Lattice QCD computations in flavour physics

Benoît Blossier

CNRS/Laboratoire de Physique des 2 Infinis Irène Joliot-Curie

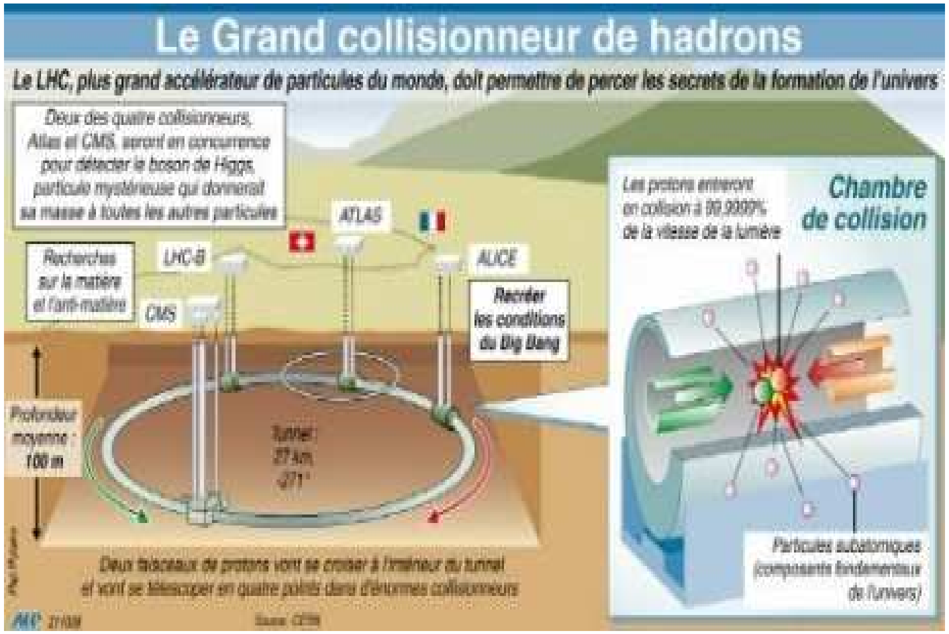
“Taming hadronic uncertainties in and beyond the Standard Model"
Orsay, 22 – 24 October 2025

- General status
- Exclusive decays
- Inclusive decays
- Outlook

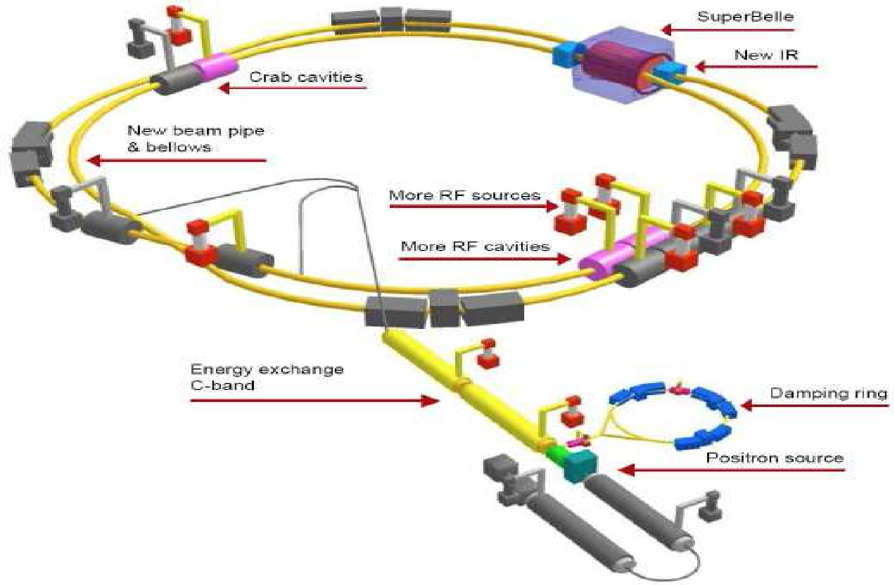
General status

An intense on-going experimental activity in flavour physics...

LHC(CERN)



Super KEKB (KEK)

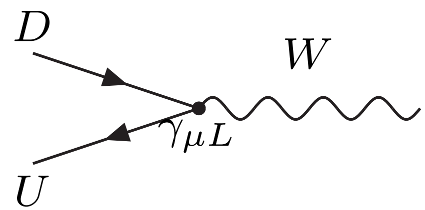


Physics case at future facilities in discussion. For instance: workshop on flavour physics at the FCC, November 2025 at CERN [https://indico.cern.ch/event/1588013/]

Standard Model in the quark sector

3 families of quarks: $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$; strong hierarchy among quark masses

Quarks coupled to charged weak bosons by a left-handed current.



Quark flavour eigenstates $\neq$ quark weak eigenstates; flavour mixing described by the Cabibbo-Kobayashi-Matrix mechanism, source of weak CP violation.

$$\begin{pmatrix} \text{d'} \\ \text{s'} \\ \text{b'} \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} \text{d} \\ \text{s} \\ \text{b} \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} \text{d} \\ \text{s} \\ \text{b} \end{pmatrix}$$

