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Diego Guadagnoli
CNRS, LAPTh Annecy

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An alternative way to test the existing $b \to s\, \mu\mu$ tensions:

- *in a different q^2 region*

&

- *with a decay “contained” in the $B_s \to \mu^+ \mu^-$ dataset*

b → s data tensions

- *$b \to s\ \mu^+ \mu^-$ still discrepant w.r.t. SM in several independent channels*

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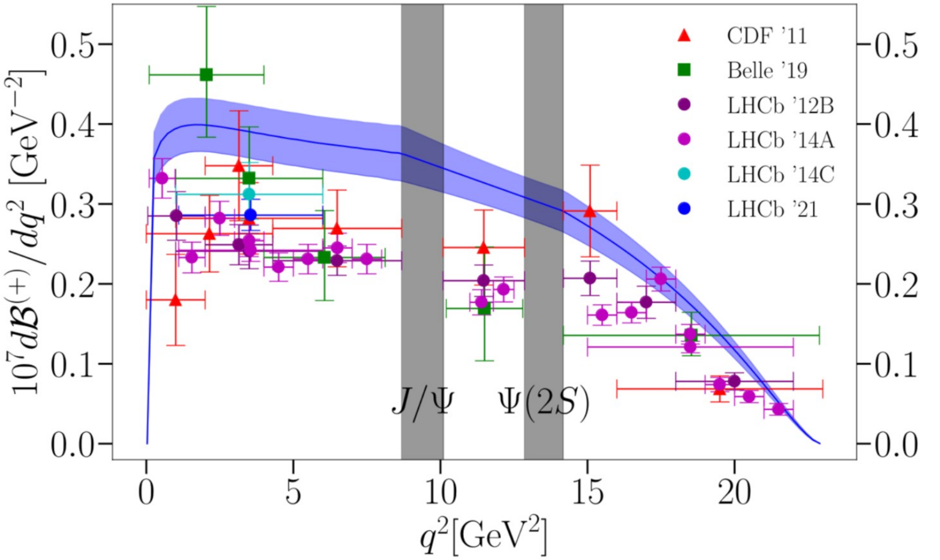

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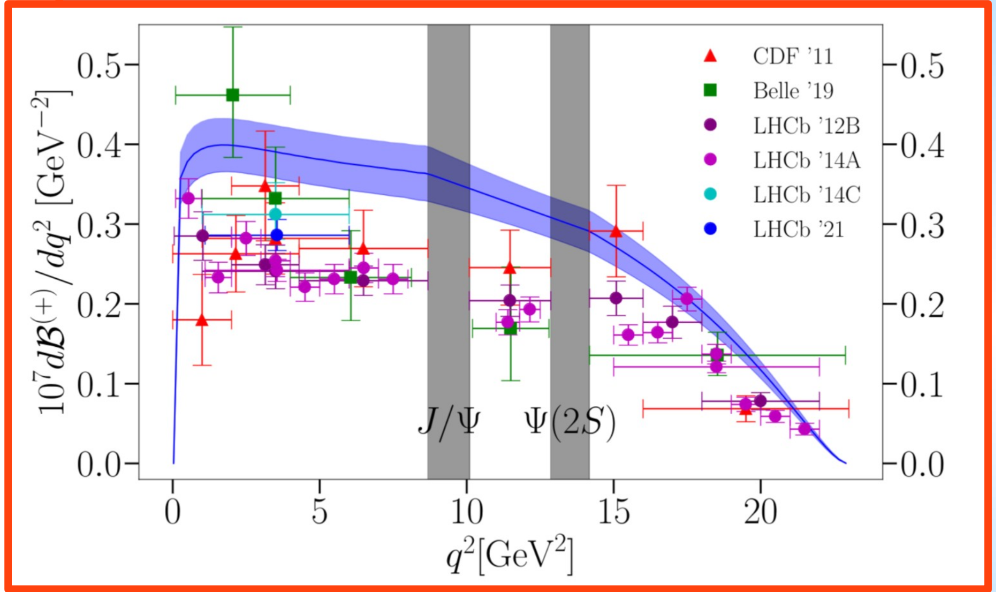
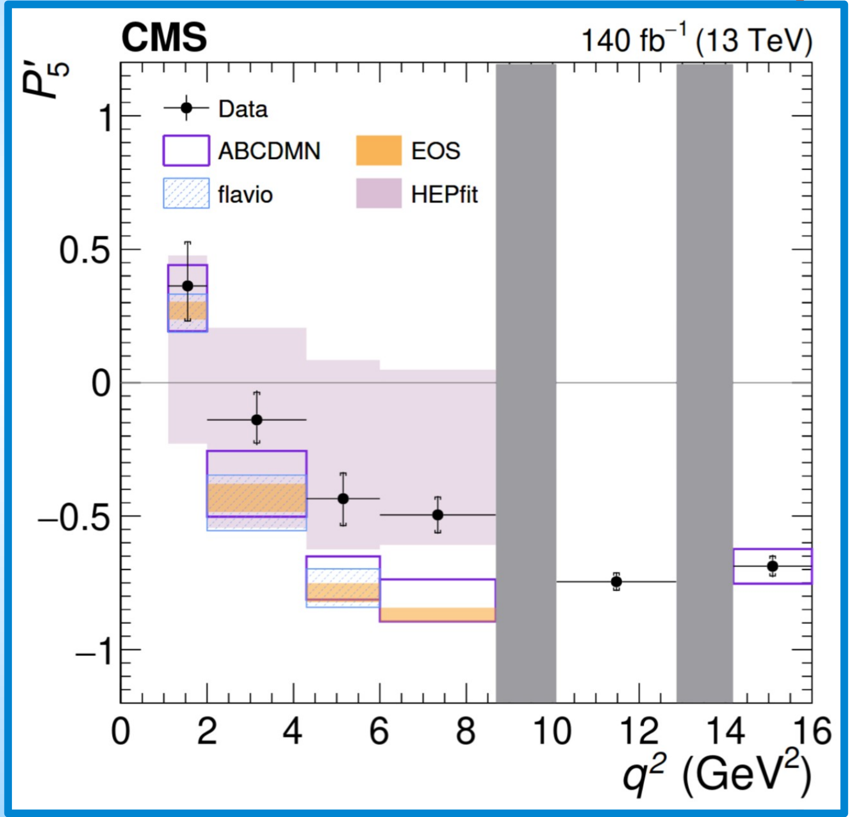


