Experimental $b \to c\ell\nu$: how to deal with form-factors uncertainties?

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Taming hadronic uncertainties in and beyond the Standard Model
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DESY.

Overview

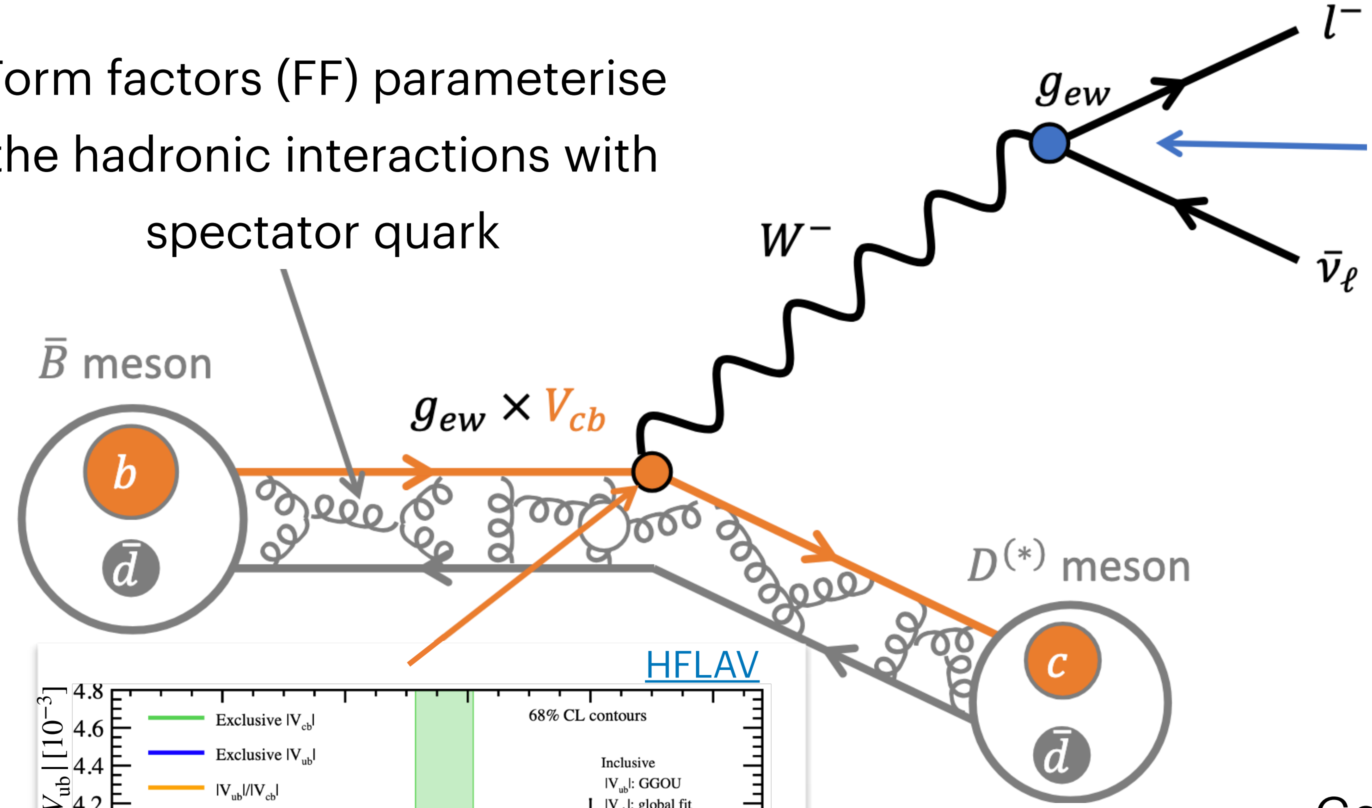
- Brief overview of $b \to c\ell\nu$ decays.
- Form factors for $B \to D\ell\nu$ and $B \to D^*\ell\nu$ and current methods for estimating their uncertainties.
- Assessment of the impact of these uncertainties on current measurements of $|V_{\rm cb}|$, $R(D^{(*)})$...
- Applications and future prospects for ongoing/upcoming measurements, including strategies to reduce or mitigate these uncertainties using:

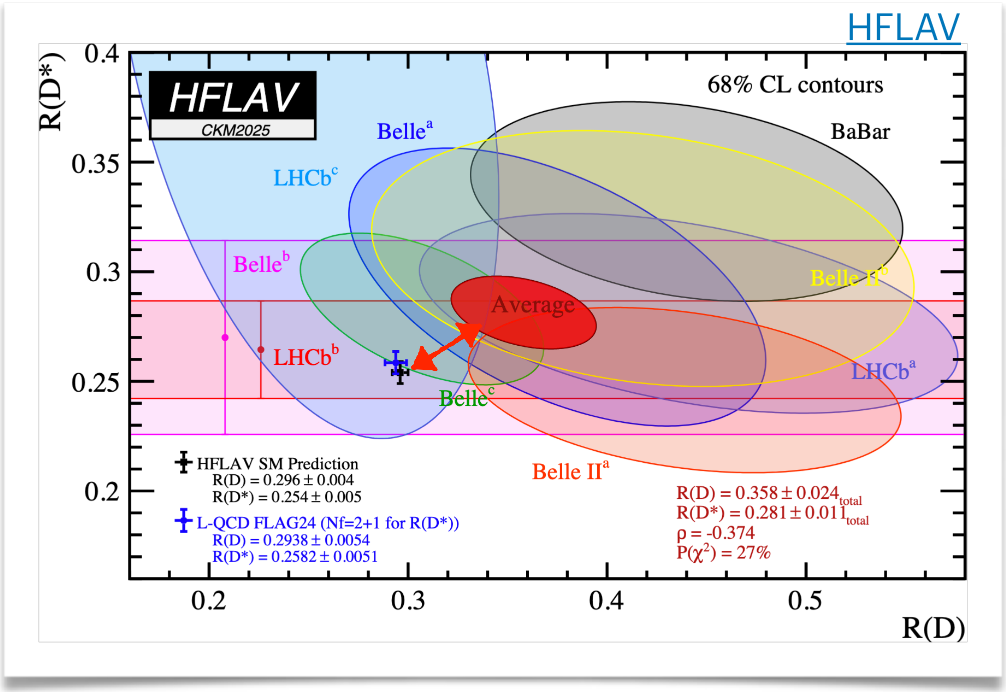
1. Data-driven approach: e.g. first combined $B \to D\ell\nu$ and $B \to D^*\ell\nu$ analysis at Belle II.

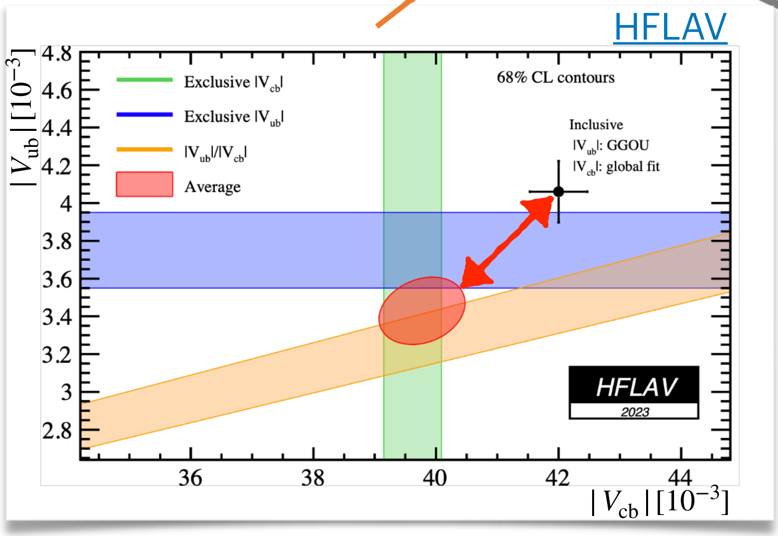
2. Optimised variables for next $R(D)$-$R(D^*)$ measurements.

The $b \to c\ell\nu$ decays

Form factors (FF) parameterise the hadronic interactions with spectator quark



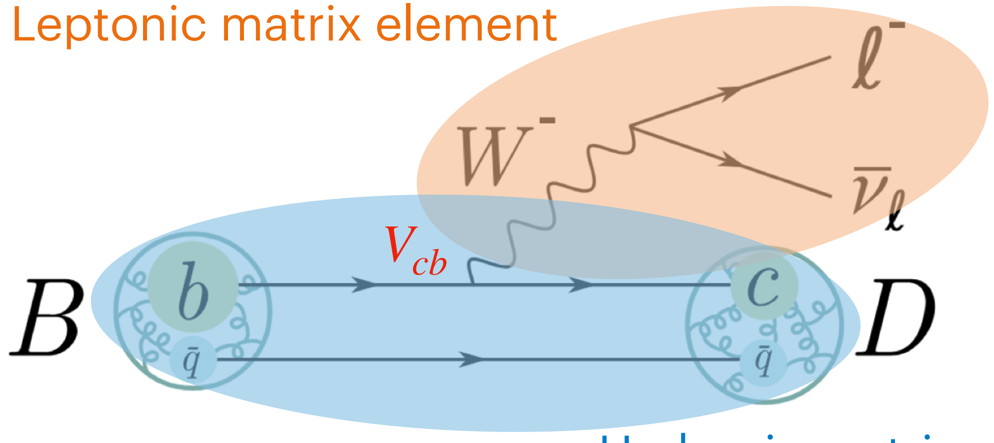




Good understanding of the form factors is crucial for precise predictions and determinations of observables

$R(D^{(*)})$, $|V_{cb}|$, A_{FB}, $P_\tau(D^{(*)})$, $F_{L,\tau}(D^{(*)})$

Exclusive $B \to D^{(*)}\ell\nu$ decays



Semileptonic decays of $B \to D^{(*)}$:

easy probe to measure $|V_{cb}|$.