$V_{ij} \sim \mathcal{O}(1)$

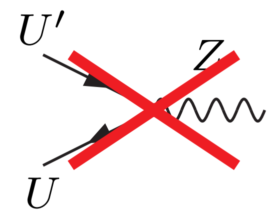
$V_{ij} \sim \mathcal{O}(\lambda)$

$V_{ij} \sim \mathcal{O}(\lambda^2)$

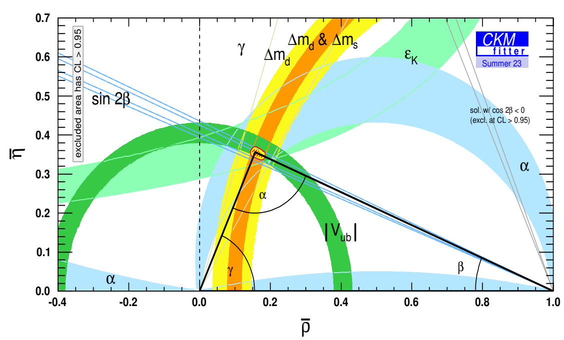
$V_{ij} \sim \mathcal{O}(\lambda^3)$

$\lambda \sim 0.22$

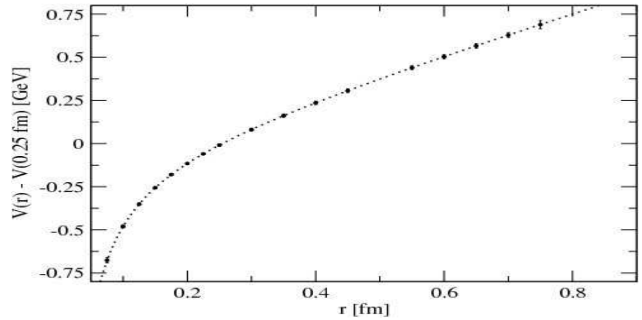
Unitarity of the CKM matrix: Glashow - Iliopoulos - Maiani mechanism, no Flavour Changing Neutral Current at tree level.



6 unitarity triangles: flavour physics constraints on sides and angles.



Quark confinement within hadrons:
a formidable theoretical challenge in flavour physics.



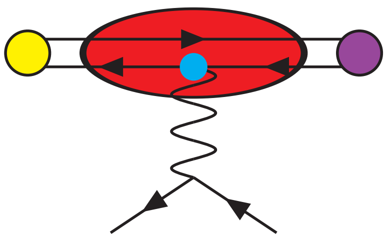
$\Gamma = Q \times T$

experimental measurement

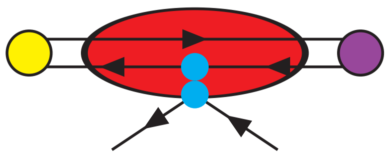
QCD contribution

parameters of the theory

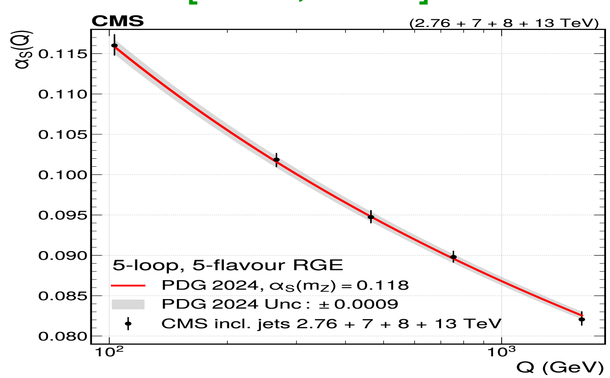
full theory



EFT

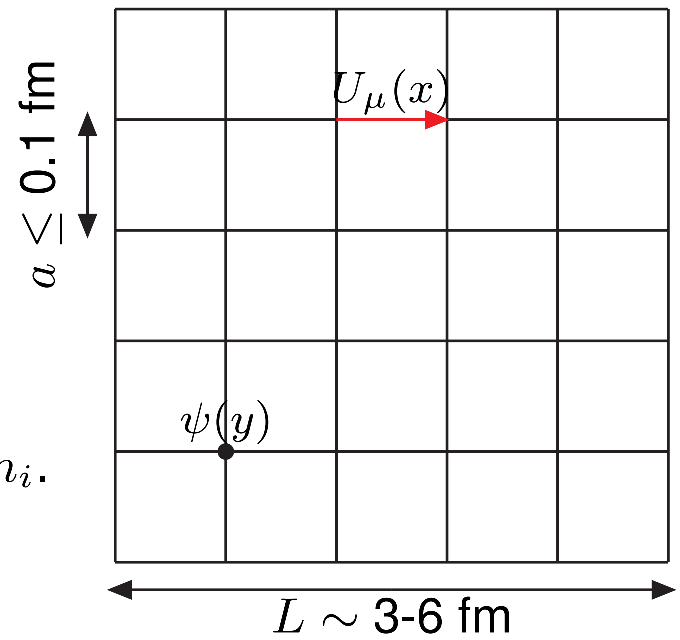


[CMS, 2025]



$Q = \sum_n q_n \alpha_s^n$: asymptotic series

Monte-Carlo simulation of QCD



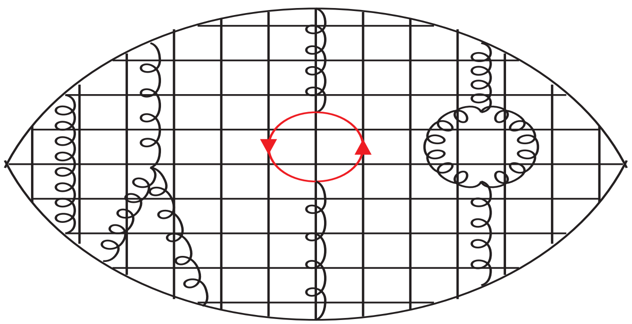
Discretisation of QCD in a finite volume of Euclidean space-time.

The lattice spacing a is a non perturbative UV cut-off of the theory.

Fields: $\psi^i(x)$, $U_\mu(x) \equiv e^{iag_0 A_\mu(x+\frac{a\hat{\mu}}{2})}$.

Inputs: bare coupling $g_0(a) \equiv \sqrt{6/\beta}$, bare quark masses m_i.