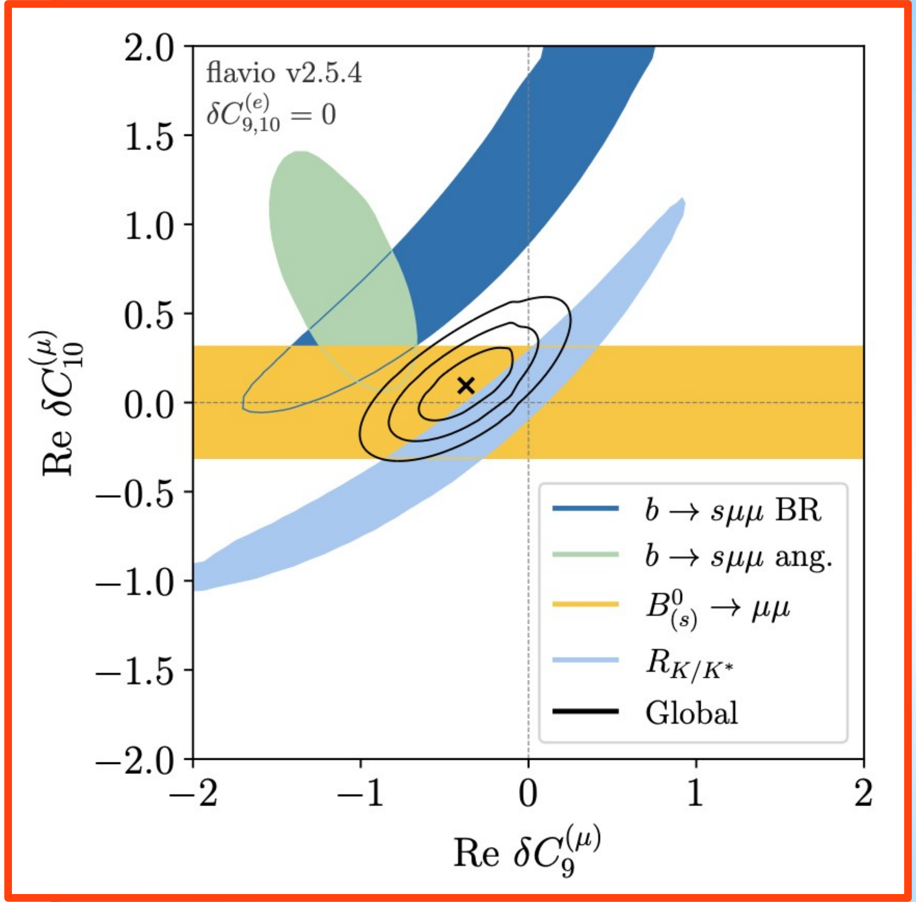
***Angular Obs.**: $P_5'(B \to K^*\mu\mu)$*



Main tension & possible solution

[DG, Normand, Simula, Vittorio, 2023]

$\delta C_{9\,(10)}$: vector (axial) leptonic current



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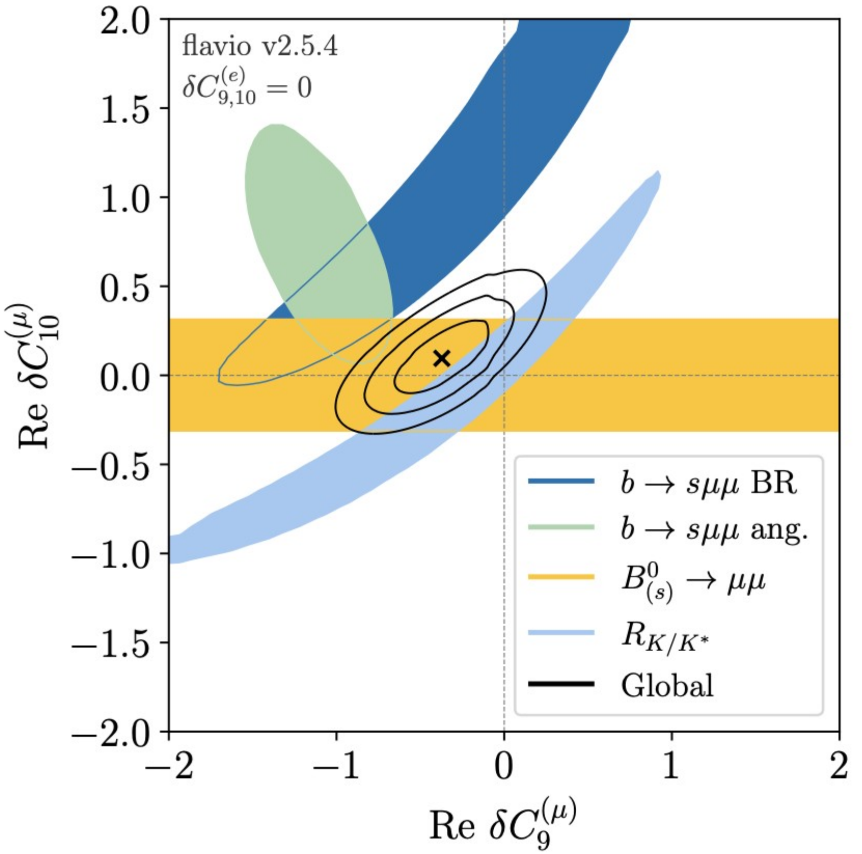

tension ⚡

- *$R_{K(*)}$ & $B_s \to \mu\mu$ SM-like*
- *$b \to s\,\mu\mu$ BR & ang. <u>not</u> SM-like*

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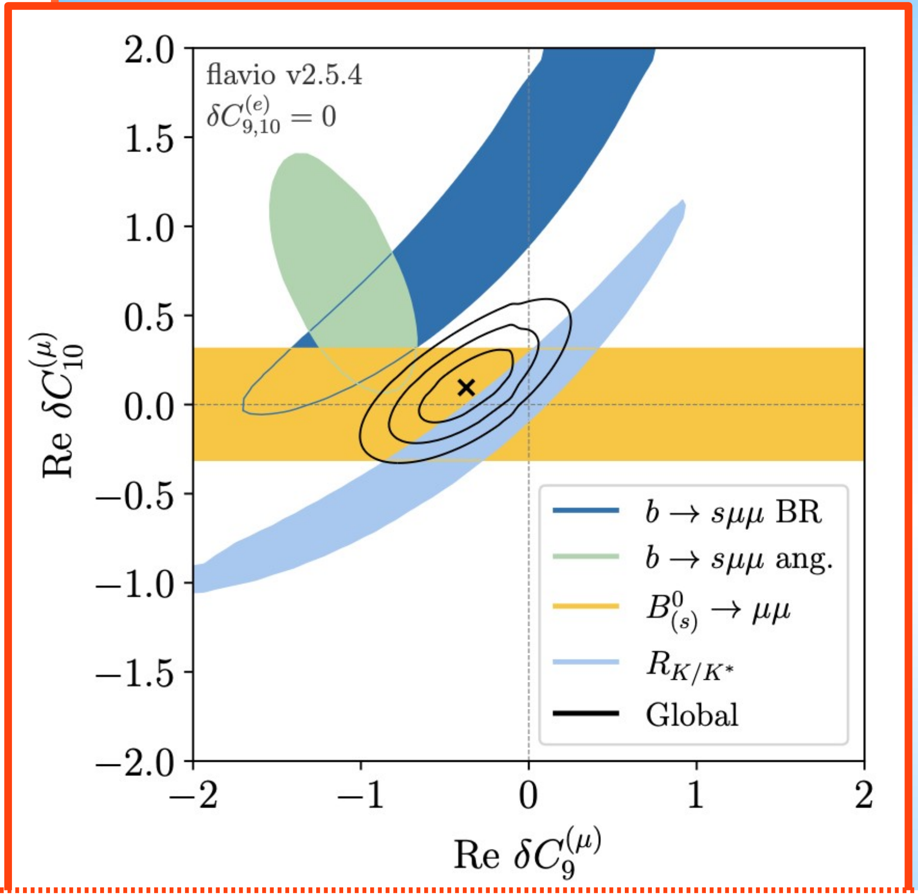

