M}(s)_{ab} = \xi_a(s)\, \mathcal{T}(s)_{ab}\, \xi_b(s)\,,$$

with

$$\begin{aligned} \rho(s) &= \text{phase space factor},\\ \xi(s) &= \beta(s)B(s) = \text{centrifugal barrier factor},\\ \beta(s) &= \text{interaction structure function},\\ B(s) &= \text{taming factor}. \end{aligned}$$

We are using the two-potential formalism and divide $\mathcal{T}$ into resonant and non-resonant (background) parts

$$\mathcal{T}_{ab} = \mathcal{T}^{\text{B}}_{ab} + \mathcal{T}^{\text{R}}_{ab}$$

Background potential

As background we use a contact interaction term

$$V_{\text{B}}(s) = f_0 \qquad \Longrightarrow \qquad \mathcal{T}_{\text{B}} = (1 - V_{\text{B}}\Pi)^{-1}\, V_{\text{B}}$$

The loop integral of two non-interacting particles is dispersively defined as

$$\Pi(s) = \Pi(s_0) + \frac{s - s_0}{2\pi i} \int_{s_{\text{thr}}}^{\infty} \frac{\mathrm{d}s'}{s' - s_0} \frac{\text{disc}\,\big(\Pi(s')\big)}{s' - s}$$
$$\text{with disc}\,\big(\Pi(s)\big) = 2i\xi(s)\rho(s)\xi(s)\,.$$

Two particle interaction is dressed by the background interaction

$$\gamma^{\dagger}_{\text{in}} = 1 + \Pi\mathcal{T}_{\text{B}}\,, \qquad \gamma_{\text{out}} = 1 + \mathcal{T}_{\text{B}}\Pi\,,$$
$$\text{with disc}\,\big(\gamma(s)\big) = 2i\mathcal{M}^*_{\text{B}}(s)\rho(s)\gamma(s)\,.$$

Resonance potential

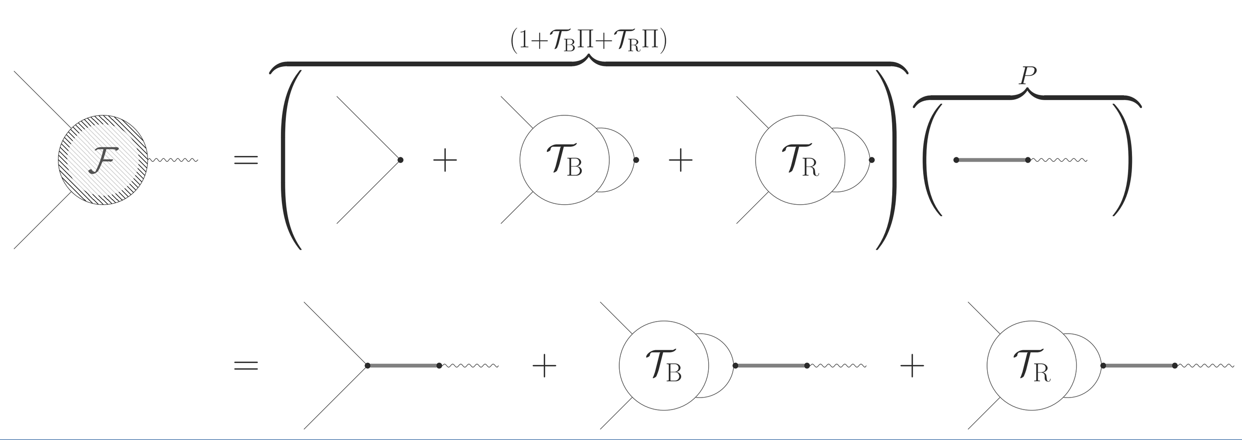
The resonance interaction is defined via

$$V_{\text{R}}(s) = -g^T G_{\text{R}}(s) g\,, \quad G^{kl}_{\text{R}}(s) = \frac{\delta_{kl}}{s - m_k^2} \quad \Longrightarrow \quad \mathcal{T}_{\text{R}} = \gamma_{\text{out}}\,(1 - V_{\text{R}}\Sigma)^{-1}\, V_{\text{R}}\gamma^{\dagger}_{\text{in}}$$

$$\Sigma(s)_{ij} = \Sigma(s_0)_{ij} + \frac{s - s_0}{2\pi i} \int_{s}^{\infty} \frac{\mathrm{d}s'}{s' - s_0} \frac{\text{disc}\,\big(\Sigma(s')\big)_{ij}}{s' - s}$$
$$\text{with disc}\,\big(\Sigma(s)\big) = 2i\xi(s)\gamma(s)^*\rho(s)\xi(s)\gamma(s).$$

