Model-independent parameterization of $B \to \pi\pi\ell\nu$ decays

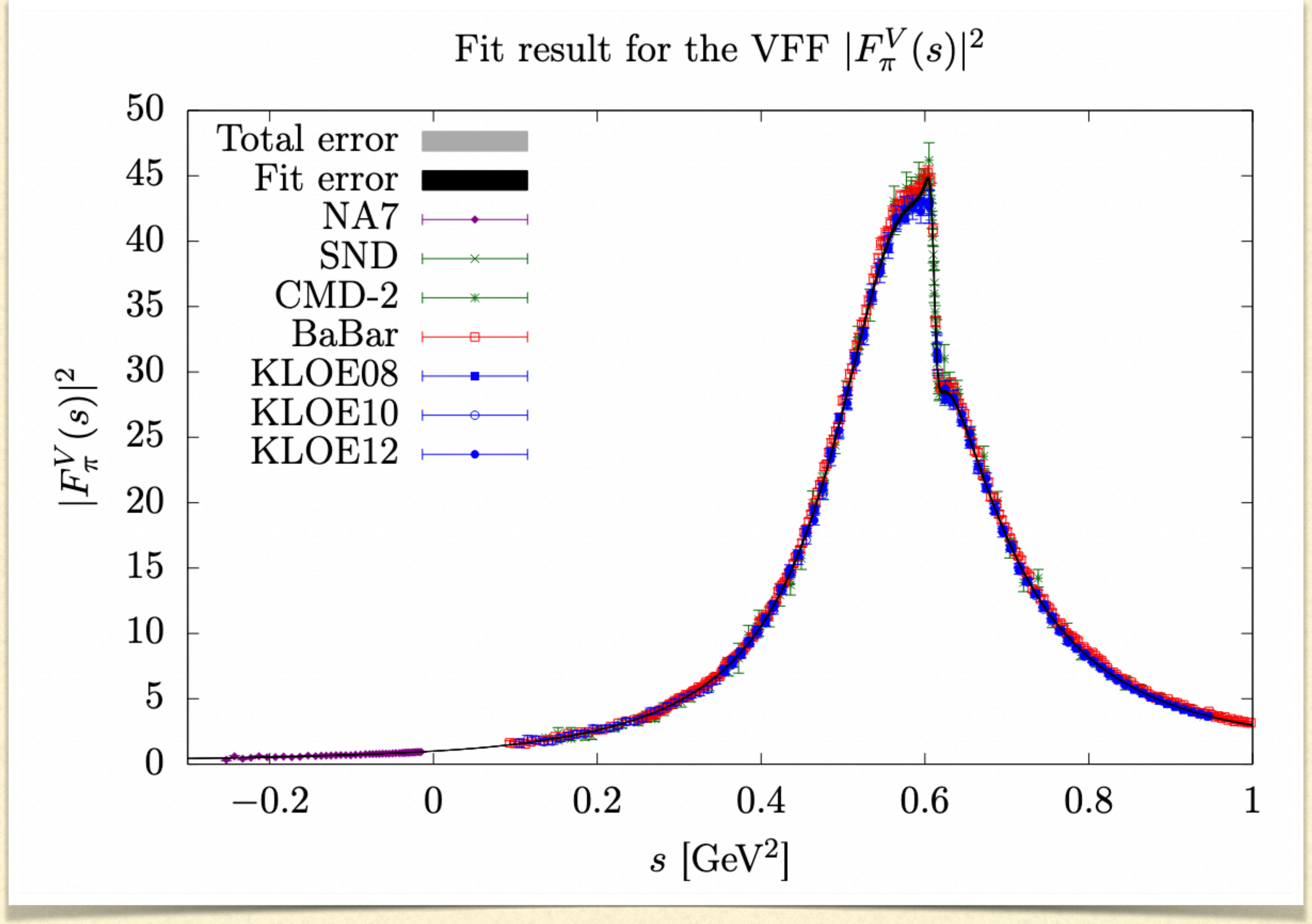
PRD 112 (2025) 1, 1 - in collaboration with Bastian Kubis & Raynette van Tonder

Universität Zürich UZH

Outline

- The need to go beyond the narrow ρ
- A new form factor parameterization
- Phenomenological implications

Real resonances have finite width!



Colangelo, Hoferichter, Stoffer JHEP 02 (2019) 006

- The ρ is neither narrow, nor described by a Breit-Wigner lineshape
- Don't even get me started on the f_0
- There is significantly more physics in the full $B \to \pi\pi\ell\nu$ decay than just the ρ
- Tensions between different excl. $|V_{ub}|$ determinations
- Tensions between different $B \to \rho\ell\nu$ measurements

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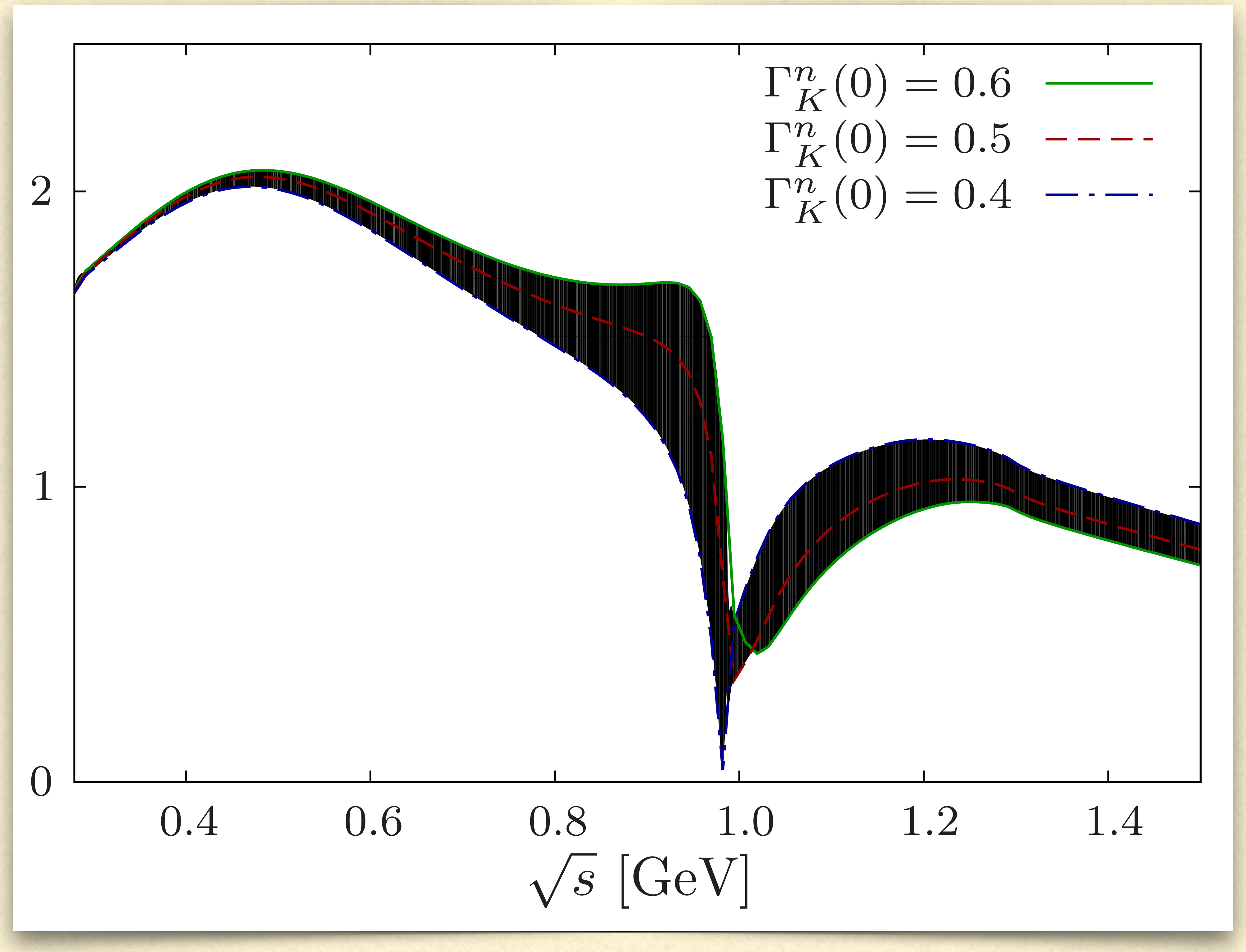

Taken from: Daub, Hanhart, Kubis JHEP 02 (2016) 009

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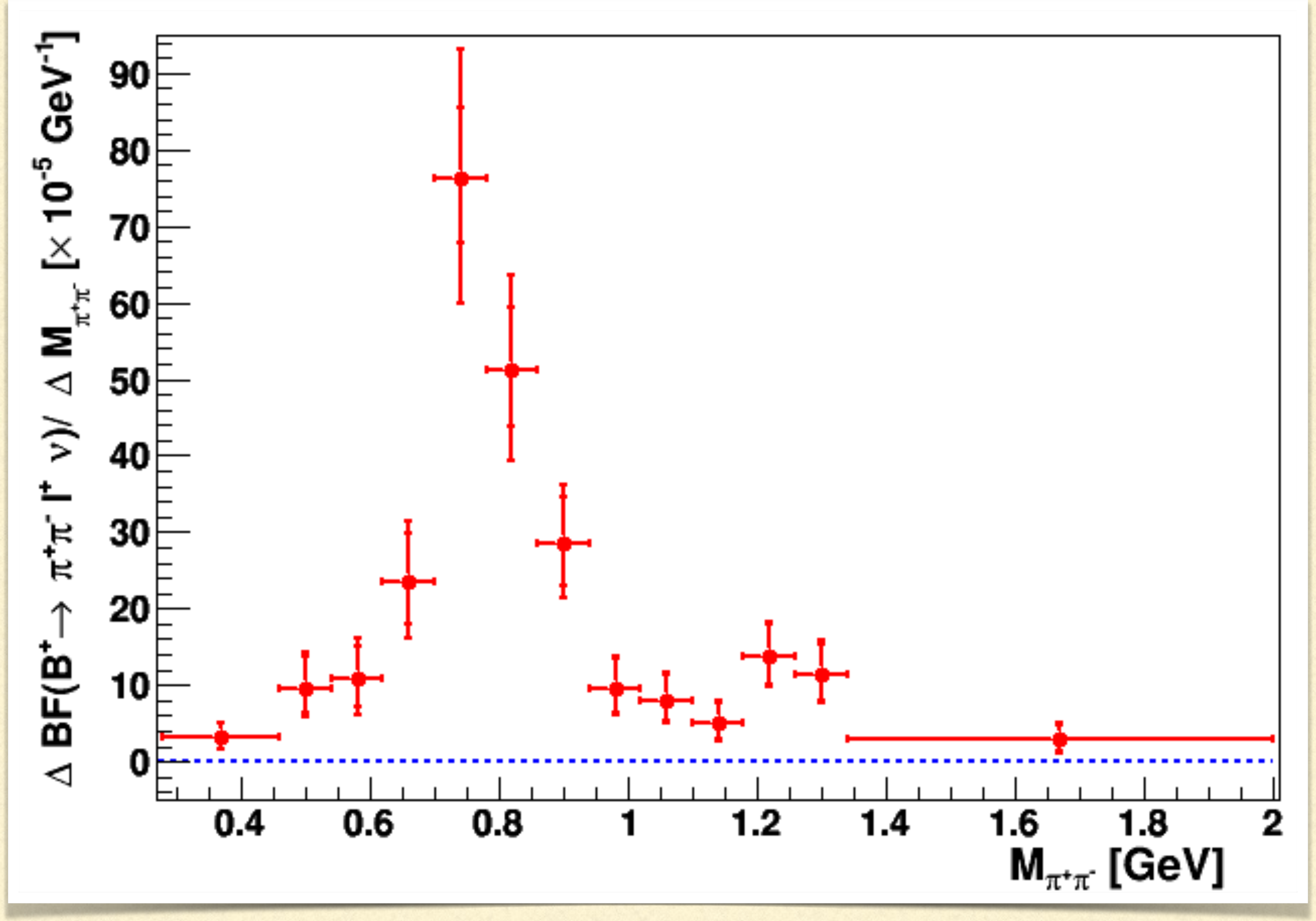

Beleño et al. PRD 103 (2021) 11, 112001

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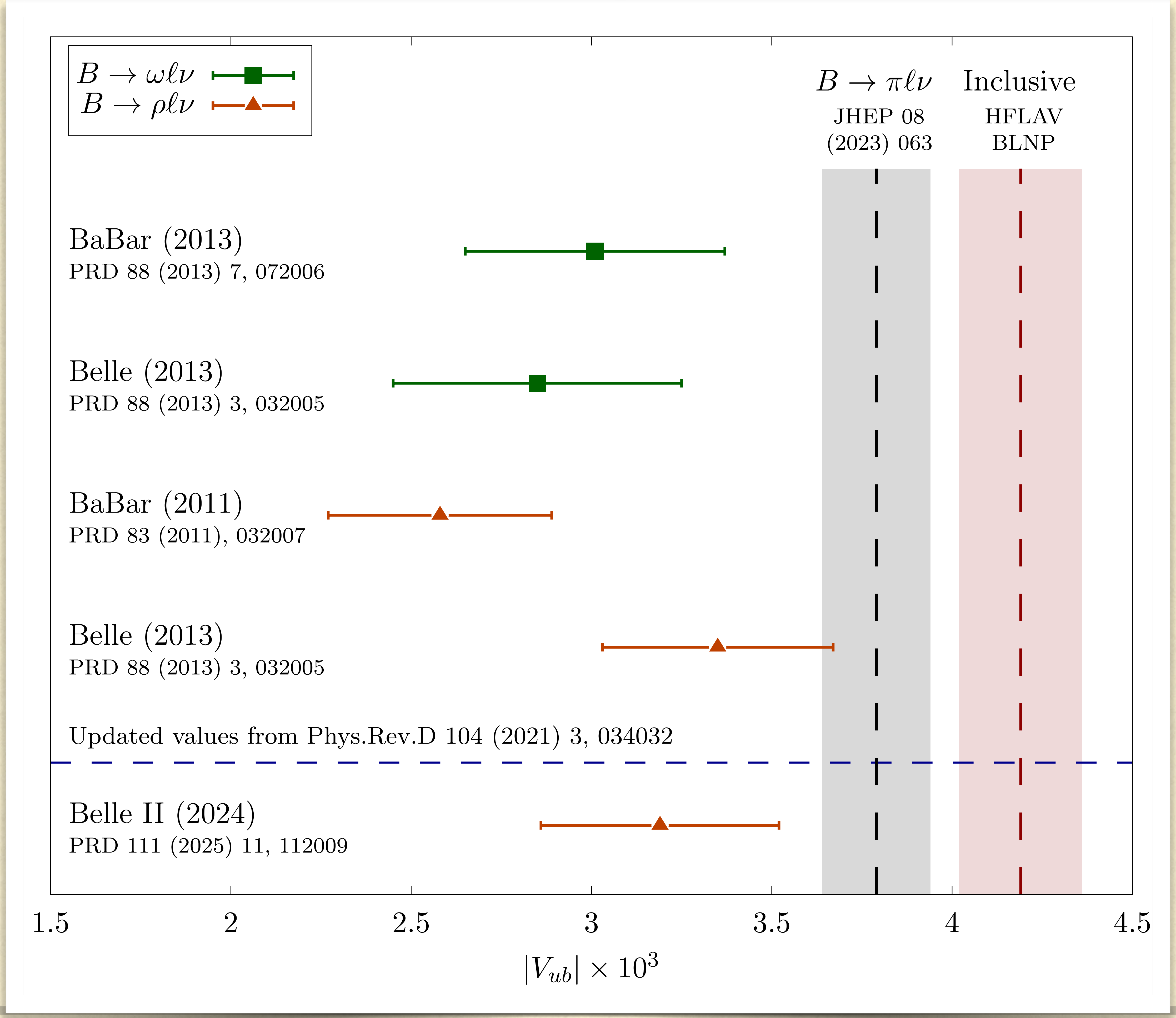