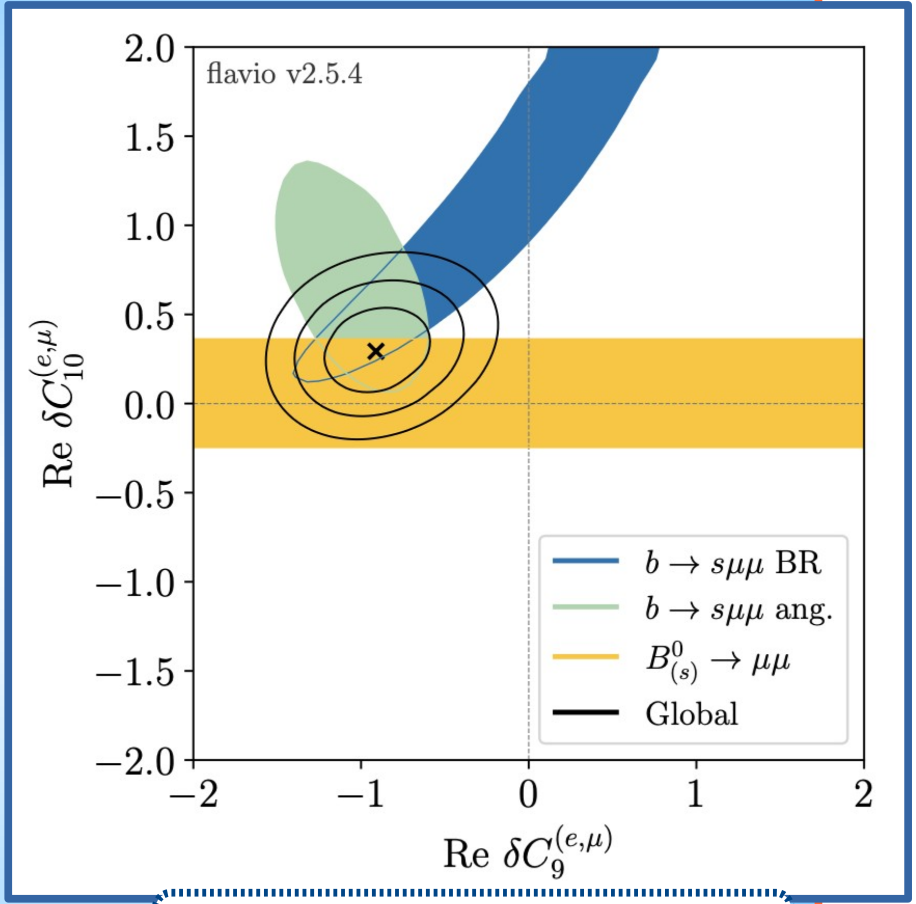

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Possible solution 1

Lepton-univ. shift to C_9

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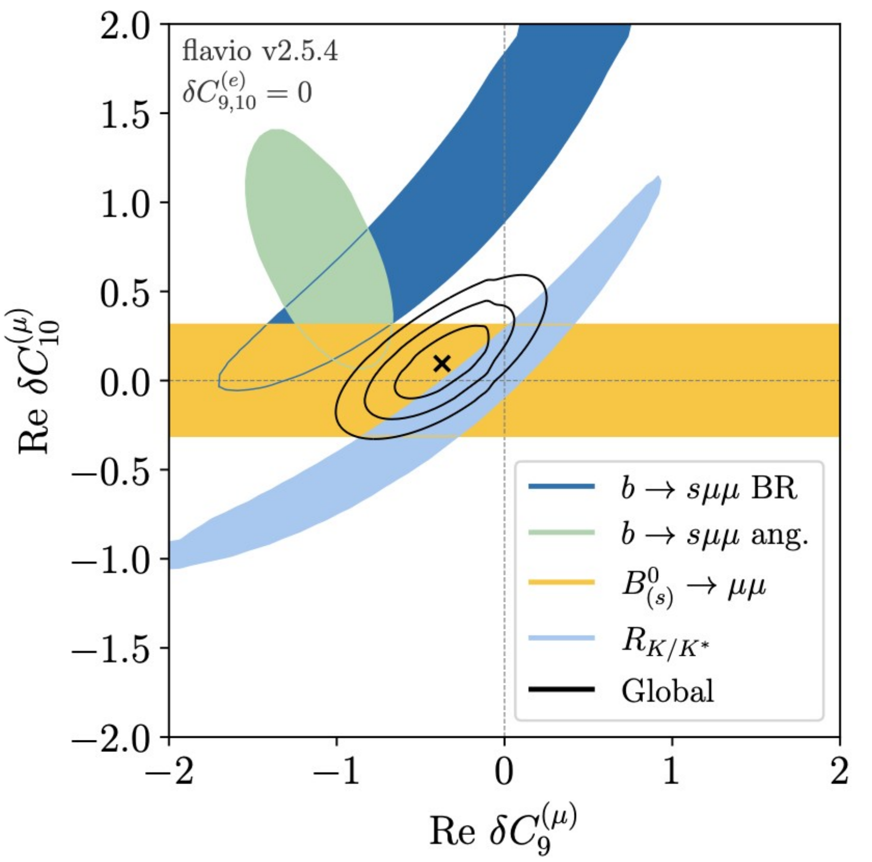
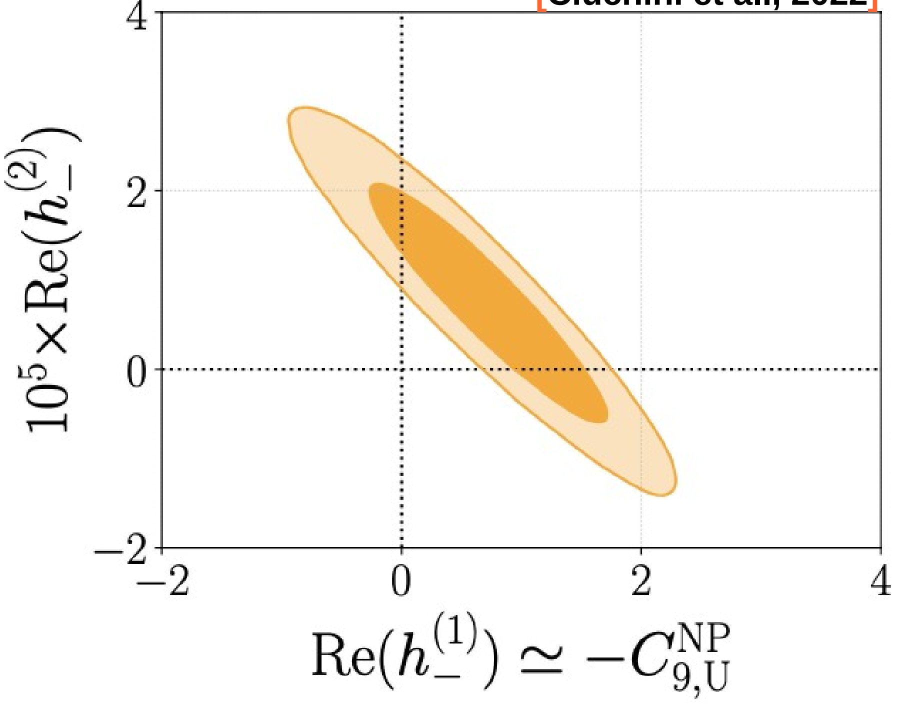


