Hadronic uncertainties in $b \to c$ transitions

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Taming Hadronic Uncertainties in and Beyond the Standard Model

IJCLab

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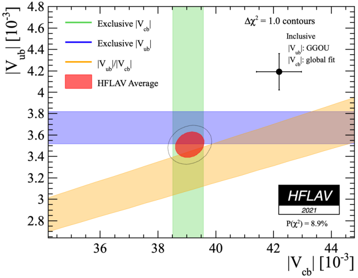
Long-standing puzzles in semileptonic decays

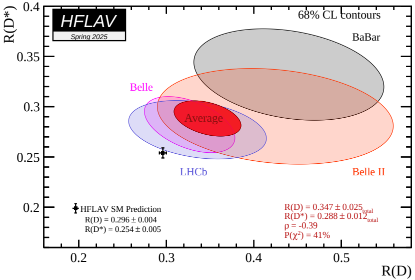


- Inclusive determination: $B \to X_c \ell \bar{\nu}$
  - $\Rightarrow$ Stable against various datasets
- Exclusive decays: $B \to D^{(*)} \ell \bar{\nu}$, $\Lambda_b \to \Lambda_c \ell \bar{\nu}$
  - $\Rightarrow$ Lattice QCD results are in tension
  - $\Rightarrow$ Experimental measurement show various disagreements

Lepton flavour universality

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)} \tau \bar{\nu})}{\mathcal{B}(B \to D^{(*)} \ell \bar{\nu})}$$

- Current discrepancy at the order of 3.3σ
- HFLAV theory prediction has arithmetic average of various determinations



Inclusive decays

Theory framework for $B \to X_c \ell \bar{\nu}$

Double expansion in $1/m$ and α_s

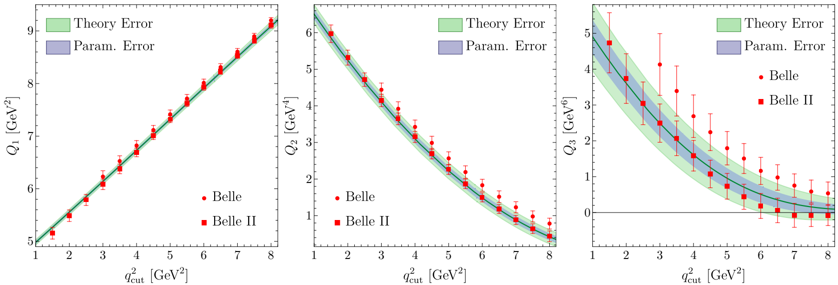
$$\Gamma_{sl} = \Gamma_0 f(\rho) \Big[1 + a_1 \left(\frac{\alpha_s}{\pi}\right) + a_2 \left(\frac{\alpha_s}{\pi}\right)^2 + a_3 \left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1 \left(\frac{\alpha_s}{\pi}\right)\right) \frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(g_0 + g_1 \left(\frac{\alpha_s}{\pi}\right)\right) \frac{\mu_G^2(m_b)}{m_b^2} + d_0 \frac{\rho_D^3}{m_b^3} - g_0 \frac{\rho_{LS}^3}{m_b^3} + \dots \Big]$$

- The coefficients a_i, p_i, g_i, d_i are known
- $\mu_\pi^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v (i\vec{D})^2 b_v | B \rangle_\mu$ $\quad$ $\mu_G^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v | B \rangle_\mu$
- Efforts ongoing to extract information from Lattice QCD [P. Gambino, S. Hashimoto, '20]
  - $\Rightarrow$ Physical results for $D_s \to X \ell \bar{\nu}$ [ETMC, '25]
  - $\Rightarrow$ Ongoing efforts for $B_{(s)} \to X_c \ell \bar{\nu}$ [Barone et al., '23]
- α_s^3 corrections are known [Fael, Schönwald, Steinhauser, '20]
- Two ways:
  - Hadronic and lepton mass moments
  - q^2 moments [Fael, Mannel, Vos, '18, Bernlochner at al., '22]

Global fit

[MB, Capdevila, Gambino, '21, Finauri, Gambino, '23]