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A new form factor parameterization

- Model-independent parameterizations like BGL and its modifications such as BCL have played a crucial role in the past three decades
- Surprisingly simple form (although issues with truncation or subthreshold cuts)
- Allow to connect theoretical & experimental information from different kinematical regions
- General, but still allow to impose symmetry constraints (e.g. HQET)
- In use beyond semileptonic B-decays: Pion VFF, Lepton-Nucleon scattering, …

➡ We want a model-independent parameterization for two-hadron final states that has the same strengths as the BGL expansion

A new form factor parameterization

Ingredient 1: Process-independent lineshapes

Ingredient 2: Three-body rescattering

$$F^{(l)}_{(s)}(s,q^2) = \Omega^{(l)}_{(s)}(s)\left(\frac{1}{B_{(s)}(q^2)\tilde{B}^{(l)}_{(s)}(s)\phi^{(l)}_{(s)}(q^2)\tilde{\phi}^{(l)}_{(s)}(s)}\sum_{i,j} b^{(l)}_{ij,(s)} z^i_{(s)} y^j_{(s)} + \frac{s^n}{\pi}\int \frac{\mathrm{d}x}{x^n}\frac{\sin\delta^{(l)}_{(s)}(s)\hat{F}^{(l)}_{(s)}(x,q^2)}{|\Omega^{(s)}_l(x)|(x-s)}\right)$$

Ingredient 3: Model-independent double-expansion

Ingredient 1: $2 \to 2$ scattering

$$\left\langle p_3 p_4; b \,\middle|\, \mathscr{S} - 1 \,\middle|\, p_1 p_2; a \right\rangle = i(2\pi)^4 \delta^{(4)}\left(\sum p_i\right) \mathscr{M}_{ba}(\{p_i\})$$

$$\mathscr{M}_{ab} - \mathscr{M}^*_{ba} = i(2\pi)^4 \sum_c \int \mathrm{d}\Phi_c \mathscr{M}_{ca} \mathscr{M}^*_{cb}$$

$$\mathscr{A}_a - \mathscr{A}^*_a = i(2\pi)^4 \sum_c \int \mathrm{d}\Phi_c \mathscr{M}^*_{ca} \mathscr{A}_c$$

- Simplest scattering process with nontrivial kinematic dependence
- Described by unitary operator $\mathscr{S}$
- Scattering amplitude $\mathscr{M}$ depends on 2 independent Mandelstam variables
- $\mathscr{M}$ real below lowest threshold, imaginary part constrained by Unitarity above
- Two-particle production amplitude $\mathscr{A}$ shares phase with $\mathscr{M}$, e.g. pion production in lepton collisions

Ingredient 1: Partial waves

$$\mathcal{M}_{ba}(s,t) = \sum_l P_l(\cos\theta)\sqrt{\rho_b}^{-1} f_{ba}(s)\sqrt{\rho_a}^{-1}$$

$$f^l_{aa}(s) = \frac{\eta_l(s)e^{2i\delta_l(s)} - 1}{2i}$$

- Resonances have well-defined spin, their poles only occur in a specific partial wave of $\mathcal{M}$
- Partial-wave expansion conveniently separates different resonances, e.g. in pion scattering: $\rho, f_0(500), f_0(980), f_2(1270)$
- Diagonal elements can be expressed through scattering phase δ_l and inelasticity η_l

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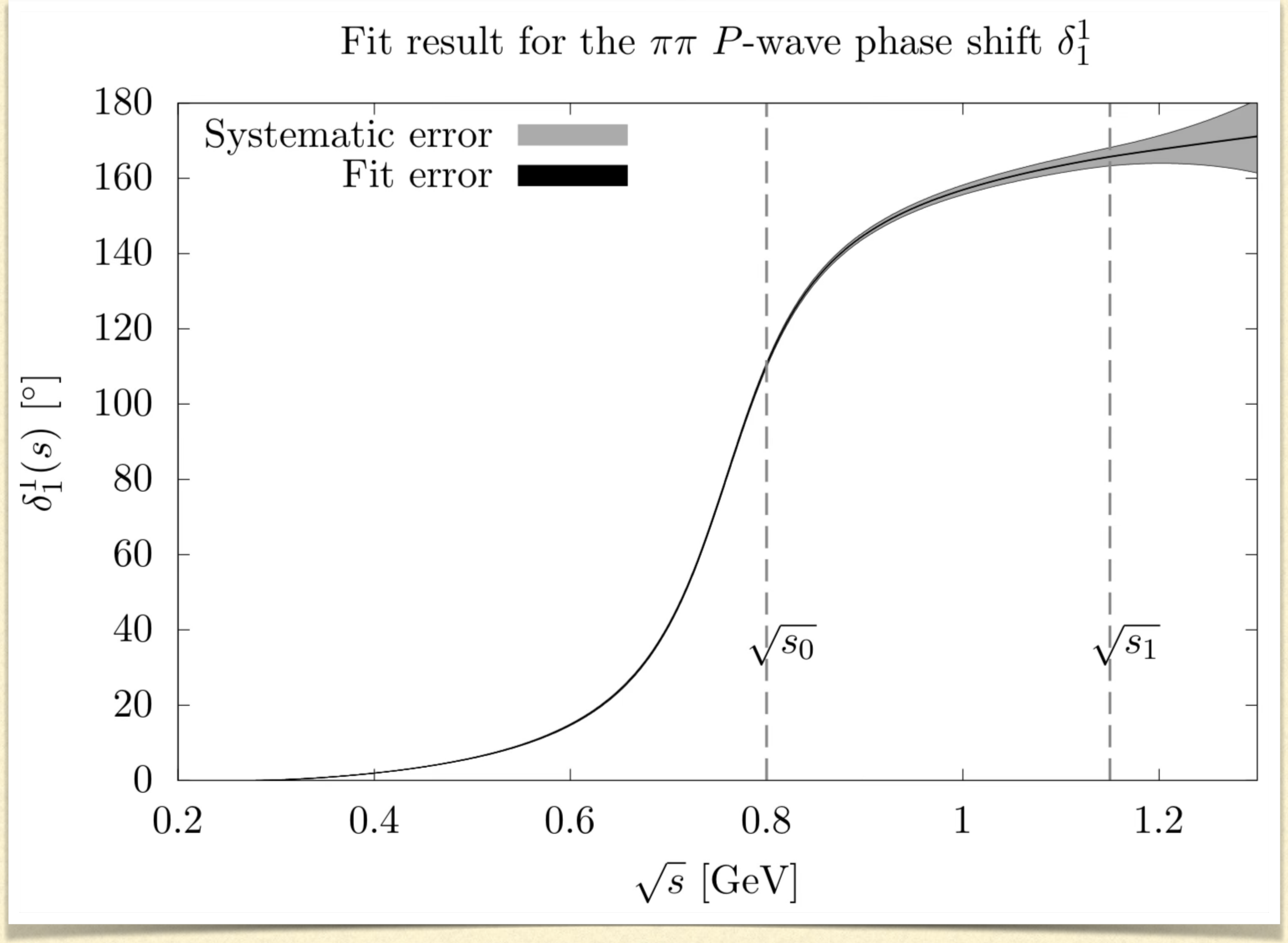

Colangelo, Hoferichter, Stoffer JHEP 02 (2019) 006

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Ingredient 1: Phase shifts

$$\ln \Omega_l(s) = \frac{s}{\pi} \int \mathrm{d}s' \frac{\delta_l(s')}{s'(s'-s)}$$

$$F(s) = \Omega(s) \sum a_i z(s, s_{in})^i$$

$$\mathrm{Im}\Omega(s + i\epsilon) = \frac{1}{\pi} \int_{s_{\mathrm{thr}}}^{\infty} \frac{T^*(s')\Sigma(s')\Omega(s')}{s' - s - i\epsilon} \mathrm{d}s'$$

- Watson's theorem: Below the first inelastic threshold, the elastic scattering phase is universal
- Omnès function is a model-independent way to transport this information
- Common treatment of lineshapes in $e^+e^- \to \pi^+\pi^-$, $\tau \to \pi^-\pi^0\nu_\tau, K \to \pi\pi\ell\nu, B_{(s)} \to J/\Psi\pi^+\pi^-, \ldots$
- Works best for light mesons, $\pi\pi$, $K\pi$, but also S-wave $D\pi$: see Raynette's talk tomorrow!
- Extensions beyond first inelastic threshold clear
- For novel ideas: see Nienke's talk tomorrow!

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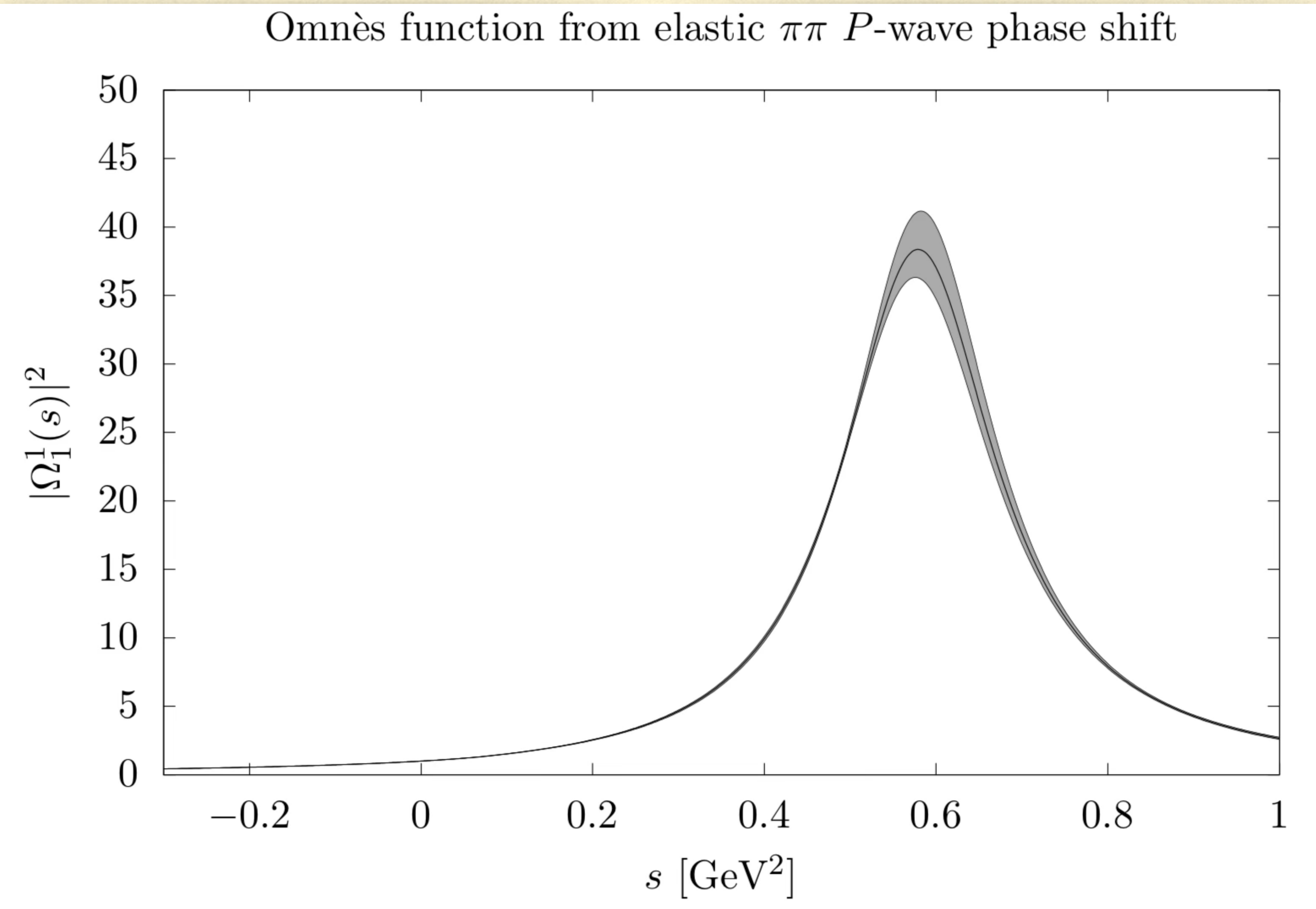