[Ciuchini et al., 2022]



$$H_V^- \propto \frac{m_B^2}{q^2}\left[\frac{2m_b}{m_B}\left(C_7^{\mathrm{SM}} + h_-^{(0)}\right)\widetilde{T}_{L-} - 16\pi^2 h_-^{(2)}\, q^4\right] + \left(C_9^{\mathrm{SM}} + h_-^{(1)}\right)\widetilde{V}_{L-}\,,$$

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Possible solution 2

Hadronic effects
difficult to assess by direct calculation

Motivation in short

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- *With Run 3 (⇨ comparable e and μ efficiencies), $B_s \to ee\,\,\gamma$ becoming realistic (with challenges)*

$B_s \to \mu\mu\,\gamma$ from $B_s \to \mu\mu$

$B_s \to \mu\mu\gamma$: "indirect" method

[Dettori, DG, Reboud, 2017]

Basic Idea *Extract* $B_s \to \mu\mu\gamma$ *from* $B_s \to \mu\mu$ *event sample, by enlarging* $m_{\mu\mu}$ *below* B_s *peak*

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- *… to access $B_s \to \mu\mu\gamma$, that probes any $\mu\mu$ "anomaly"*
  - *via a larger set of EFT couplings*
  - *in a different, not well tested, q^2 region*
  - *with a completely different exp approach*

Exp side

[thanks F. Dettori]

***Pros** (besides those already stated)*

- No need to reconstruct the γ (factor-of-20 loss in efficiency)

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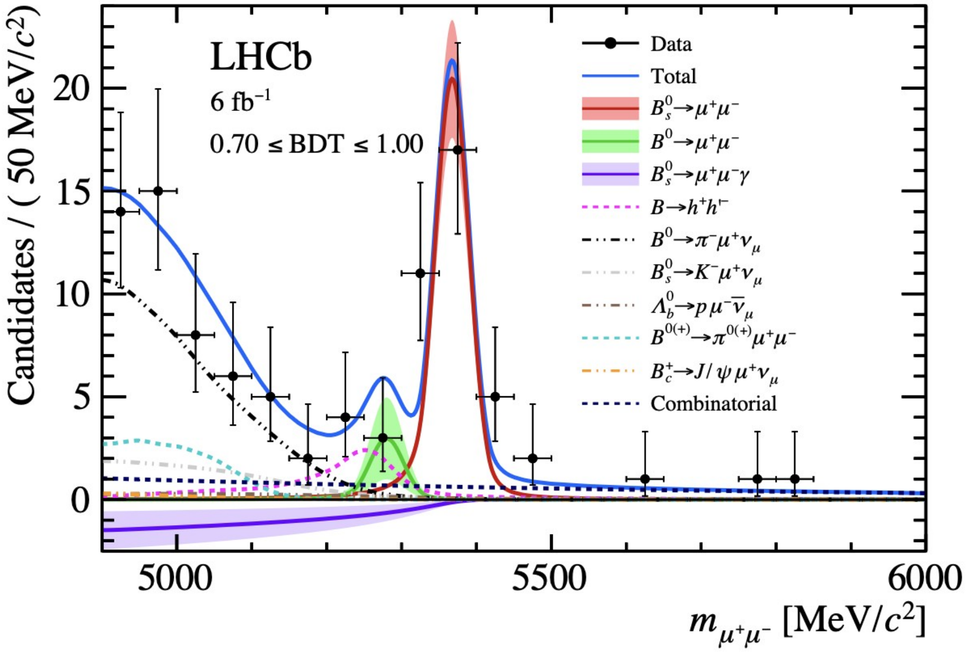
- Calibration not trivial – no "analogous" channel

Results

[thanks F. Dettori]



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$$\mathcal{B}(B^0 \to \mu^+\mu^-) = \left(1.2\,^{+\,0.8}_{-\,0.7}\pm 0.1\right)\times 10^{-10}$$

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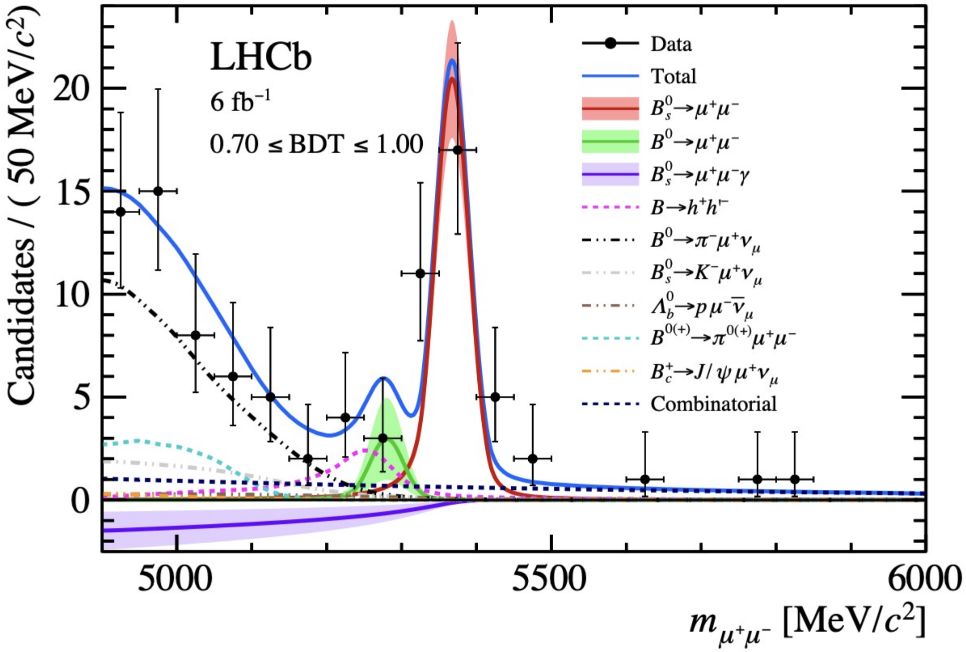

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First world limit (new PDG entry)