$$d\Gamma \propto G_F^2 |V_{cb}|^2 |L^\mu H_\mu|^2$$

Determine $|V_{cb}|$ by removing the hadronic interaction effects

Hadronic matrix element can not be calculated from first principles:

→ can be parameterised with form factors and extracted from data

Differential distributions [PRD, 108, 012002]
Angular coefficients [PRL, 133, 131801]

→ theory must provide (at least) inputs on their normalisation

Essential to provide measurements that are as independent as possible from the FF model

$B \to D^{(*)}\ell\nu$ form factors

- $B \to D\ell\nu$ decay rate in the massless-lepton limit is

$$\frac{d\Gamma}{dw} \propto \Gamma_0(w)\,|V_{cb}|^2\,|G(w)|^2$$

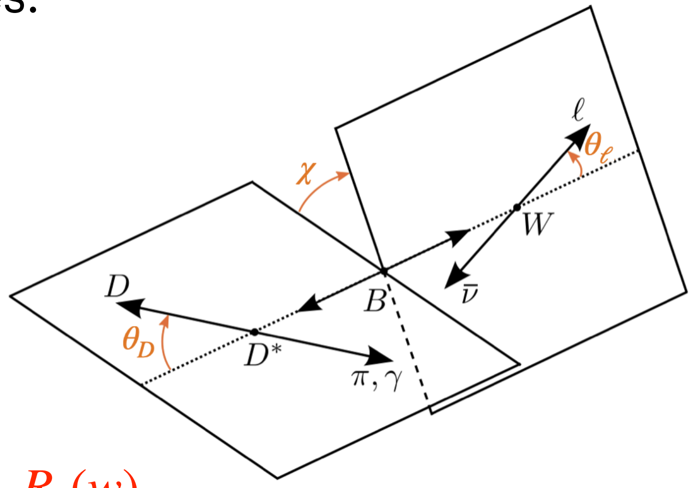
where $w = E_D/m_D$ and $G(w)$ encapsulates the form factor: $G(w) = \frac{\sqrt{4r}}{1+r^2} f_+(w)$, with $r = m_D/m_B$

one form factor

- $B \to D^*\ell\nu$ decay rate depends on $w = E_{D^*}/m_{D^*}$ and three helicity angles:

angular functions

$$\frac{d^4\Gamma}{dw\,d\cos\theta_\ell\,d\cos\theta_D\,d\chi} \propto \Gamma_0'(w)\,|V_{cb}|^2 \sum_{i=1}^{6} H_i(w) k_i(\theta_\ell, \theta_D, \chi)$$



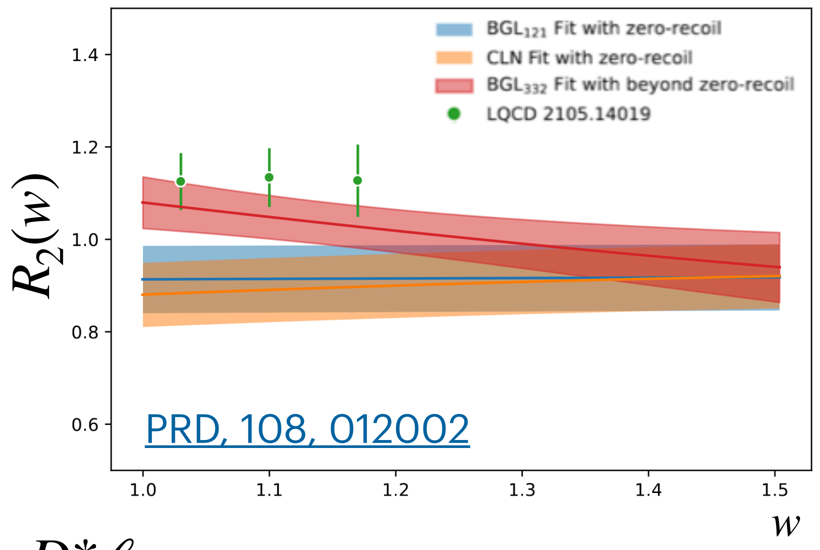
helicity amplitudes encapsulate the three form factors $h_{A_1}(w), R_1(w), R_2(w)$

Form-factors, experimentally

Two different FF parametrisations:

1. Caprini-Lellouch-Neubert (CLN):
   few parameters but strong theoretical assumptions.

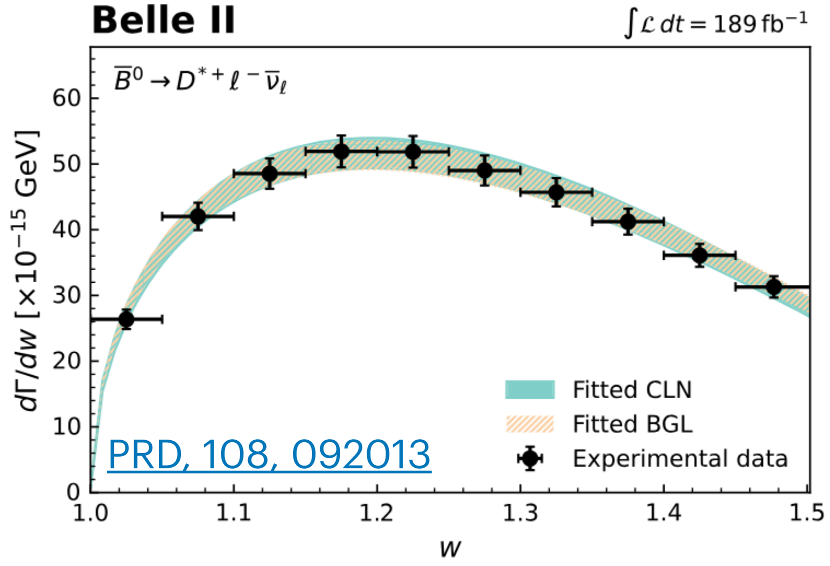
2. Boyd-Grinstein-Lebed (BGL):
   less restrictive, recommended by theorists.



Might lead to model-dependent results, a concern especially for $B \to D^* \ell \nu$.

Current approach: unfolded (1D) distributions of $w, cos\theta_l, cos\theta_D, \chi$ and fitted with different models.

$$\frac{d^4\Gamma}{dw\,d\cos\theta_D\,d\cos\theta_\ell\,d\chi} = \frac{3}{16\pi}\Gamma_0(w)|V_{cb}|^2\Big\{H_+^2(w)\sin^2\theta_D\,(1-\cos\theta_\ell)^2 + H_-^2(w)\sin^2\theta_D\,(1+\cos\theta_\ell)^2 + 4\,H_0^2(w)\cos^2\theta_D\,\sin^2\theta_\ell - 2\,H_-(w)H_+(w)\sin^2\theta_D\,\sin^2\theta_\ell\,\cos 2\chi - 2\,H_+(w)H_0(w)\sin 2\theta_D\,\sin\theta_\ell\,(1-\cos\theta_\ell)\,\cos\chi + 2\,H_-(w)H_0(w)\sin 2\theta_D\,\sin\theta_\ell\,(1+\cos\theta_\ell)\,\cos\chi\Big\}$$



How do we deal with a choice of FF model?

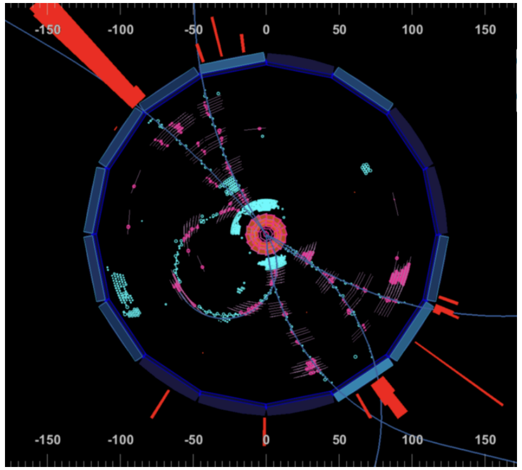
Dealing with form factors

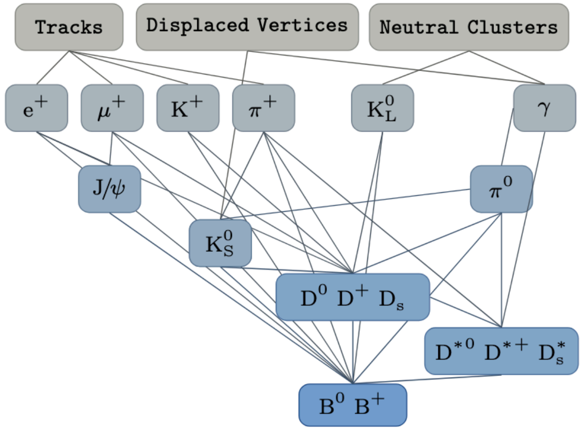
Fitting the form factors

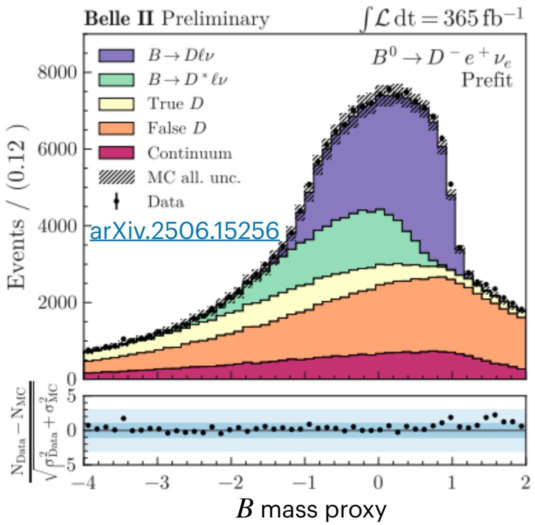
Form factors determine the shape and behaviour of our signal models. Experimentally, we adjust them to better match the experimental data, allowing the signal shapes to adapt to observations.