Computation of Green functions of the theory from first principles:



$$\langle O(U,\psi,\bar{\psi})\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, O(U,\psi,\bar{\psi}) e^{-S(U,\psi,\bar{\psi})}$$

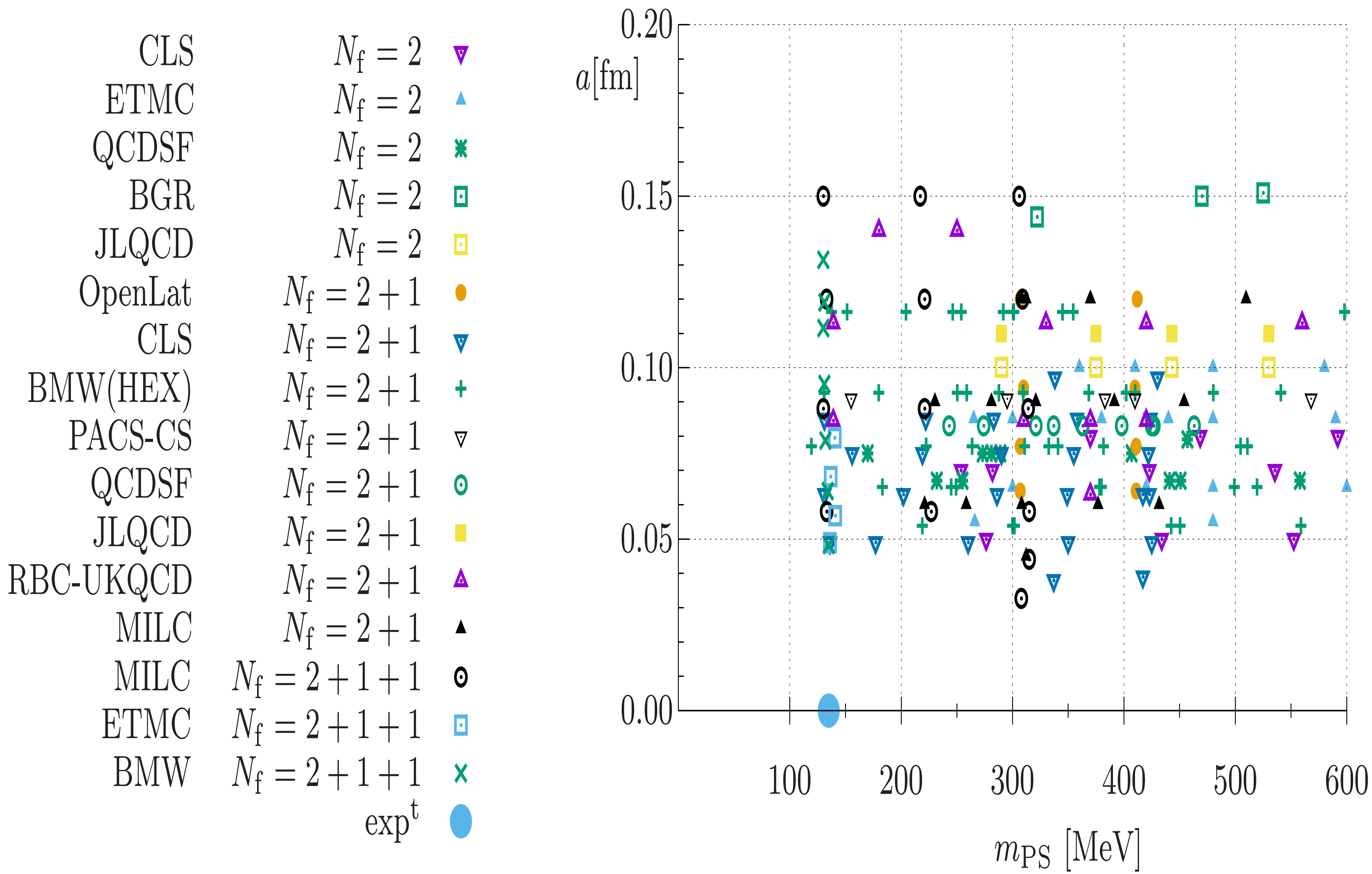
$$\mathcal{Z} = \int \mathcal{D}U\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi} e^{-S(U,\psi,\bar{\psi})}$$

$$S(U,\psi,\bar{\psi}) = S^{\rm YM}(U) + \bar{\psi}^i_x M^{ij}_{xy}(U) \psi^j_y$$

$$\mathcal{Z} = \int \mathcal{D}U\, \mathrm{Det[M(U)]} e^{-S^{\rm YM}(U)} \equiv \int \mathcal{D}U e^{-S_{\rm eff}(U)}$$

Monte Carlo simulation: $\langle O \rangle \sim \frac{1}{\mathrm{N_{conf}}} \sum_i O(\{U\}_i)$: statistical sample $\{U\}_i$ in function of the Boltzmann weight $e^{-S_{\rm eff}}$. Vacuum polarisation effects encoded in $\mathrm{Det}[M(U)]$, particularly expensive in computer time.

Lattice simulations set up



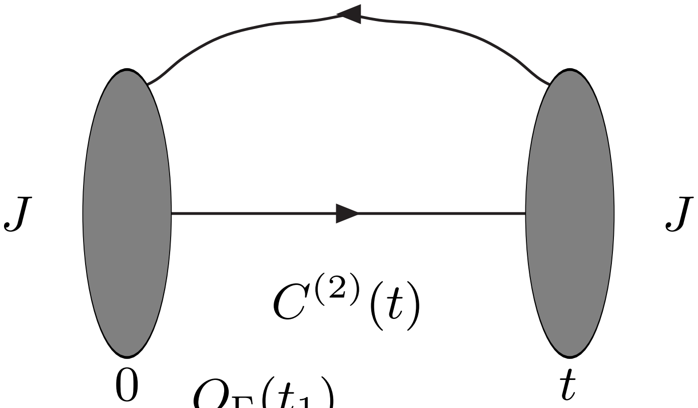
Several collaborations have built gauge ensembles at the physical point and compute with them flavour physics observables.

Strong isospin breaking and QED effects taken into account (BMW, ETMC, MILC, RC*).

Sharing of samples through the International Lattice Data Grid (ILDG).