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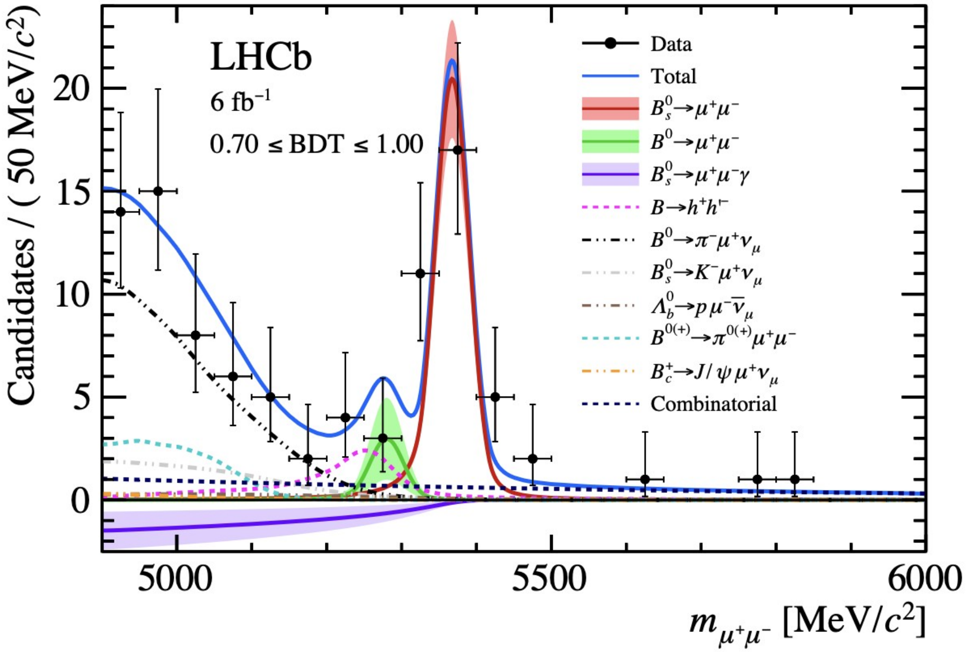

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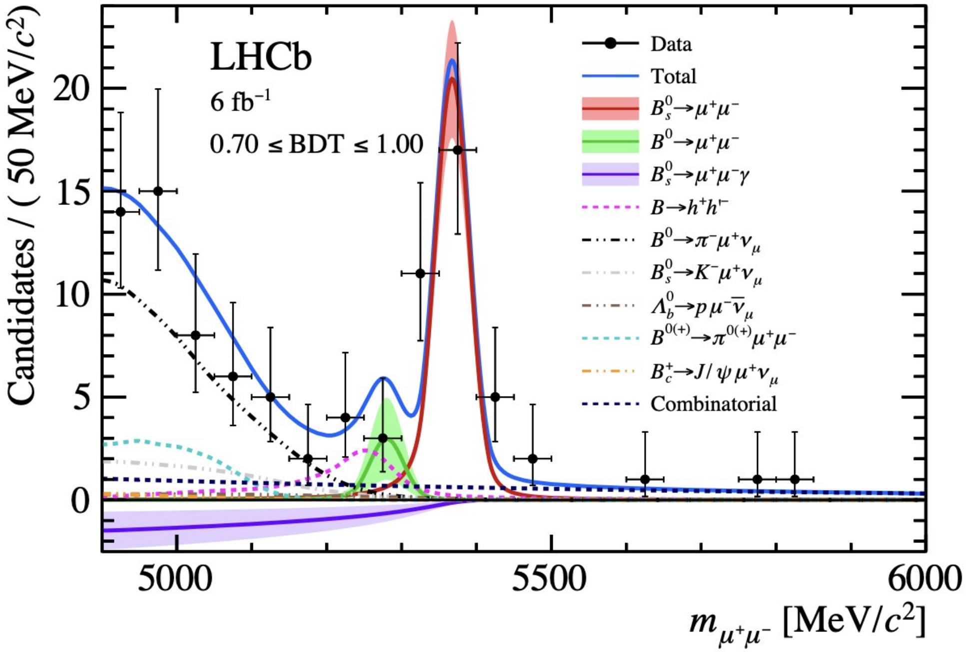

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- *Run 3 analyses will dramatically increase sensitivity*

  For ref: Run 3 data thus far = 20 fb = 2x Run 1 & 2 combined

The elephant in the room (FFs)

Radiative leptonic FFs in LQCD

Novel ideas & applications, both at low q^2 (large E_γ) and high q^2 (small E_γ)

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Total width w/ either 0 or 1 γ

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IR-safe ***IR-safe***

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Continuum, scalar-QED $O(\alpha)$ $\ell\ell'\gamma$ width

Requirement

E_γ^{max} small enough to justify scalar-QED approach in Γ_1

FFs at low q^2

within factorization

$B_s \to \mu\mu\gamma$ *with energetic* γ

[Beneke-Bobeth-Wang, *'20*]

- *For low $q^2 \leq 6$ GeV, $B_s \to \gamma^*$ f.f.'s can be calculated in a systematic expansion in $1/m_b$, $1/E_\gamma$*

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  - *non-local NLP*
    - ○ *actually dominant contribution by far*
    - ○ *escapes first-principle description*

similar to $B_u \to \ell\,\nu\,\gamma$

resonance paramet'n

Bottom line

[Beneke-Bobeth-Wang, *'20*]

- *Dominant parametric error,* $^{+70\%}_{-30\%}$ *, from* λ_B *(as expected)*

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[Beneke-Bobeth-Wang, '20]

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- *Prediction*

$$\langle \mathcal{B} \rangle_{[4m_\mu^2,\, 6.0]} = \left(12.51^{+3.83}_{-1.93}\right) \cdot 10^{-9}, \qquad \langle \mathcal{B} \rangle_{[2.0,\, 6.0]} = \left(0.30^{+0.25}_{-0.14}\right) \cdot 10^{-9}$$

i.e. ϕ region gives 97.6% of the BR

FFs within LCSRs

[Janowski, Pullin, Zwicky, *'21*]
see also [Pullin, Zwicky, *'21;* Albrecht *et al.*, *19*]

- *FFs fitted to a z-expansion ansatz*

$$F_n^{\bar{B}\to\gamma}(q^2) = \frac{1}{1 - q^2/m_R^2}\left(\alpha_{n0} + \sum_{k=1}^{N} \alpha_{nk}(z(q^2) - z(0))^k\right)$$

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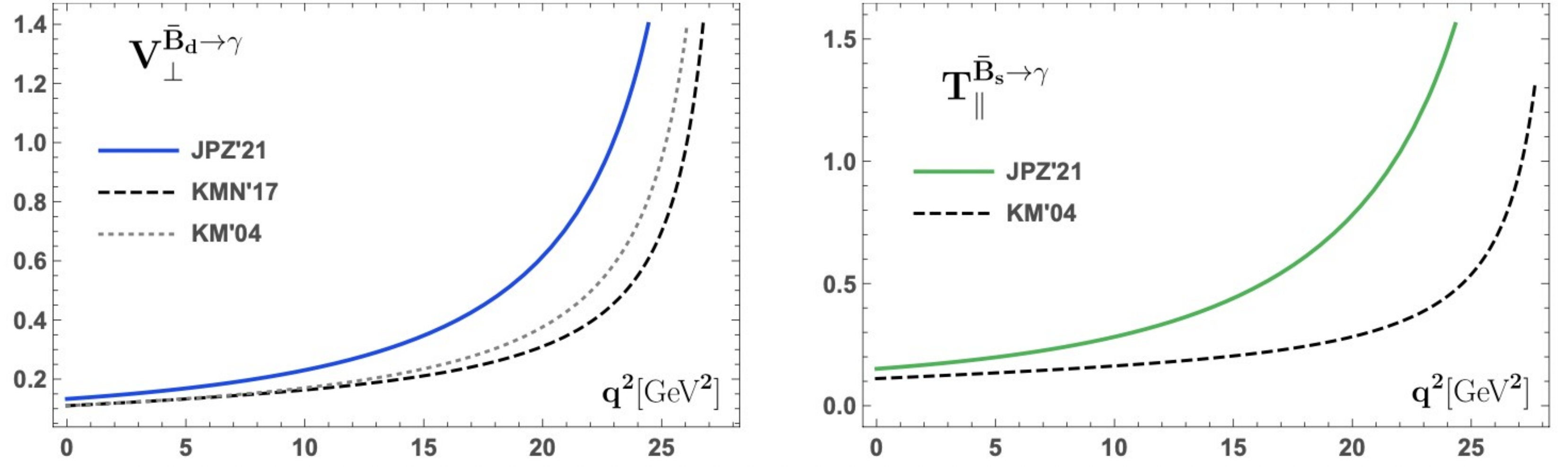
- *Comparison with the quark-model FF parameterizations in*

