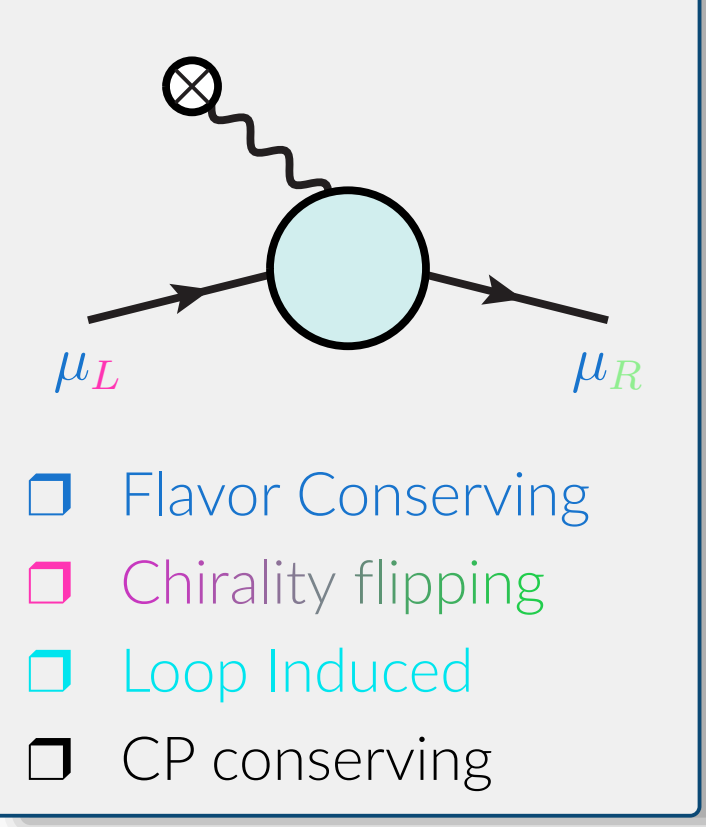


Structure of Δa_μ

Key Properties



BSM with generic couplings $c_{L,R}$ of the left- and right-handed muons:

① loop preserves muon chirality

$$\mu_R \rightarrow \mu_L \sim \frac{|c_{L,R}|^2 m_\mu^2}{16\pi^2 M^2}$$

② loop flips muon chirality

$$\mu_R \rightarrow \mu_L \sim R_\chi \times \frac{c_{L,R} m_\mu^2}{16\pi^2 M^2}$$

⇒ chiral enhancement R_χ

Relationships to other observables

Other dipole observables

In SM: no CP or flavour violation in lepton sector

$$\Rightarrow d_{e,\mu,\tau}^{\text{SM}} \sim 10^{-40} e \cdot \text{cm}$$

$$\text{BR}(\mu \rightarrow e\gamma)_{\text{SM}} \sim 10^{-50}$$

BSM scenarios often CPV / LFV
⇒ prediction of clear signal

Observables related via dipole operator

$$\mathcal{L} \supset L_{e\gamma} \bar{e} \sigma^{\alpha\beta} e_{Rj} F_{\alpha\beta} + h.c.$$

CPV phase:

$$\phi_\mu = \arg L_{e\gamma 22}$$

LFV parameter:

$$\theta_{ij} = L_{e\gamma ij} / L_{e\gamma 22}$$

$$\frac{d_\mu [e \cdot \text{cm}]}{4.1 \times 10^{-30}} \approx \left(\frac{\Delta a_\mu}{10^{-9}} \right) \left(\frac{\tan \phi_\mu}{2000} \right)$$

$$\frac{\text{BR}(\mu \rightarrow e\gamma)}{4.2 \times 10^{-13}} \approx \left(\frac{\Delta a_\mu}{10^{-9}} \right) \frac{\theta_{12}^2 + \theta_{21}^2}{(5 \times 10^{-5})^2}$$

⇒ large CPV still possible, but LFV strongly constrained

Muon-Higgs coupling $\lambda_{\mu\mu}$

flavour-conserving and chirality flipping $\leftrightarrow$ similar chiral enhancement

In SMEFT: $\mathcal{L} \supset C_{\mu\gamma} (\bar{L}_L \sigma^{\alpha\beta} \mu_R \Phi) F_{\alpha\beta} + C_{\mu\Phi} (\bar{L}_L \mu_R \Phi) |\Phi|^2$

$$\Delta a_\mu = \frac{4m_\mu v}{e\sqrt{2}} \text{Re}\{C_{\mu\gamma}\}$$

$$\lambda_{\mu\mu} = \frac{m_\mu}{v} - C_{\mu\Phi} \frac{v^2}{\sqrt{2}}$$

Correlation:

$$\left| \frac{\lambda_{\mu\mu}}{\lambda_{\mu\mu}^{\text{SM}}} \right|^2 = \left| 1 - \frac{|\kappa|}{728} \left(\frac{\Delta a_\mu}{10^{-9}} \right) \right|^2 = \underbrace{1.21 \pm 0.35}_{\text{experiment}}$$