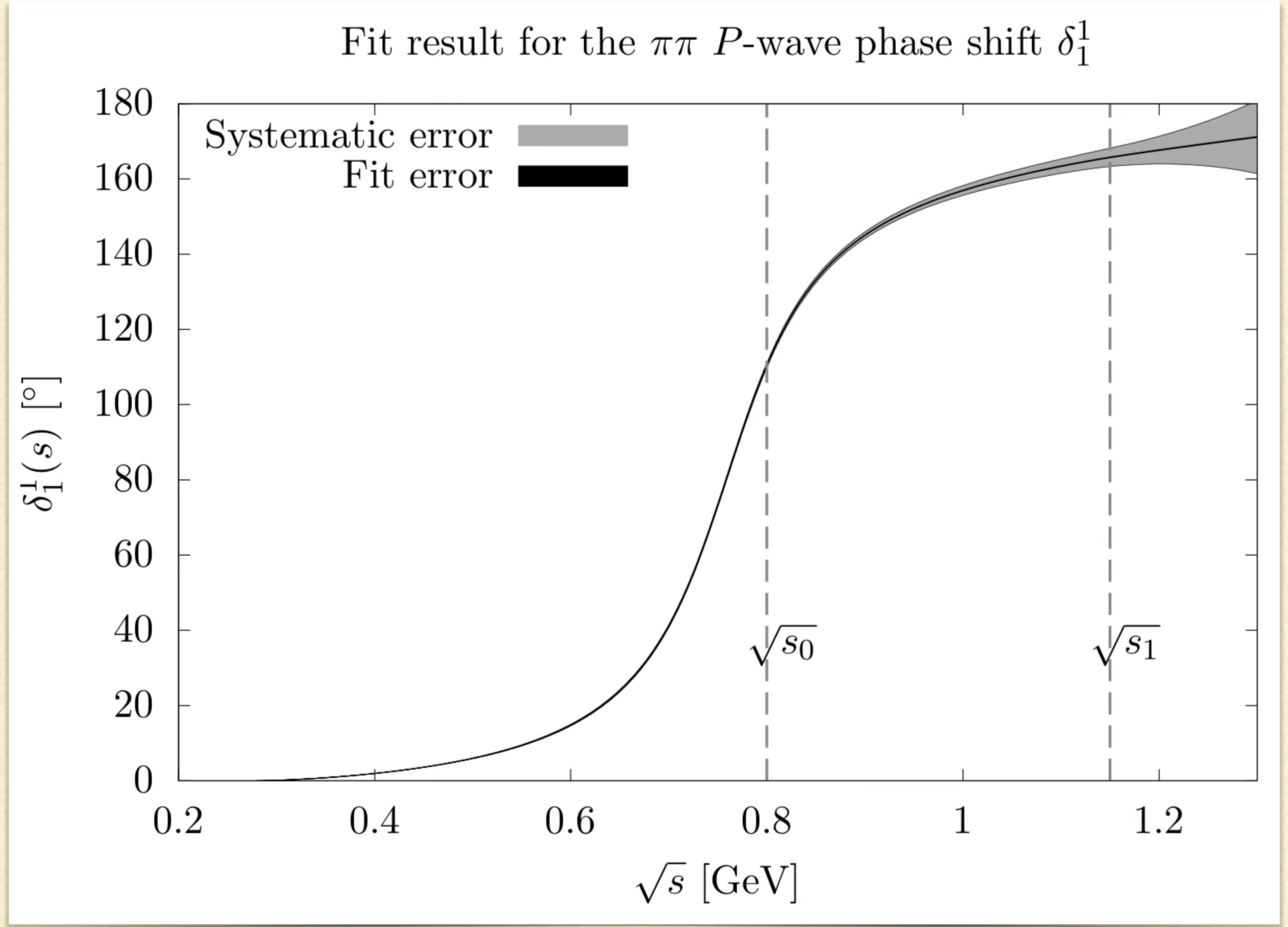

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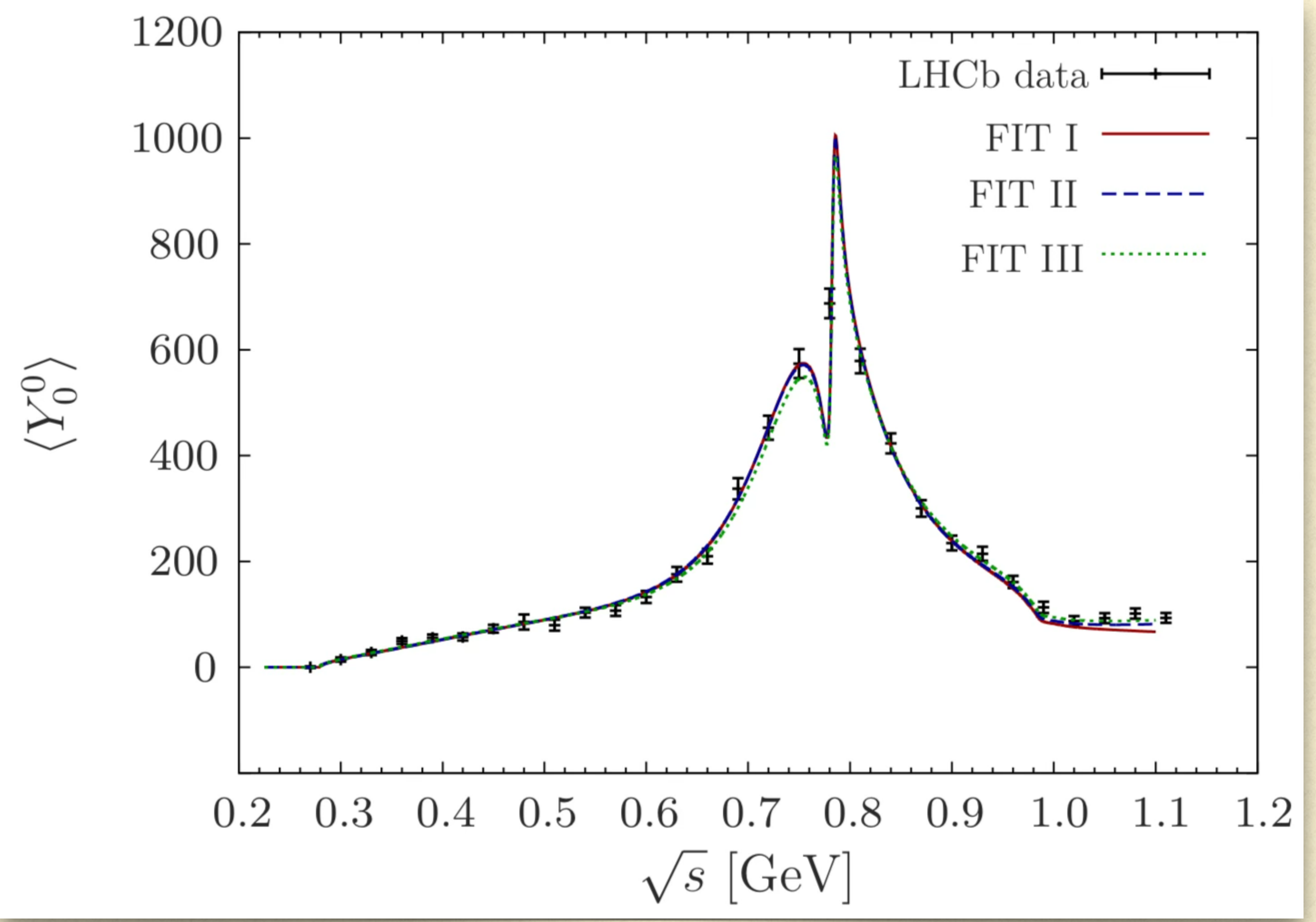

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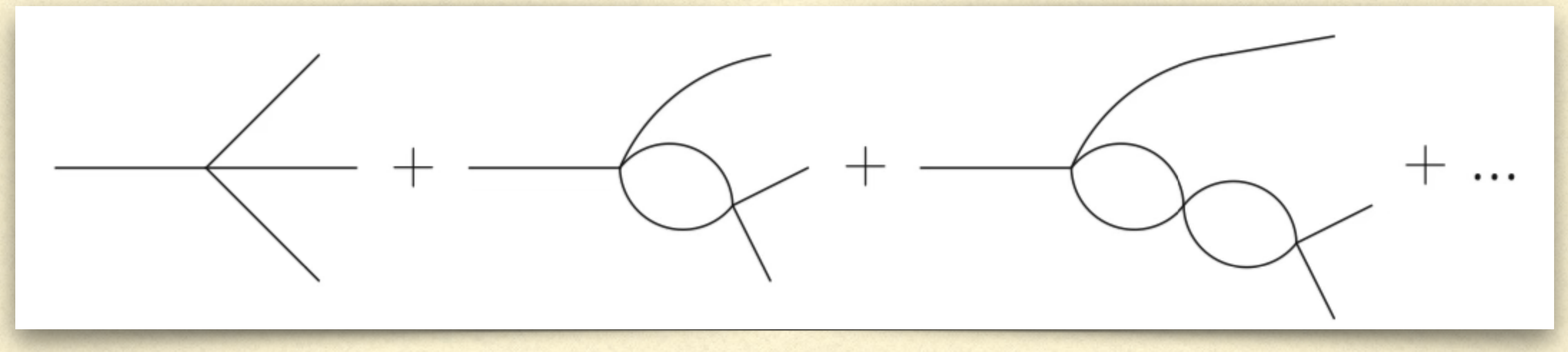
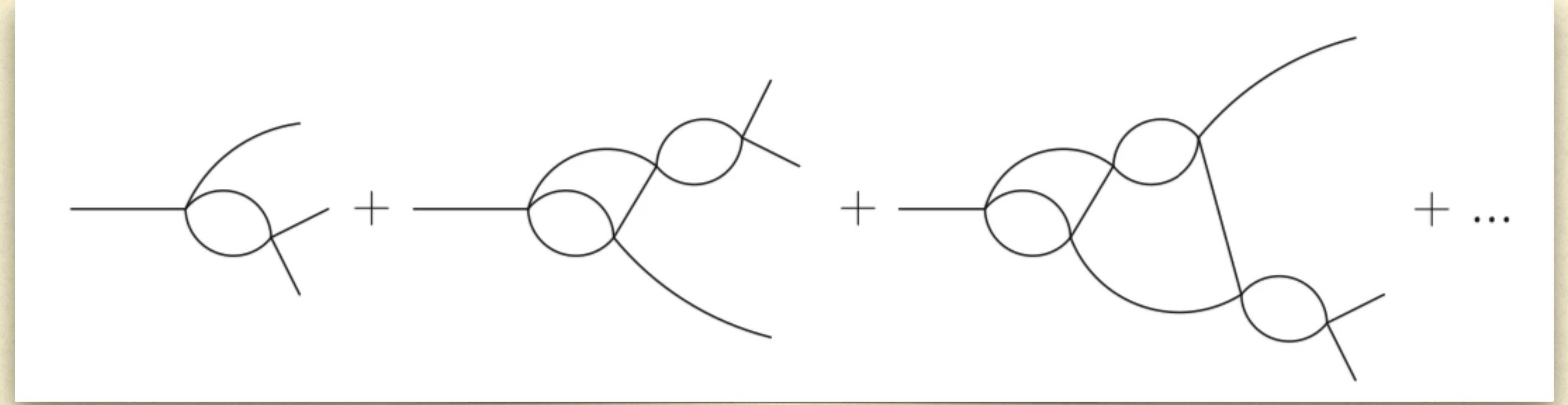
Ingredient 2: Three-body decays





Taken from: EPJC 83 (2023) 6, 510

$$F^{(l)}_{(s)}(s) = \Omega^{(l)}_{(s)}(s)\left(Q^{(l)}_{(s)}(s) + \frac{s^n}{\pi}\int \frac{\mathrm{d}x}{x^n}\frac{\sin\delta^{(l)}_{(s)}(s)\hat{F}^{(l)}_{(s)}(x)}{|\Omega^{(l)}_{(s)}(x)|(x-s)}\right)$$

- Amplitudes relevant for Unitarity bounds are $1 \to n$ amplitudes of particle with mass q^2
- Khuri-Treiman formalism allows to reconstruct three-body rescattering (PR 119 1115-1121 (1960))
- Write decay amplitude as sum of 3 partial-wave expanded amplitudes
- Fixed s, t & u dispersion-relations lead to coupled system of integral equations
- The two other channels enter via hat functions (B^* exchange)

Ingredient 2: Three-body decays

$$\mathcal{F}(s,t,u) = \sum_{x\in\{s,t,u\}} \sum_l F^{(l)}_{(x)}(x,q^2) P_l(\cos\theta_x)$$

- Amplitudes implicitly depend on mass
- s-dependence not polynomial above inelastic thresholds
- The hat functions depend on $B^* \to \pi$ FFs

$$F^{(l)}_{(s)}(s,q^2) = \Omega^{(l)}_{(s)}(s)\left(f^{(l)}_{(s)}(s,q^2) + \frac{s^n}{\pi}\int \frac{\mathrm{d}x}{x^n} \frac{\sin\delta^{(l)}_{(s)}(s)\hat{F}^{(l)}_{(s)}(x,q^2)}{|\Omega^{(s)}_l(x)|(x-s)} \right)$$

Ingredient 3: Unitarity bounds

$$\Pi^{L/T}_{(J)}(q) \equiv i \int d^4x \, e^{iq\cdot x} \langle 0 \big| \, J^{L/T}(x) \, J^{L/T}(0) \big| 0 \rangle$$

$$\chi^{L}_{(J)}(Q^2) \equiv \frac{\partial \Pi^{L}_{(J)}}{\partial q^2}\bigg|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty \mathrm{d}q^2 \frac{\mathrm{Im}\Pi^{L}_{(J)}(q^2)}{(q^2-Q^2)^2}$$

$$\chi^{T}_{(J)}(Q^2) \equiv \frac{1}{2}\frac{\partial^2 \Pi^{T}_{(J)}}{\partial (q^2)^2}\bigg|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty \mathrm{d}q^2 \frac{\mathrm{Im}\Pi^{T}_{(J)}(q^2)}{(q^2-Q^2)^3}$$

$$\mathrm{Im}\Pi^{T/L}_{(J)} = \frac{1}{2}\sum_X \int \mathrm{dPS}\, P^{\mu\nu}_{T/L} \left\langle 0 \left| J_\mu \right| X \right\rangle \left\langle X \left| J_\nu \right| 0 \right\rangle \delta^{(4)}(q - p_X)$$

$$\mathrm{Im}\Pi^{T}_{(V)}\,|_{BD} = K(q^2)\,|f_+(q^2)\,|^2$$

- Starting point: once and twice subtracted dispersion relations [Boyd, Grinstein, Lebed; Caprini; …]
- Susceptibilities perturbatively computable for large space-like Q^2 or at $Q^2 = 0$ if heavy quarks involved; also on the Lattice! (Martinelli, Simula, Vittorio; Harrison)
- Optical theorem allows to write the imaginary part as sum over all possible final states
- Neglecting a final state leads to an inequality

