Error Scaling of Sea Quarks Isospin Breaking Effects

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Motivations

Lattice simulations take place at a finite lattice spacing and in a finite volume; the continuum and infinite volume limits are two primary sources of systematic error in precise lattice computations. We present a study of the error of sea-quark IBE to understand the scaling towards the infinite volume and continuum limits. We used $N_f = 3$ O(a)-improved Wilson fermions with C* b. c. in space and $M_\pi \approx 400$ MeV.

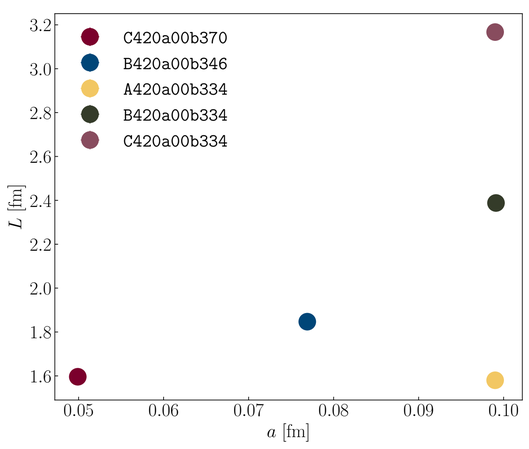

Fig. 1: Set of our ensembles used for the work.

Sea-Quarks IBE

Sea-Quarks IBE come from the expansion of the fermionic determinant in the e.m. coupling constant e^2 and the up and down quark mass difference δm_{ud}:

$$\delta\mathcal{O}_s = -\delta\beta\langle S_g\mathcal{O}\rangle_{\mathrm{Iso},c} + \sum_f \delta m_f \langle \overset{f}{\bigcirc}\ \mathcal{O}\rangle_{\mathrm{Iso},c}$$

mass

$$+e^2\left[\sum_f q_f^2\left(\langle \overset{f}{\bigcirc\!\!\!\!\!\!\bigcirc}\ \mathcal{O}\rangle_{\mathrm{Iso},c} + \langle \overset{f}{\bigcirc\!\!\!\!\!\!\bigcirc}\ \mathcal{O}\rangle_{\mathrm{Iso},c}\right) + \sum_{fg} q_f q_g \langle \overset{f}{\bigcirc}\!\!\sim\!\!\overset{g}{\bigcirc}\ \mathcal{O}\rangle_{\mathrm{Iso},c}\right].$$

tadpole, lightbulb, lanterns

Factorization of the Error

The statistical error of the insertions of sea-quarks IBE factorises, in the continuum limit:

$$\sigma\left[\langle\mathcal{D}\mathcal{O}\rangle_{\mathrm{Iso},c}\right] \underset{a\to 0}{\sim} \sqrt{\langle\mathcal{D}^2\rangle_{\mathrm{Iso},c}\langle\mathcal{O}^2\rangle_{\mathrm{Iso},c}}.$$

We then study the scaling in the infinite volume and continuum limit of the error of the diagrams:

$$\sigma\left[\langle \bigcirc\ \mathcal{O}\rangle_{\mathrm{Iso},c}\right] \underset{a\to 0}{\sim} \sigma_{\mathcal{O}} a^{-1}\sqrt{V}$$

$$\sigma\left[\langle\mathcal{D}\mathcal{O}\rangle_{\mathrm{Iso},c}\right] \underset{a\to 0}{\sim} \sigma_{\mathcal{O}} a^{-2}\sqrt{V} \text{ for } \mathcal{D} = \bigcirc\!\!\!\!\!\!\bigcirc,$$

$$\bigcirc\!\!\sim\!\!\bigcirc,\ \bigcirc\!\!\!\!\!\!\bigcirc.$$

References

[1] G. M. de Divitiis *et al.* [RM123], "Leading isospin breaking effects on the lattice," Phys. Rev. D **87** (2013) no.11, 114505.

[2] G. S. Bali *et al.* [RQCD], "Scale setting and the light baryon spectrum in $N_f = 2 + 1$ QCD with Wilson fermions," JHEP **05** (2023), 035.

[3] T. Harris *et al.*, "Efficiently unquenching QCD+QED at O(α)," PoS **LATTICE2022** (2023), 013.

[4] T. Harris, "Beyond the electroquenched approximation," PoS **EuroPLEx2023** (2024), 011.

[5] A. Altherr *et al.*, "Error Scaling of Sea Quark Isospin-Breaking Effects," PoS **LATTICE2024** (2025), 116.

Volume Scaling

The expected volume scaling of $\sqrt{V}$ is respected by the gauge error of each sea diagram.