N-pt correlators

Extraction of masses, decay constants of bound states, local and non local hadronic matrix elements; informations on spectral functions:

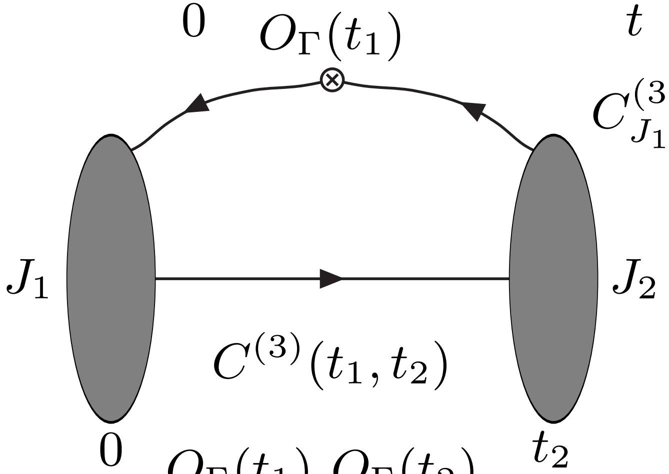


$$C^{(2)}_{JJ}(t,\vec{p}) = \sum_{\vec{x}} \langle \Omega | \mathcal{T}[J(\vec{x},t) J^\dagger(0)] | \Omega \rangle e^{-i\vec{p}\cdot\vec{x}}$$

$$= \sum_n \frac{\mathcal{Z}_n^2 e^{-E_n\, t}}{2E_n}$$

$$\mathcal{Z}_n = \langle \Omega | J | n \rangle \qquad \langle n | m \rangle = 2E_n \delta_{mn}$$

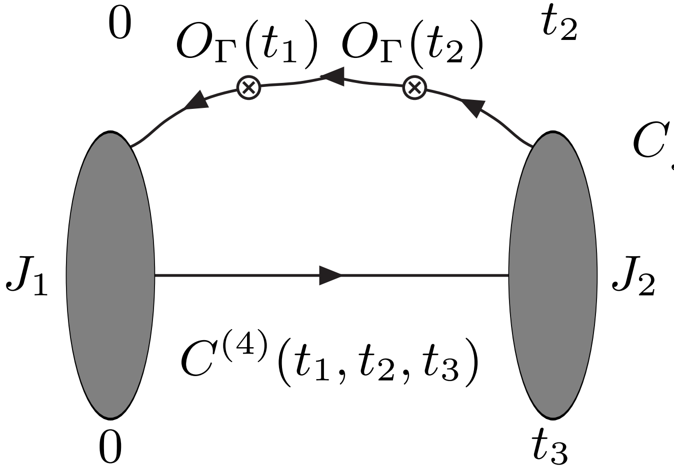
$$C^{(2)}_{JJ}(t,\vec{p}) \xrightarrow{(E_1-E_0)t \gg 1} \frac{\mathcal{Z}_0^2 e^{-E_0\, t}}{2E_0}$$



$$C^{(3)}_{J_1,J_2,O_\Gamma}(t_1,t_2,\vec{p},\vec{p}\,') = FT(\vec{p},\vec{p}\,')[\langle \Omega | \mathcal{T}[J_2(t_2) O_\Gamma(t_1) J_1^\dagger(0)] | \Omega \rangle]$$

$$\xrightarrow{t_1, t_2-t_1 \gg 0} \frac{\sqrt{\mathcal{Z}_{0,J_1}}\sqrt{\mathcal{Z}_{0,J_2}}}{2E_{0,J_1}\, 2E_{0,J_2}} e^{-E_{0,J_1}\, t_1} e^{-E_{0,J_2}\,(t_2-t_1)}$$

$$\times \langle H_0^{J_2}(\vec{p}) | O_\Gamma | H_0^{J_1}(\vec{p}\,') \rangle$$



$$C^{(4)}_{J_1,J_2,O_\Gamma}(t_1,t_2,t_3,\vec{p},\vec{p}\,',\vec{q})$$

$$= FT(\vec{p},\vec{p}\,',\vec{q})[\langle \Omega | \mathcal{T}[J_2(t_3) O_\Gamma^\dagger(t_2) O_\Gamma(t_1) J_1^\dagger(0)] | \Omega \rangle]$$

$$\xrightarrow{t_1, t_3-t_2 \gg 0} \frac{\sqrt{\mathcal{Z}_{0,J_1}}\sqrt{\mathcal{Z}_{0,J_2}}}{2E_{0,J_1}\, 2E_{0,J_2}} e^{-E_{0,J_1}\, t_1} e^{-E_{0,J_2}\,(t_3-t_2)}$$

$$\times FT(\vec{q})[\langle H_0^{J_2}(\vec{p}) | O_\Gamma^\dagger(t_2) O_\Gamma(t_1) | H_0^{J_1}(\vec{p}\,') \rangle]$$

Challenges

Lattice computation is related to phenomenology after the extrapolation to the continuum limit and the physical point of renormalized quantities.

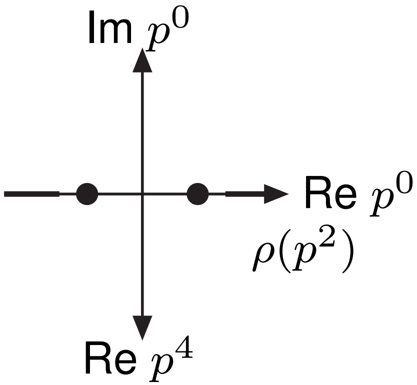
$$\langle \vec{O}^{[d]}_{\rm cont}\rangle(\mu, x_{\rm phys}) \quad = \quad \lim_{a\to 0, x\to x_{\rm phys}} Z(a\mu) \Bigg[\langle \vec{O}^{[d]}_{\rm latt}\rangle(a,x) + \sum_{i,n\geq 0} f^{(i)}_n(a,x) a^n \langle \vec{O}^{[d+n]}_{i,\rm latt}\rangle(a,x)$$

