Improved π^0, η, η' transition form factors in resonance chiral theory (2409.10503)

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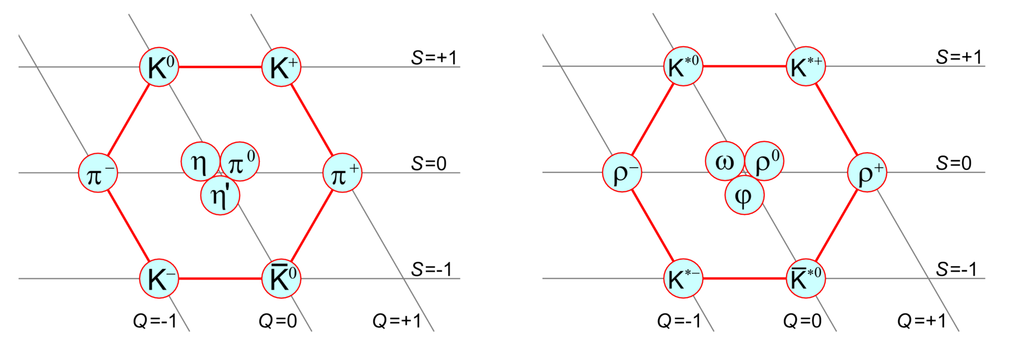
Introduction

In this work, we improve the on-shell transition form factor ($P \to \gamma^*\gamma^*$) within the context of Resonance Chiral Theory (RχT). By including a second multiplet of vector meson resonances, it is possible to fulfill all short-distance constraints(SDCs) stemming from pQCD. After imposing the high-energy behavior, the still free parameters and couplings were obtained from a global analysis of the experimental TFF data, the radiative decay widths, and LQCD calculations for the DV sector, together with stabilization points. We obtained a consistent and competitive determination for all the P-pole contributions to the HLbL piece of a_μ.

Effective Lagrangian

We used an RχT effective Lagrangian, which extends the applicability of χPT to energies above 1GeV [1]. For matching the SDCs, we required P+P'+V+V'.

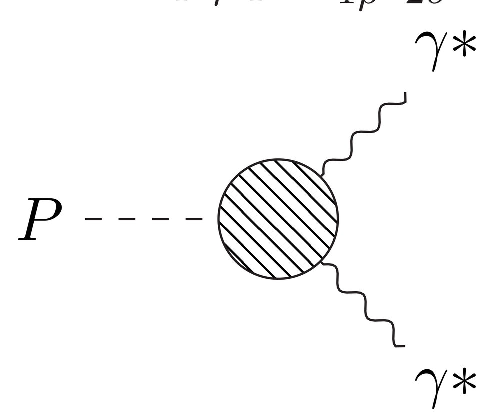


$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}(C_\pi\pi^0 + C_q\eta + C'_q\eta') & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{2}}(-C_\pi\pi^0 + C_q\eta + C'_q\eta') & K^0 \\ K^- & \bar{K}^0 & -C_s\eta + C'_s\eta' \end{pmatrix},$$

$$V_{\mu\nu} = \begin{pmatrix} (\rho+\omega)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (-\rho+\omega)/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_{\mu\nu}.$$

Transition Form Factor

$$\mathcal{M}_{P\gamma^*\gamma^*} = ie^2\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon^*_{1\rho}\epsilon^*_{2\sigma}\mathcal{F}_{P\gamma^*\gamma^*}(q_1^2, q_2^2)\,,$$



Satisfy the SDCs:

$$\lim_{Q^2\to\infty} Q^2|\mathcal{F}(-Q^2,-Q^2)| = \frac{2F_\pi}{3},$$

$$\lim_{Q^2\to\infty} Q^2|\mathcal{F}(-Q^2,0)| = 2F_\pi,$$

for the $\eta^{(\prime)}$ there is a flavor factor of $\frac{5C_q-\sqrt{2}C_s}{3}\left(\frac{5C'_q+\sqrt{2}C'_s}{3}\right)$. In our model, after imposing the TFFs, 12 parameters were free: 3 mass parameters, 4 mixing parameters, and 5 couplings (2SV+3DV).