[Melikhov, Nikitin, ‘04; Kozachuk, Melikhov, Nitikin, ‘17]



FFs at high q^2

A phenomenological approach
using LQCD and heavy-quark symmetry

Our approach

[DG, Normand, Simula, Vittorio, *'23*]

① *Use available $D_s \to \gamma$ LQCD data*

(directly computed in very range of interest)

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[• *Validate as much as possible*]

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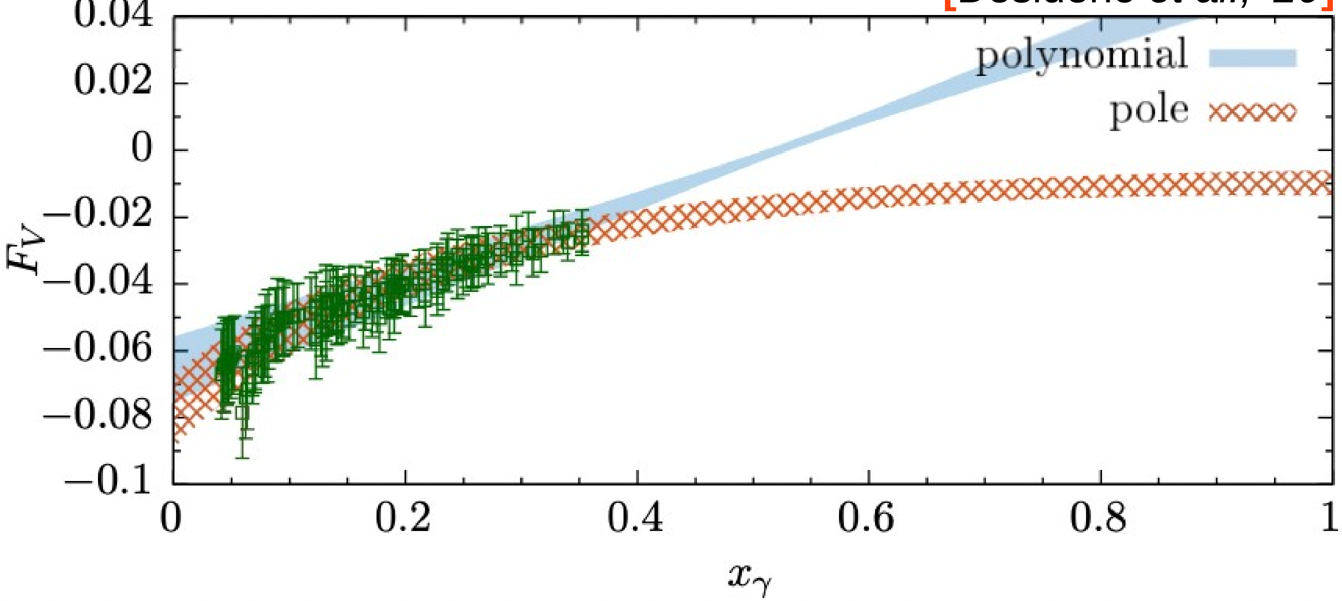
- High q^2 means low $x_\gamma \equiv 1 - q^2 / m^2_{Ds}$

$q^2 \in [4.2, 5.0]^2$ GeV^2 ⇔ $x_\gamma \in [0.39, 0.13]$

[Desiderio *et al.*, ‘20]



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High q^2 means small E_γ

⇨ The nearest vector (or axial) meson dominates

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⇨ One can thus relate the (fitted) residue to the (otherwise unknown) tri-coupling

$$r_\perp = \frac{m_{B_s} f_{B_s^*}}{m_{B_s^*}} g_{B_s^* B_s \gamma}$$

② *VMD: fit ansaetze*

FFs are described as a sum of poles + cuts

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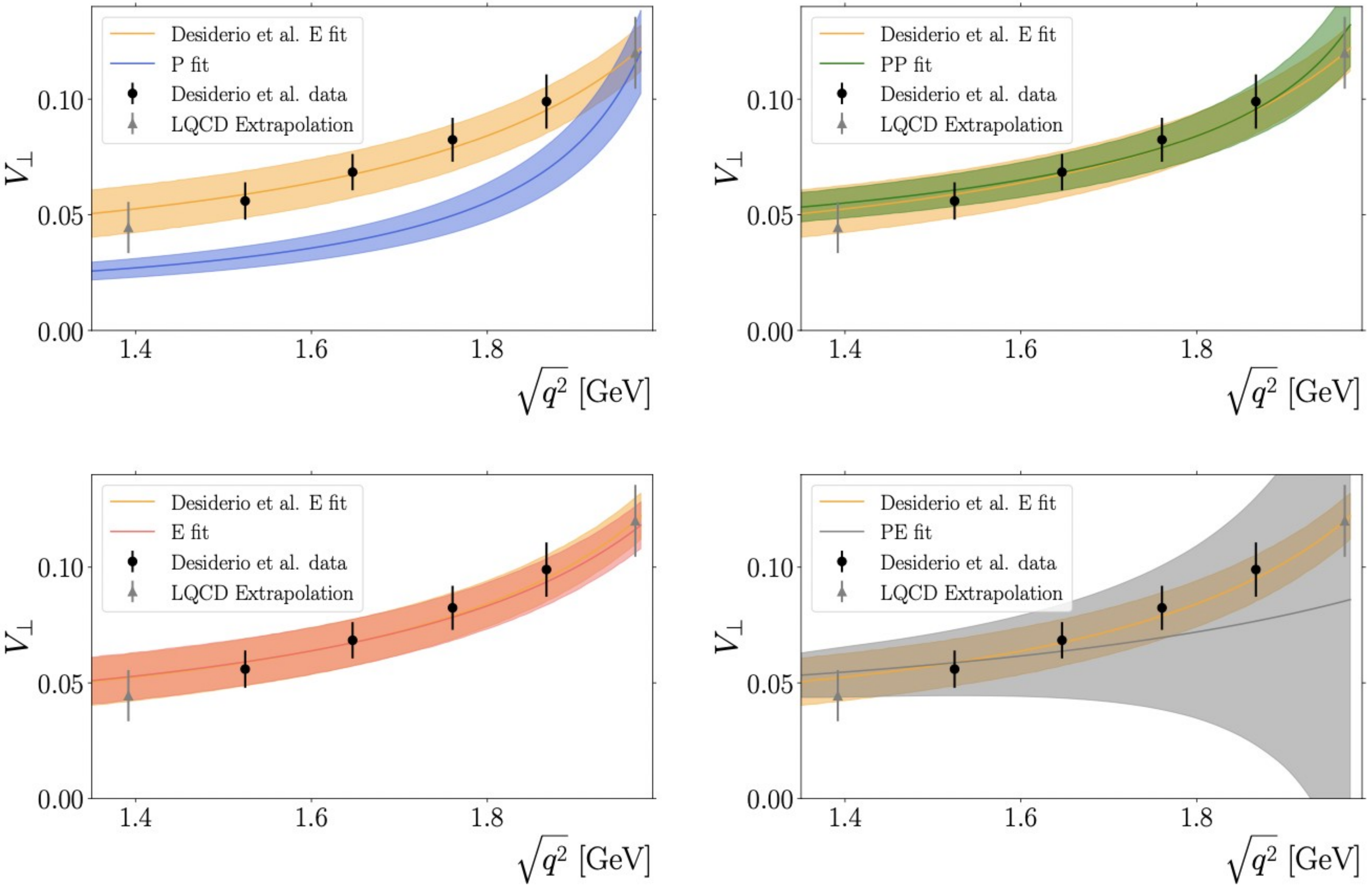
PE fit	One phys & one eff pole		