	$m_b^{\rm kin}$	$\overline{m}_c$	μ_π^2	μ_G^2	ρ_D^3	ρ_{LS}^3	$10^2{\rm BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$	$\chi^2_{\rm min}(/{\rm dof})$
without	4.573	1.092	0.477	0.306	0.185	−0.130	10.66	42.16	22.3
q^2-moments	0.012	0.008	0.056	0.050	0.031	0.092	0.15	0.51	0.474
Belle II	4.573	1.092	0.460	0.303	0.175	−0.118	10.65	42.08	26.4
	0.012	0.008	0.044	0.049	0.020	0.090	0.15	0.48	0.425
Belle	4.572	1.092	0.434	0.302	0.157	−0.100	10.64	41.96	28.1
	0.012	0.008	0.043	0.048	0.020	0.089	0.15	0.48	0.476
Belle &	4.572	1.092	0.449	0.301	0.167	−0.109	10.65	42.02	41.3
Belle II	0.012	0.008	0.042	0.048	0.018	0.089	0.15	0.48	0.559



About QED effects in inclusive decays

Why do we care about QED Effects?

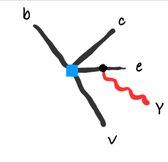
- We want to match the theory description with the experimental measurements that are always affected by photon emissions
- The MC PHOTOS accounts for QED effects, reporting results which can be compared with the non-radiative theory predictions
- PHOTOS knows only about real emission and obtains the virtual part by normalisation

$$\frac{d\Gamma}{dzdx} = \mathcal{F}^{(0)}(\omega_{\text{virtual}} + \omega_{\text{real}}) \Rightarrow \int dx(\omega_{\text{virtual}} + \omega_{\text{real}}) = 1$$

Are virtual corrections under control?

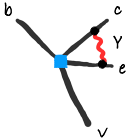
QED effects for inclusive V_{cb}

1. **Collinear logs**: captured by splitting functions



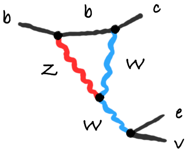
$$\sim \frac{\alpha_e}{\pi} \log^2\left(\frac{m_b^2}{m_e^2}\right)$$

2. **Threshold effects** or Coulomb terms



$$\sim \frac{2\pi\alpha_e}{3}$$

3. **Wilson Coefficient**



$$\sim \frac{\alpha_e}{\pi}\left[\log\left(\frac{M_Z^2}{\mu^2}\right) - \frac{11}{6}\right]$$

Branching ratio

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- The total branching ratio is not affected by large logs due to KLN theorem
- The large corrections are from the Wilson Coefficient and the threshold effects

$$\frac{\Gamma}{\Gamma^{(0)} g(\rho)} = 1 + \frac{\alpha}{\pi}\left[\ln\left(\frac{M_Z^2}{m_b^2}\right) - \frac{11}{6} + 5.516(14)\right]$$

$$= 1 + 1.43\% - 0.44\% + 1.32\% = 1 + 2.31\%$$

Wilson Coefficient ($1.43\% - 0.44\%$) — Threshold effects (1.32%)

- Large shift of the branching ratio of the same order of the current error on V_{cb}
- How do we incorporate in the current datasets?
  - $\Rightarrow$ At the moment possible only for BaBar data
  - $\Rightarrow$ The global fit changes minimally

Finauri, Gambino, '23
Carvunis, Finauri, Gambino, Jung, Mächler

- Moments are less sensitive because they are normalised

Exclusive decays

Exclusive $b \to c$ matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

Exclusive $b \to c$ matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

$\longleftarrow$ form factor

scale Λ_{QCD}

independent

Lorentz structures

Exclusive $b \to c$ matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

scale Λ_{QCD}

independent Lorentz structures

form factor

Form factors determinations

- Lattice QCD
- QCD SR, LCSR

only points at specific kinematic points

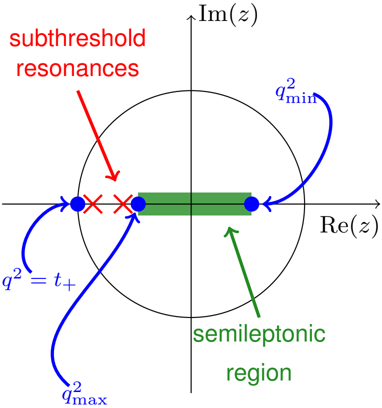
Form factors parametrisations

- HQET (CLN + improvements) $\Rightarrow$ reduce independent degrees of freedom
- Analytic properties $\to$ BGL

data points needed to fix the coefficients of the expansion

The z-expansion and unitarity

[Boyd, Grinstein, Lebed, '95, Caprini, Lellouch, Neubert, '98]