Form factor and Observables

$$F = \gamma_{\text{out}}\,(1 - V_R\Sigma)\, P \quad \text{with} \quad P_i = \sum_R \frac{\alpha^R g_i^R}{s - m_R^2}$$



Channels

PP channel:

$$\langle P(q_1)\bar{P}(q_2)|\, j^\mu(0)\, |0\rangle = (q_1 - q_2)^\mu F_P^V(s)\,,$$
$$\beta_{PP}(s)^2 = \frac{1}{3}(q_1 - q_2)^\mu g_{\mu\nu}(q_1 - q_2)^\nu\,,$$
$$\beta_{PP}(s) = \sqrt{\frac{s - 4M_P^2}{3}}\,.$$

VP channel:

$$\langle P(q_0)V(q_V,\lambda)|\, j^\mu(0)\, |0\rangle = \epsilon^{\mu\nu\alpha\beta} n^\lambda_\nu q_{0\alpha} k_\beta f_{VP}(s)\,,$$
$$\beta_{VP}(s)^2 = \frac{1}{3}\sum_\lambda \left(\epsilon^{\mu\nu\alpha\beta} n^\lambda_\nu q_{0\alpha} k_\beta\right)^* g_{\mu\rho} \left(\epsilon^{\rho\gamma\eta\delta} n^\lambda_\gamma q_{0\eta} k_\delta\right)\,,$$
$$\beta_{VP}(s) = \sqrt{\frac{(s - (M_P + M_V)^2)(s - (M_P - M_V)^2)}{6}}\,.$$

AP channel:

$$\langle a_1(q_1,\lambda)\pi(q_2)|\, j^\mu(0)\, |0\rangle = n^\lambda_\nu \left(T_1^{\mu\nu} F_1(s) + T_2^{\mu\nu} F_2(s)\right) \text{ [3]}$$

Definition of structure function more involved

Resonances in loop
$\rho\eta$, $K^*\bar{K}$ and $a_1\pi$ channel have resonances (R) with non-negligible width. In construction of Π_i we use spectral density to smear out discontinuity