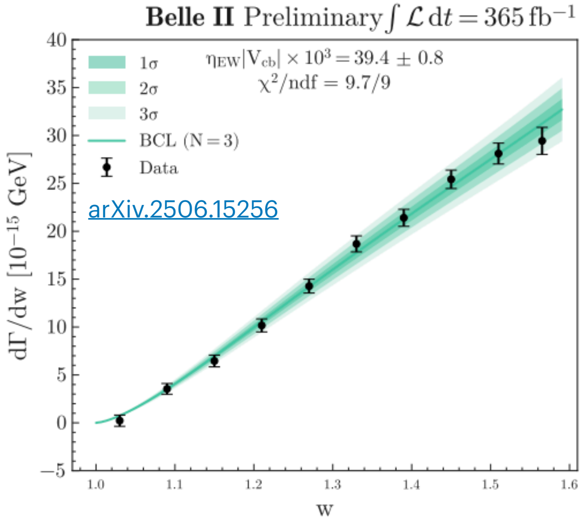
correct procedure at each fit iteration

Generate MC according to a FF model → **Reconstruction and selection** → **Model signal/bkg template** → **Extract parameters**









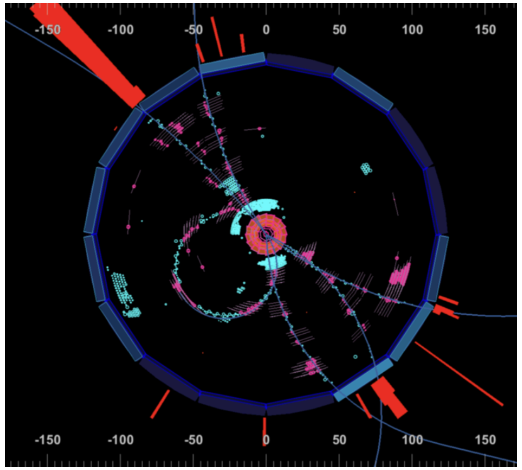
This is not pragmatic!

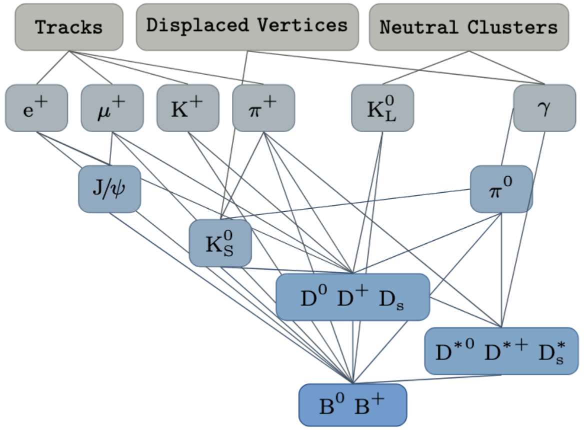
Dealing with form factors

Fitting the form factors

Practical approach

Generate MC according to a FF model → Reconstruction and selection → Model signal/bkg template → At each fit iteration, reweigh truth information to a different FF model → Extract parameters





Assume reconstruction and selection are not heavily model dependent

Techniques developed using different tools: Hammer, eFFORT2...

Dealing with form factors

Reweighting procedure

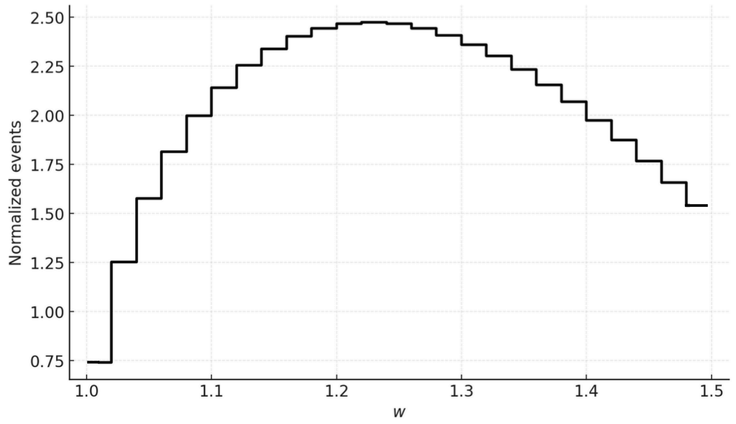
- Dependence on the form factors included in the fit through a reweighting of the reference templates.
- At each call of the fit minimisation, the signal templates are rebuilt using a weight:

$$p_i = \frac{\Gamma_{ref}}{\Gamma_{new}} \frac{d\Gamma_{new}/d\Omega}{d\Gamma_{ref}/d\Omega}$$

$d\Gamma_{ref}$ = reference model, what is assumed in simulation to generate the sample.

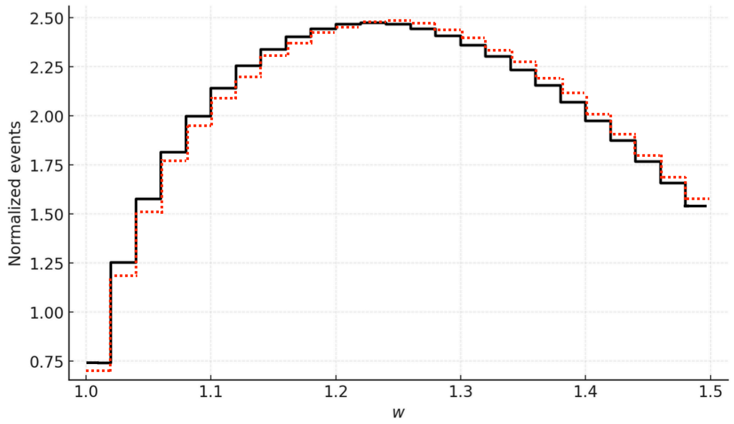
$d\Gamma_{new}$ = new model used in the fit to measure the form-factors.

$\Omega = w$ for $D\ell\nu$, $\Omega = (w, \cos\theta_\ell, \cos\theta_D, \chi)$ for $D^*\ell\nu$ ....



HAMMER (MC)

Helicity Amplitude Module
for Matrix Element Reweighting



Assign a systematic uncertainty due to the choice of the form-factor model by evaluating the impact of differences between the alternative and nominal fit results on physical observables

Form-factors uncertainties

$|V_{cb}|$ and $R(D)$-$R(D^*)$ measurements

Form-factor uncertainties are not always negligible:

$|V_{cb}|$ at Belle II

Source		Uncertainty [%]
Statistical		0.9
Systematic		1.5
	$B^{0/+}$ lifetime	0.1
	Signal form factor	0.1
	$B \to D^* \ell \nu$ form factor	0.1
	$\mathcal{B}(B \to X_c \ell \nu)$	0.3
	$\mathcal{B}(D \to K\pi(\pi))$	0.5
	Tracking efficiency	0.5
	$N_{\Upsilon(4S)}$	0.7
	f_{00}/f_{+-}	0.1
	$f_{\not{B}}$	0.4
	Background w modelling	0.3
	(E^*_Y, m_Y) reweighting	0.3
	Lepton identification	0.3
	Kaon identification	0.6
	Vertex fit χ^2 correction	0.3
	Simulation sample size	0.5
Theoretical		1.3
to set the normalisation	Lattice QCD inputs	1.2
	Long-distance QED	0.5
Total		2.1

arXiv.2506.15256 **submitted to PRD**

$R(D)$-$R(D^*)$ at Belle II

Source	$R(D^*)$	$R(D)$	ρ
Simulation sample size	4.8%	8.4%	−0.44
gap-mode branching fraction	2.6%	2.6%	0.00
$\bar{B} \to D^{**}\tau^-/(\ell^-)\bar{\nu}_\ell$ branching fractions	0.3%	1.3%	0.25
Hadronic B decay branching fractions	1.6%	1.5%	−0.26
Form factors	0.5%	0.9%	−0.70
Fraction of misreconstructed $D^{(*)}$	0.5%	1.2%	0.00
Continuum background	2.4%	2.1%	0.93
Fit biases	0.3%	1.2%	0.00
Low-momentum π^0, γ efficiency	2.2%	2.4%	0.99
Other efficiency corrections	0.7%	1.4%	0.92
B-tagging efficiency of data	0.9%	1.8%	−1.00
B-tagging efficiency of $B \to D\tau\nu$	0.1%	1.8%	1.00
M^2_{miss} resolution	0.5%	0.8%	0.48
Total systematic uncertainty	6.7%	10.2%	−0.20
Statistical uncertainty	8.3%	16.3%	−0.40

To be submitted to PRL

$R(D)$-$R(D^*)$ at LHCb

Source	$R(D^+)$	$R(D^{*+})$
Form factors	0.023	0.035
$\bar{B} \to D^{**}[D^+X]\mu/\tau\nu$ fractions	0.024	0.025
$\bar{B} \to D^+X_cX$ fraction	0.020	0.034
Misidentification	0.019	0.012
Simulation size	0.009	0.030
Combinatorial background	0.005	0.020
Data vs simulation agreement	0.016	0.011
Muon identification	0.008	0.027
Multiple candidates	0.007	0.017
Total systematic uncertainty	0.047	0.085
Statistical uncertainty	0.043	0.081

PRL, 134, 061801 (2025)

Can we go beyond this?

Application:
first combined $B \to D^{(*)}\ell\nu$ at Belle II

Exclusive measurements at Belle II

$B \to D\ell\nu$ and $B \to D^*\ell\nu$

Exclusive approach: $B \to D\ell\nu$ and $B \to D^*\ell\nu$ decays. They have been analysed independently so far at Belle II.