- in the complex plane form factors are real analytic functions
- q^2 is mapped onto the conformal complex variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- q^2 is mapped onto a disk in the complex z plane, where $|z(q^2, t_0)| < 1$

$$F_i = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

$$\sum_{k=0}^{n_i} |a_k^i|^2 < 1$$

How to apply unitarity

- Penalty function in the χ^2 or likelihood [P. Gambino, M. Jung, S. Schacht, '19]

$$\chi^2 \to \chi^2(a_k^i, a_k^i|_{\mathrm{data}}) + w_i \theta\left(\sum_{k=0}^{n_i} |a_k^i|^2 - 1\right)$$

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$$\chi^2 \to \chi^2(a_k^i, a_k^i|_{\text{data}}) + w_i\theta\left(\sum_{k=0}^{n_i}|a_k^i|^2 - 1\right)$$

- Dispersive Matrix Method [M. Di Carlo, G. Martinelli, M. Naviglio, F. Sanfilippo, S. Simula, L. Vittorio, '21] [G. Martinelli, S. Simula, L. Vittorio, '21,'23]

$$\mathbf{M} = \begin{pmatrix} \chi & \phi f & \phi_1 f_1 & \phi_2 f_2 & \ldots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-zz_1} & \frac{1}{1-zz_2} & \ldots & \frac{1}{1-zz_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \frac{1}{1-z_1 z_2} & \ldots & \frac{1}{1-z_1 z_N} \\ \phi_2 f_2 & \frac{1}{1-z_2 z} & \frac{1}{1-z_2 z_1} & \frac{1}{1-z_2^2} & \ldots & \frac{1}{1-z_2 z_N} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \frac{1}{1-z_N z_2} & \ldots & \frac{1}{1-z_N^2} \end{pmatrix}$$

$$\det\mathbf{M} > 0 \Rightarrow \beta - \sqrt{\gamma} \leq f_0 \leq \beta + \sqrt{\gamma}$$

How to apply unitarity

- Penalty function in the χ^2 or likelihood

[P. Gambino, M. Jung, S. Schacht, '19]

$$\chi^2 \to \chi^2(a_k^i, a_k^i|_{\rm data}) + w_i \theta\left(\sum_{k=0}^{n_i} |a_k^i|^2 - 1\right)$$

- Dispersive Matrix Method

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$$\det \mathbf{M} > 0 \Rightarrow \beta - \sqrt{\gamma} \leq f_0 \leq \beta + \sqrt{\gamma}$$

- Bayesian inference

[J. Flynn, A. Jüttner, T. Tsang, '23]

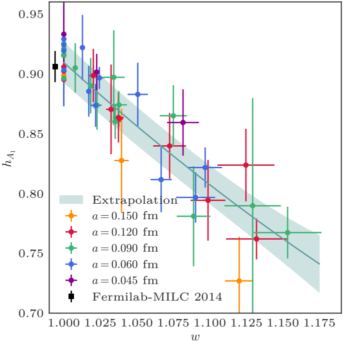
$$\langle g(\mathbf{a})\rangle = \mathcal{N} \int d\mathbf{a}\, g(\mathbf{a})\, \pi(\mathbf{a}|\mathbf{f}, C_{\mathbf{f}}) \pi_{\mathbf{a}}$$

$\theta(1 - |\mathbf{a}|^2)$

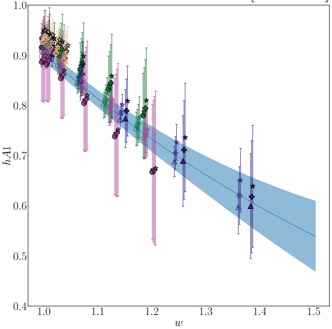
contains the lattice χ^2

$B \to D^*$ from lattice away from zero recoil

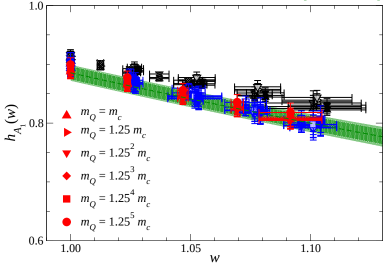
[2105.14019]



[2304.03137]

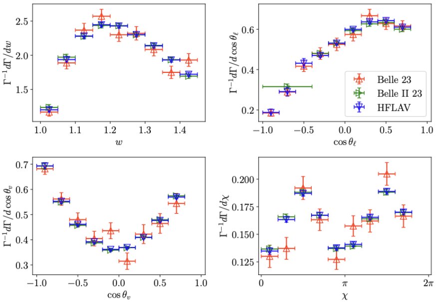


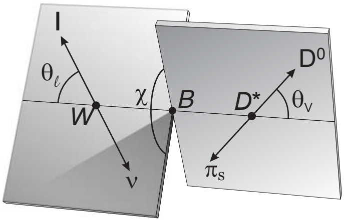
[2306.05657]



- Are these results compatible with each other?
- Are they compatible with experimental data?