Global Fit to TFF data

Available Data

- $\Gamma(P \to \gamma\gamma)$, related to TFFs for real photons(PDG)

$$\Gamma(P\to\gamma\gamma) = \frac{(4\pi\alpha)^2}{64\pi} m_P^3 |\mathcal{F}_{P\gamma\gamma}(0,0)|^2$$

- Single Virtual TFF data (BaBar, Belle, CLEO, LEP).
- Double Virtual TFF data for η' (BaBar) complemented by LQCD calculations [2] at $Q_1^2 = Q_2^2 = 0.1, 1.0$ and 4.0 GeV^2 for all 3 mesons.
- Stabilization points for the vector meson multiplets mass parameters and the mixings.
- Sub-leading asymptotic behavior of π^0 DV TFF given by $\delta_\pi^2 = 0.20(2)$GeV^2.

Cost Function

A modified χ^2 was used as a cost function (correlation between LQCD points was considered):

$$\chi^2_{\text{Global}} = \chi^2_{\text{P}_{\text{SV}}} + \chi^2_{\eta'_{\text{DV}}\,\text{Exp}} + \chi^2_{\text{P}^{\text{LQCD}}_{\text{DV}}} + \sum_{\text{D}}^{\text{Extra Points}} \left(\frac{D_{\text{exp}} - D_{\text{model}}}{\Delta D_{\text{exp}}}\right)^2$$

Equivalence with CAs

These results were mapped to a rational approximant:

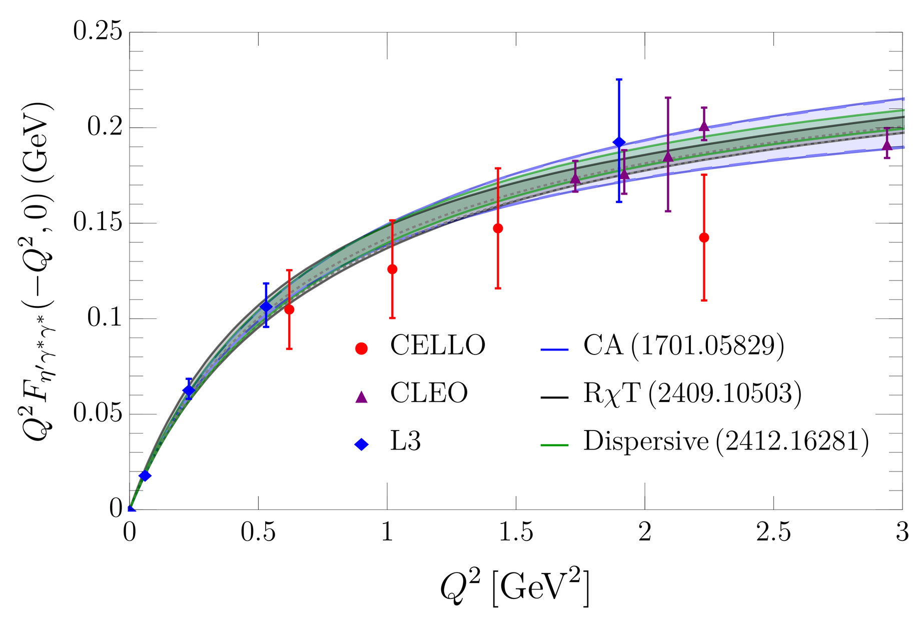
$$C^N_M(x,y) = \frac{R_N(x,y)}{Q_M(x,y)} = \frac{\sum_{i,j=0}^{N} a_{i,j}x^iy^j}{\sum_{i,j=0}^{M} b_{i,j}x^iy^j},$$

resulting in a C_2^2 in the chiral limit and C_4^4 for $\eta - \eta'$.