- $B \to D^*\ell\nu$ measurement:

  limited by systematic uncertainties:

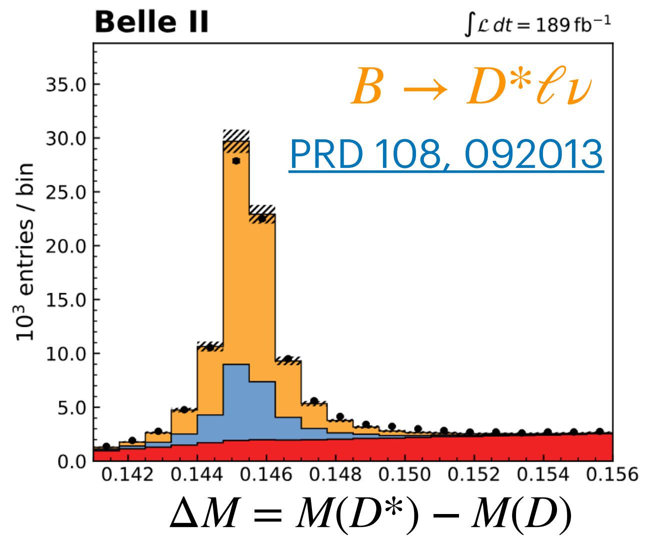
  pion reconstruction efficiency in the $D^{*+} \to \overline{D}^0\pi^+$ transition.

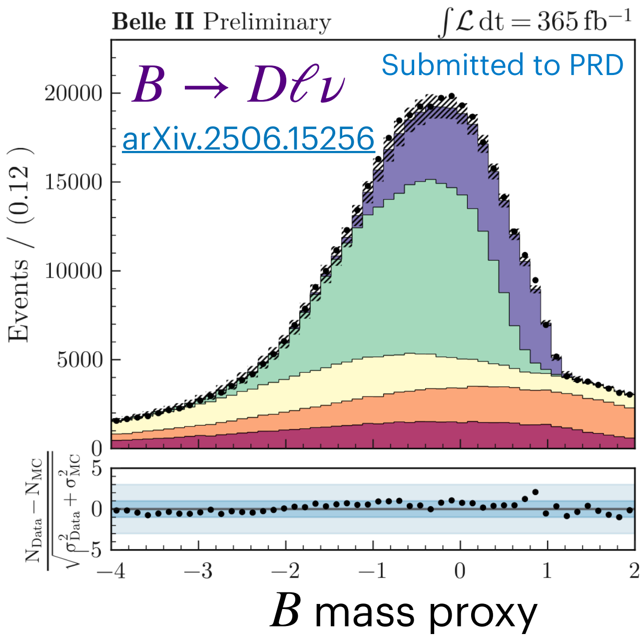
  (Impact ~1.5% on $|V_{cb}|$).

- $B \to D\ell\nu$ measurement:

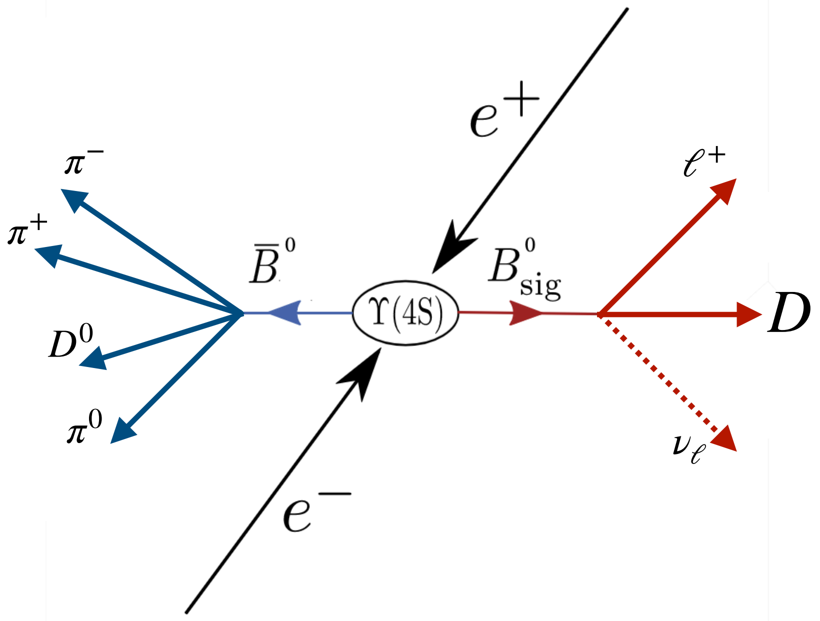
  provide smaller samples compared to the $B \to D^*\ell\nu$ decays.

  Large feed-down contribution from $B \to D^*\ell\nu$ decays.



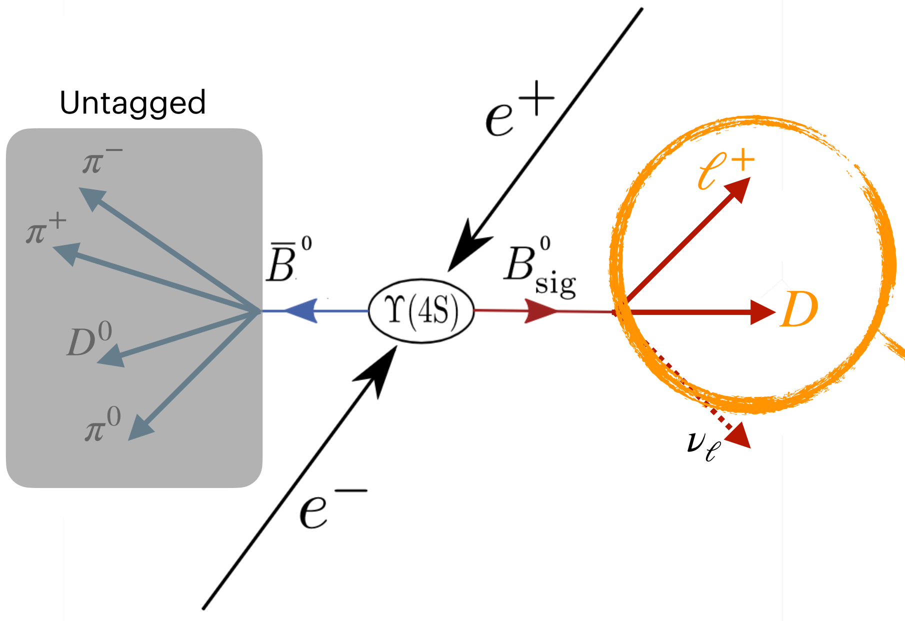


Production at Belle II



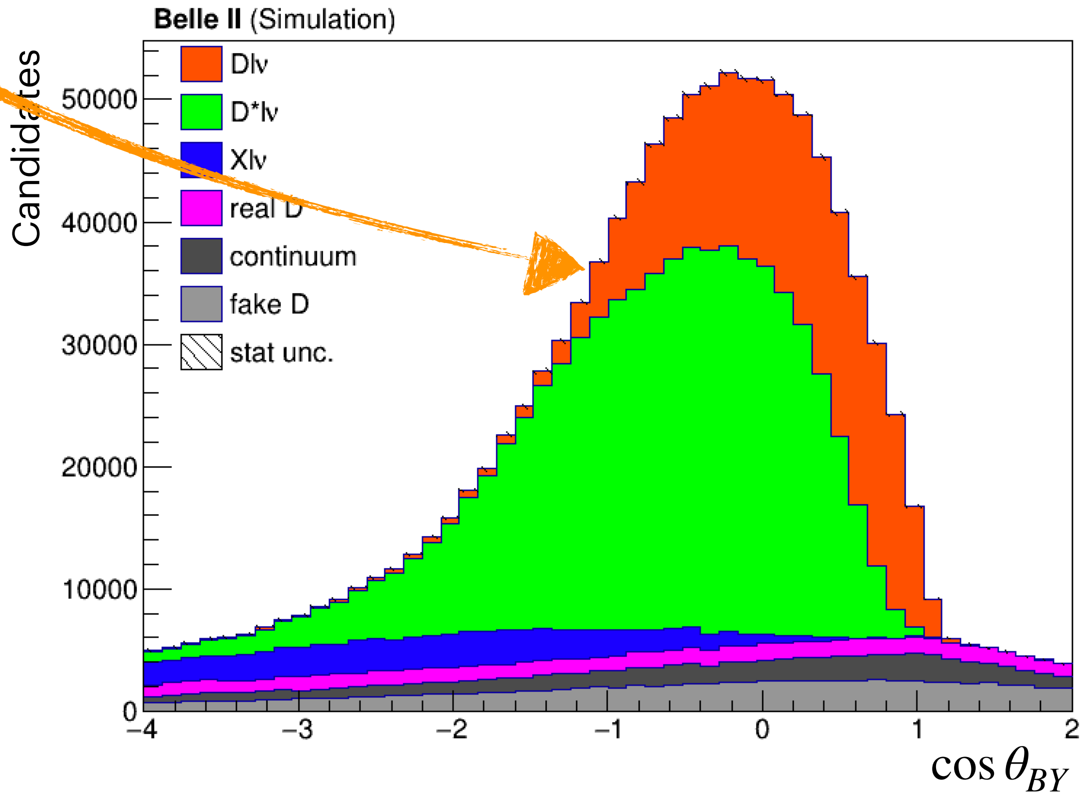
- e^+e^- collisions at 10.6 GeV energy.
- BB mesons pairs from $\Upsilon(4S)$ decays.