Ingredient 3: Unitarity bounds

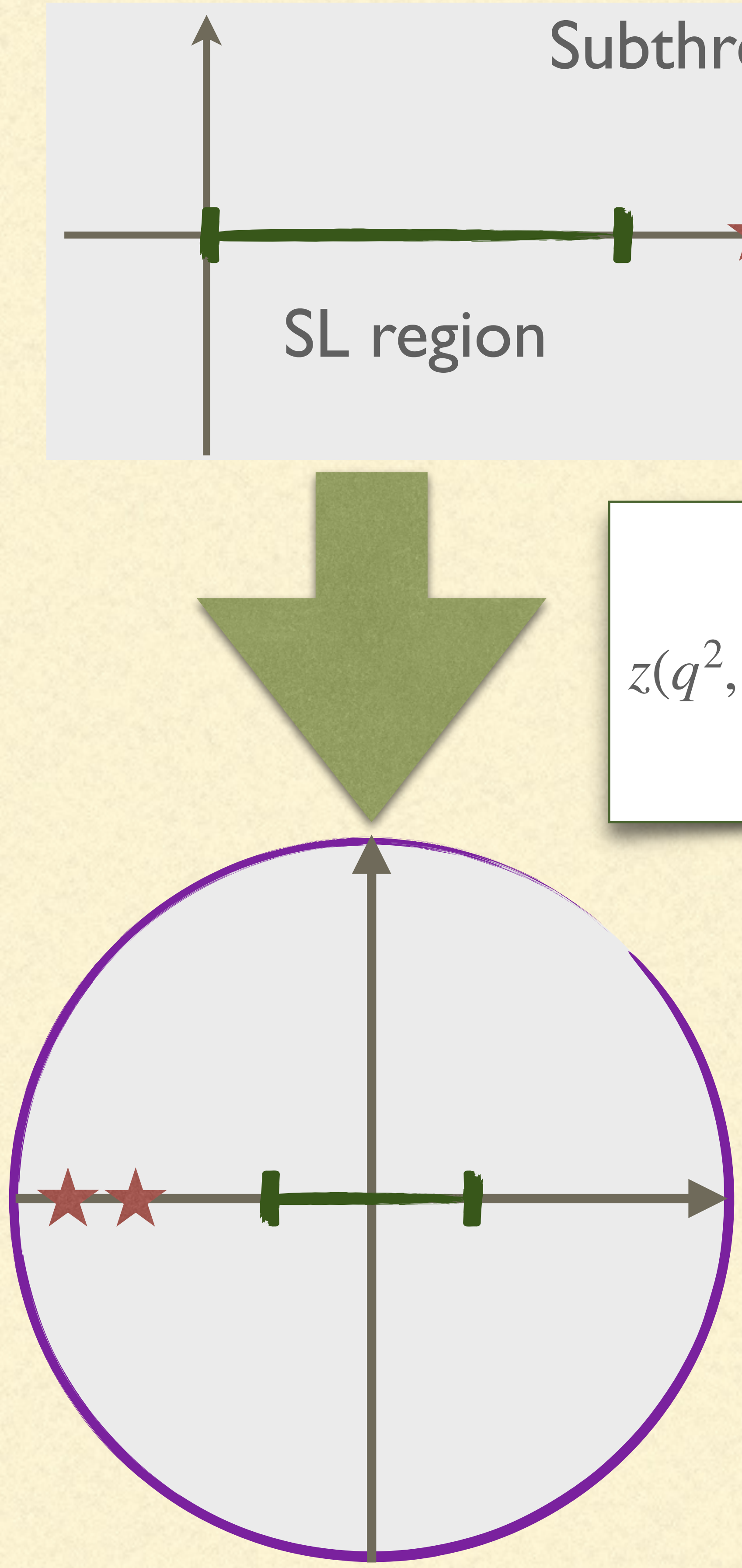
$$\mathrm{Im}\Pi(q^2)\Big|_{M_1M_2M_3} = \sum_x \int_{x_+}^{(\sqrt{q^2}-m_y)^2} \mathrm{d}x \sum_l \frac{K_l(q^2,x)}{2l+1} |F_{(x)}^{(l)}(x,q^2)|^2$$

$$\chi \geq \frac{1}{\pi}\int_0^\infty \mathrm{d}q^2 \int_{s_+}^{s_-(q^2)} \mathrm{d}s \frac{K(s,q^2)}{q^{2n}} |\Omega(s) f(s,q^2)|^2$$

$$\chi \geq \frac{1}{\pi}\int_{s_+}^\infty \mathrm{d}s \hat{K}(s) \int_{q_+^2(s)}^\infty \mathrm{d}q^2 \frac{\tilde{K}(s,q^2)}{q^{2n}} |f(s,q^2)|^2$$

- Unitarity bounds in general off-diagonal
- Off-diagonal terms small, ignore for derivation of parameterization
- Similar to KT treatment: ignore left-hand cuts and add them back later
- Crucial: change integration order!
- In NWA: $\hat{K}(s) \to \delta(s - M_R^2)$

Ingredient 3: Unitarity bounds



$$z(q^2, q_0^2) = \frac{\sqrt{q_+^2 - q^2} - \sqrt{q_+^2 - q_0^2}}{\sqrt{q_+^2 - q^2} + \sqrt{q_+^2 - q_0^2}}$$

$$1 \geq \frac{1}{2\pi i} \oint \frac{\mathrm{d}z}{z} \left| B(z)\Phi(z)f(z) \right|^2$$

$$f(z) = \frac{1}{\Phi(z)B(z)} \sum_{i=0}^{\infty} a_i z^i \qquad 1 \geq \sum_{i=0}^{\infty} |a_i|^2$$

- Mapping q^2 to the dimensionless variable z transforms integration region to unit circle
- Insert Blaschke products to get rid of subthreshold poles and zeroes in kinematic factors
- Series expansion (or orthogonal polynomials)
- Semileptonic region: $|z| < 1$

Ingredient 3: Double-expansion

$$f(s, q^2) = \frac{1}{B(q^2)\phi(q^2; s)} \sum_i a_i(s) p_i(z(q^2), q_+^2(s))$$

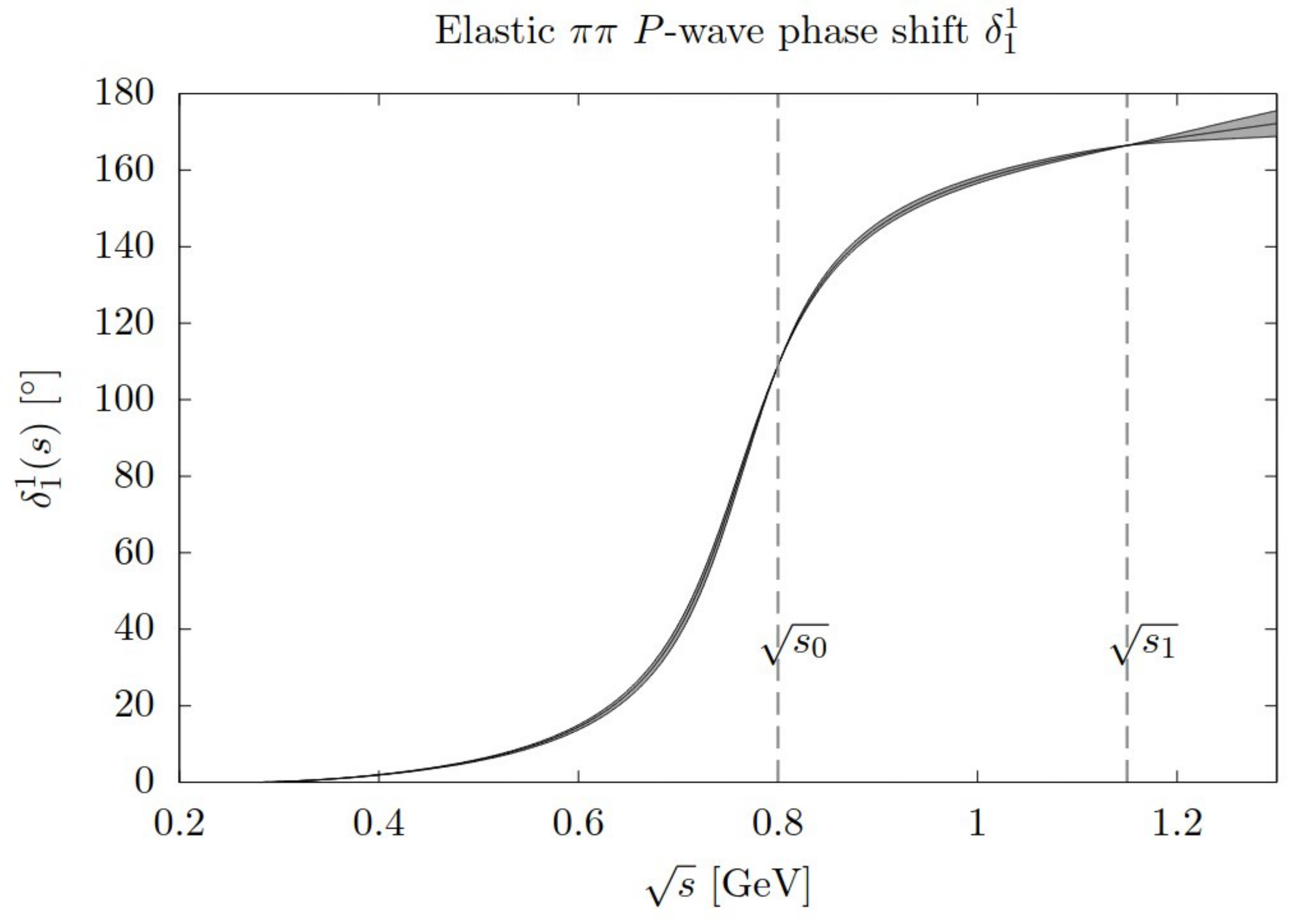
$$\chi \geq \frac{1}{\pi} \sum_i \int_{s_+}^{\infty} \mathrm{d}s \hat{K}(s) \, |\, a_i(s) \,|^2$$

$$a_i(s) = \frac{1}{\tilde{B}(s)\tilde{\phi}(s)} \sum_j b_{ij} y^j$$

- q^2-integration as in standard BGL
- If $q_+^2(s)$ larger than lowest two-body threshold: use of orthogonal polynomials
- Now we can treat every a_i as an s-dependent FF
- Since we pulled out the Omnès function: follow Caprini's treatment of pion VFF, (EPJ C 13 471-484 (2000))
- Alternative: BCL-like expansion

$$y = \frac{\sqrt{s_{in} - s} - \sqrt{s_{in}}}{\sqrt{s_{in} - s} + \sqrt{s_{in}}}$$

Phenomenology



Taken from Colangelo, Hoferichter & Stoffer, JHEP 02 (2019) 006

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- D-wave sufficiently understood
- S-wave funny
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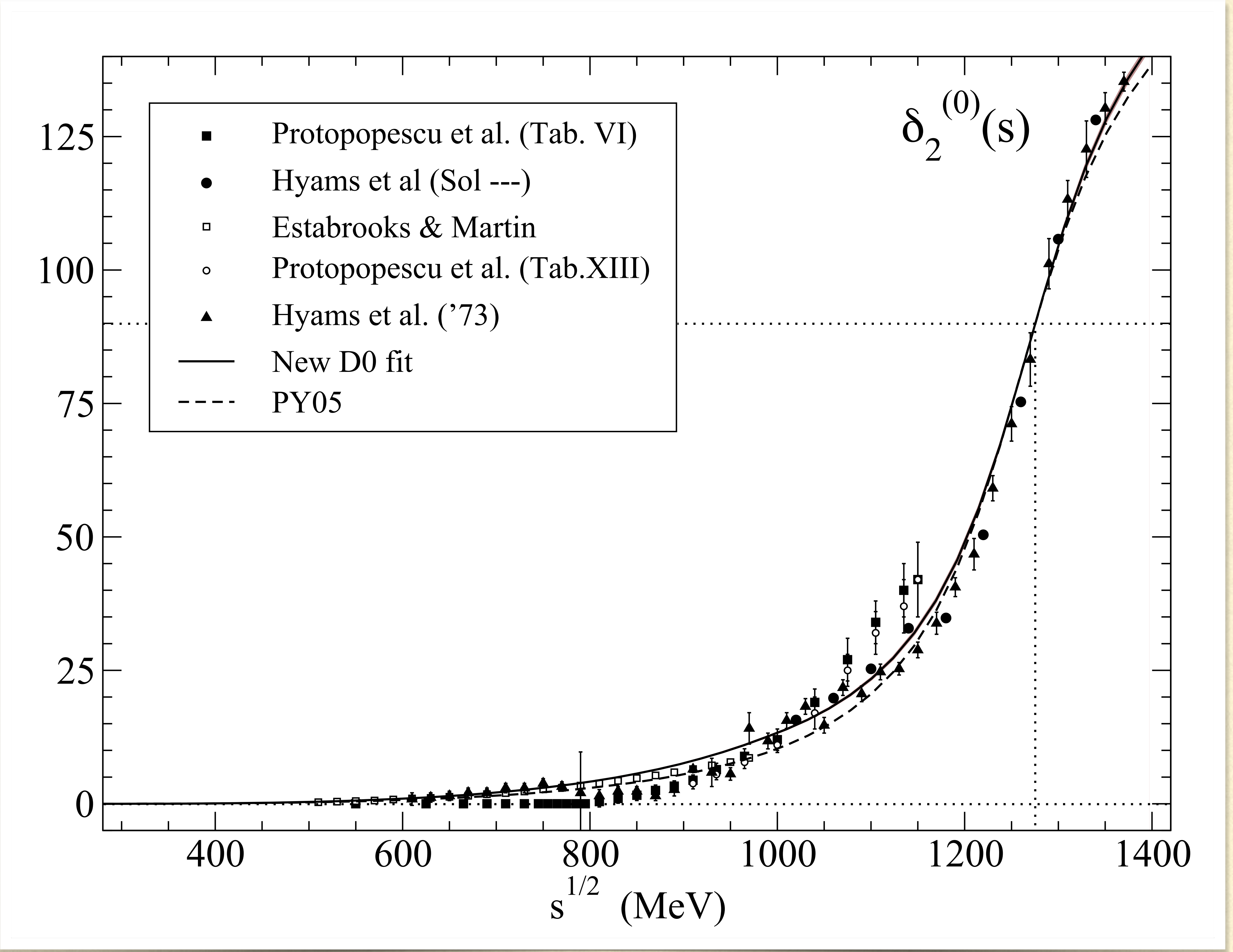