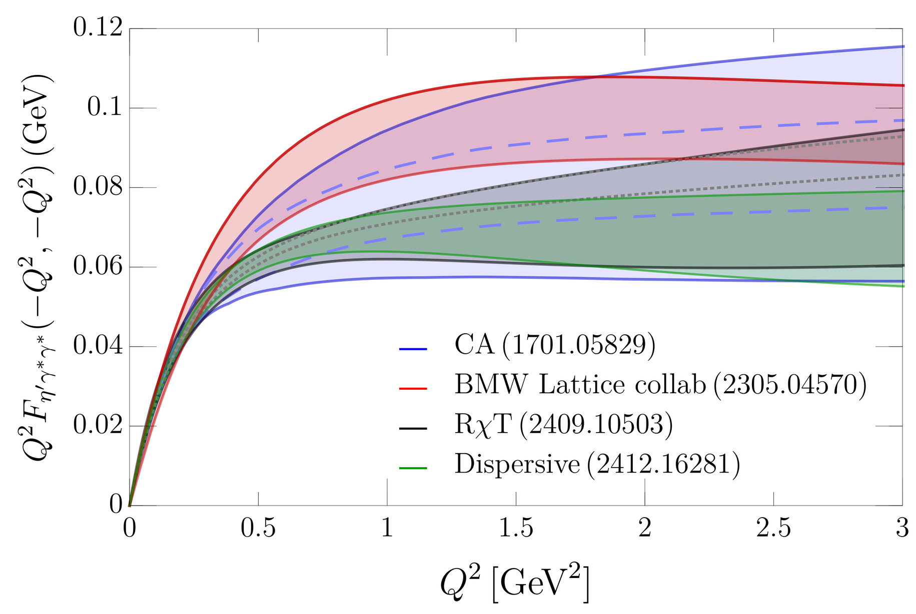
Global Fit Results

We obtained a good description of the SV data for all 3 P-mesons. We show the results for η'. The results for the other two vector mesons show similar results and agreement with other approaches. The bands represent the 1σ error bands. In our case, including systematic and statistical errors.

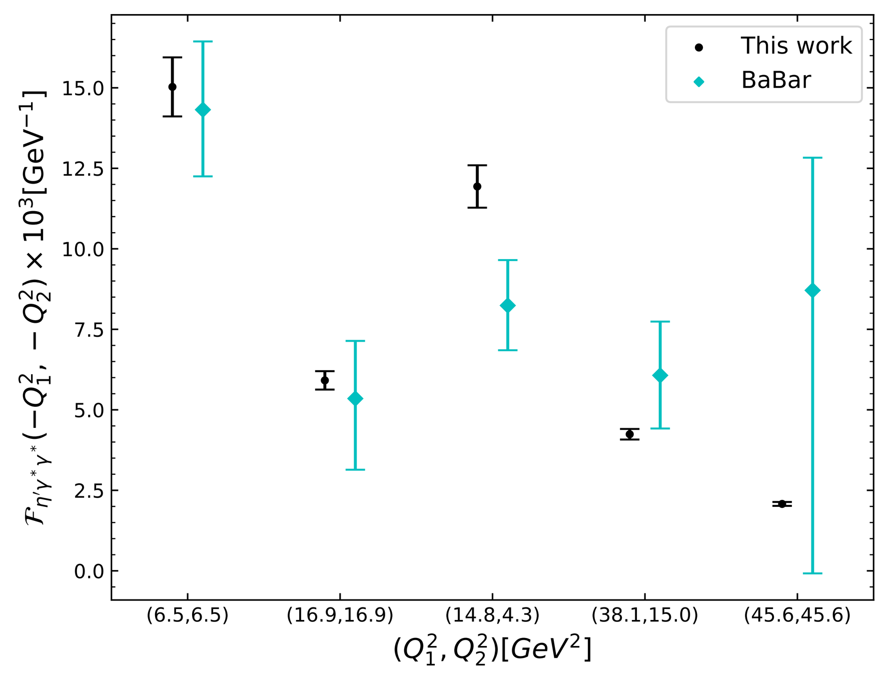
Single virtual η' TFF:



Double virtual η' TFF:



Double Virtual η' TFF compared with measurements from BaBar:



Systematic Error Analysis

For the systematic errors we considered:

- Use of all available experimental data. ($\sim 0.3\%$)
- Truncation of the infinite tower of resonances. ($\sim 3\%$)
- Sub-leading corrections in $1/N_C$. ($\sim 2\%$)
- Possible bias by Hybrid Analysis. ($\sim 0.6\%$)
- Asymptotic behavior for asymmetric double virtualities. ($\sim 0.01\%$)

$g-2$ contribution

Given the global analysis for the TFFs, we found the results (in units of 10^{-11}):

$$a_\mu^{\pi^0\text{-pole}} = (61.9 \pm 0.6^{+2.4}_{-1.5}) = (61.9^{+2.5}_{-1.6})\,,$$

$$a_\mu^{\eta\text{-pole}} = (15.2 \pm 0.5^{+1.1}_{-0.8}) = (15.2^{+1.2}_{-0.9})\,,$$

$$a_\mu^{\eta'\text{-pole}} = (14.2 \pm 0.7^{+1.4}_{-0.9}) = (14.2^{+1.6}_{-1.1})\,,$$

adding up to

$$a_\mu^{\pi^0+\eta+\eta'\text{-pole}} = (91.3 \pm 1.0^{+3.0}_{-1.9}) = (91.3^{+3.2}_{-2.1})\,,$$

which is in agreement with other approaches [3–7].

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