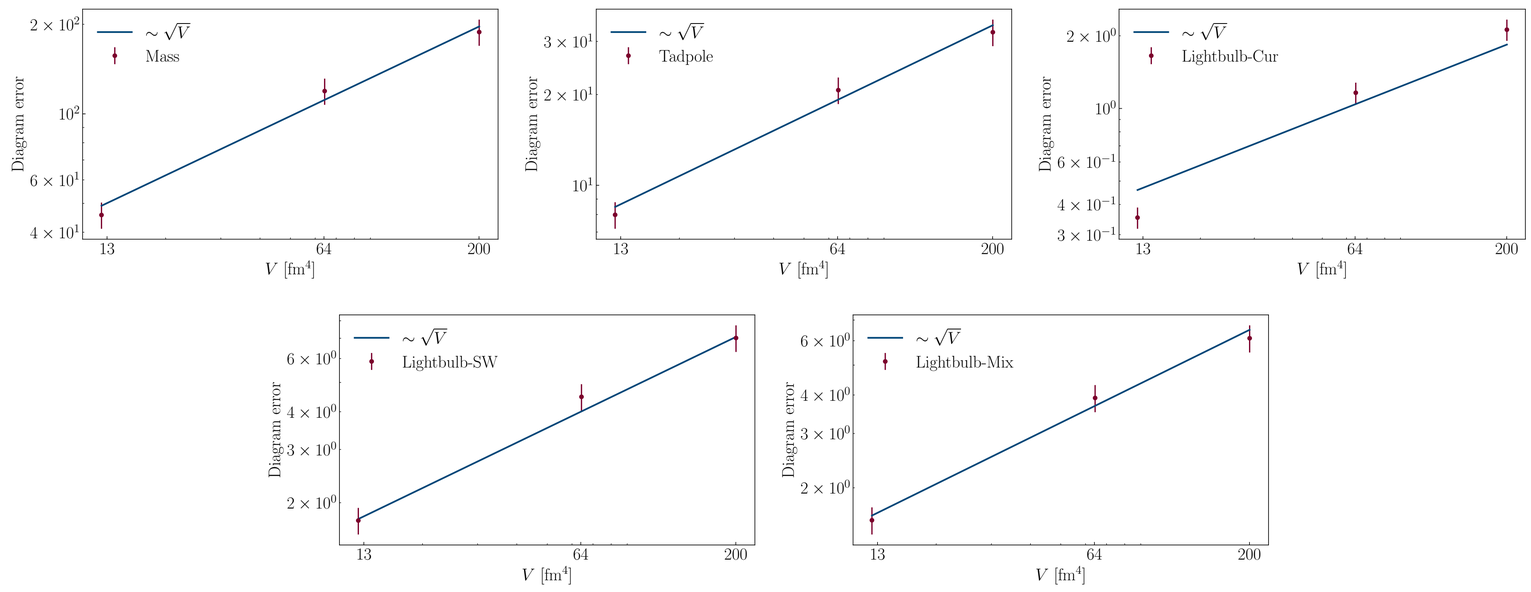

Fig. 2: Error scaling with the volume for the mass, tadpole and lightbulb diagrams (All the diagrams but the current lightbulb reach the gauge noise). A linear regression with the expected scaling is added as a reference.

Continuum Scaling

The continuum scaling of the gauge error does not respect the expected scaling.

