Tau IB corrections

- Dispersive and lattice isospin-breaking corrections – time line?
  Great progress, but far from complete! (Bruno/Parrino and Holz/Cottini talks)

- Interplay of lattice and phenomenology: What can lattice contribute to dispersive/pheno approach? And vice-versa? ("Inverse problem")

- Matching between LD and SD effects, and between lattice and phenomenology?
  How can inclusive lattice results be compared with pheno/dispersive 2π results?

- Significance of rho parameters (Zhang/Toledo talks); normalization (charge conservation)

A few slides by Martin Hoferichter and Mattia Bruno in this discussion session

Towards an improved short-distance matching for $\tau \to \pi\pi\nu_\tau$

- Standard factor $S_{\text{EW}} = 1 + \frac{2\alpha}{\pi} \log \frac{M_Z}{m_\tau} + \cdots$ encodes **universal corrections** to all semi-leptonic decays, known at NLL (in a particular scheme)
- However: not accounted for at present is **scheme dependence** of the short-distance Wilson coefficient at NLL

  $\hookrightarrow$ corresponds to choice of evanescent operator Cirigliano et al. 2023
- Needs to cancel in the **matching at the hadronic scale**

  $\hookrightarrow$ organized at the level of the chiral low-energy constant
- Matching can be performed, following Descotes-Genon 2005, Cirigliano et al. 2023, based on lattice-QCD input for γW box correction Feng et al. 2020, Yoo et al. 2023
- Further cross check: in addition to evanescent scheme, need to show that all scale dependence cancels at the order considered work in progress
  - Chiral renormalization scale μ_χ
  - LEFT renormalization scale μ
- Related question: how to make the connection to lattice-QCD calculations for isospin breaking in $\tau \to \pi\pi\nu_\tau$? slides by M. Bruno

A similar example: radiative corrections to neutron β decay

- Start from **LEFT Lagrangian**

$$\mathcal{L}_{\text{LEFT}} = -2\sqrt{2}G_F \bar{e}_L\gamma_\rho\mu_L\,\bar{\nu}_{\mu L}\gamma^\rho\nu_{eL} - 2\sqrt{2}G_F V_{ud} C(a,\mu)\bar{e}_L\gamma_\rho\nu_{eL}\,\bar{u}_L\gamma^\rho d_L + \text{h.c.} + \cdots$$

$\hookrightarrow$ scheme for G_F defined by muon decay

- Wilson coefficient for the semileptonic operator in $\overline{\text{MS}}$ + NDR for γ_5

$$C(a,\mu) = 1 + \frac{\alpha}{\pi}\log\frac{M_Z}{\mu} + \underbrace{\frac{\alpha}{\pi}\bigg(\frac{a}{6} - \frac{3}{4}\bigg)}_{\equiv B(a)} - \frac{\alpha\alpha_s}{4\pi^2}\log\frac{M_W}{\mu} + \mathcal{O}(\alpha\alpha_s, \alpha^2)$$

$$\gamma^\alpha\gamma^\rho\gamma^\beta P_L \otimes \gamma_\beta\gamma_\rho\gamma_\alpha P_L = 4\big[1 + a(4-d)\big]\gamma^\rho P_L \otimes \gamma_\rho P_L + E(a)$$

- Dependence on a and μ needs to cancel in observables (at the order considered)
- **Matching to ChPT**: relevant low-energy constant is $g_V(\mu_\chi) = 1 + \mathcal{O}(\alpha)$

$\hookrightarrow$ corrections correspond to $\frac{4}{3}X_1 + X_6^r(\mu_\chi) - 4K_{12}^r(\mu_\chi)$ here

A similar example: radiative corrections to neutron β decay

- Master formula Cirigliano et al. 2023

$$g_V(\mu_\chi) = \bar{C}(\mu)\left[1 + \bar{\Box}^V_{\text{had}}(\mu_0) - \frac{\alpha(\mu_\chi)}{2\pi}\left(\frac{5}{8} + \frac{3}{4}\log\frac{\mu_\chi^2}{\mu_0^2} + \left(1 - \frac{\alpha_s(\mu_0)}{4\pi}\right)\log\frac{\mu_0^2}{\mu^2}\right)\right]$$

$$C(a,\mu) \equiv \bar{C}(\mu)\left(1 + \frac{\alpha(\mu)}{\pi}B(a)\right)$$

$$\bar{\Box}^V_{\text{had}}(\mu_0) \equiv -ie^2\int\frac{d^4q}{(2\pi)^4}\frac{\nu^2+Q^2}{Q^4}\left[\frac{T_3(\nu,Q^2)}{2m_N\nu} - \frac{2}{3}\frac{1}{Q^2+\mu_0^2}\left(1 - \frac{\alpha_s(\mu_0)}{\pi}\right)\right]$$

- $T_3(\nu, Q^2)$ is a two-current matrix element of the nucleon, μ_0 another factorization scale
- Dependence on a, μ, μ_χ, μ_0 drops out from decay rate at the considered order
- For $\tau \to \pi\pi\nu_\tau$, the analog of $T_3(\nu, Q^2)$ can be extracted from existing lattice calculations Feng et al. 2020, Yoo et al. 2023

Consequences for the combination with lattice QCD

- Following the above procedure, $S_{\text{EW}} G_{\text{EM}}(s)$ will be scale and scheme independent
- The ratio $|F_\pi^V(s)/f_+(s)|$ is then defined as follows:
  - $|F_\pi^V(s)|$ follows from the inclusive $e^+e^- \to \pi\pi(\gamma)$ cross section by dividing out $\eta(s)$ (or some improved version thereof)
  - $|f_+(s)|$ follows from the inclusive decay rate $\tau \to \pi\pi\nu_\tau(\gamma)$ by dividing out $S_{\text{EW}} G_{\text{EM}}(s)$
  - IR safe, but defines a scheme for the separation of long-distance contributions
  - Dispersive calculation of the corresponding matrix elements well defined talk by M. Cottini
  - Information should be contained in $\delta G_{11}(t,\mu)$, but proper matching will require some thought slides by M. Bruno
- Comments on data-driven determination of $|F_\pi^V(s)/f_+(s)|$
  - Fits to e^+e^- data and τ spectrum can help disentangle differences in shape
  - Normalization needs to be fixed by theory, usually, via charge conservation
  - If this constraint is dropped, energy-independent isospin-breaking effects can no longer be separated from experimental issues

    $\hookrightarrow$ uncontrolled effects of $\mathcal{O}(\alpha)$, possibly enhanced by resonance physics