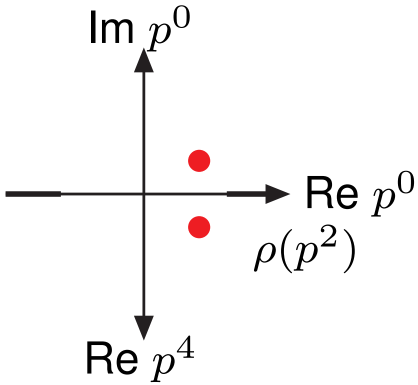
$$+ \sum_{j,m<0} g^{(j)}_m(a,x) a^m \langle \vec{O}^{[d+m]}_{j,\rm latt}\rangle(a,x) \Bigg]$$

Regularization on the lattice breaks O(4) symmetry to H(4):
– Cut-off effects to be subtracted perurbatively or non-perturbatively
– Complicated renormalization pattern, non vanishing off-diagonal elements in Z
– Eigenstates of the transfer matrix are mixing of states with different J^P spin-parity numbers
– Subtracting power-law divergent terms in a very difficult

Many theoretical works to simplify this and get reliable results:
– Symanzik programm up to $O(a^2)$
– regularisations with partially restored symmetries
– interpolators projection on a single H(3) irreducible representation
– smoothing of fields techniques like the gradient flow

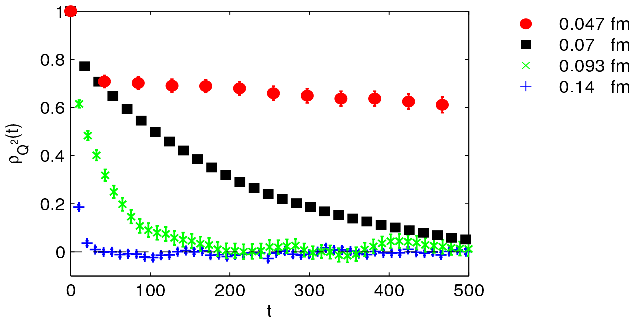
Analytical continuation of Euclidean to Minkowskian spacetime is straightforward if poles of the spectral functions are located on the real axis. Further, much harder, work is required to tackle spectral functions with complex poles. [talk by F. Erben]. In first approximation, states like D^* are considered stable.



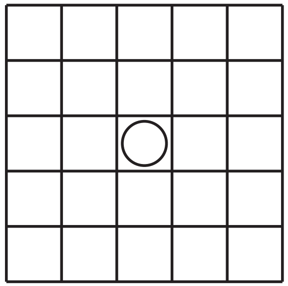


Simulations close to the continuum limit affected by the critical slowing down. Topological charge not properly sampled. Long standing issue, no optimal solution yet (open boundary conditions, metadynamics).

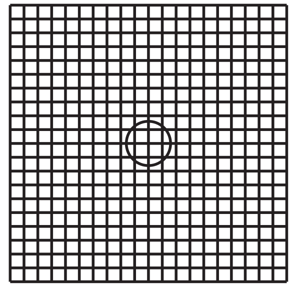
[S. Schaefer and F. Virotta, 2010]



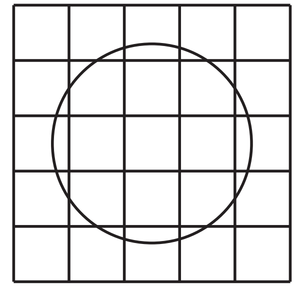
B physics affected by systematic effects: excited states, noisy data, $am_b \geq 1$: solutions based on effective field theory framework, aggressive subtraction of some cut-off effects,



Cut-off Effects



cut-off effects

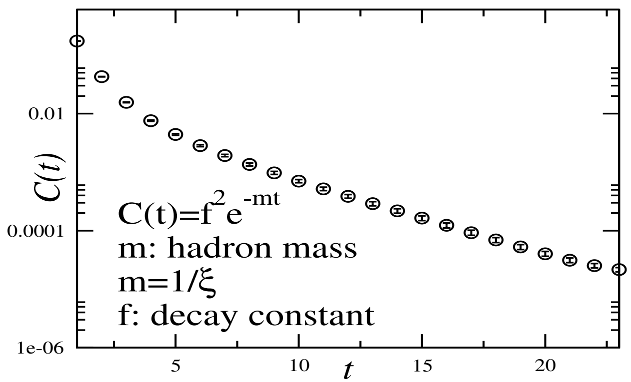


cut-off effects

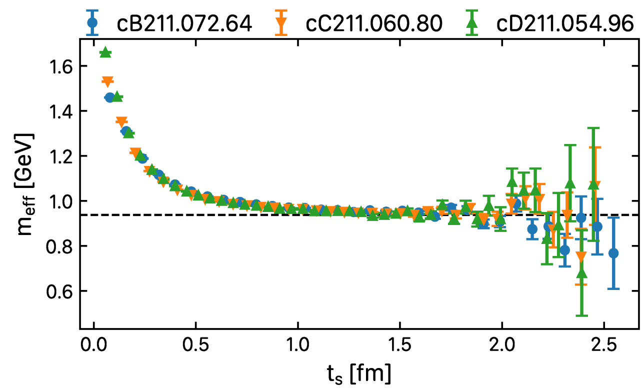
Lattice data for baryons will never be competitive due to the noise.

pion: $C^{(2)}(t)/\sqrt{[(\delta C^{(2)}(t))^2]} \sim 1$

nucleon: $C^{(2)}(t)/\sqrt{[(\delta C^{(2)}(t))^2]} \sim e^{-(m_N - 3/2 m_\pi)t}$



[C. Alexandrou *et al*, 2023]



Degradation of the signal for mesons correlators at non zero momentum. Example for a pion: $C^{(2)}(t,\vec{p})/\sqrt{[(\delta C^{(2)}(t,\vec{p}))^2]} \sim e^{-(E_\pi - m_\pi)t}$

Flavour Lattice Averaging Group (FLAG) [flag.itp.unibe.ch/2024/]

Collection of useful results after a careful survey of the world-wide work among the lattice community.