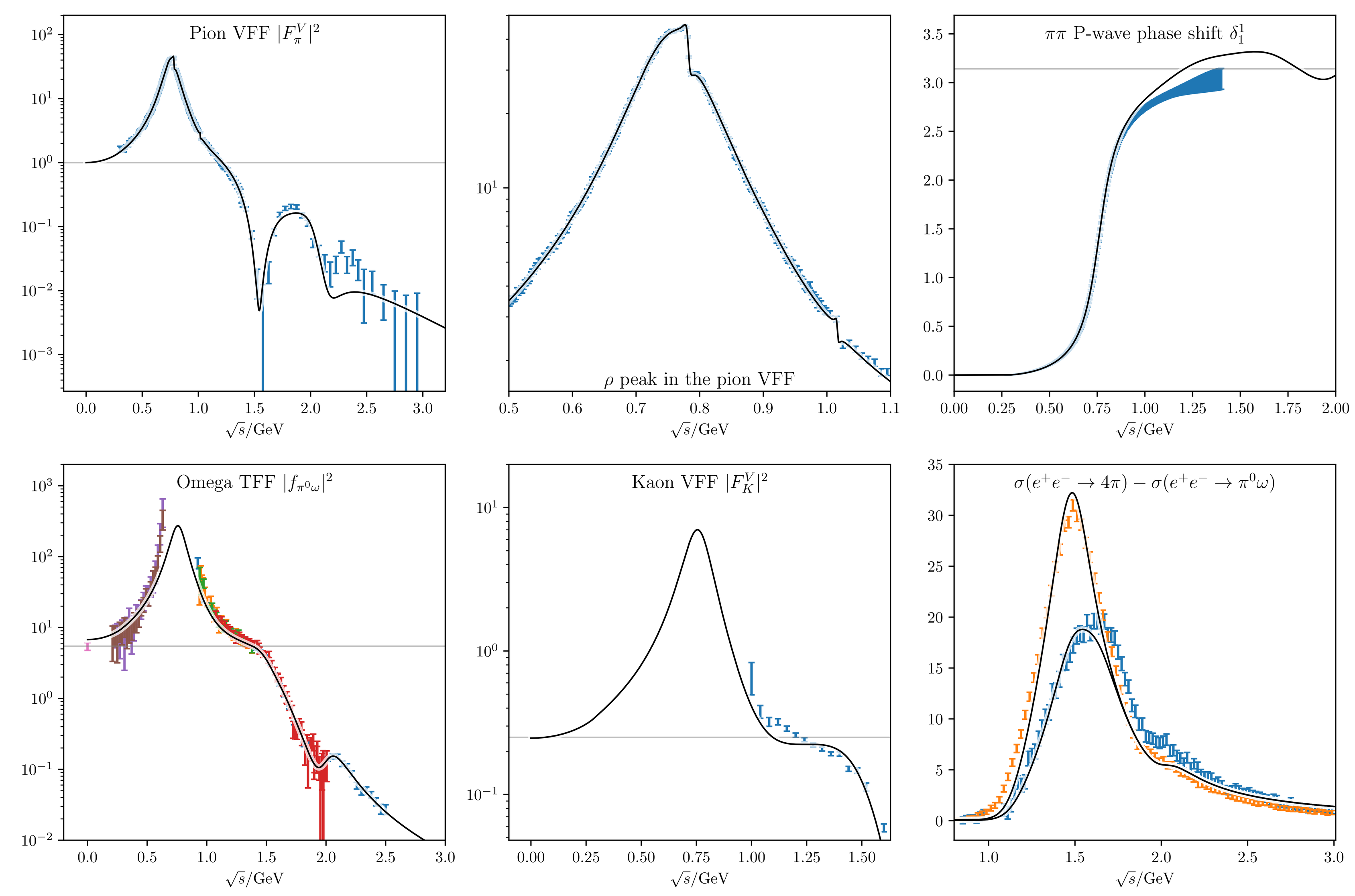
$$\Pi_{RP}(s) = \frac{s - s_0}{\pi} \int_{(\sqrt{s_{\text{decay}}}+m_P)^2}^{\infty} \frac{\int_{s_{\text{decay}}}^{(\sqrt{s'}-m_P)^2} \rho(s', m_R^2, m_P^2)\,\xi^2_{RP}(s', m_R^2, m_P^2)\,\sigma_R(m_R^2)\,\mathrm{d}m_R^2}{(s' - s)(s' - s_0)}\,\mathrm{d}s'$$

Loops have complicated analytic structure [4]

Effective 4π
We use the decay chain $a_1\pi \to \rho\pi\pi \to 4\pi$

- decouple structures into two different channels via change of basis
- have to take care of symmetry factors and charge states
  $\to$ simulated different interference effects in charged and partially charged final states
- smearing out complicated: pole positon of a_1 not well-known
  $\to$ use τ-decay data for fit of spectral density of a_1

Preliminary results



References

[1] P. Stoffer, G. Colangelo, and M. Hoferichter, JINST **18**, C10021 (2023) [arXiv:2308.04217 [hep-ph]].

[2] L. A. Heuser, G. Chanturia, F. K. Guo, C. Hanhart, M. Hoferichter, and B. Kubis [arXiv:2403.15539 [hep-ph]].

[3] G. Colangelo, M. Hoferichter, M. Procura, and P. Stoffer, JHEP **09**, 074 (2015) [arXiv:1506.01386 [hep-ph]].

[4] M. Döring, C. Hanhart, F. Huang, S. Krewald, and U.-G. Meissner, Nucl. Phys. A **829**, 170 (2009) [arXiv:0903.4337 [nucl-th]].

HISKP bcgs bctp Dispersion Relations

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