$B \to D\ell\nu$ decays at Belle II



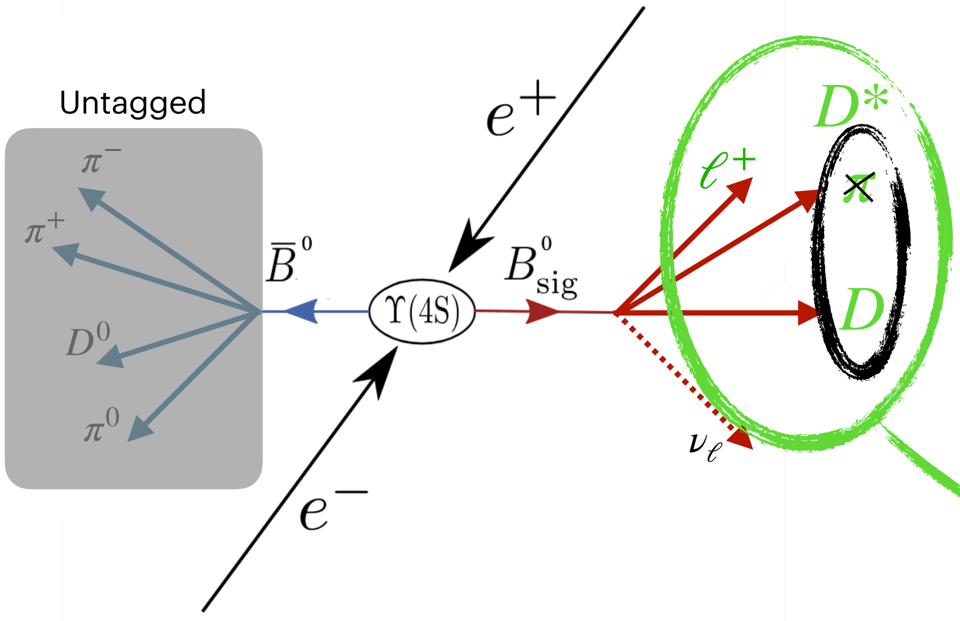
- Reconstruct the ℓ and D meson.
- Cannot detect the ν_ℓ, broad peak.

$\cos\theta_{BY}$ = angle between the B mesons and $D\ell$ system in the CMS.

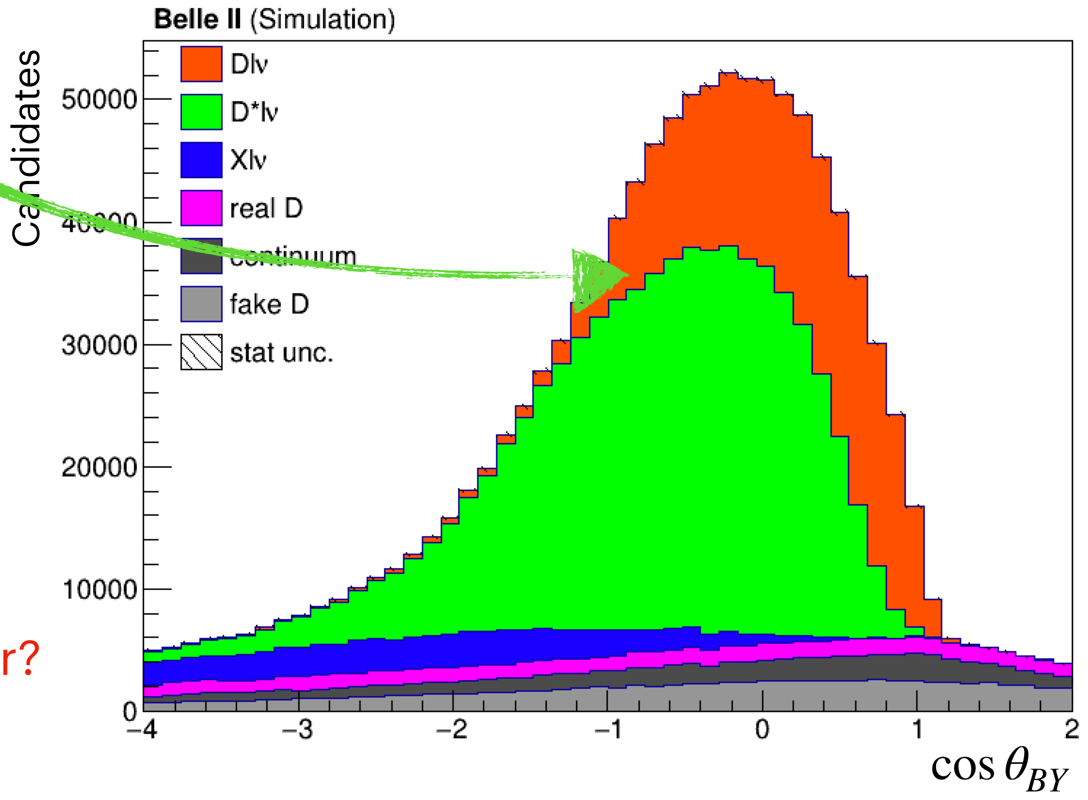


- e^+e^- collisions at 10.6 GeV energy.
- BB mesons pairs from $\Upsilon(4S)$ decays.

Feed-down contribution from $B \to D^*\ell\nu$



- Reconstruct the ℓ and D meson.
- $B \to D^*\ell\nu$ decays peaks at lower values: missing also the π.



- e^+e^- collisions at 10.6 GeV energy.
- BB mesons pairs from $\Upsilon(4S)$ decays.

Can we analyse $B \to D\ell\nu$ and $B \to D^*\ell\nu$ together?

First combined $B \to D\ell\nu$ and $B \to D^*\ell\nu$ at Belle II

Perform the first simultaneous analysis of $B \to D\ell\nu$ and $B \to D^*\ell\nu$ at Belle II where D^* is partially reconstructed. Measure:

1. $\mathscr{B}(B \to D\ell\nu)$ and $\mathscr{B}(B \to D^*\ell\nu)$
2. $|V_{cb}|$ and the decay form factors **[main focus of this talk]**
3. $f_{+-}/f_{00} = \mathscr{B}(\Upsilon(4S) \to B^+B^-)/\mathscr{B}(\Upsilon(4S) \to B^0\bar{B}^0)$

Variables to further characterise the $B \to D^*\ell\nu$ decay:

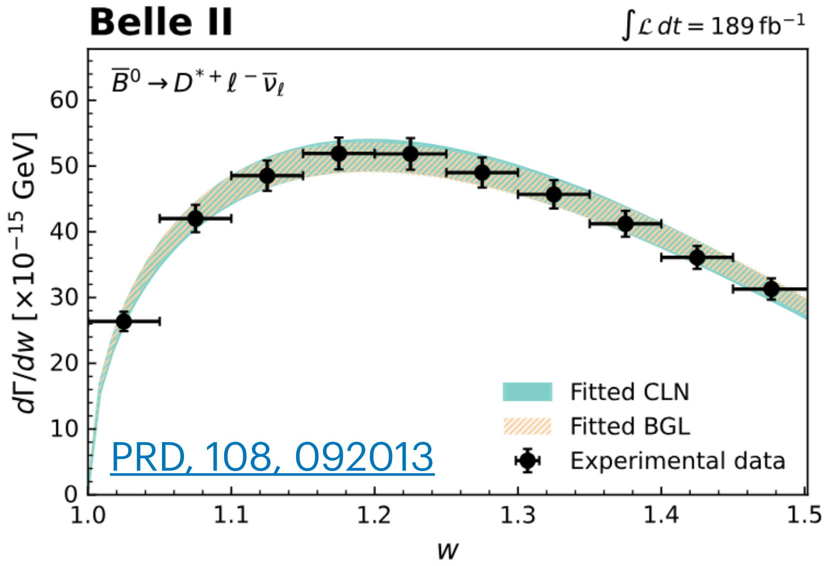
3. Forward-backward asymmetry (A_{FB})
4. Longitudinal D^* polarisation ($F_L^{D^*}$)

A novel approach

Current approach

Unfolded (1D) distributions of $w, cos\theta_l, cos\theta_D, \chi$ and fitted with different models.

$$\frac{d^4\Gamma}{dw\,d\cos\theta_v\,d\cos\theta_\ell\,d\chi} = \frac{3}{16\pi}\Gamma_0(w)|V_{cb}|^2\Big\{H_+^2(w)\sin^2\theta_v\,(1-\cos\theta_\ell)^2 + H_-^2(w)\sin^2\theta_v\,(1+\cos\theta_\ell)^2 + 4\,H_0^2(w)\cos^2\theta_v\,\sin^2\theta_\ell - 2\,H_-(w)H_+(w)\sin^2\theta_v\,\sin^2\theta_\ell\,\cos2\chi - 2\,H_+(w)H_0(w)\sin2\theta_v\,\sin\theta_\ell\,(1-\cos\theta_\ell)\cos\chi + 2\,H_-(w)H_0(w)\sin2\theta_v\,\sin\theta_\ell\,(1+\cos\theta_\ell)\cos\chi\Big\}$$



New approach

Measure $|V_{cb}|^* H_i$ in bins of w w/o assuming a FF parametrisation.

Helicity amplitudes H_i depend on the form-factors: obtain a model-independent measurement.