Quantities under study:

- – Quark masses
- – V_{ud} and V_{us}
- – Low Energy Constants
- – Kaon mixing bag parameter B_K
- – V_{cd}, V_{cs}, V_{ub} and V_{cb}
- – Strong coupling constant α_s
- – $B_{(s)}$ and $D_{(s)}$ meson decay constants
- – B mixing bag parameter B_B
- – form factors of $B_{(s,c)}$, $D_{(s)}$ and Λ_c SL decays
- Nucleon form factors

Technicalities and systematic issues are difficult to address by non experts. FLAG is performing global averages of results, after a selection according to several quality criteria:

– continuum limit extrapolation

★ 3 or more lattice spacings, 2 smaller than 0.1 fm, $a^2_{\max}/a^2_{\min} \geq 2$

○ 2 or more lattice spacings, 1 smaller than 0.1 fm, $a^2_{\max}/a^2_{\min} \geq 1.4$

■ otherwise

– renormalization and matching:

★ absolutely renormalized or non-pertubative

○ 1-loop perturbation theory or higher with an estimate of truncation error

■ otherwise

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Nucleon form factors

– finite-volume for QCD

★ $(m^2_{\pi\,\mathrm{min}}/m^2_{\pi\,\mathrm{fid}})\exp(4 - m_{\pi\,\mathrm{min}}L) < 1$ or 3 volumes at fixed simulation parameters

○ $(m^2_{\pi\,\mathrm{min}}/m^2_{\pi\,\mathrm{fid}})\exp(3 - m_{\pi\,\mathrm{min}}L) < 1$ or 2 volumes at fixed simulation parameters

■ otherwise

$m_{\pi\,\mathrm{fid}} = 200$ MeV

– finite-volume for QED

★ $1/(Lm_{\pi\,\mathrm{min}})^{n_{\mathrm{min}}} \leq 0.02$ or at least 4 volumes

○ $1/(Lm_{\pi\,\mathrm{min}})^{n_{\mathrm{min}}} \leq 0.04$ or at least 3 volumes

■ otherwise

$n_{\mathrm{min}} \leq 3$

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Nucleon form factors

– finite-volume for IB
★ all effects included in the lattice computation
○ effects taken into account in the electro-quenched approximation
■ otherwise

– chiral extrapolation
★ $m_{\pi\,\mathrm{min}} \lesssim 200$ MeV with 3 pion masses or 2 pion masses with $m_{\pi\,\mathrm{min}} = 135 \pm 10$ MeV and $m_{\pi\,\mathrm{max}} \leq 200$ MeV
○ 3 pion masses with 200 MeV $\lesssim m_{\pi\,\mathrm{min}} \lesssim 400$ MeV or 2 pion masses with $m_{\pi\,\mathrm{min}} < 200$ MeV
■ otherwise

– Heavy quark:
★ tree-level $\mathcal{O}(1/m_b)$ matching and full $\mathcal{O}(a)$ improvement in the case of EFT (HQET, NRQCD), at least tree-level $\mathcal{O}(a)$ improvement in the case of relativistic heavy qu
■ otherwise

Exclusive decays

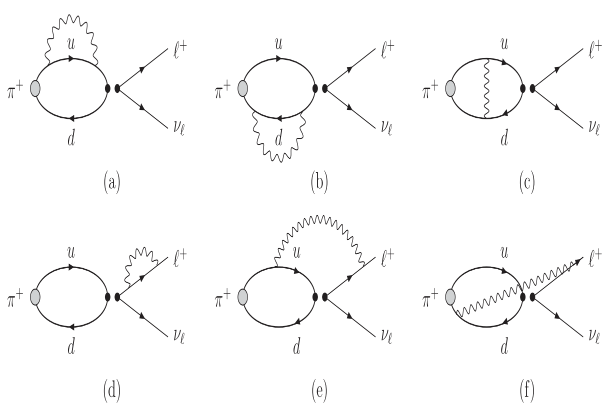
Extracting decay constants of pseudoscalar mesons in the isosymmetric limit is a mature subject

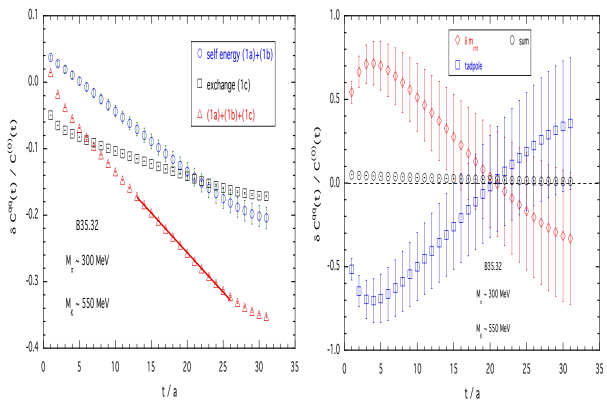
$f_K/f_\pi \Longrightarrow$ constraint on V_{us}/V_{ud}

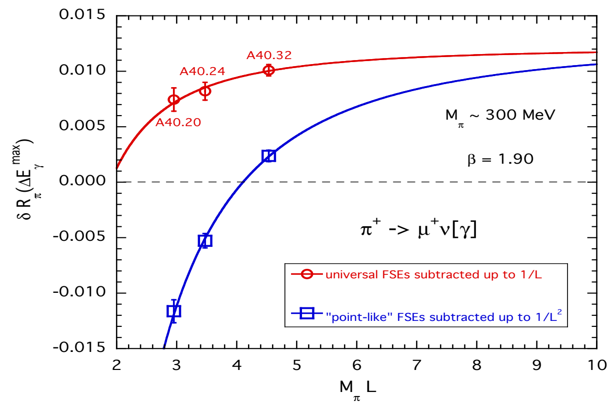
A source of uncertainty on $f_{D_{(s)}}$ and $f_{B_{(s)}}$ difficult to reduce is the scale setting: lattice spacing a usually fixed from f_π or m_Ω