New $B \to D^* \ell \bar{\nu}$ Belle and Belle II data



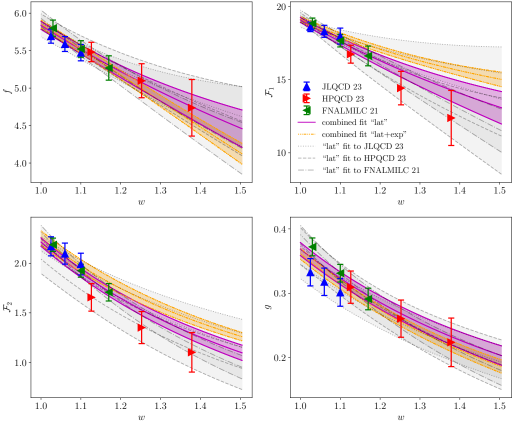


$$\frac{d\Gamma}{dw d\cos(\theta_\ell) d\cos(\theta_v) d\chi} = \frac{3G_F^2}{1024\pi^4} |V_{cb}|^2 \eta_{EW}^2 M_B r^2 \sqrt{w^2-1} q^2$$
$$\times \{ (1-\cos(\theta_\ell))^2 \sin^2(\theta_v) H_+^2(w) + (1+\cos(\theta_\ell))^2 \sin^2(\theta_v) H_-^2(w)$$
$$+ 4\sin^2(\theta_\ell)\cos^2(\theta_v) H_0^2(w) - 2\sin^2(\theta_\ell)\sin^2(\theta_v)\cos(2\chi) H_+(w) H_-(w)$$
$$- 4\sin(\theta_\ell)(1-\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi) H_+(w) H_0(w)$$
$$+ 4\sin(\theta_\ell)(1+\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi) H_-(w) H_0(w) \}$$

- Between 7 to 10 bins per kinematic variable
- Available on HEPData with correlations
- Angular observables analysis are available, data just newly released

Fits to Lattice and Experimental data

[MB, A. Jüttner, '24]



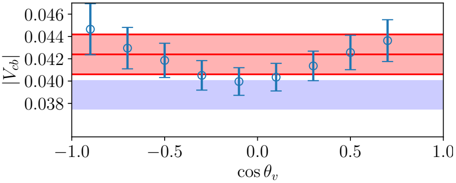
- Good fit quality for fits to LQCD data or LQCD + experimental data
- Adding experimental data reduces the uncertainties, especially at large w
- Especially for $\mathcal{F}_1$ and $\mathcal{F}_2$, the shape changes between two datasets

for similar results see also Martinelli, Simula, Vittorio, '23

$|V_{cb}|$ extraction

$$|V_{cb}|_{\alpha,i} = \left(\Gamma_{\rm exp}\left[\frac{1}{\Gamma}\frac{d\Gamma}{d\alpha}\right]^{(i)}_{\rm exp} \Big/ \left[\frac{d\Gamma_0}{d\alpha}(\mathbf{a})\right]^{(i)}_{\rm lat}\right)^{1/2}$$

[MB, A. Jüttner, '24]



Blue band

- Frequentist fit $p-$value $\sim 0\%$
- Affected by d'Agostini Bias
  - $\Rightarrow$ Severe especially for FNAL/MILC and HPQCD data

Red band

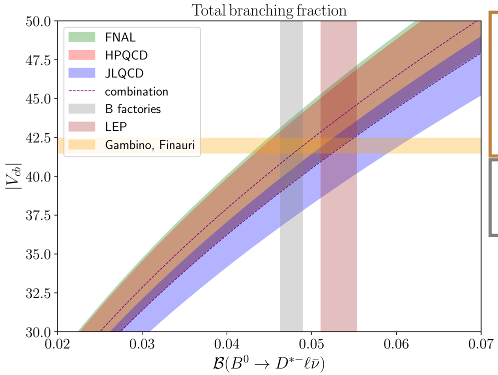
- Frequentist fit $p-$value $\sim 0\%$
- Akaike-Information-Criterion analysis: average over all possible fits with at least two data points and then weighted average