...

② VMD: the vector-FF example



③ *From the D_s to the B_s*

Basic idea:

$$\text{Tri-coupling} = \sum_{i\,=\,\text{valence quarks}} (\pm\ \text{e.m. charge})_i \times \underbrace{(\text{magn. moment})_i}_{\propto\ 1\,/\,m_i}$$

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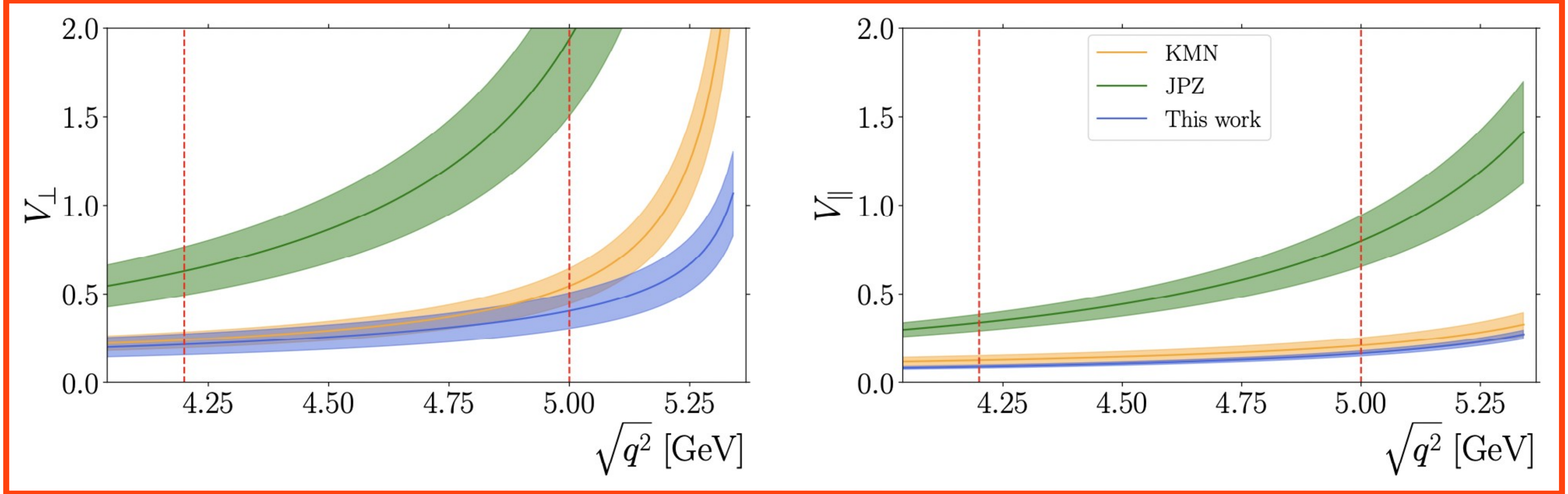
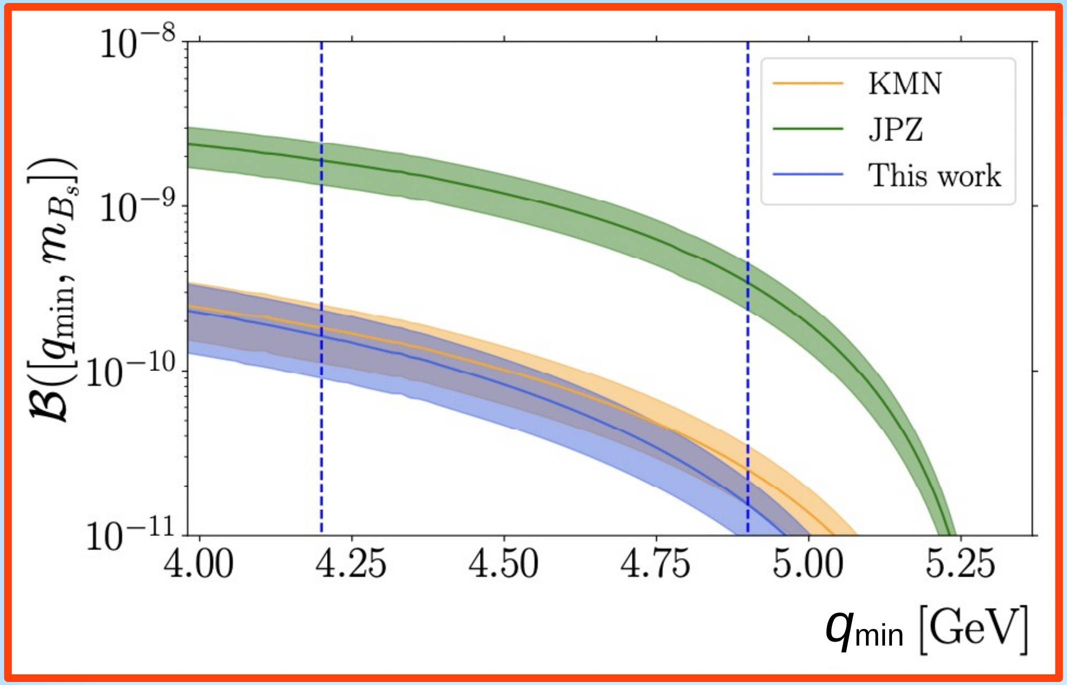