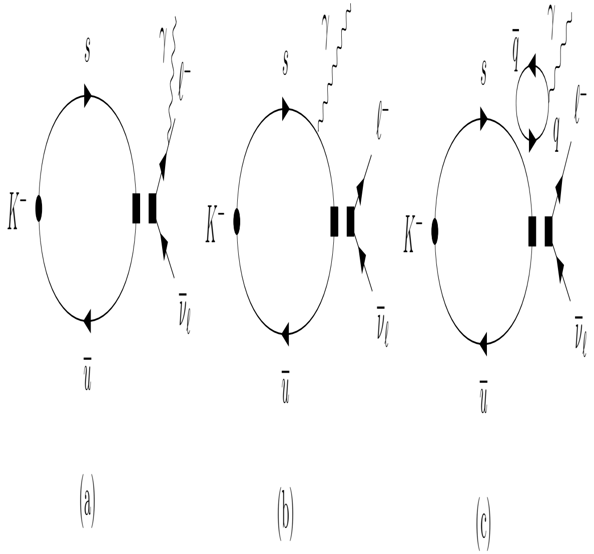
Isospin breaking correction $\sim$ a few %: $m_u \neq m_d$ effects computed by inserting extra operators in isosymmetric correlators. Corrections from virtual photon exchanges and radiative decays examined by combining non-perturbative computation of QCD matrix elements with an elegant subtraction of infrared divergences.

[V. Lubicz *et al*, 2015, 2016]

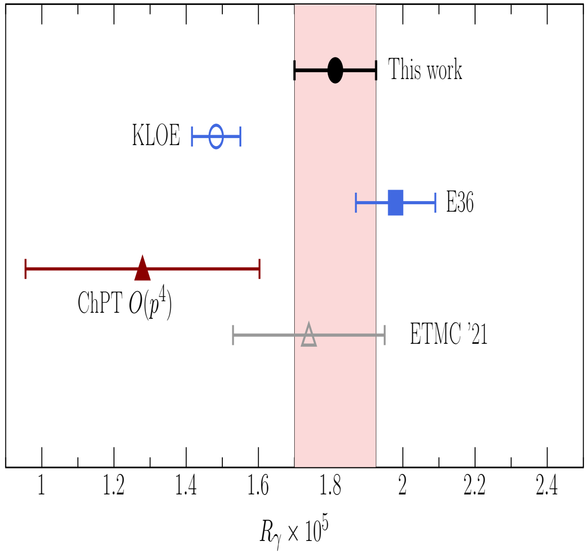


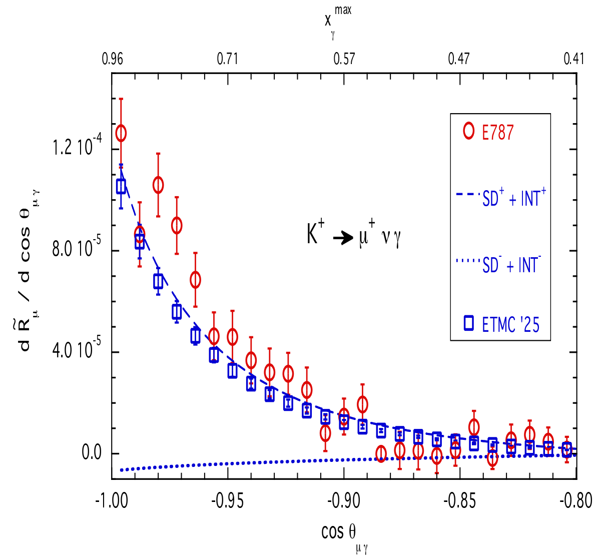






[R. Di Palma *et al*, 2025]



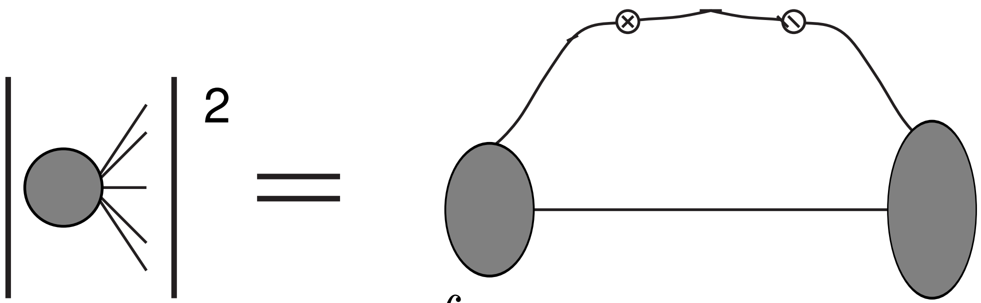


Form factors associated to $\langle H_1|J|H_2\rangle$, in the approximation of narrow H_1 and H_2 states, are subject to more systematic effects. Critical in the B sector, due to a strong contamination from excited states if $t_{\rm sink} - t_{\rm source} \lesssim 1.5$ fm (almost every calculations of $B \to D^{(*)}$!)

Strong correlation among form factors of $\langle P|V, A|V\rangle$

Systematic errors reported in a Bayesian inference framework

Inclusive decays



$$\Gamma = \int dQ \rho^{\mu\nu}(Q) K_{\mu\nu}(Q) \equiv \rho \otimes K$$

K is a non-QCD “kernel", ρ is the QCD spectral function

Spectral functions and 2n-pt Euclidean correlators are related through a Laplace transform:
$C(t) = \int d\omega \rho(\omega) e^{-\omega t}$

Direct extraction of $\rho(\omega)$: a finite number of noisy and correlated date $\Longrightarrow$ ill-posed inverse problem

A first method: expand $K(\omega)$ in a sum of orthogonal polynomials, for instance, Chebyshev

$$K(\omega) = \sum_j \lambda_j T_j(e^{-\omega}) = \sum_j \lambda_j e^{-\omega j} \quad \Gamma = \sum_j \lambda_j C(j)$$