This new approach can be applied to any decay

*only 2D differential decay rate accessible
(D^* partially reconstructed)

Model-independent observables

Define, for the first time, the following model-independent observables:

$D\ell\nu$, one FF, differential rate depends on the recoil variable w:

$$\frac{d\Gamma}{dw} \propto \Gamma_0(w)\,|V_{cb}|^2\,|G(w)|^2$$

FF

$G'(w)$ measured in 7 bins of w

$D^*\ell\nu$, richer structure, rate depends on decay angles too:

combinations of FF

$$\frac{d^2\Gamma^*}{dw\,d\cos\theta_\ell} \propto \Gamma_0'(w)\,|V_{cb}|^2 \left\{ a(w) + b(w)\cos\theta_\ell + c(w)\cos^2\theta_\ell \right\}$$

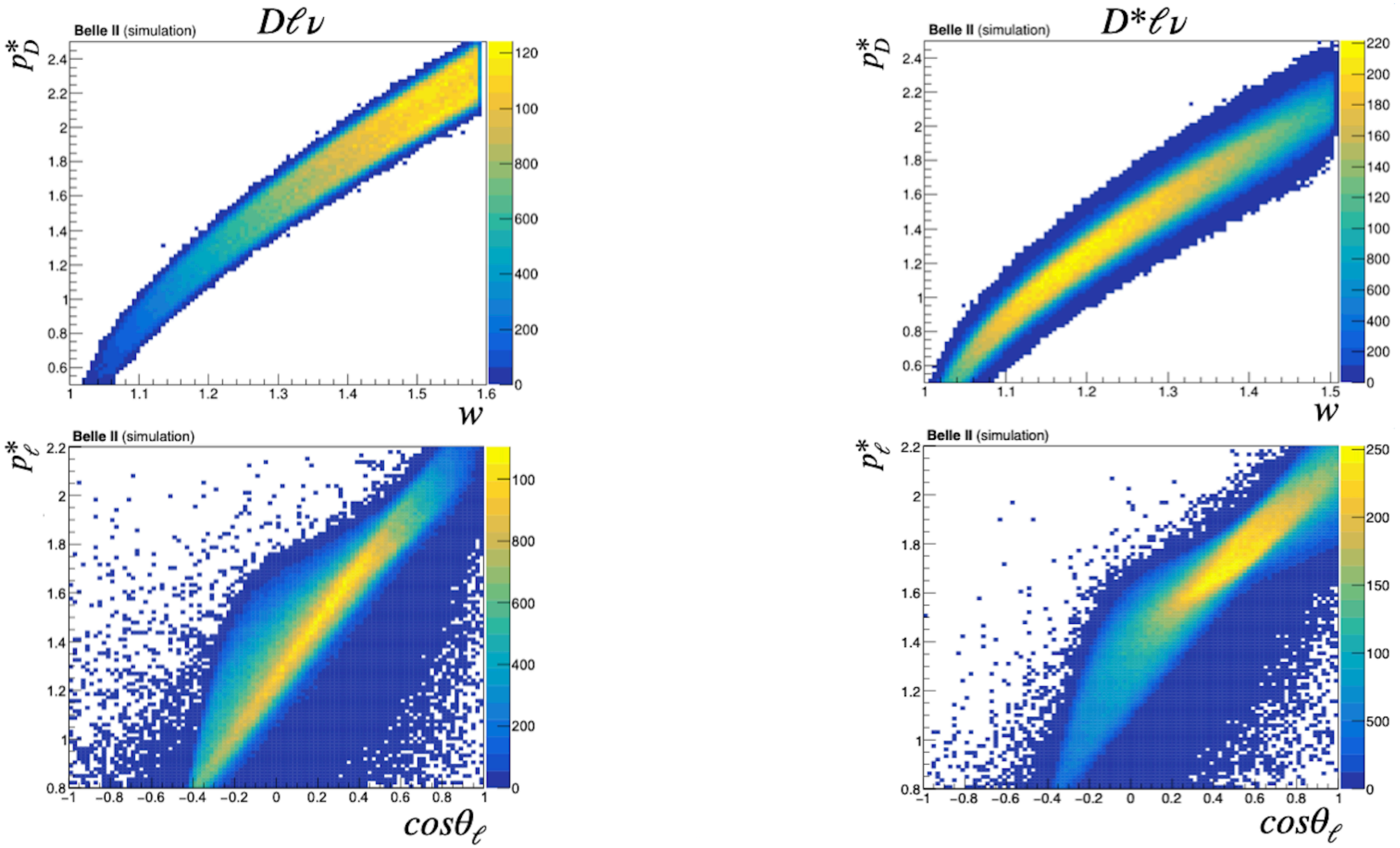
$a'(w), b'(w), c'(w)$, measured in 5 bins of w

Measuring these observables directly using data w/o assuming any FF parametrisation

→ get rid of FF systematic uncertainties

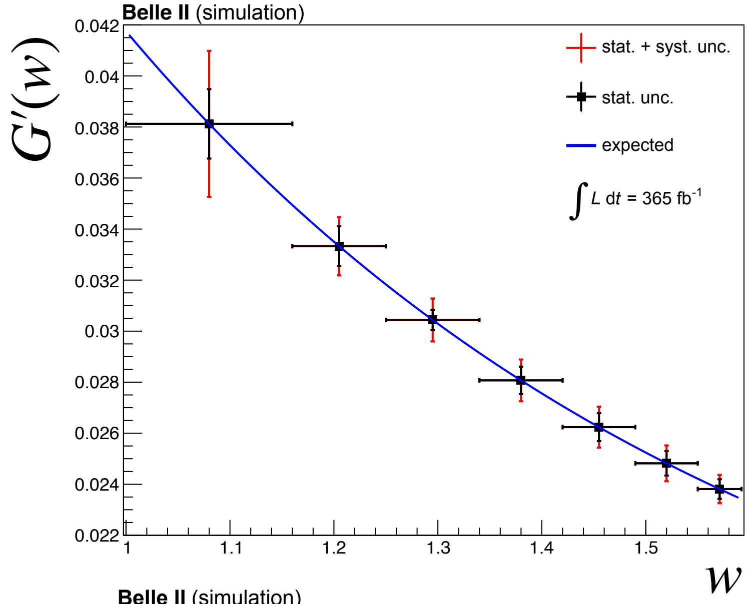
Proxy variables

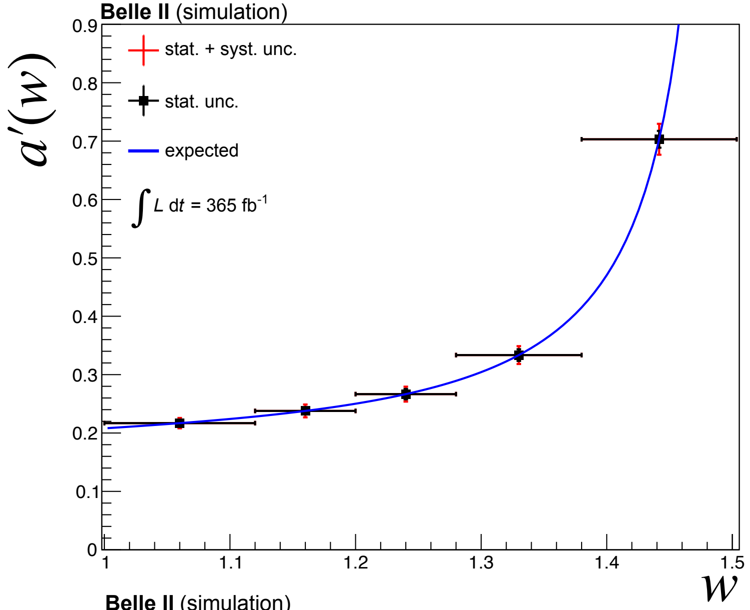
- p_D^* momentum of the D in the CMS, encapsulates w for both $D\ell\nu$ and $D^*\ell\nu$.
- p_ℓ^* momentum of the lepton in the CMS, encapsulates $\cos\theta_\ell$ for $D^*\ell\nu$.

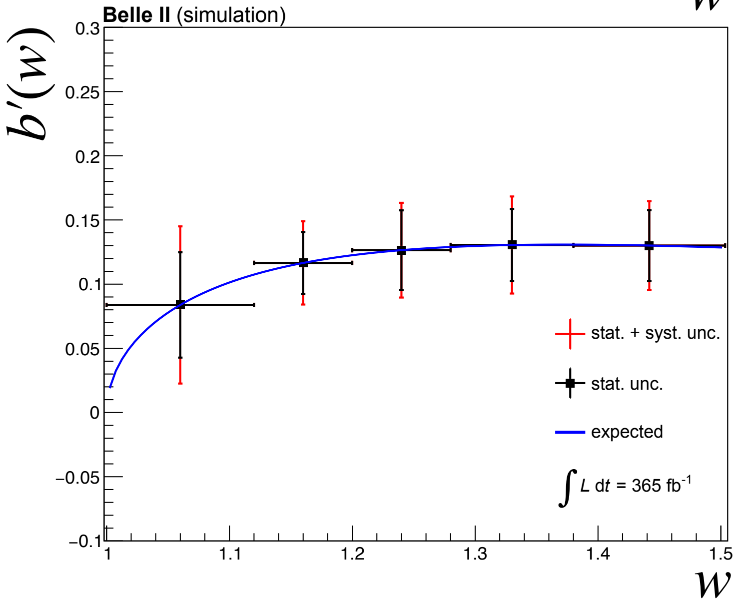


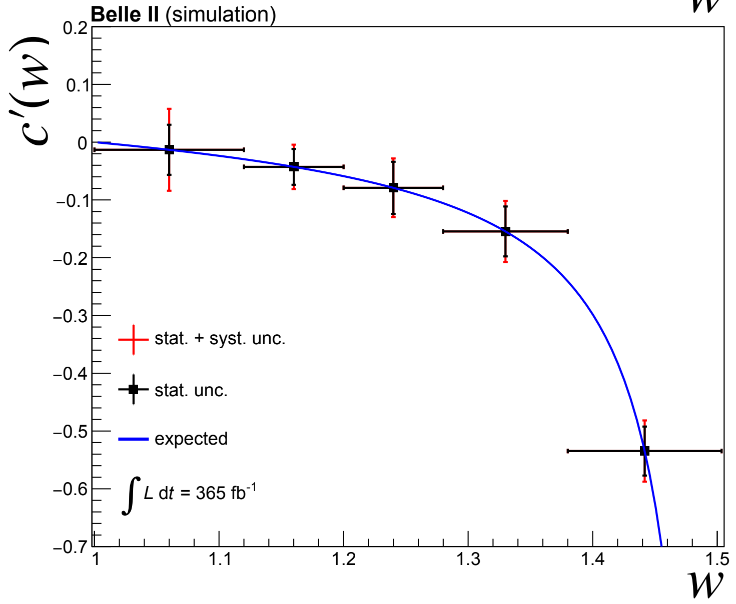
p_D^* and p_ℓ^* allow the measurements of G', a', b', c'

Expected impact on model-independent observables









Obtained the same generator values

$|V_{cb}|$ extraction

Extract $|V_{cb}|$ simultaneously from $B \to D\ell\nu$ and $B \to D^*\ell\nu$ using a χ^2 fit of the measured model-independent observables:

Model-independent observables					Need to assume a model for the form factors
	$G'(w)$	=	$|V_{cb}|$	x	$G(w)$
	$a'(w)$	=	$|V_{cb}|^2$	x	$a(w)$
	$b'(w)$	=	$|V_{cb}|^2$	x	$b(w)$
	$c'(w)$	=	$|V_{cb}|^2$	x	$c(w)$

Form factors must be known at least in one value of w: use lattice QCD calculations.