Taken from: Kaminski, Pelaez, Yndurain PRD 74 (2006) 014001

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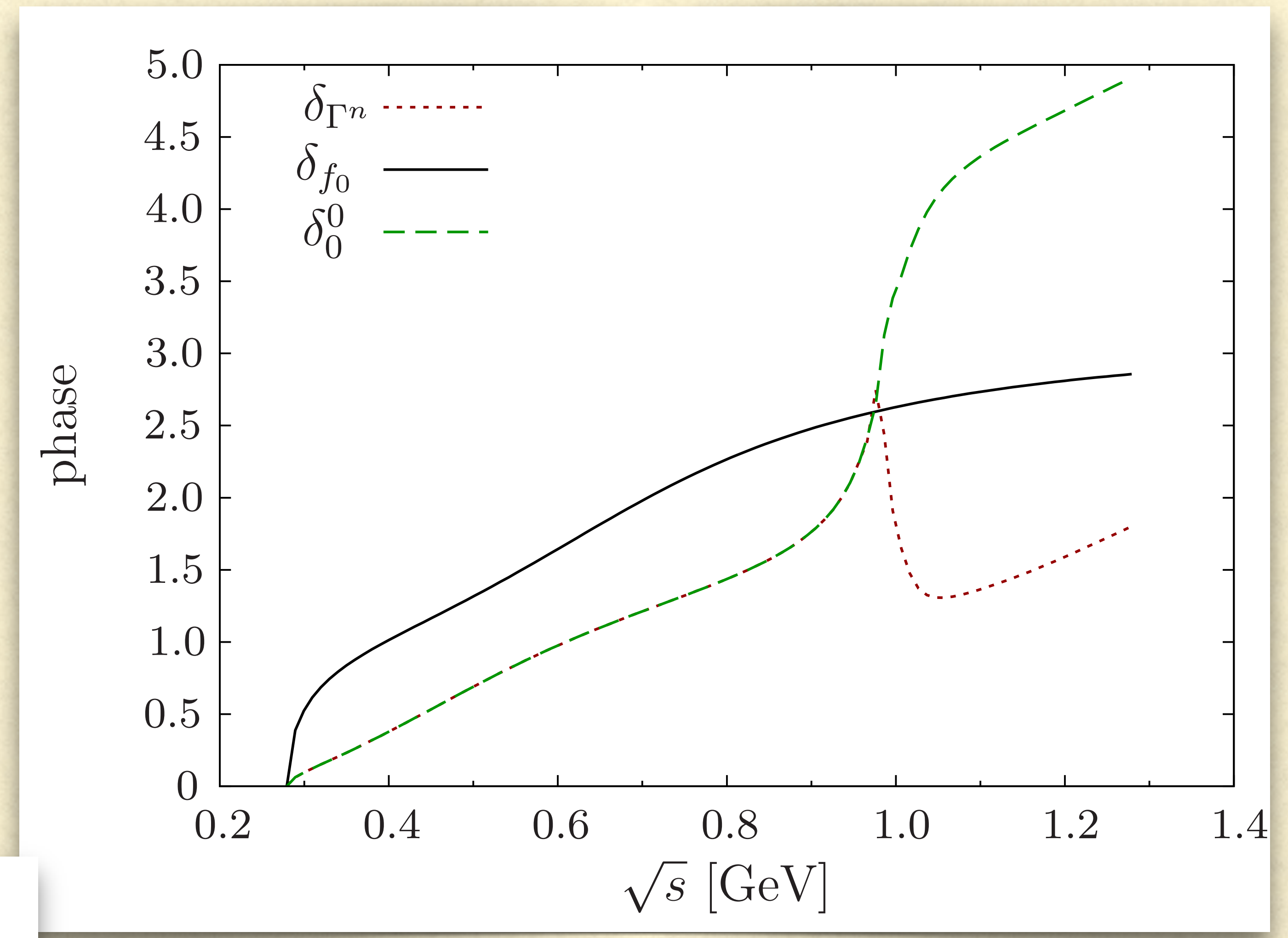

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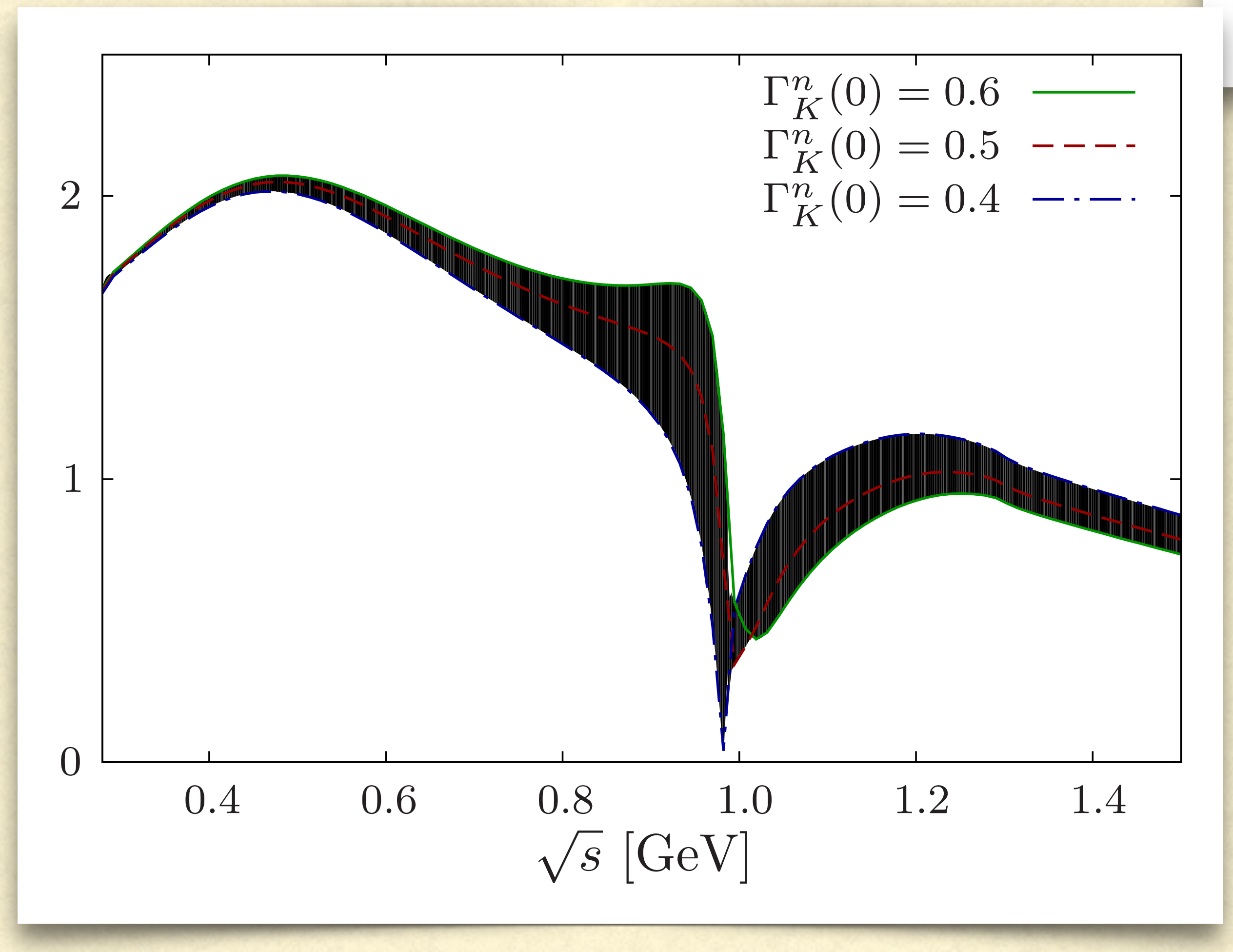

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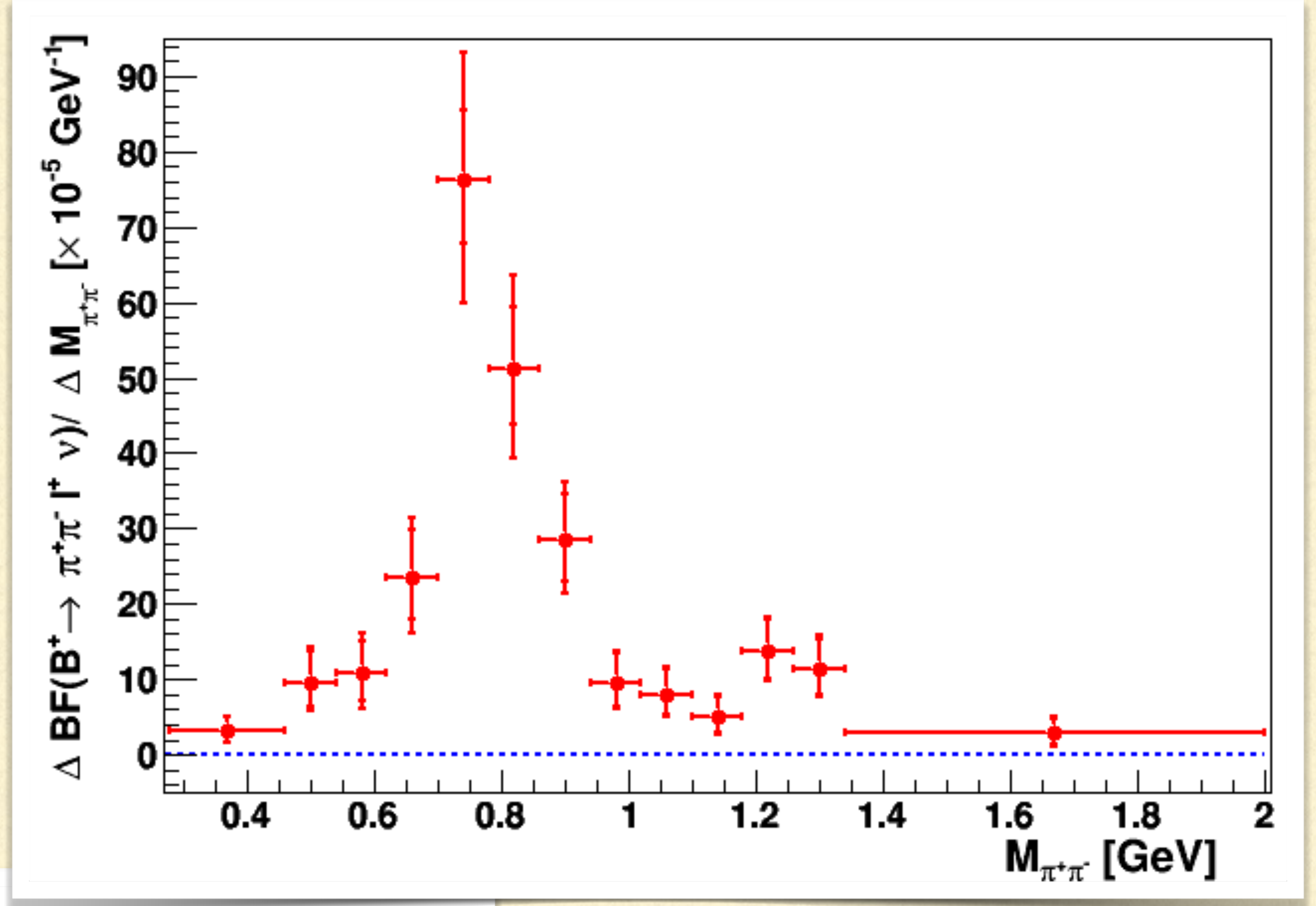