$$w_{\{\alpha,i\}} = \mathcal{N}^{-1}\exp\left(-\frac{1}{2}(\chi^2_{\{\alpha,i\}} - 2N_{{\rm dof},\{\alpha,i\}})\right) \qquad \text{where} \quad \mathcal{N} = \sum_{\text{sets }\{\alpha,i\}} w_{\rm set}$$

$$|V_{cb}| = \langle |V_{cb}|\rangle \equiv \sum w_{\rm set}|V_{cb}|_{\rm set}$$

Exclusive V_{cb} from total branching fraction

MB, A. Jüttner, '23 + WIP



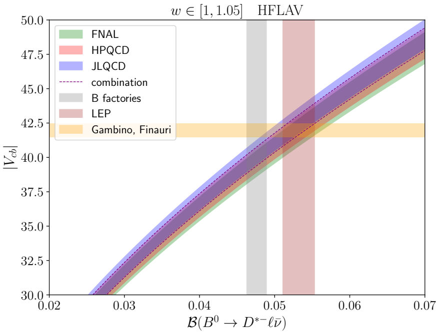
Experiment	BF (rescaled) [%]	Parameters	
ALEPH	5.56 +/- 0.27 +/- 0.33	input	parameters
OPAL incl	6.13 +/- 0.28 +/- 0.57	input	parameters
OPAL excl	5.17 +/- 0.20 +/- 0.36	input	parameters
DELPHI incl	4.96 +/- 0.14 +/- 0.35	input	parameters
DELPHI excl	5.23 +/- 0.20 +/- 0.42	input	parameters
CLEO	6.17 +/- 0.19 +/- 0.37	input	parameters
BELLE untagged	4.90 +/- 0.02 +/- 0.16	input	parameters
BELLE tagged	4.95 +/- 0.11 +/- 0.22	input	parameters
BABAR untagged	4.52 +/- 0.04 +/- 0.33	input	parameters
BABAR tagged	5.26 +/- 0.16 +/- 0.31	input	parameters
Average	**5.06 +/- 0.02 +/- 0.12**	**chi2/dof = 16.0/9 (CL = 0.0661)**	

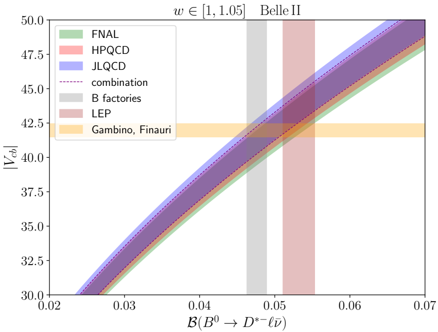
- Shape information shifts the total branching fraction prediction

Thanks to C. Schwanda for the averages!

Exclusive V_{cb} close to zero-recoil

MB, A. Jüttner, ’23 + WIP





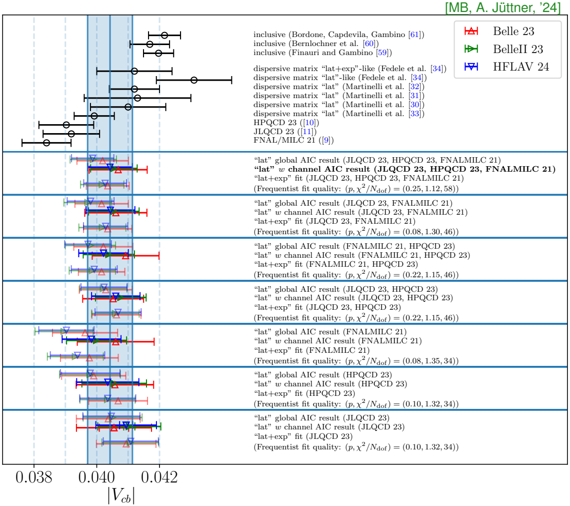
B factories: $|V_{cb}| = 40.07 \pm 0.86$

LEP: $|V_{cb}| = 42.37 \pm 1.09$

B factories: $|V_{cb}| = 41.24 \pm 1.15$

LEP: $|V_{cb}| = 43.60 \pm 1.35$

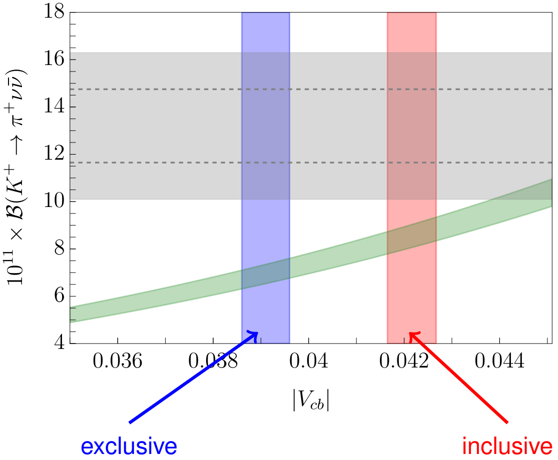
$|V_{cb}|$ - Summary



- Residual 2σ difference with inclusive
- The AIC produces slightly larger uncertainties, overall all results are quite consistent