Lattice points at $w = 1$:

- $G(1) : 1.054 \pm 0.009$ for $D\ell\nu$
- $h_{A_1}(1) : 0.908 \pm 0.013$ for $D^*\ell\nu$

$|V_{cb}|$ extraction

BGL parametrisation

- Assume a BGL parametrisation: series expansions in a variable z(w)<<1, with coefficients to be determined experimentally, or computed from e.g. lattice QCD.

$$G'(w) = |V_{cb}| \times G(w) \quad D\ell\nu, \text{ 1 series}$$

$$a'(w) = |V_{cb}|^2 \times a(w)$$

$$b'(w) = |V_{cb}|^2 \times b(w) \quad D^*\ell\nu, \text{ combination of 3 series}$$

$$c'(w) = |V_{cb}|^2 \times c(w)$$

- Form-factor expansion are infinite series: must be truncated at a specific order. Some truncation introduces a model dependency into the measurement.

- To extract $|V_{cb}|$, important to check the stability of the results on the truncation order.

Adopted a choice, but the measured model-independent observables can be reinterpreted with any future advanced form-factor parametrisation for a new $|V_{cb}|$ determination from data analysis also provides measurements of $F_L(D^*)$, A_{FB} and f_{+-}/f_{00}

$|V_{cb}|$ and BRs expected uncertainties

Simultaneous measurement of $B \to D\ell\nu$ and $B \to D^*\ell\nu$ avoids major systematics of separate analyses (e.g. f_{+-}/f_{00}, slow pion efficiency reconstruction), but introduces complementary systematics.

	Expected results	Best measurements
$\mathcal{B}(B^+ \to \overline{D}^0\ell^+\nu)$	$XXX \pm 0.01(stat) \pm 0.07(syst)$	BaBar $2.34 \pm 0.03(stat) \pm 0.13(syst)$ PRD, 79, 012002
$\mathcal{B}(B^+ \to \overline{D}^{*0}\ell^+\nu)$	$XXX \pm 0.02(stat) \pm 0.14(syst)$	BaBar $5.40 \pm 0.02(stat) \pm 0.21(syst)$ PRD, 79, 012002
$\mathcal{B}(B^0 \to D^-\ell^+\nu)$	$XXX \pm 0.01(stat) \pm 0.06(syst) \pm 0.02(th)$	Belle $2.31 \pm 0.03(stat) \pm 0.11(syst)$ PRD, 93, 032006
$\mathcal{B}(B^0 \to D^{*-}\ell^+\nu)$	$XXX \pm 0.02(stat) \pm 0.13(syst) \pm 0.05(th)$	Belle $4.90 \pm 0.02(stat) \pm 0.16(syst)$ PRD, 100, 052007
$\|V_{cb}\|$ $[10^{-3}]$	$XXX \pm 0.29(stat) \pm 0.70(syst) \pm 0.45(latt)$	Latest Belle II results $39.2 \pm 0.4(stat) \pm 0.6(syst) \pm 0.5(latt)$ arXiv.2506.15256

submitted to PRD

Analysis is still under review, showing the potential of the measurement using simulation

Model-independent observables

Pros and cons

- Remove systematic uncertainty from FF model
- More model-independent measurement
- Flexibility with data
- Measurement can be reinterpreted with any future FF model improvements

- Add more fit parameters
- Increase uncertainty on physical parameters
- $|V_{cb}|$ extraction still requires:
  1. An assumed FF parametrisation
  2. At least one theoretical point for normalisation

Model-independent observables are useful but their advantage depends on whether the analysis is statistics- or systematics-limited

Application:
$R(D)$ - $R(D^*)$

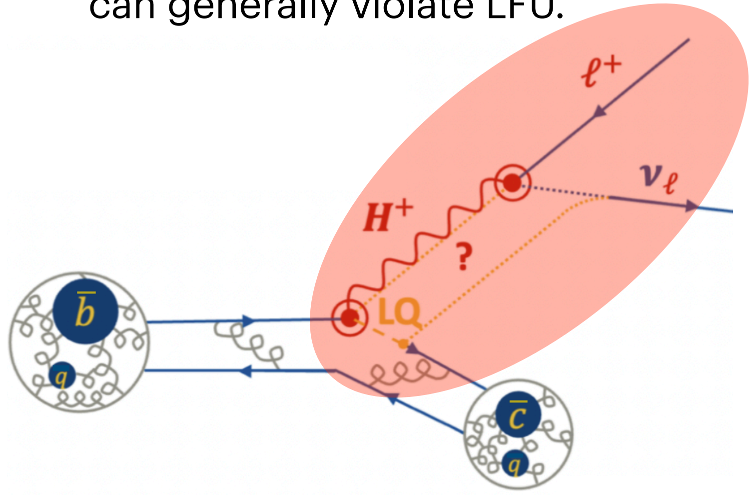
$R(D)$-$R(D^*)$

Current status

One way to investigate LFU in semileptonic B decays is to define the ratio:

$$R(D^{(*)})_{\tau/\ell} = \frac{\mathscr{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathscr{B}(B \to D^{(*)}\ell\nu_\ell)}$$

Non-SM contributions (H^+, LQ, SUSY...) can generally violate LFU.



Long-standing tension between $R(D^{(*)})$ value and the SM prediction.

With the latest Belle II measurement this tension increases to 3.8σ

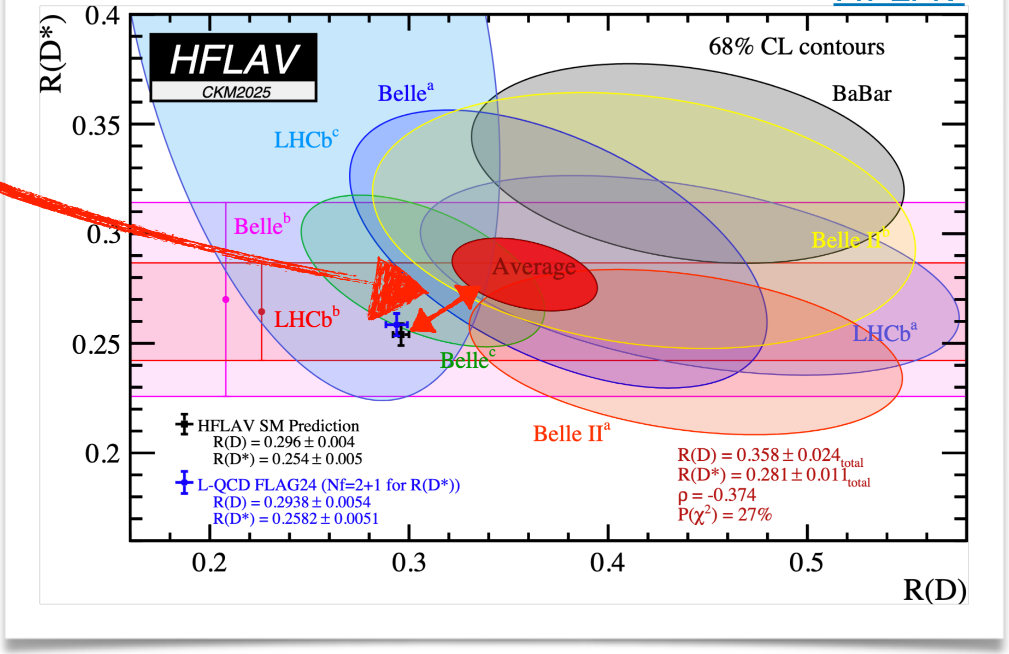
Measurements statistically limited but

Syst. unc. $\frac{R(D^*_{\tau/\ell})}{R(D_{\tau/\ell})}$: form factors $\frac{8.7\%}{9.2\%}$ [LHCb]

Syst. unc. $\frac{R(D^*_{\tau/\ell})}{R(D_{\tau/\ell})}$: form factors $\frac{0.5\%}{0.9\%}$ [Belle II]

Reducing uncertainties on both the prediction and measurements sides is crucial to better understand this tension

HFLAV



$R(D)$-$R(D^*)$

How to improve $R(D)$-$R(D^*)$ predictions/measurements?

- Can use model-independent observables → increase total uncertainty on $R(D^{(*)})$.
- Alternative: slightly redefine the ratio to reduce FF uncertainties in the predictions and measurements.