with model-dependent κ

Collider Limits

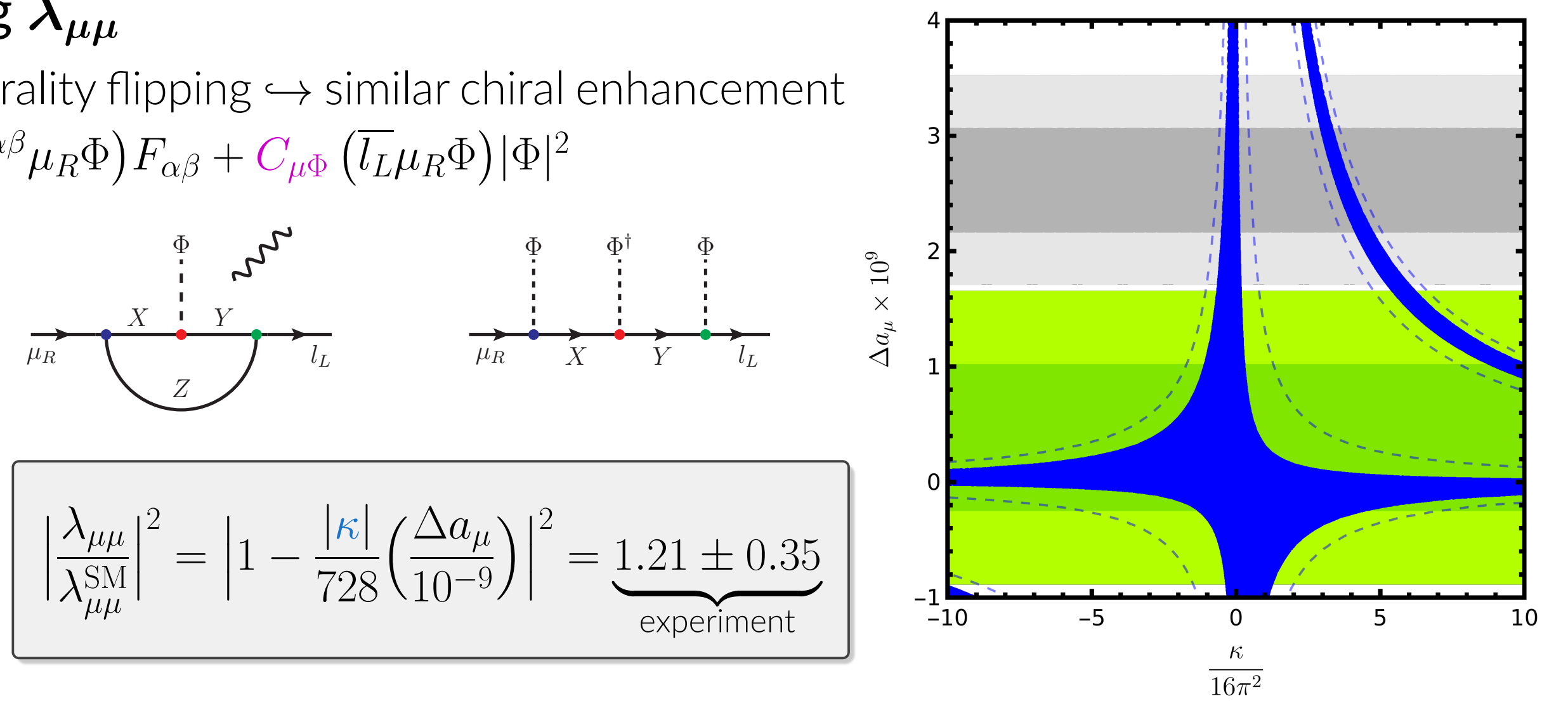
Generic chiral enhancement with $c_{L,R} \sim \mathcal{O}(1)$:

positive Δa_μ

$$M \gtrsim \sqrt{R_\chi} \times 206 \text{ GeV}$$

negative Δa_μ

$$M \gtrsim \sqrt{R_\chi} \times 282 \text{ GeV}$$



Specific BSM Scenarios

Light gauge bosons

⇒ additional (broken) $U(1)_X$ gauge group with massive vector boson

□ Z' : SM leptons charged, e.g. $L_\mu - L_\tau$

$$\mathcal{L} \supset -g_{Z'} (\bar{\mu} \gamma^\alpha \mu - \bar{\tau} \gamma^\alpha \tau + \bar{\nu}_\mu \gamma^\alpha \nu_\mu - \bar{\nu}_\tau \gamma^\alpha \nu_\tau) Z'_\alpha$$

constraints from cosmology (N_{eff}) and $Z' \rightarrow \nu \bar{\nu}$ (NA64- ϵ/μ)