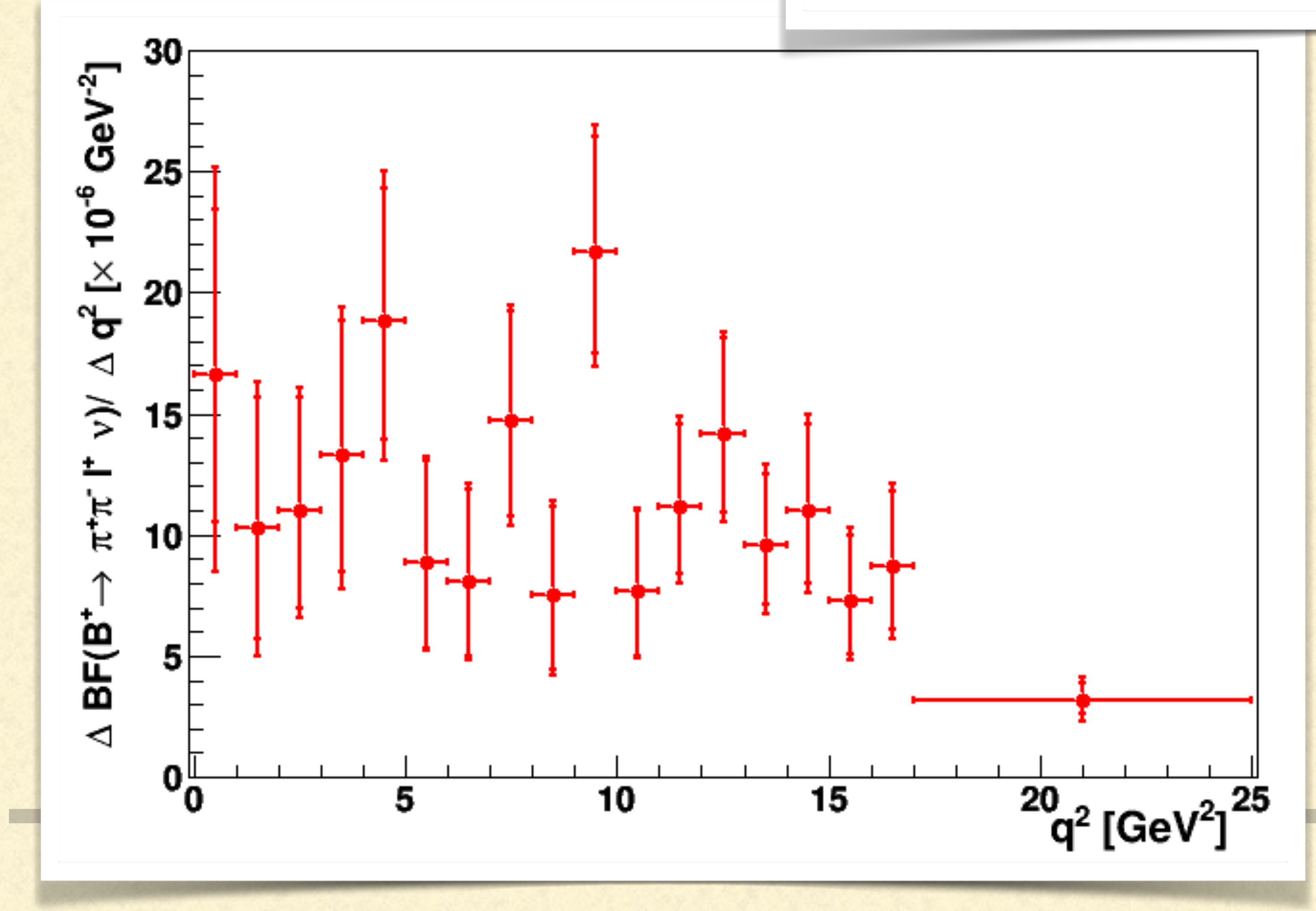

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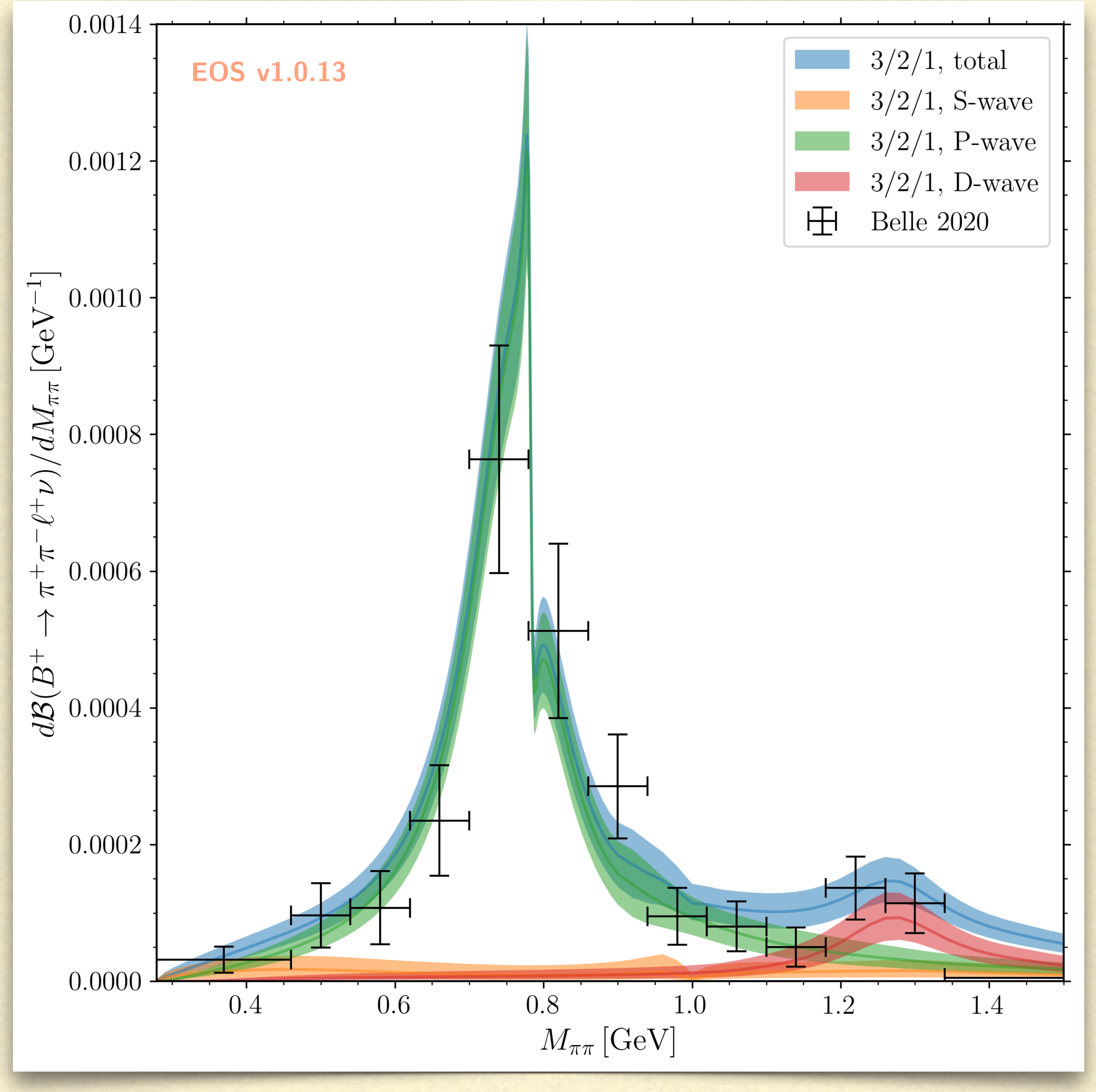

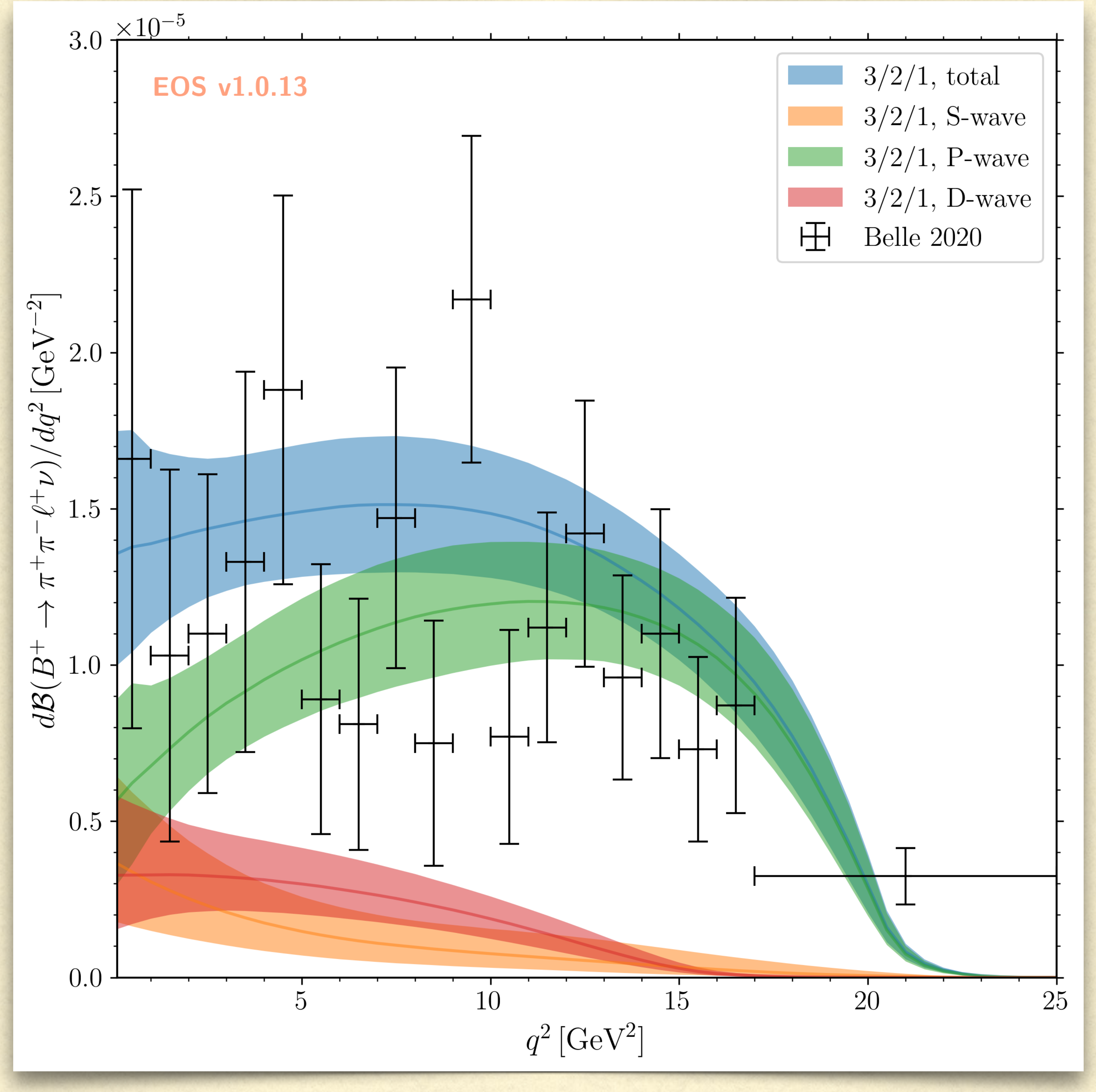

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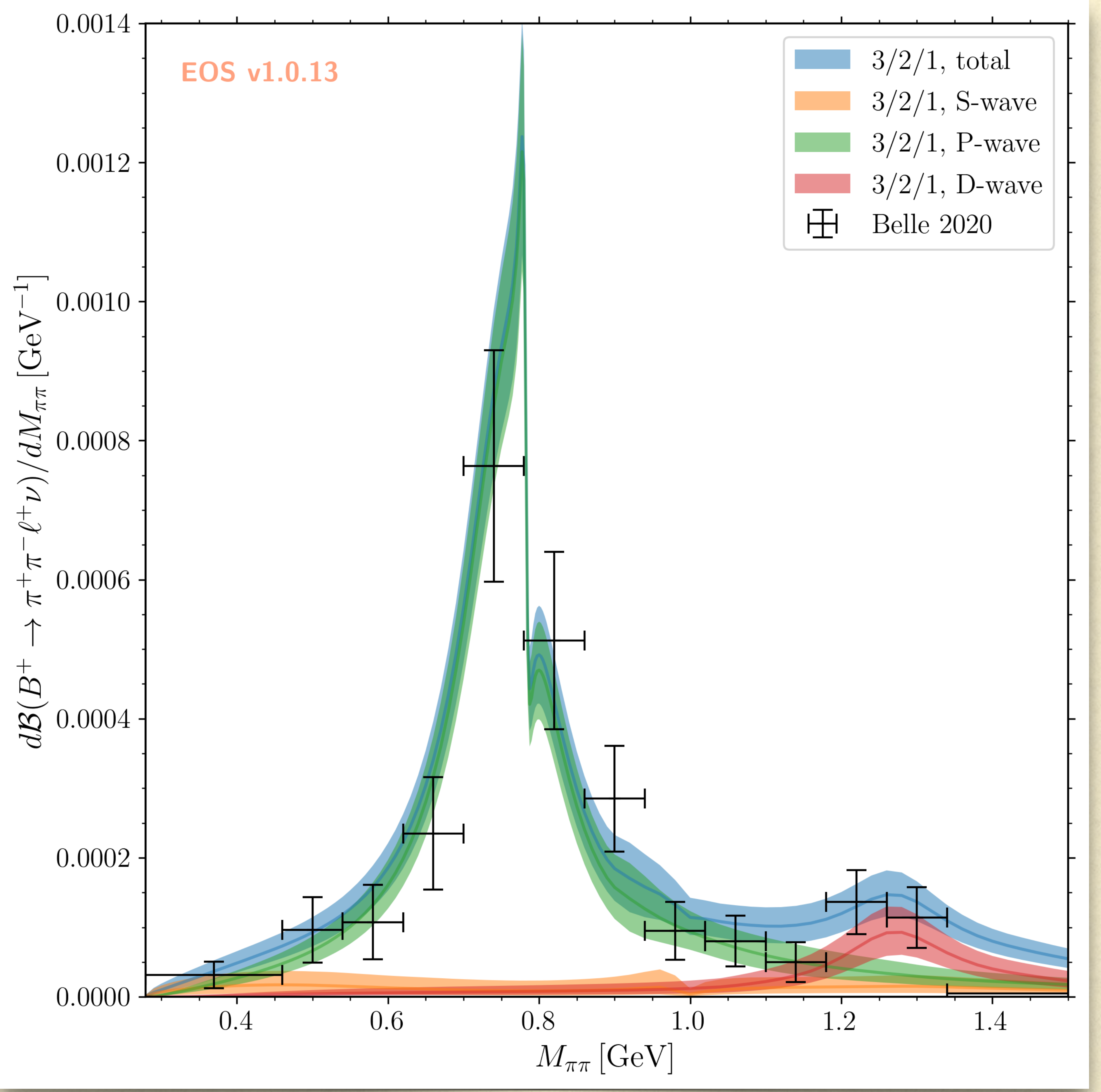

- Small S- and D-wave contribution in the ρ signal region → BaBar and Belle II likely overestimate "non-resonant" contribution
- 2σ evidence for $B \to f_2(1270)\ell\nu$ decays
- $\mathrm{Br}(B^+ \to \pi^0\pi^0\ell^+\nu) = 2.9^{+0.9}_{-0.7} \times 10^{-5}$
- Currently no direct determination of $|V_{ub}|$ until compatible form factor calculations become available