Classic definition

$$R(D^{(*)})_{\tau/\ell} = \frac{\int_{m_\tau^2}^{q^2_{max}} dq^2 \frac{d\Gamma}{dq^2}(B \to D^{(*)}\tau\nu_\tau)}{\int_{m_\ell^2}^{q^2_{max}} dq^2 \frac{d\Gamma}{dq^2}(B \to D^{(*)}\ell\nu_\ell)}$$

Rel. unc. ~5% using Latt. (FNAL)

→

New definition

$$R(D^{(*)})_{\tau/\ell} = \frac{\int_{m_\tau^2}^{q^2_{max}} dq^2 \frac{d\Gamma}{dq^2}(B \to D^{(*)}\tau\nu_\tau)}{\int_{m_\tau^2}^{q^2_{max}} dq^2 \frac{d\Gamma}{dq^2}(B \to D^{(*)}\ell\nu_\ell)}$$

Rel. unc. ~2% using Latt. (FNAL)

→

New definition + reweighting

$$R(D^{(*)})_{\tau/\ell} = \frac{\int_{m_\tau^2}^{q^2_{max}} dq^2 \frac{d\Gamma}{dq^2}(B \to D^{(*)}\tau\nu_\tau)}{\int_{m_\tau^2}^{q^2_{max}} dq^2 \frac{w_\tau(q^2)}{w_\ell(q^2)} \frac{d\Gamma}{dq^2}(B \to D^{(*)}\ell\nu_\ell)}$$

Rel. unc. ~0.4% using Latt. (FNAL)

Achieve roughly a 10x improvement in form factor uncertainty making this approach particularly useful for both predictions and measurements

Summary

- Discussed form factors for $B \to D\ell\nu$ and $B \to D^*\ell\nu$ decays, including current methods for estimating their uncertainties.
- Assessed the impact of form-factor uncertainties on current measurements of $|V_{cb}|$ and $R(D^{(*)})$.
- Explored strategies to reduce or mitigate these uncertainties, including:

1. Data-driven approach, illustrated through a simulation study of the first combined $B \to D\ell\nu$ and $B \to D^*\ell\nu$ analysis at Belle II.
2. Optimised variables, aimed at improving predictions and measurements for next analyses enabling a significant reduction in form-factor uncertainties.

Reducing form-factor uncertainties is crucial for improving precision in many flavour observables, and ongoing approaches show promising results

Backup

Sources of systematic uncertainty

	Sources	Description
external inputs	NBB	Gaussian constraint: $(387 \pm 6) \cdot 10^6$
	BR(D decays)	Gaussian constraints: $\mathscr{B}(D \to K\pi) = (3.95 \pm 0.03)\,\%$, $\mathscr{B}(D \to K\pi\pi) = (9.38 \pm 0.16)\,\%$, $\mathscr{B}(D^{*+} \to D^0 X) = (67.7 \pm 0.5)\,\%$
	Lifetime ratio	Gaussian constraint: 0.929 ± 0.004
	BR(D** + gap)	Include D** and gap modes as a Gaussian constraints with their uncertainties.
efficiencies	track efficiency	$\epsilon(1 + 0.0024K)^N$ with N=#tracks and K Gaussian constrained to a Normal distribution.
	efficiency corrections	Gaussian constraints: $0.995 \pm 0.003\,(D^0)$, $1.007 \pm 0.007\,(D^-)$
	HadronID/LeptonID	Apply 100 weight variations for both hadronID and leptonID to the simulated samples. Perform 100 Asimov fits to the data set.
bkg modeling	Fake D +continuum	Normalization as Gaussian constraints for both signal and control regions. Generated toys with the MC templates in the signal region, fit with the template obtained from the strategy of D mass sidebands.
	Real D (prim.)	Simulated pseudo-experiments with a variation of 20% assumed yield from PDG BR uncertainty.
	Real D (sec.)	Simulated pseudo-experiments with a variation of 20% assumed yield from PDG BR uncertainty.
	Real D (fake)	Simulated pseudo-experiments with a variation of 20% assumed yield from PDG BR uncertainty.
	$D^{(*)*}\tau\nu_\tau, D^{(*)}\ell_{misID}\nu_\ell$	Simulated pseudo-experiments with a variation of 30% assumed yield from PDG BR uncertainty.
	Form factors D**	Fit Asimov data set with 100 different FF variations**.** Take the average residuals as systematic uncertainties.
bias	Fit bias	Take the average residuals as systematic uncertainties.
theoretical unc.	Isospin breaking	Include a factor $(1 + \alpha\pi\theta)$ in the lifetime ratio, Gaussian constraint on θ with a normal distribution.
statistics	MC stat	Obtain from the difference of covariance matrices between a fit w/ and w/o the uncertainties on the templates.
	stat	Obtain from a fit w/o any of the previous sources.

Expected uncertainties on BRs and f_{+-}/f_{00}

	Relative unc. [%] on $\mathcal{B}(B \to D\ell\nu)$	Relative unc. [%] on $\mathcal{B}(B \to D^*\ell\nu)$	Relative unc. [%] on f_{+-}/f_{00}
NBB	1.5	1.5	< 0.1
BR(D decays)	1.0	0.7	1.9
Lifetime ratio	0.2	0.2	0.4
track efficiency	0.8	0.8	0.2
Efficiency corrections	0.5	0.3	0.7
BR(D** + gap)	1.3	1.2	1.1
Form factors (D**)	0.4	0.4	0.4
Backgr. model	0.8	0.9	0.4
HadronID	1.4	1.0	0.8
LeptonID	0.3	0.3	< 0.1
Fit bias	< 0.1	0.1	< 0.1
MC stat.	0.3	0.2	0.3
Isospin breaking (th.)	1.0	1.1	2.3
TOTAL SYST	2.9 (syst) + 1.0 (th)	2.6 (syst) + 1.1 (th)	2.5 (syst) + 2.3 (th)
STAT	0.5	0.4	0.7

$|V_{cb}|$ extraction

BGL parametrisation

Extract $|V_{cb}|$ simultaneously from $B \to D\ell\nu$ and $B \to D^*\ell\nu$ using a χ^2 fit of the measured model-independent observables:

Predicted values using BGL model

cov. matrix

$$\chi^2 = \sum_{i,j}^{22} (x_i - x_i^{BGL}) C_{ij}^{-1} (x_j - x_j^{BGL})$$

Measured values of the model-independent observables

Form factors must be known at least in one value of w: use lattice QCD calculations.

Lattice points at $w = 1$:

- $G(1) : 1.054 \pm 0.009$ for $D\ell\nu$
- $h_{A_1}(1) : 0.908 \pm 0.013$ for $D^*\ell\nu$

Expected uncertainties on $|V_{cb}|$ and FFs

	Rel. unc. [%] on	Uncertainty [10^{-2}] on						
	$\|V_{cb}\|$	$a_1^{f_+}$	$a_2^{f_+}$	a_0^g	a_1^g	a_1^f	a_1^F	a_2^F
NBB	0.7	< 0.01	< 0.1	< 0.01	< 0.1	< 0.1	< 0.01	< 0.1
BR(D decays)	0.4	0.10	1.1	< 0.01	0.2	< 0.1	< 0.01	0.1
Lifetime ratio	0.1	< 0.01	< 0.1	< 0.01	< 0.1	< 0.1	< 0.01	< 0.1
track efficiency	0.4	0.01	0.1	< 0.01	< 0.1	< 0.1	< 0.01	< 0.1
eff. corrections	0.1	0.03	0.4	< 0.01	0.1	< 0.1	< 0.01	0.1
BR(D + gap)**	0.6	0.05	0.7	0.02	0.3	0.1	0.02	0.3
Form factors (D)**	0.5	0.58	3.4	0.06	1.7	0.3	0.11	2.5
Backgr. modelling	1.0	0.43	5.5	0.16	2.5	1.0	0.22	3.5
HadronID	0.7	0.09	1.9	< 0.01	0.3	0.2	0.02	0.2
LeptonID	0.4	0.09	0.8	< 0.01	1.8	0.7	0.09	1.8
Fit bias	0.1	0.01	0.2	< 0.01	0.1	< 0.1	0.01	0.3
MC stat.	0.5	0.24	3.2	0.12	2.5	1.0	0.19	3.2
TOTAL SYST	1.8	0.76	7.3	0.23	4.1	1.5	0.30	5.2
Isospin breaking (th. unc.)	0.5	< 0.01	< 0.1	< 0.01	< 0.1	< 0.1	< 0.01	< 0.1
STAT	0.7	0.39	5.5	0.18	4.2	1.5	0.27	4.6
Lattice points	1.2	0.57	6.5	0.03	0.5	0.2	0.01	0.2
TOTAL	2.3	1.03	11.2	0.29	5.9	2.1	0.40	6.9

A_{FB} and $F_L^{D^*}$

From the coefficient a', b' and c', one can also obtain the lepton forward-backward asymmetry and longitudinal polarisation:

$$A_{FB}(w) = \frac{\int_0^1 \frac{d^2\Gamma}{dw d\cos\theta_\ell} d\cos\theta_\ell - \int_{-1}^0 \frac{d^2\Gamma}{dw d\cos\theta_\ell} d\cos\theta_\ell}{\int_{-1}^1 \frac{d^2\Gamma}{dw d\cos\theta_\ell} d\cos\theta_\ell} = \frac{3b'(w)}{6a'(w) + 2c'(w)}$$

$$F_L^{D^*}(w) = \frac{H_0^2(w)}{H_0^2(w) + H_+^2(w) + H_-^2(w)} = \frac{a'(w) - c'(w)}{3a'(w) + c'(w)}$$

where:

$$a'(w) = |V_{cb}|^2 a(w), \quad b'(w) = |V_{cb}|^2 b(w), \quad c'(w) = |V_{cb}|^2 c(w)$$

with:

$$a(w) = H_+^2(w) + H_-^2(w) + 2H_0^2(w), \quad b(w) = 2H_-^2(w) - 2H_+^2(w), \quad c(w) = H_+^2(w) + H_-^2(w) - 2H_0^2(w)$$

Expected uncertainties on A_{FB} and F_L^{D*}

	$A_{FB}[10^{-2}]$	$F_L^{D*}[10^{-2}]$
NBB	< 0.01	< 0.01
BR(D decays)	0.03	0.03
Lifetime ratio	< 0.01	< 0.01
track efficiency	< 0.01	0.01
efficiency corrections	0.03	0.01
BR(D** + gap)	0.10	0.13
Form factors (D**)	0.42	0.78
Backgr. model	1.71	1.57
HadronID	0.05	0.02
LeptonID	0.28	0.60
Fit bias	0.61	0.64
MC stat.	1.01	0.91
Isospin breaking (th.)	< 0.01	< 0.01
TOTAL SYST	2.15 (syst + th)	2.16 (syst + th)
STAT	1.96	1.81