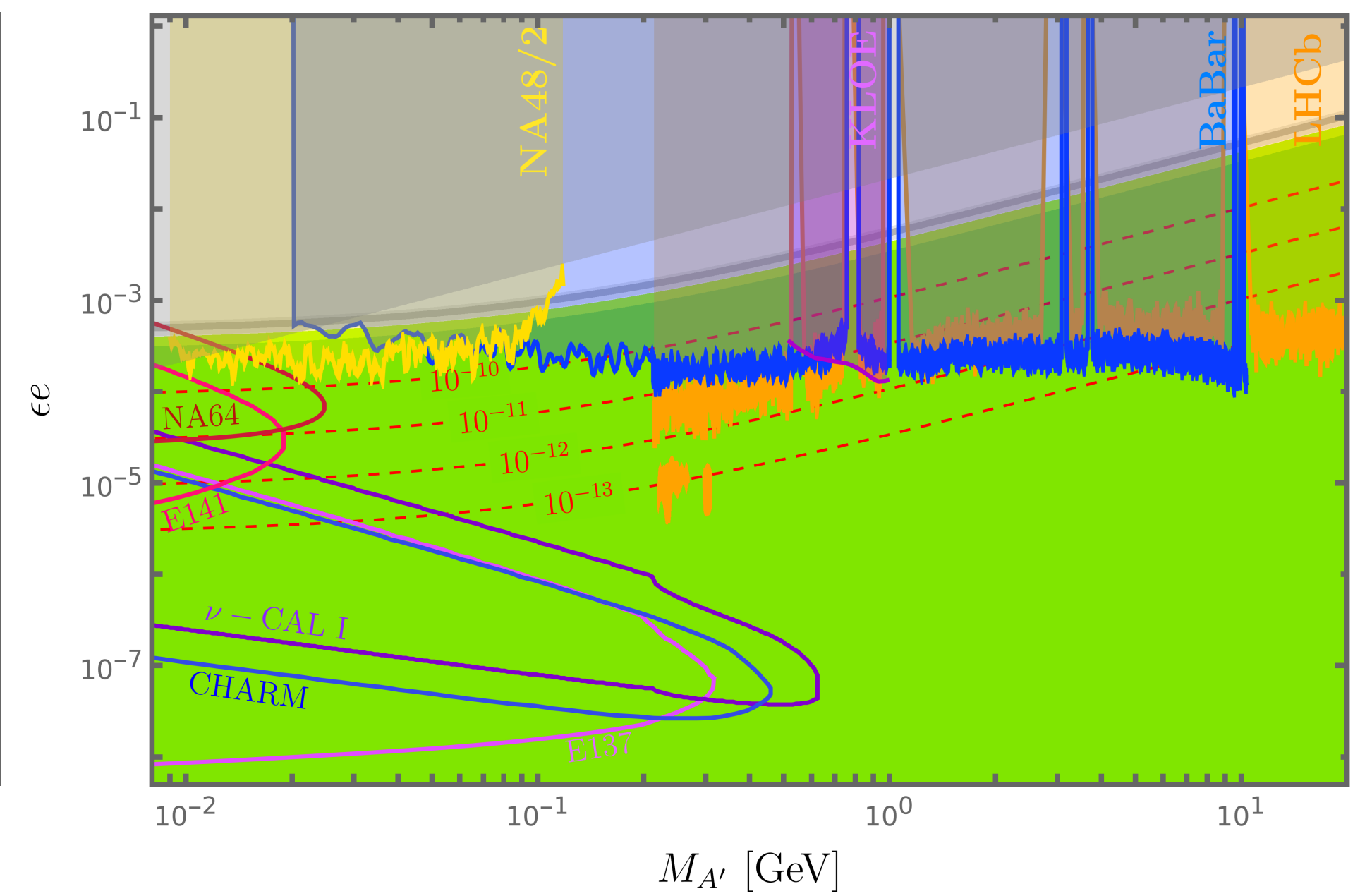
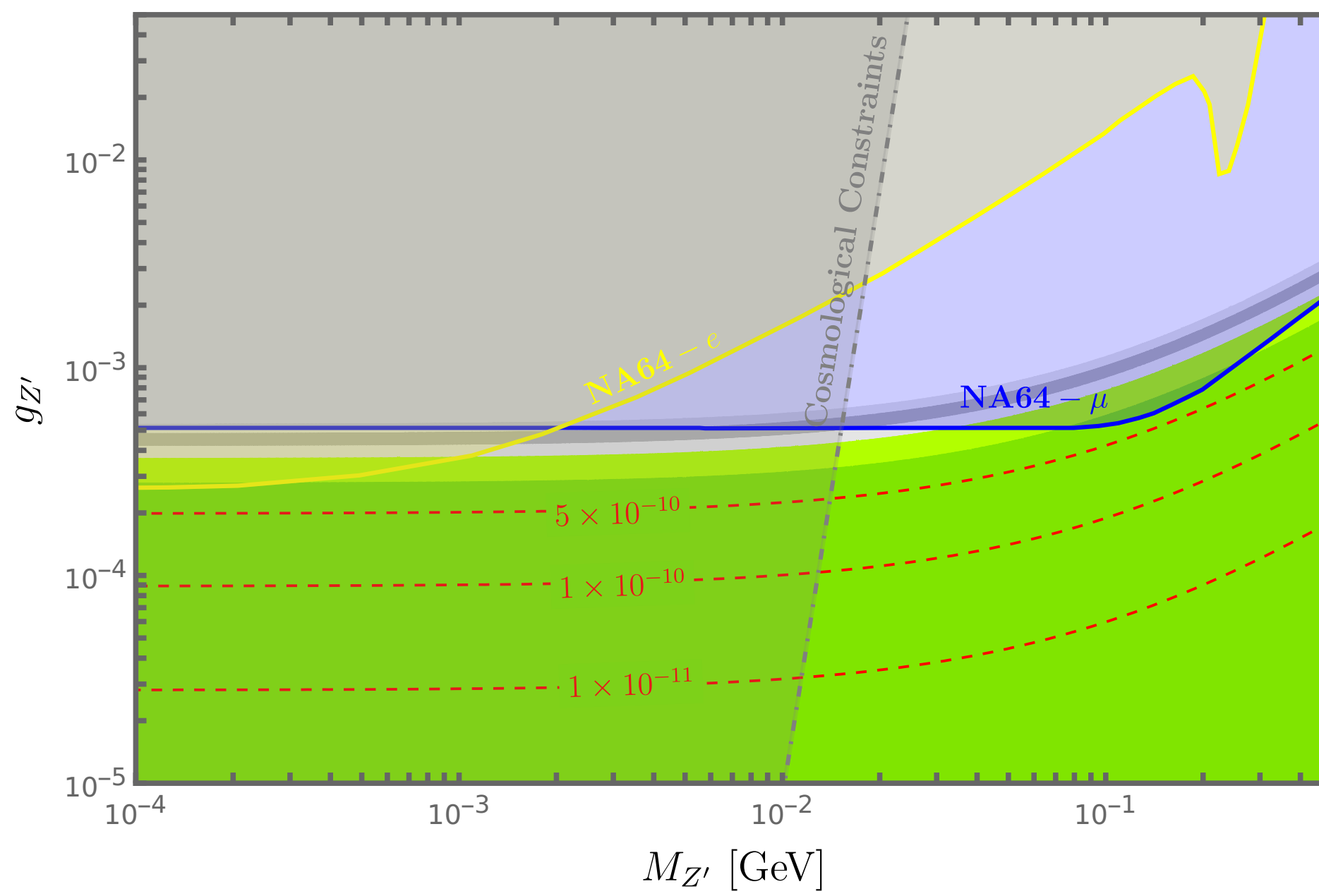
→ maximum possible $\Delta a_\mu^{Z'} \lesssim 2 \times 10^{-9}$ ($M_{Z'} \approx 20 \text{ MeV}$)