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \propto |\lambda_{ts}|^2 \quad \lambda_{ts} \equiv \lambda |V_{cb}|^2 \left[(\bar{\rho} - 1) \left(1 - \frac{\lambda^2}{2} \right) + i\bar{\eta} \left(1 + \frac{\lambda^2}{2} \right) \right] + \mathcal{O}(\lambda^4)$$

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \propto |\lambda_{ts}|^2 \quad \lambda_{ts} \equiv \lambda |V_{cb}|^2 \left[(\bar{\rho} - 1) \left(1 - \frac{\lambda^2}{2} \right) + i\bar{\eta} \left(1 + \frac{\lambda^2}{2} \right) \right] + \mathcal{O}(\lambda^4)$$



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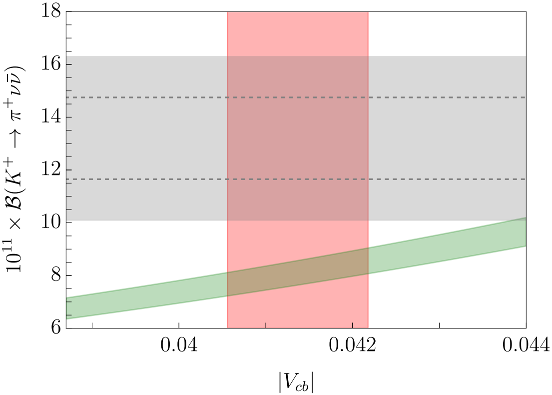


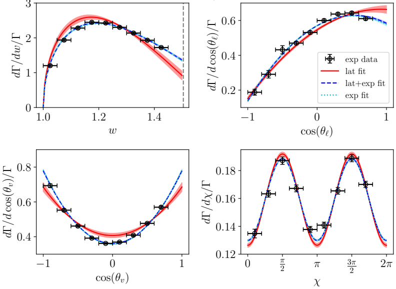
$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})^{\rm SM} = (8.09 \pm 0.63) \times 10^{-11}$$

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})^{\rm SM} = (2.58 \pm 0.30) \times 10^{-11}$$

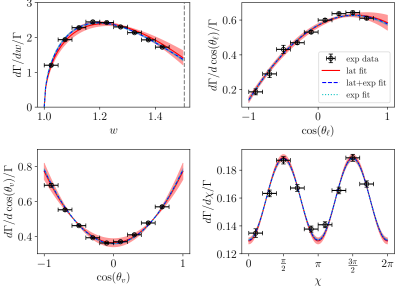
Comparison with experimental data

FNAL/MILC + HPQCD +HFLAV

[MB, A. Jüttner, '24]



JLQCD +HFLAV



- Fit to HPQCD and FNAL/MILC misses experimental points
- BGL fit to experimental and lattice data has $p-$value $\sim 18\%$
- BGL coefficients shift of a few σ between using or not experimental data

The Heavy Quark Expansion in a nutshell

The HQE exploits the fact that the b and c quarks are heavy

- Double expansion in $1/m_{b,c}$ and α_s
- The HQE symmetries relate $B^{(*)} \to D^{(*)}$ form factors
- At $1/m_{b,c}$ drastic reduction of independent degrees of freedom