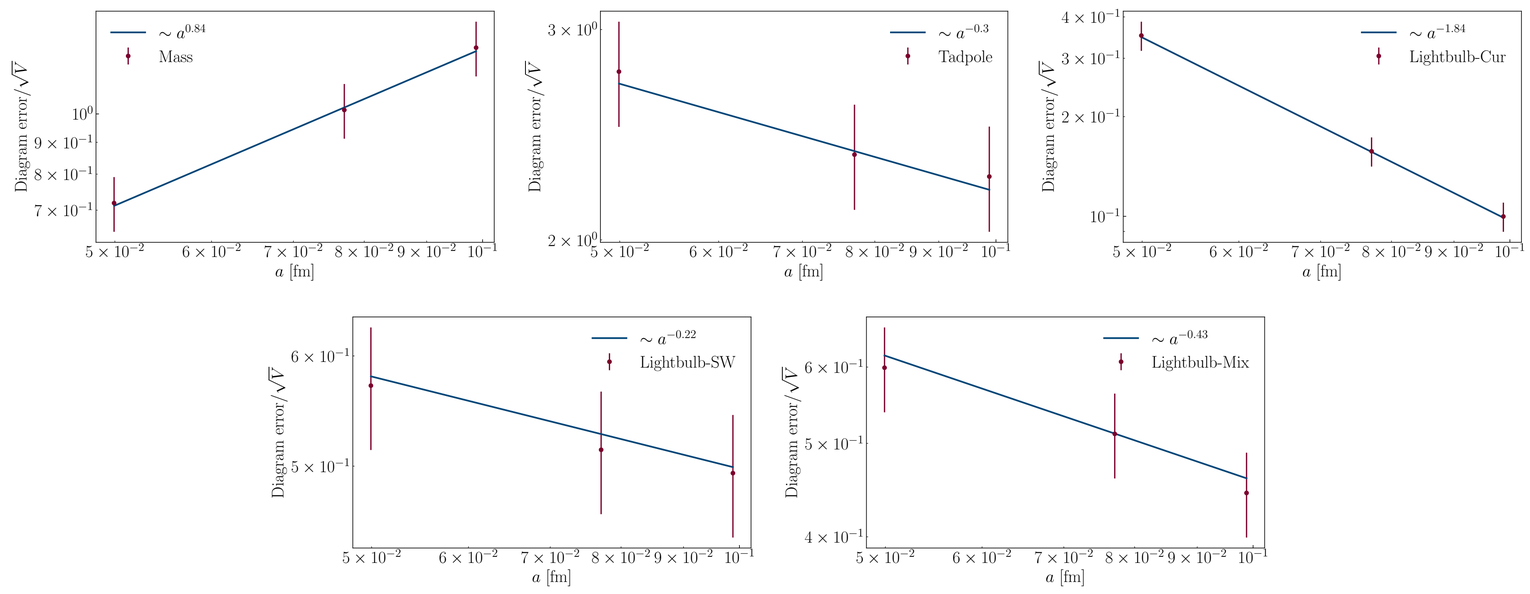

Fig. 3: Error scaling with the lattice spacing for the mass, tadpole and lightbulb diagrams (All the diagrams but the current lightbulb reach the gauge noise). A fit to a power of the lattice spacing is added as a reference.

We fitted a general power of the lattice spacing to see what the actual behaviour is instead of the a^{-1} and a^{-2} divergencies.

Correlation between Diagrams

There is a huge cancellation between the diagrams; therefore, we decided to test how it is affected by the total bare quark mass shift $\delta a\hat{m}$. We found a minimum, which is not far from the tuned shift.

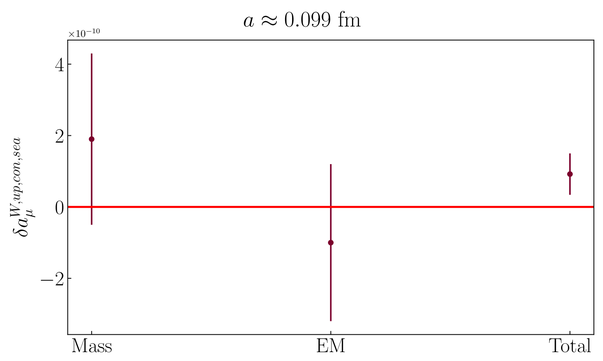

Fig. 4: The sea-quark IBE on the connected up quark intermediate window for the ensemble A420a00b334.

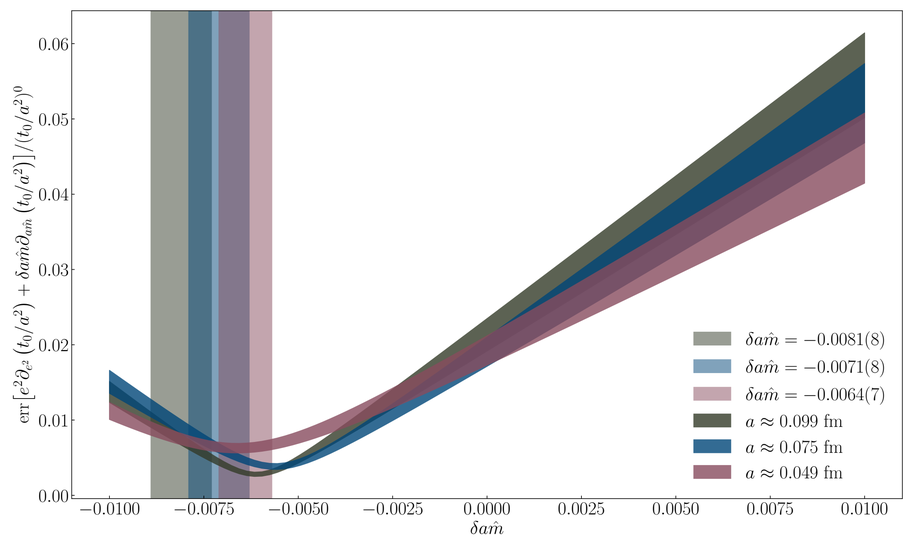

Fig. 5: Ratio between the error of the total IBE to t_0/a^2 and the isosymmetric value of t_0/a^2 for different values of the lattice spacing as a function of the bare mass shift. The vertical bands are the tuned shift with the error.

Conclusions and outlook

- The error scaling with the volume is respected, providing a serious risk of bottlenecks for the computation;
- The error scaling with the lattice spacing is not observed;
- High correlation between the diagrams suggests computing them together;
- The correlation can be used to design a new discretisation for the mass counterterm;