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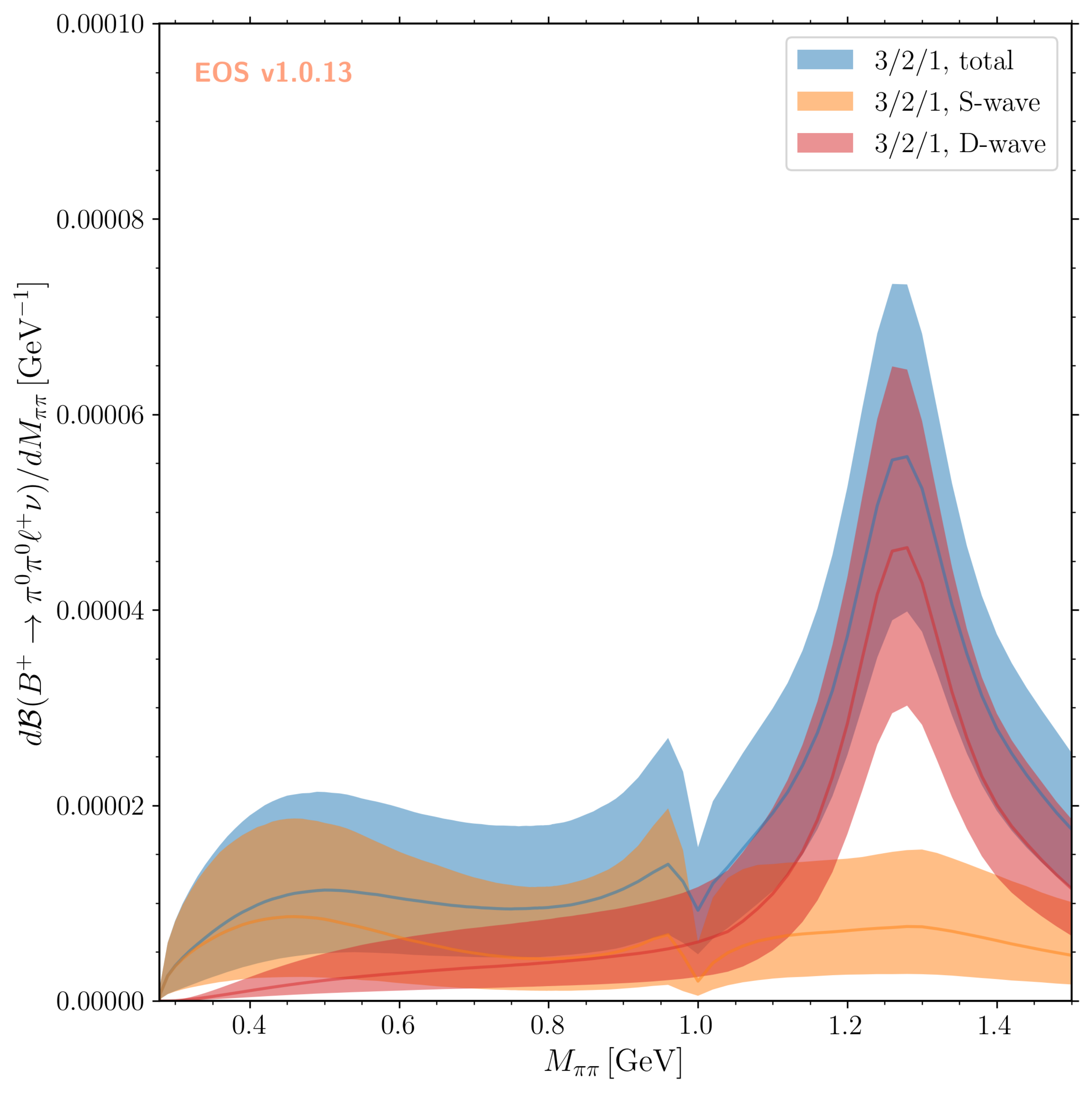
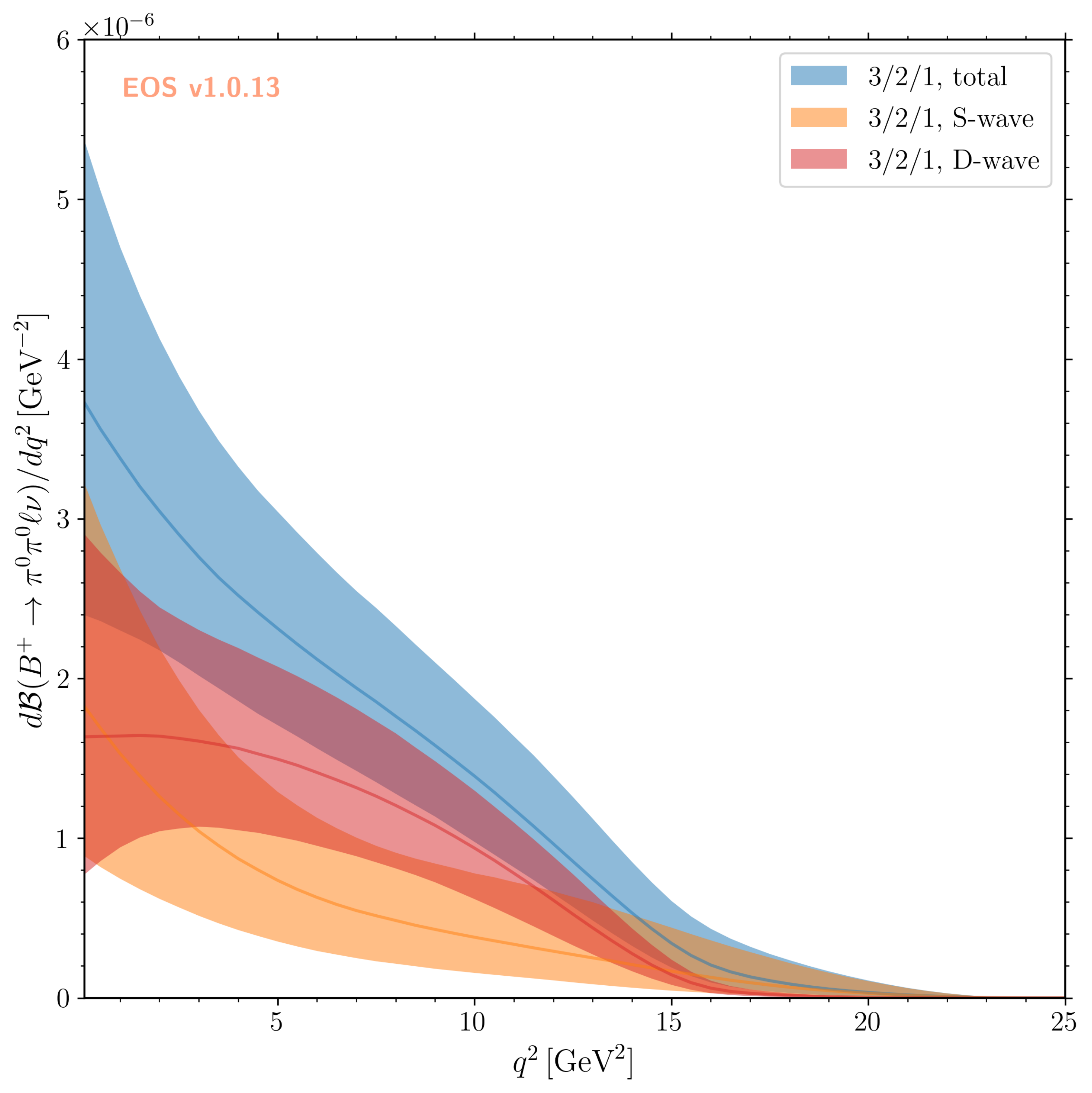
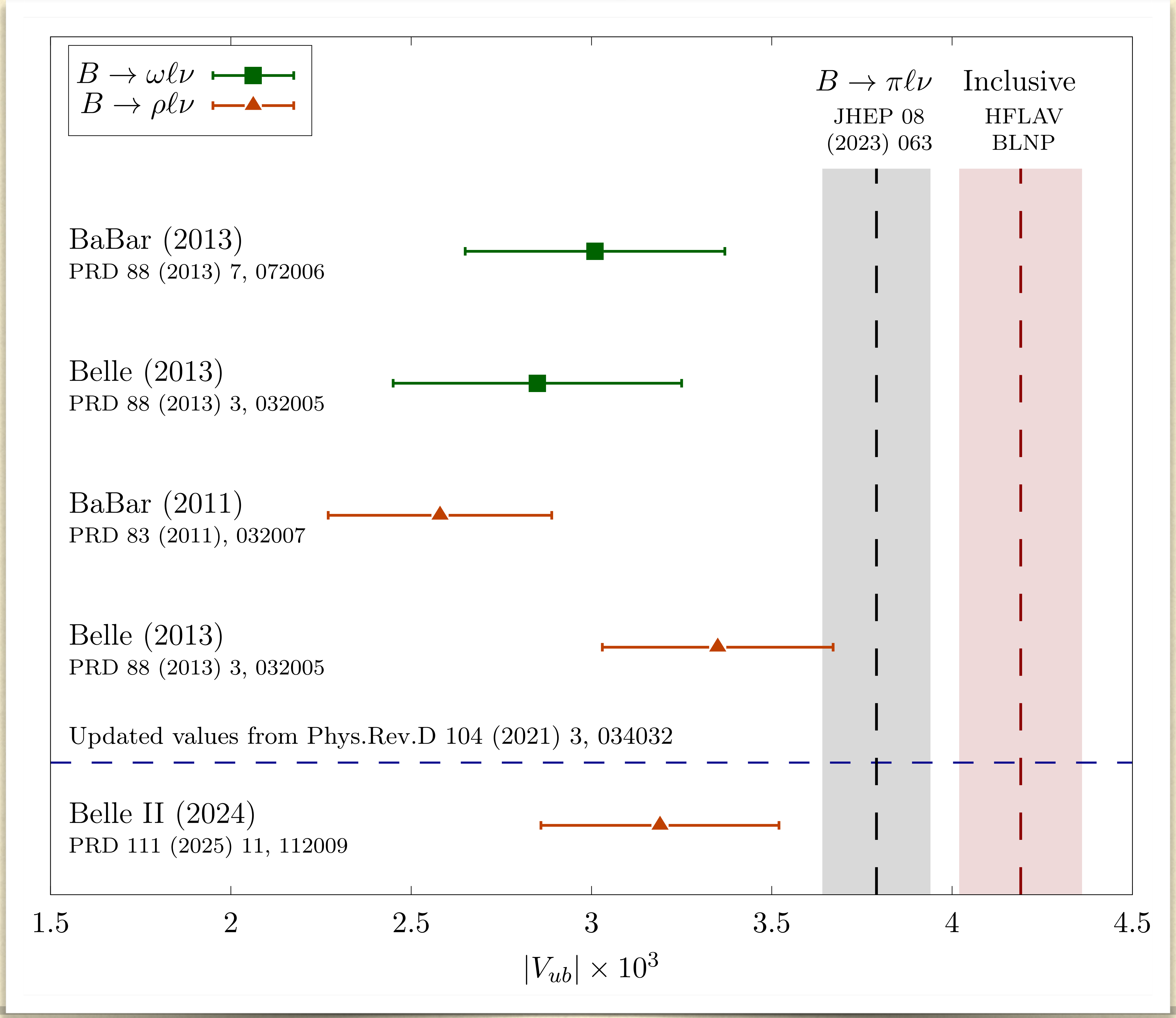




Can we resolve the ρ puzzle?



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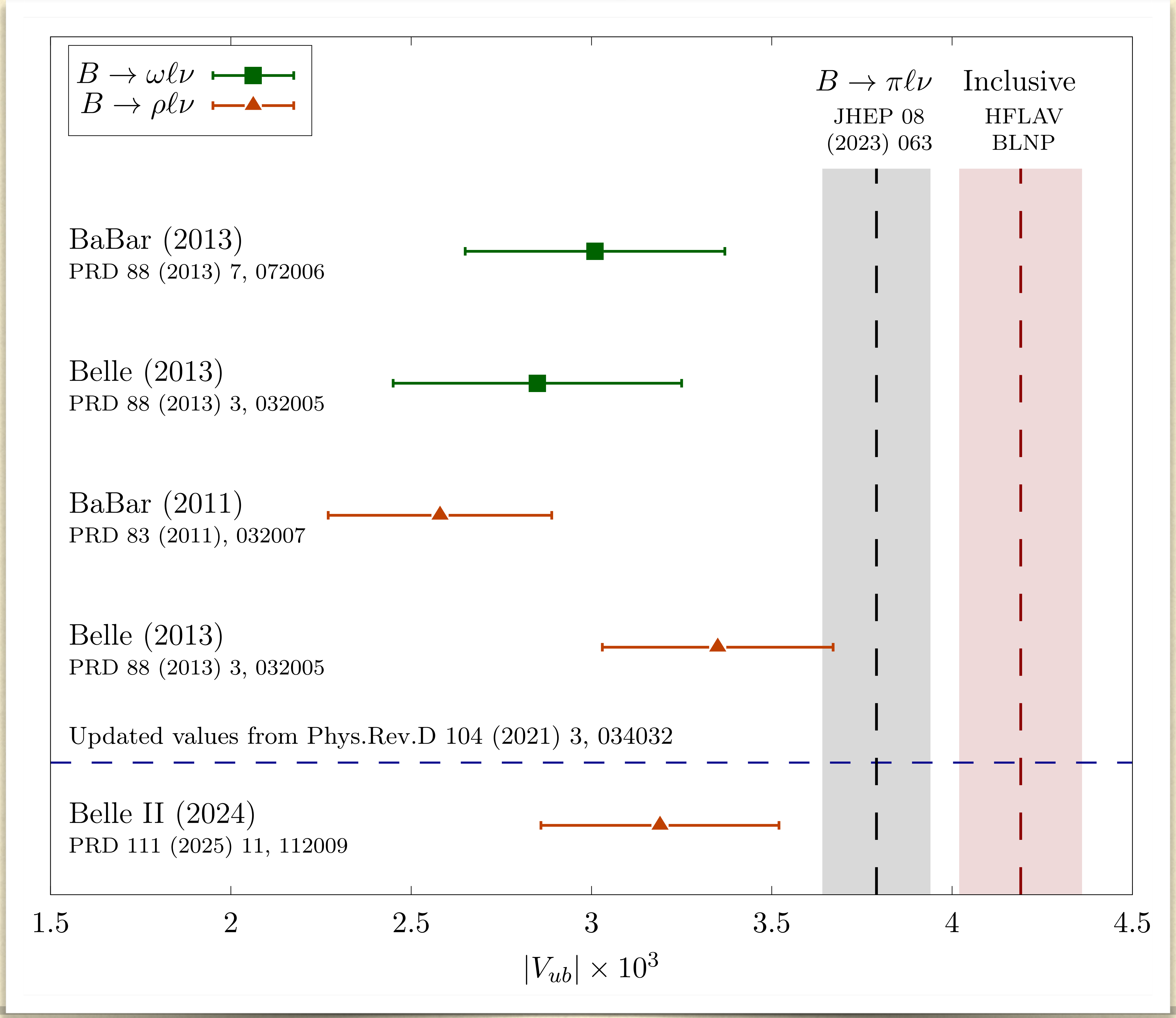

- Theory: Narrow-width LCSRs
- BaBar:

Candidate $\rho^\pm \to \pi^\pm \pi^0$ or $\rho^0 \to \pi^+\pi^-$ decays are required to have a two-pion mass within one full width of the nominal ρ mass, $0.650 < M_{\pi\pi} < 0.850\,\text{GeV}$. To re-

- Belle:

$E_{\rm ECL} < 0.7$ GeV. We select events where the invariant mass of the two pions is around the nominal ρ meson mass, requiring $|M_{\pi^+\pi^-} - m_\rho| < 2\Gamma_\rho$ where $m_\rho = 775.5\ \text{MeV}/c^2$ and $\Gamma_\rho = 149.1\ \text{MeV}/c^2$ are the nominal ρ mass and decay width, respectively.