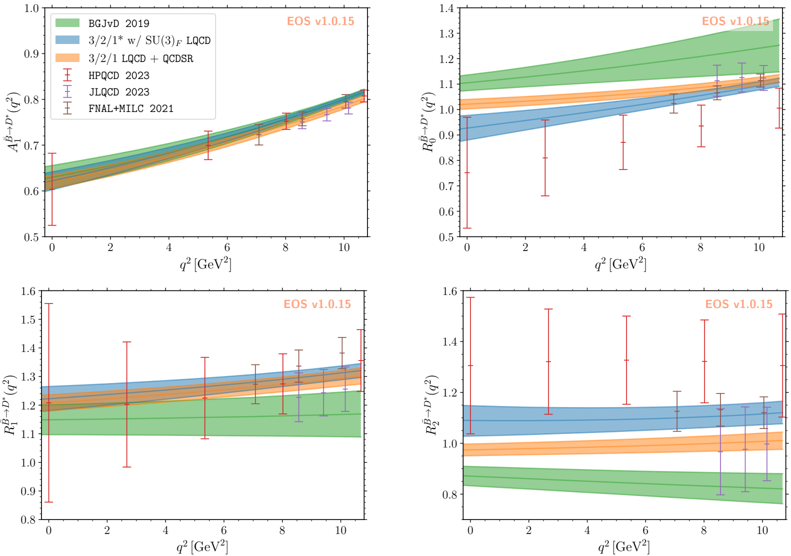
With current precision we know we have to go beyond the $1/m_{b,c}$ order and we use the following form

$$F_i = \left(a_i + b_i \frac{\alpha_s}{\pi}\right) \xi + \frac{\Lambda_{\rm QCD}}{2m_b} \sum_j c_{ij} \xi^j_{\rm SL} + \frac{\Lambda_{\rm QCD}}{2m_c} \sum_j d_{ij} \xi^j_{\rm SL} + \left(\frac{\Lambda_{\rm QCD}}{2m_c}\right)^2 \sum_j g_{ij} \xi^j_{\rm SSL}$$

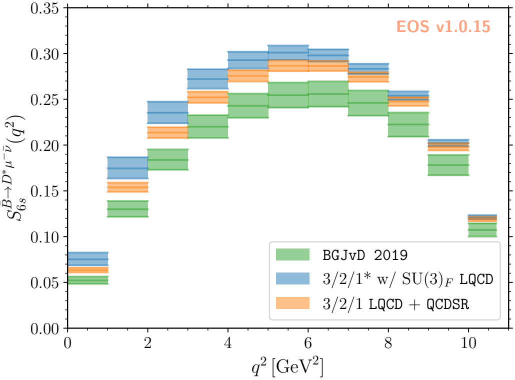
- Total of 10 independent structures to be extracted from data
- We use the conformal mapping $q^2 \mapsto z(q^2)$ to include bounds and have a well-behaved series

What happens in HQET?

[MB, N. Gubernari, M. Jung, D. van Dyk, '25]

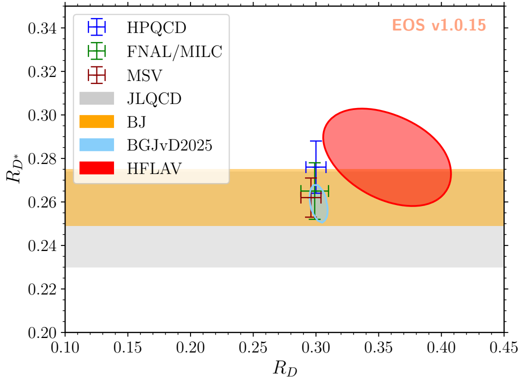


An example: $A_{\rm FB}$



- Using different datasets results in different theory predictions
- Current experimental measurements still use larger bins
  - ⇒ These effects are less evident in wider bins

Impact on lepton flavour universality



- Various predictions all agree within the 1σ range
- For BJ we apply a systematic error to account that all central values from the 3 LQCD collaborations are covered
- BGJvD has smaller uncertainties due to the correlations in the HQET framework and the inclusion of $B \to D$ data

Where do we stand and where do we go next?

Experimental data

- New measurements of inclusive and exclusive branching fractions
- New shape measurements at LHCb
- Where are the differences between Belle and Belle II coming from?

Theory

- For inclusive decays, lattice has started, and results will come
- For exclusive decays, we can try to validate the LQCD results a posteriori, but major or new reanalyses are needed

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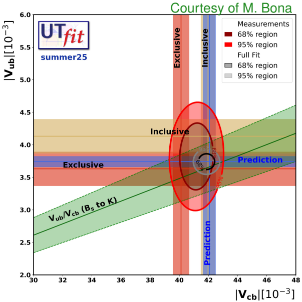
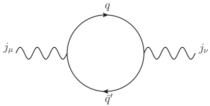


$\Rightarrow$ Unitarity fits without inputs on V_{cb} and V_{ub} predict very specific values

$\Rightarrow$ Direct determinations cannot be superseded, but maybe we can use this as a guideline to focus on specific issues

Appendix

Unitarity Bounds



$$= i \int d^4x \, e^{iqx} \langle 0 | T \left\{ j_\mu(x), j_\nu^\dagger(0) \right\} |0\rangle = (g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