□ A' : SM leptons not charged $\leftrightarrow$ interactions only from kinetic mixing

$$\mathcal{L} \supset -\frac{\epsilon}{2} F^{\mu\nu} F'_{\mu\nu} \quad A_\mu \rightarrow A_\mu + \epsilon A'_\mu \quad \mathcal{L} \supset -\epsilon A'_\mu j_{\text{em}}^\mu$$

many constraints due to uniform coupling to all SM fermions

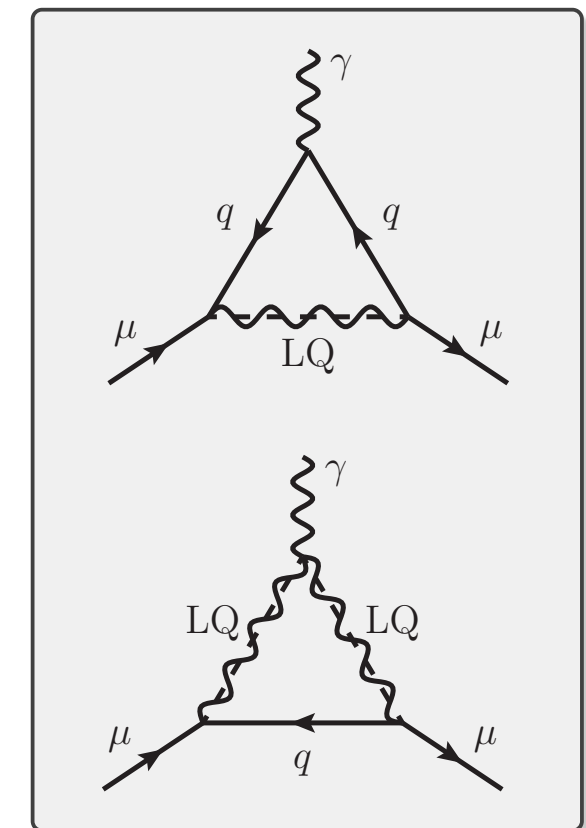
→ maximum possible $\Delta a_\mu^{A'} \lesssim 5 \times 10^{-10}$ ($M_{A'} \approx 20 \text{ MeV}$)



Leptoquarks

→ interaction with SM leptons and quarks (typical in GUT).

Chiral enhancement is possible if LQ couples to μ_L and μ_R



e.g. scalar LQ $S_1 \in (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$:

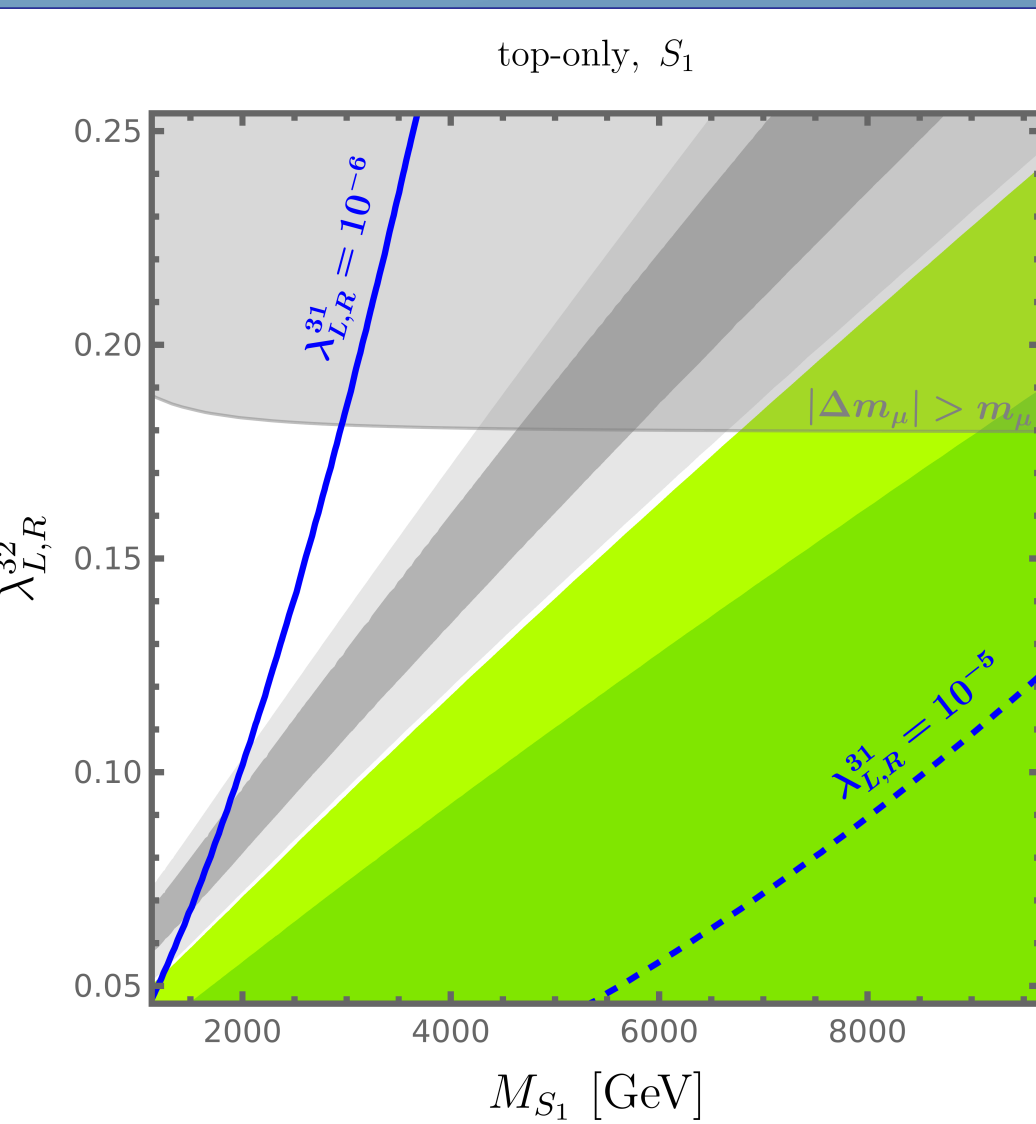
$$\mathcal{L} \supset -\lambda_L^{ij} \bar{q}_L^i \epsilon_{Lj} S_1 - \lambda_R^{ij} \bar{u}_R^i \epsilon_{Rj} S_1$$

⇒ chirality flip on heavy quark line:

$$\Rightarrow \Delta a_\mu \propto N_c \frac{\lambda_L^{32} \lambda_R^{32}}{16\pi^2} \frac{m_\mu m_t}{M_{S_1}^2}$$

Constraints:

- $M_{S_1} \gtrsim 2 \text{ TeV}$ (LHC)
- flavour-violation
- fine-tuning in $\Delta m_\mu^{1\ell}$



Δa_μ contours and exclusion from $\text{BR}(\mu \rightarrow e\gamma)$ (blue)

Vector-like leptons

Dirac fermions $F = F_L + F_R$ with same representation of $F_{L,R}$ under SM gauge group

⇒ mass term allowed *before* EWSB: $\mathcal{L} \supset M_F \bar{F}_R F_L + h.c.$

Vector-like Leptons: couple to the SM leptons and Higgs doublet at tree-level, e.g. $E \in \mathbf{1}_{-1}, L \in \mathbf{2}_{-\frac{1}{2}}$.

$$\mathcal{L} \supset -\lambda_L \bar{L}_L \mu_R \Phi - \lambda_E \bar{L}_L E_R \Phi - \bar{\lambda} \bar{L}_R E_L \Phi$$