- Belle II:

a charge opposite to that of the lepton candidate. In the $B^+ \to \rho^0 \ell^+ \nu_\ell$ mode we require that the two selected pion candidates that compose the ρ candidate have opposite charges and have a mass $m_{\pi\pi}$ in the range $[0.554, 0.996]$ GeV. This selection reduces combinatorial

What else can we do?

- Important background for leptonic B decays
- The exclusive modes we know only make up a third of all semileptonic $b \to u$ decays
- Higher multiplicities more prevalent
- Other interesting channels: $B \to K\bar{K}\ell\nu_\ell$, $B \to \eta\pi\ell\nu_\ell$
- Neutral current decays also interesting

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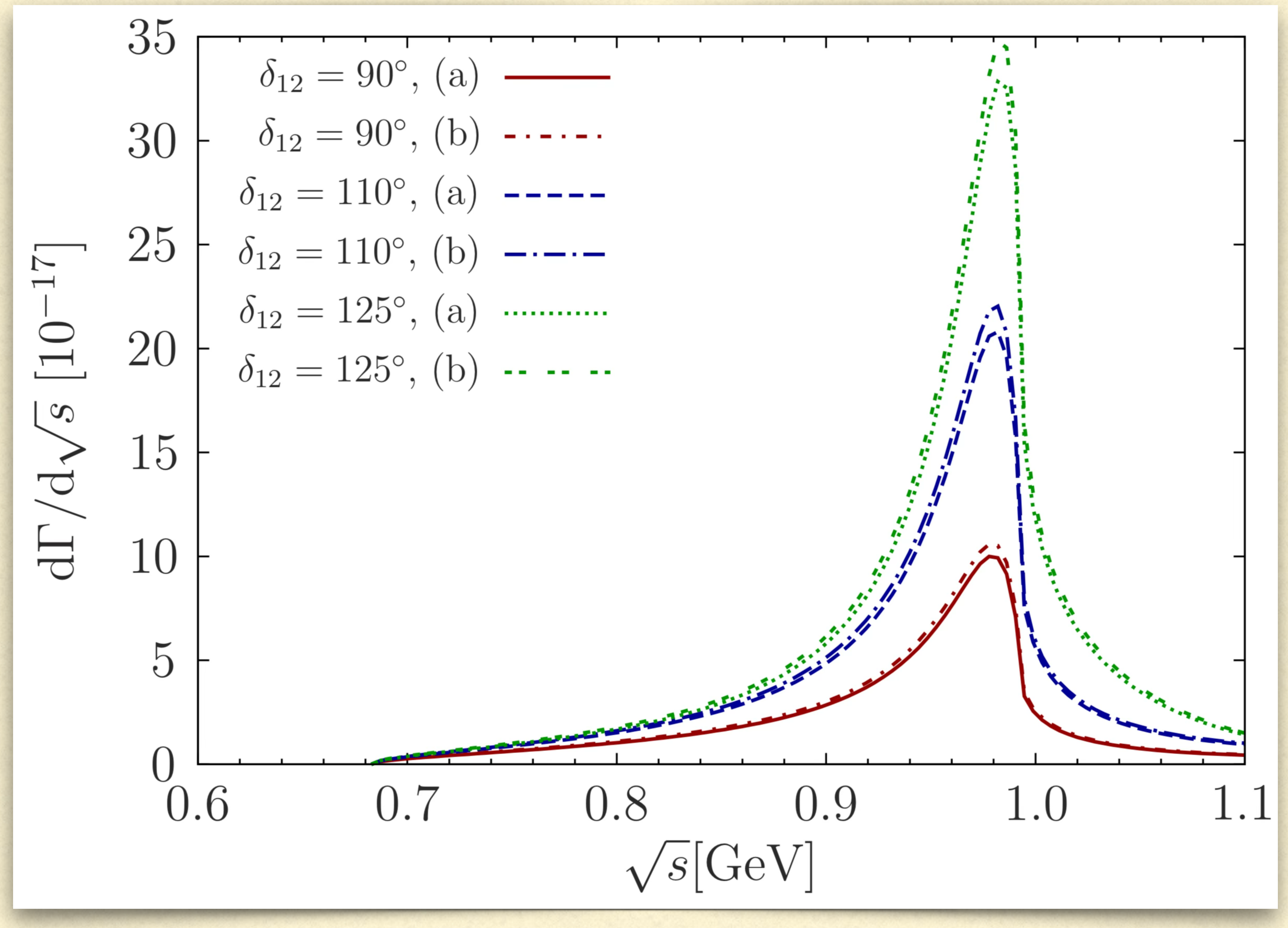

Taken from: Albaladejo, Daub, Hanhart, Kubis, Moussallam JHEP 04 (2017) 010

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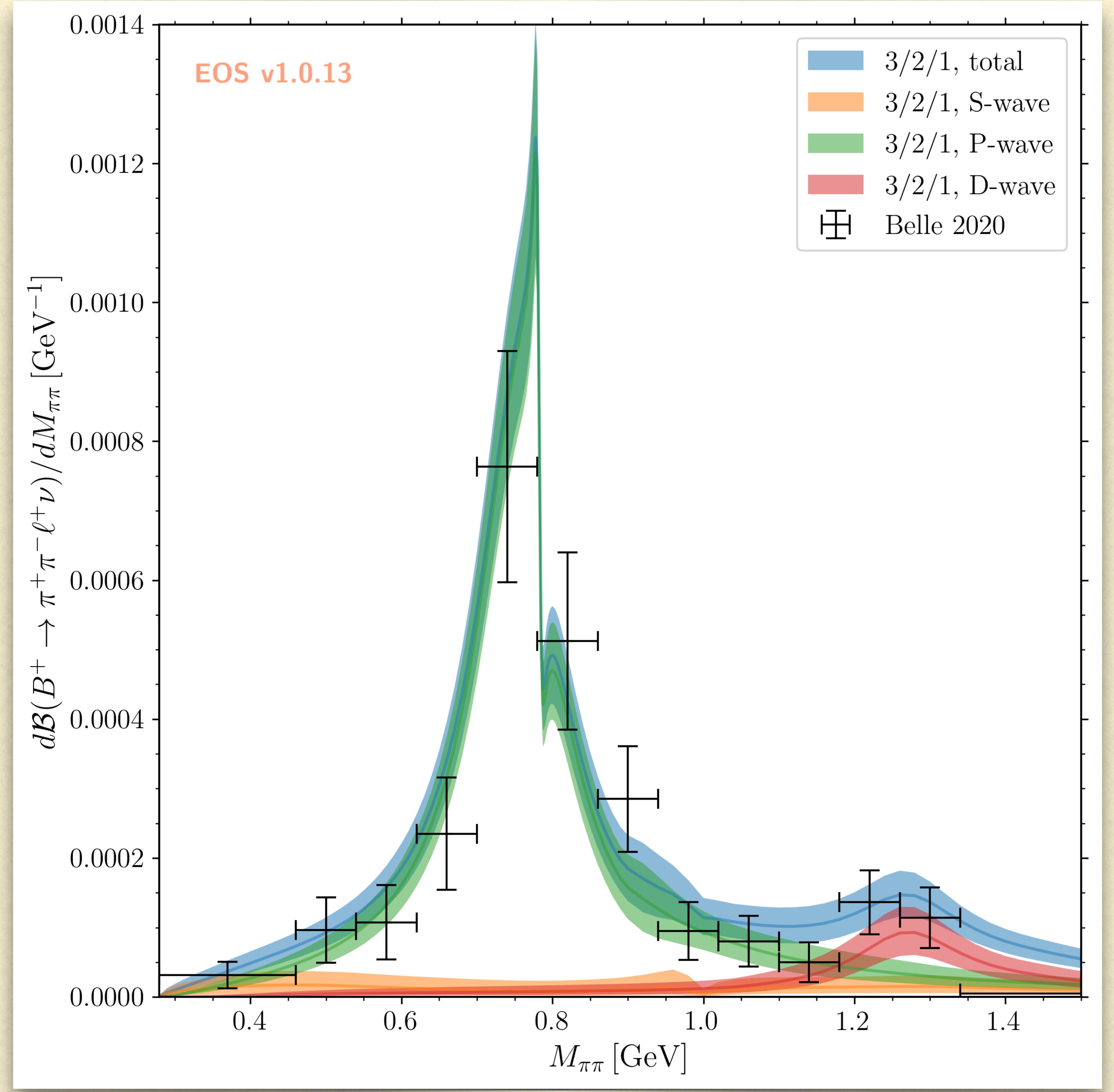
Conclusion & Outlook



- We developed a model-independent description of semileptonic decays into two hadrons
- Application to $B \to \pi\pi\ell\nu$ promising
- Realistic lineshapes help to disentangle different partial waves
- Nontrivial results on S- and D-wave
- Predictions for $B^+ \to \pi^0\pi^0\ell^+\nu_\ell$ decays
- Implementation in EOS

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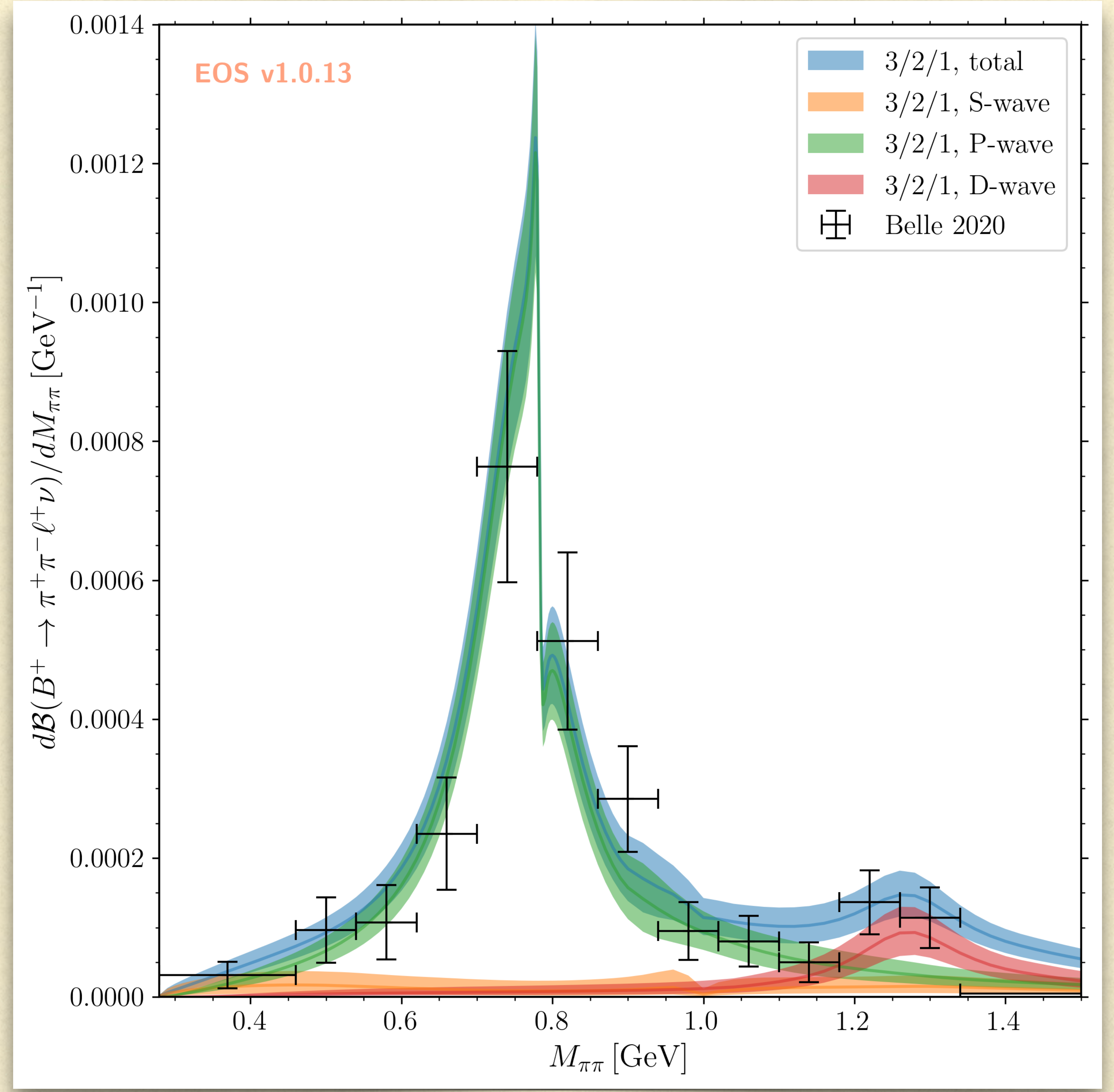

- LQCD calculations on the way, currently at unphysical pion masses: [PRL 134 (2025) 16, 161901]
- LCSR calculations can be done in a compatible manner (see Anshika's talk yesterday!)
- Please: never split into resonances and non-resonant, this is highly model-dependent and unnecessary
- We really need new measurements of $B^+ \to \pi^+\pi^-\ell^+\nu_\ell$, including angular information and improved modelling systematics!
- Other interesting channels: $B \to K\bar{K}\ell\nu_\ell, B \to \eta\pi\ell\nu_\ell$
- Let's bury the narrow-width approximation together!

Conclusion & Outlook

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Editors' Suggestion

Model-independent parametrization of $B \to \pi\pi\ell\nu$ decays

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More details in the paper