- If $q^2 \ll m_b^2$ we can calculate $\Pi(q^2)$ via perturbative techniques $\Rightarrow \chi(0)$
- Dispersion relations link Im $\left(\Pi(q^2)\right)$ to sum over matrix elements

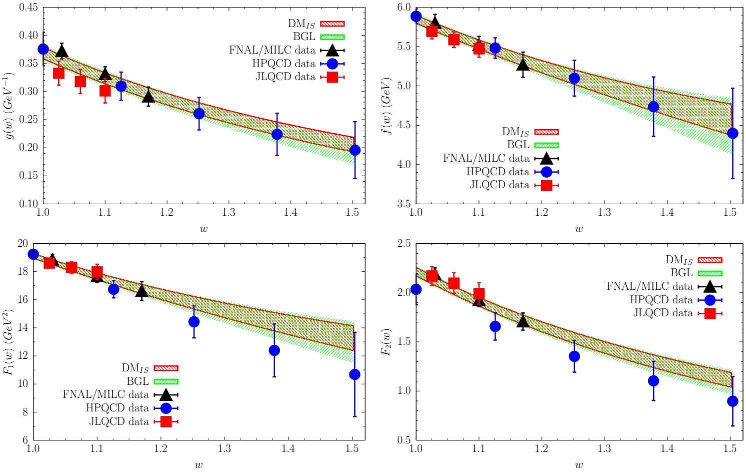
$$\sum_i |F_i(0)|^2 < \chi(0)$$

[Boyd, Grinstein,Lebed, '95
Caprini, Lellouch, Neubert, '97]

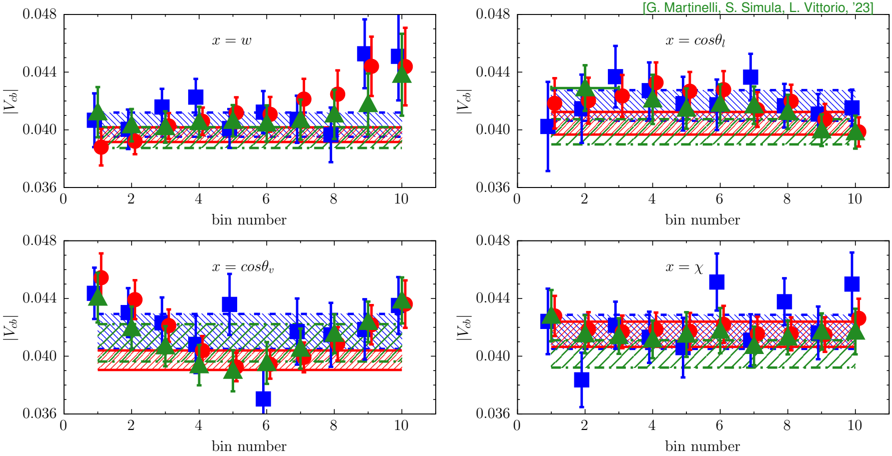
- The sum runs over **all** possible states hadronic decays mediated by a current $\bar{c}\Gamma_\mu b$
  - The unitarity bounds are more effective the most states are included in the sum
  - The unitarity bounds introduce correlations between FFs of different decays
  - $B_s \to D_s^{(*)}$ decays are expected to be of the same order of $B_{u,d} \to D_{u,d}^{(*)}$ decays due to $SU(3)_F$ simmetry

Comparison with DM

[G. Martinelli, S. Simula, L. Vittorio, '23]

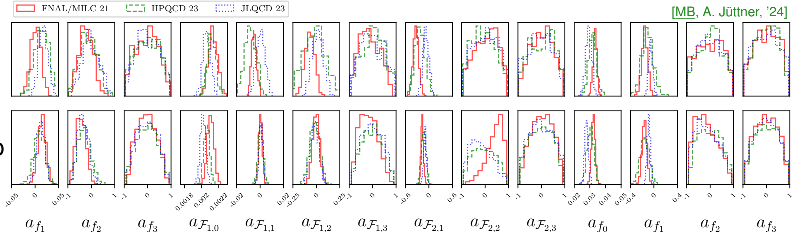


Results from the DM method



$\Rightarrow$ similar behaviour as we observe

Posterior distribution



- Small shifts between lattice only and lattice + data
- Higher order coefficients well constrained by unitarity
- $a_{\mathcal{F}_{2,2}}$ has a strange behaviour, maybe kinematic constraints?