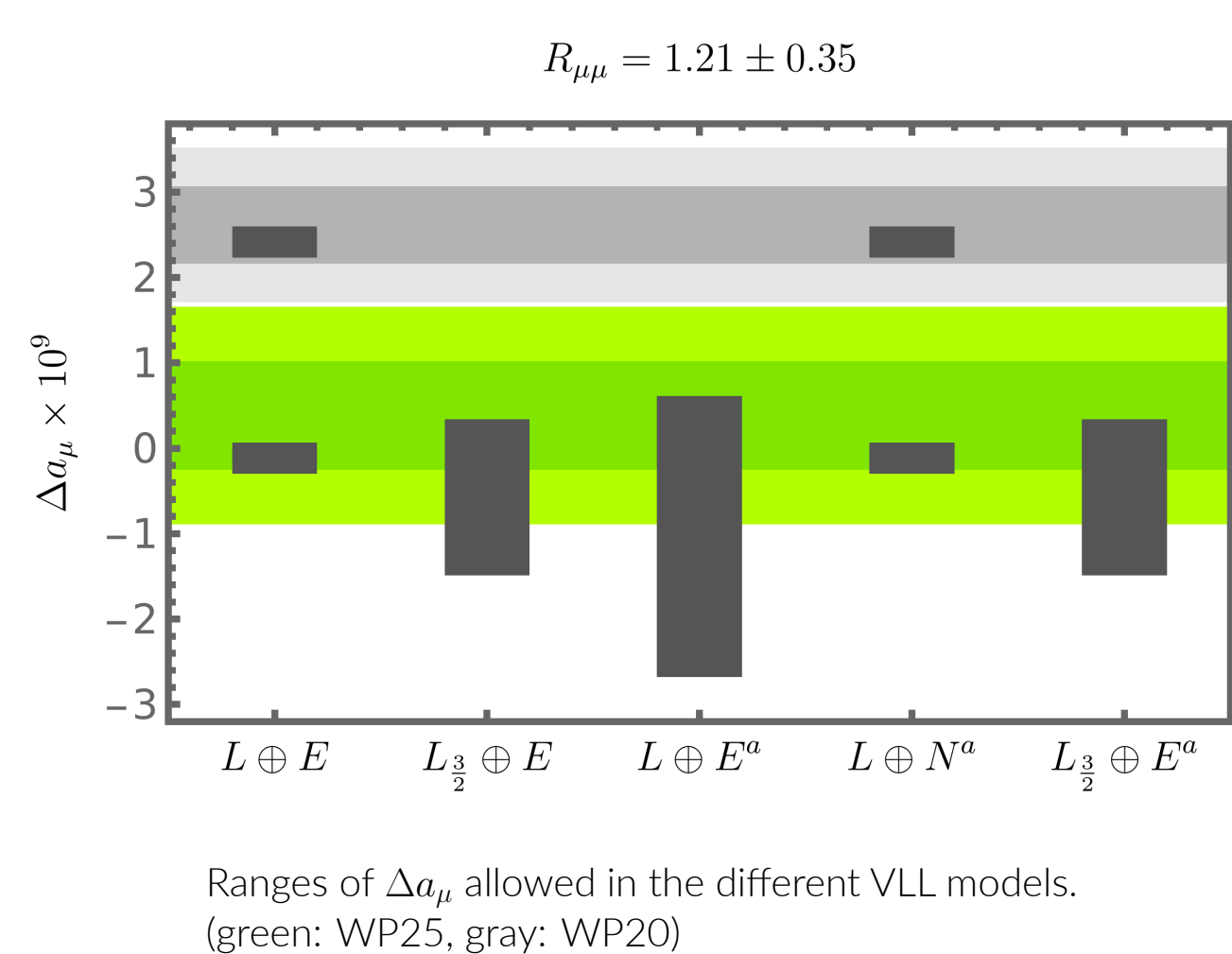
→ large chiral enhancement from VLL couplings

$$\Delta a_\mu, \lambda_{\mu\mu} \propto \lambda_L \lambda_E \bar{\lambda}$$

correlated as above with $\kappa \sim \mathcal{O}(16\pi^2)$

Constraints:

- $M_F \gtrsim 1000 \text{ GeV}$ (LHC)
- bounds on λ_i from Z-pole PO and vacuum stability



Ranges of Δa_μ allowed in the different VLL models. (green: WP25, gray: WP20)