A second method: smear the spectral function itself

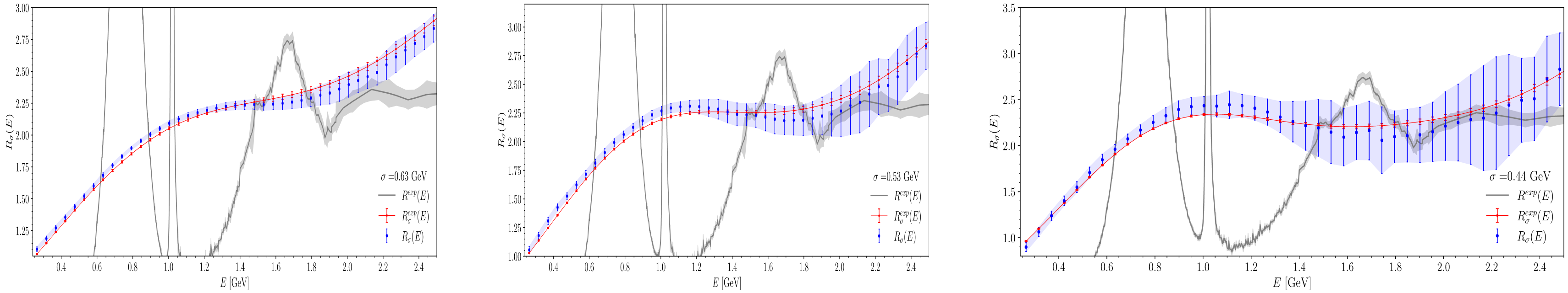
$$\rho(\omega) = \int dE \rho(E) \delta(\omega - E) = \lim_{\sigma \to 0} \int dE \rho(E) \Delta_\sigma(\omega - E) \quad \Delta_\sigma(\omega - E) = \sum_j g_j(\sigma, E) e^{-Ej}$$

$$\Gamma = \lim_{\sigma \to 0} \Gamma_\sigma = \int d\omega K(\omega) \sum_j g_j(\sigma, \omega) C(j) \quad \rho(\omega) = \lim_{\sigma \to 0} \sum_j g_j(\sigma, \omega) C(j)$$

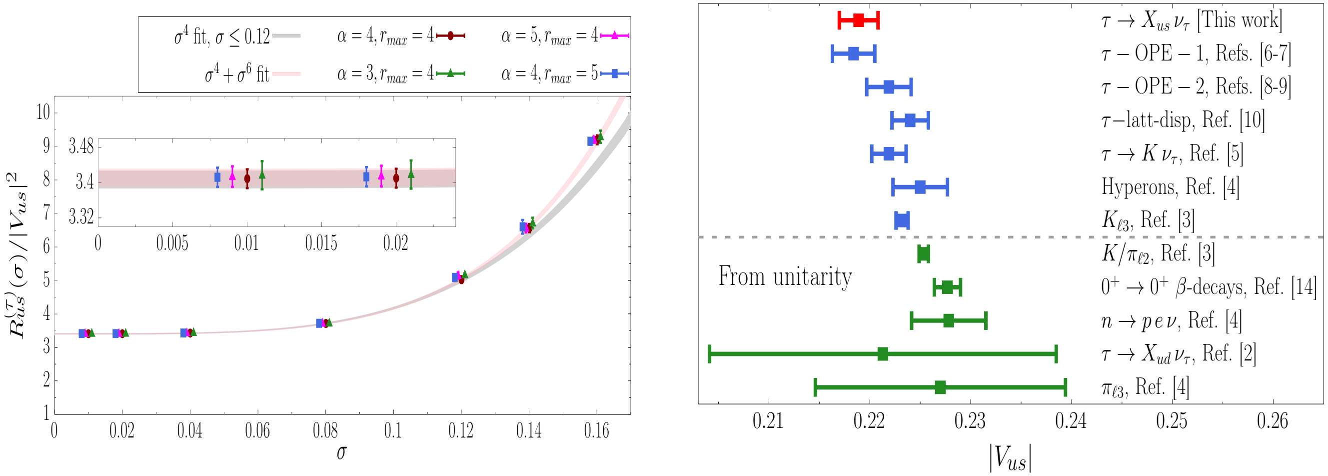
Coefficients g_j obtained in a Bayesian inference framework.

Application to the R ratio $\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$, inclusive hadronic τ and semileptonic D decays

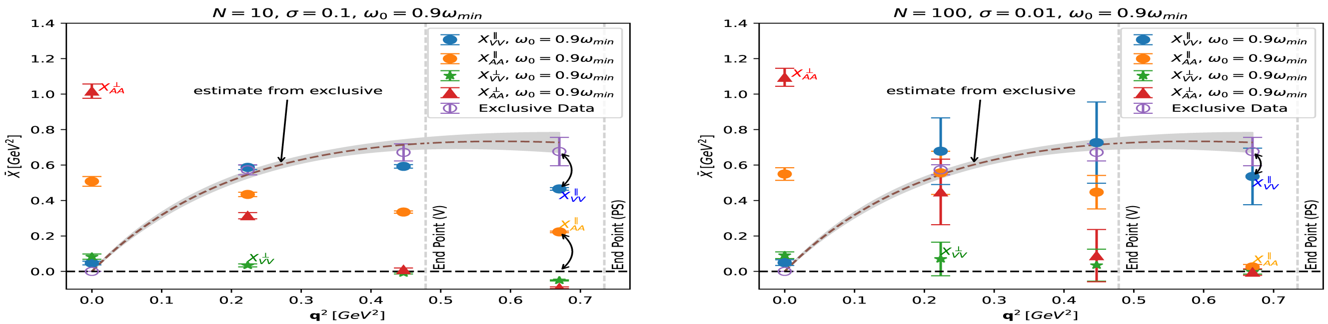
[D. Alexandrou *et al*, 2023]



[A. Evangelista *et al*, 2023]



[R. Kellermann *et al*, 2022]



Outlook

- Impressive achievements regarding simulations: QCD at the physical point, set-up with isospin breaking
- Exclusive decays: golden modes in the light, sttrange and charm sectors very well addressed
- B physics is still a challenge
- Broad states are under benchmarking
- Applied mathematics methods shared with other domains of QCD (finite temperature, hadron structure) or other fields and disciplines (condensed matter, astrophysics, geology) offer an exciting avenue to examine inclusive decays.