Axion-like Particles

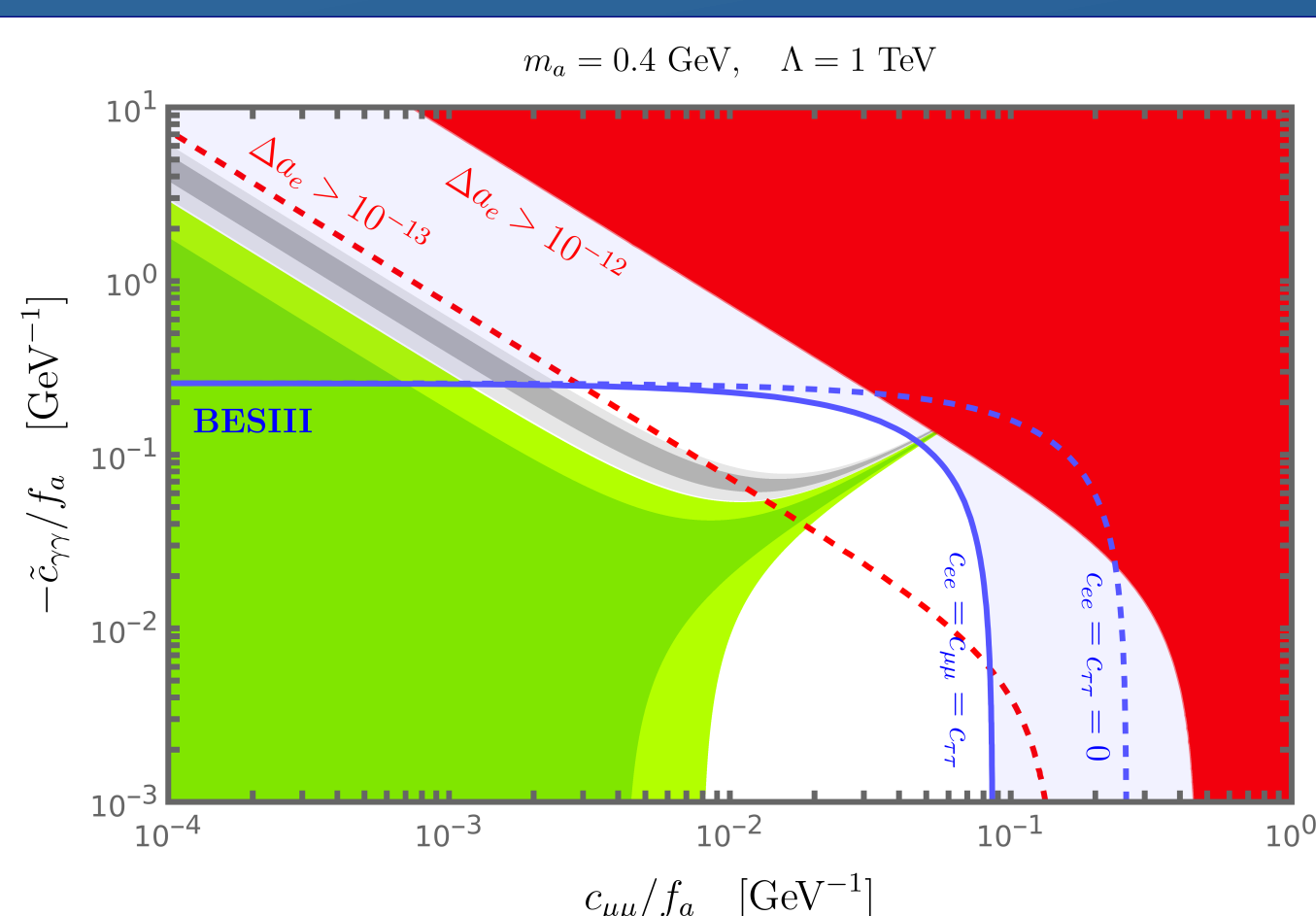
Pseudoscalar a with shift-symmetric interactions

$$\mathcal{L} \supset \frac{c_{\mu\mu}}{2} \frac{\partial_\mu a}{f_a} \bar{\mu} \gamma^\mu \gamma^5 \mu + c_{\gamma\gamma} \frac{\alpha}{4\pi f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Competing one- and two-loop contributions

$$\Delta a_\mu^{1\ell} \propto -\frac{c_{\mu\mu}^2}{16\pi^2}, \quad \Delta a_\mu^{2\ell} \propto -\frac{c_{\mu\mu} c_{\gamma\gamma} \alpha}{16\pi^3}$$

strong constraints from $J/\Psi \rightarrow \gamma^* \rightarrow a\gamma$ (BES III)



Two-Higgs-doublet Model

SM + second ($Y = \frac{1}{2}$) $SU(2)_L$ doublet scalar Φ_2 .

⇒ 5 physical scalars h ($M_h = 125 \text{ GeV}$), H , A and $H^\pm$

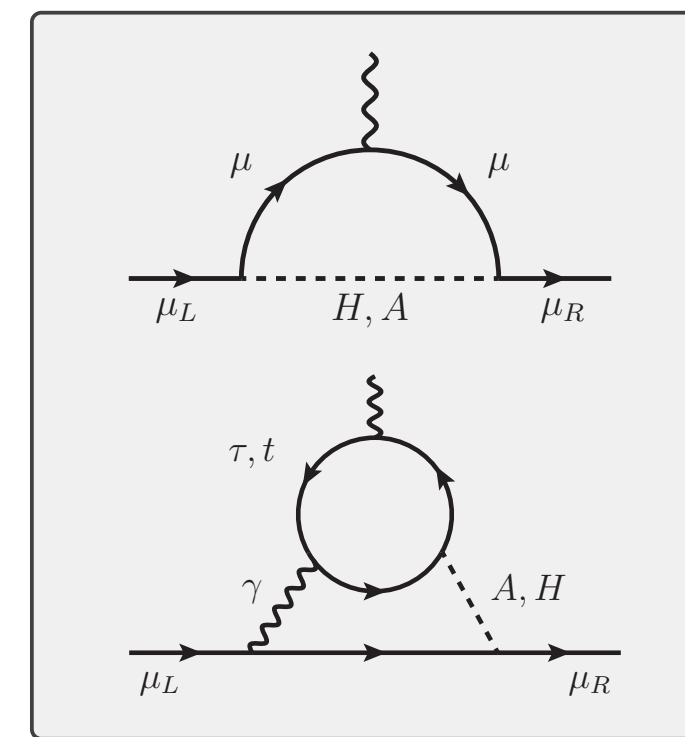
SM leptons can couple to both doublets

$$\mathcal{L} \supset \sum_{a=1,2} \bar{L}_L Y_a^l e_R \Phi_a + h.c.$$

Flavour-aligned 2HDM: $Y_2^f = \zeta_f Y_1^f$

Contribution to Δa_μ

From one-loop and two-loop Barr-Zee diagrams



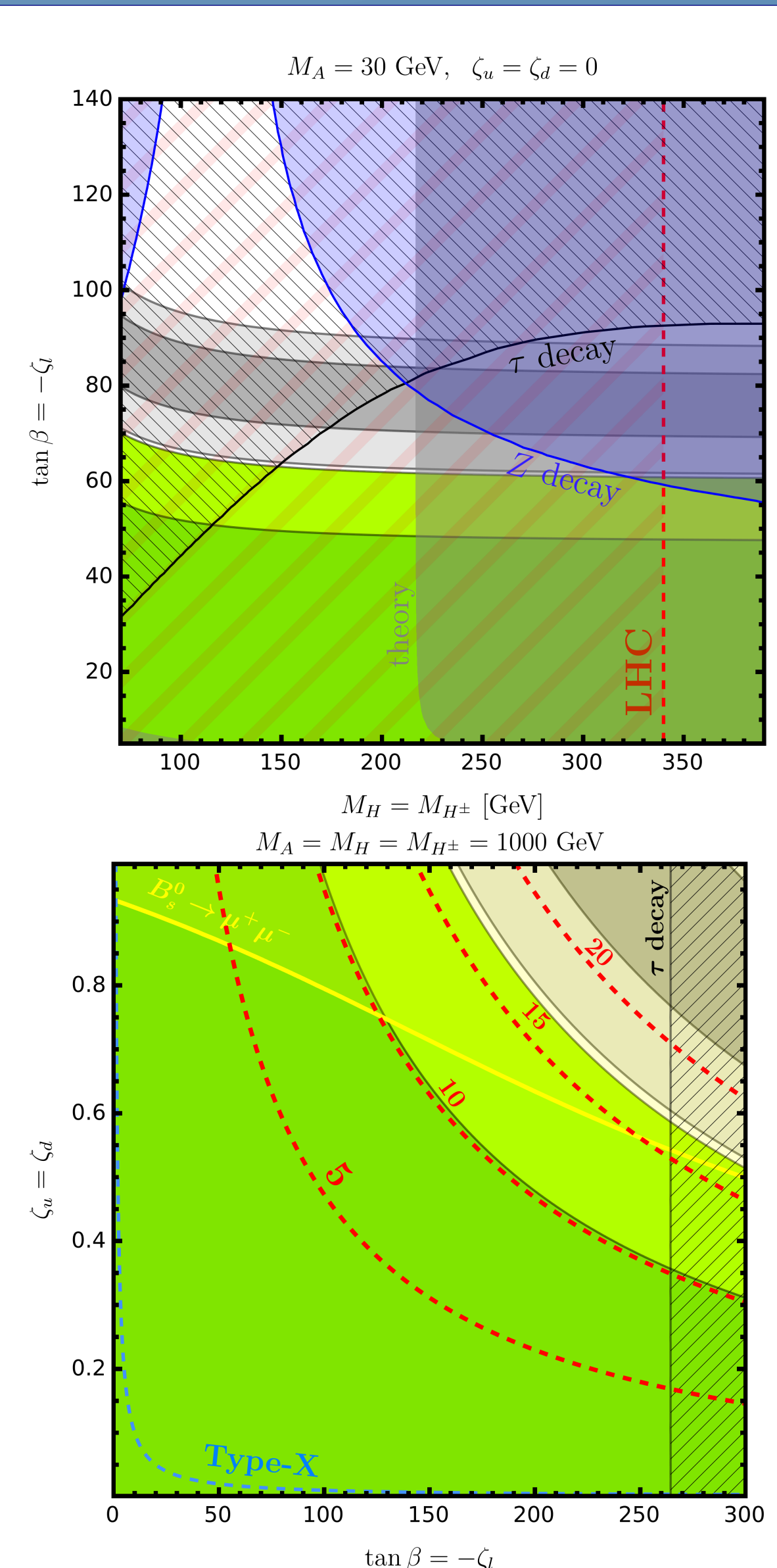
$$\Delta a_\mu^{1\ell} \sim \mp \frac{\zeta_l^2}{8\pi^2 v^2 M_S^2} \frac{m_\mu^A}{M_S^2}$$

$$\Delta a_\mu^{2\ell} \sim -\frac{\zeta_l \zeta_u m_\mu^2 m_t^2}{8\pi^2 v^2 M_S^2} \frac{\alpha}{\pi} N_c Q_t^2$$

Constraints:

light $M_A \lesssim 100 \text{ GeV}$ → strong exclusion limits from LHC

heavy $M_i \sim 1 \text{ TeV}$ → bounds on ζ_f from flavour and EW precision observables



Supersymmetry

□ Scenarios with light SUSY and dark matter still possible:

e.g. Bino LSP → DM coannihilation needs small mass splitting
→ LHC is insensitive

⇒ complementary constraint from Δa_μ

□ on the other hand... many scenarios (e.g. CMSSM, GMSB, ...) with small Δa_μ are viable again.