BR($B_s \to \mu^+ \mu^- \gamma$) prediction



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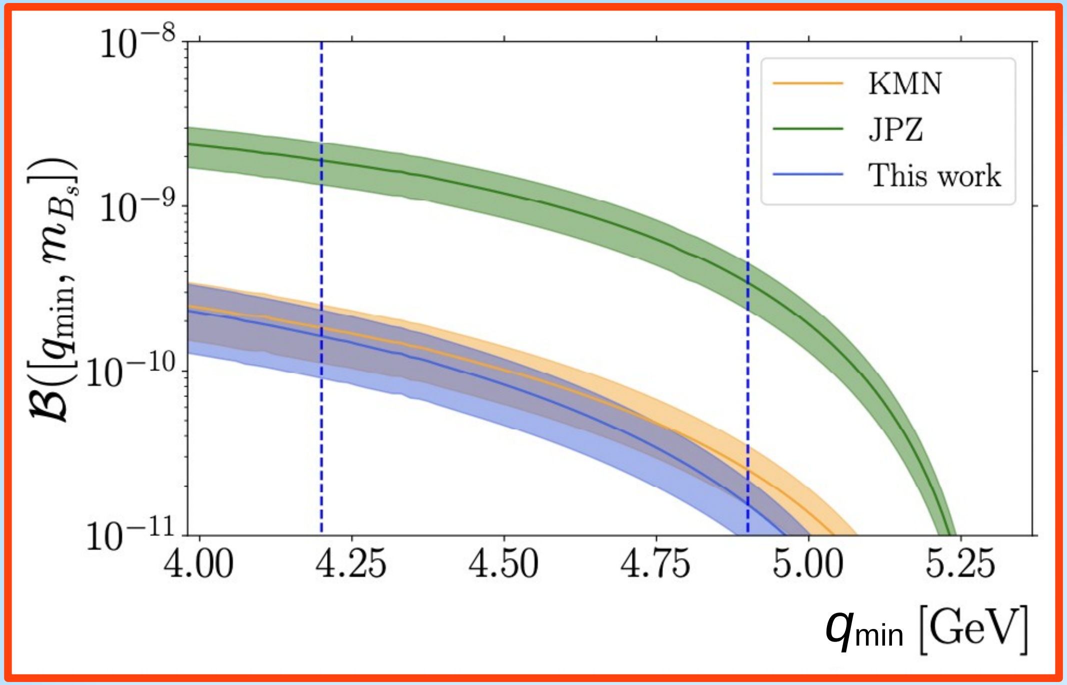

Below ~ 4.4 GeV there is broad-$c\bar{c}$ pollution

These contributions are incalculable from first principles

How large is their share of the total error?

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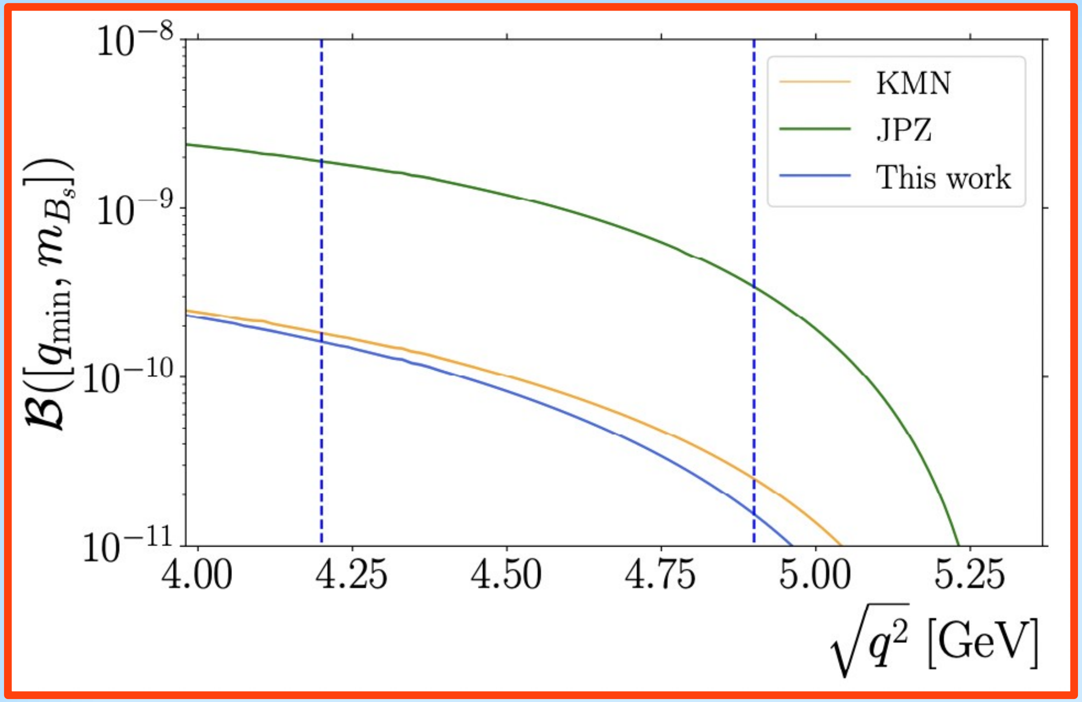

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Tiny!

- Low impact of broad $c\bar{c}$ encouraging, given that this systematics inherently escapes a rigorous description

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- f.f. uncertainty, even if still large, in principle “reducible”
- Maybe worthwhile to look for more observables with such properties

Example: the $B_s \to \mu\mu\gamma$ effective lifetime

[Carvunis et al., '21]

- *Natural exp observable: untagged rate*

$$\langle\Gamma(B_s(t) \to f)\rangle \;\equiv\; \Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f)$$

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Recalling the time dependence of the |amplitudes|^2

$$\begin{aligned} |\overset{(-)}{\mathcal{A}}_f(t)|^2 \;&=\; \frac{e^{-\Gamma_s t}}{2}\Big[\Big(|\mathcal{A}_f|^2 + |q/p|^2|\bar{\mathcal{A}}_f|^2\Big)\cosh(\Delta\Gamma_s\, t/2) \pm \Big(|\mathcal{A}_f|^2 - |q/p|^2|\bar{\mathcal{A}}_f|^2\Big)\cos(\Delta M_s\, t) \\ &-\; 2\,\mathrm{Re}\Big(q/p\,\bar{\mathcal{A}}_f\mathcal{A}_f^*\Big)\sinh(\Delta\Gamma_s\, t/2) \mp 2\,\mathrm{Im}\Big(q/p\,\bar{\mathcal{A}}_f\mathcal{A}_f^*\Big)\sin(\Delta M_s\, t) \end{aligned}$$

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- *$A_{\Delta\Gamma}$ can be extracted from (an accurate measurement of) the effective lifetime*

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[Carvunis et al., ‘21]

- $A_{\Delta\Gamma}$ looks like a natural “ratio-of-amplitudes-squared” observable

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- *NP with non-standard CPV less constrained than NP with CKM CPV*

  (For NP with non-standard CPV, also constraints on Re(WCs) get looser)

$A_{\Delta\Gamma}$ at high q^2

[Carvunis et al., '21]

- *Consider the range* $s \in [(4.1\ \text{GeV})^2,\ m_{Bs}^2] = [0.59, 1]\ m_{Bs}^2$

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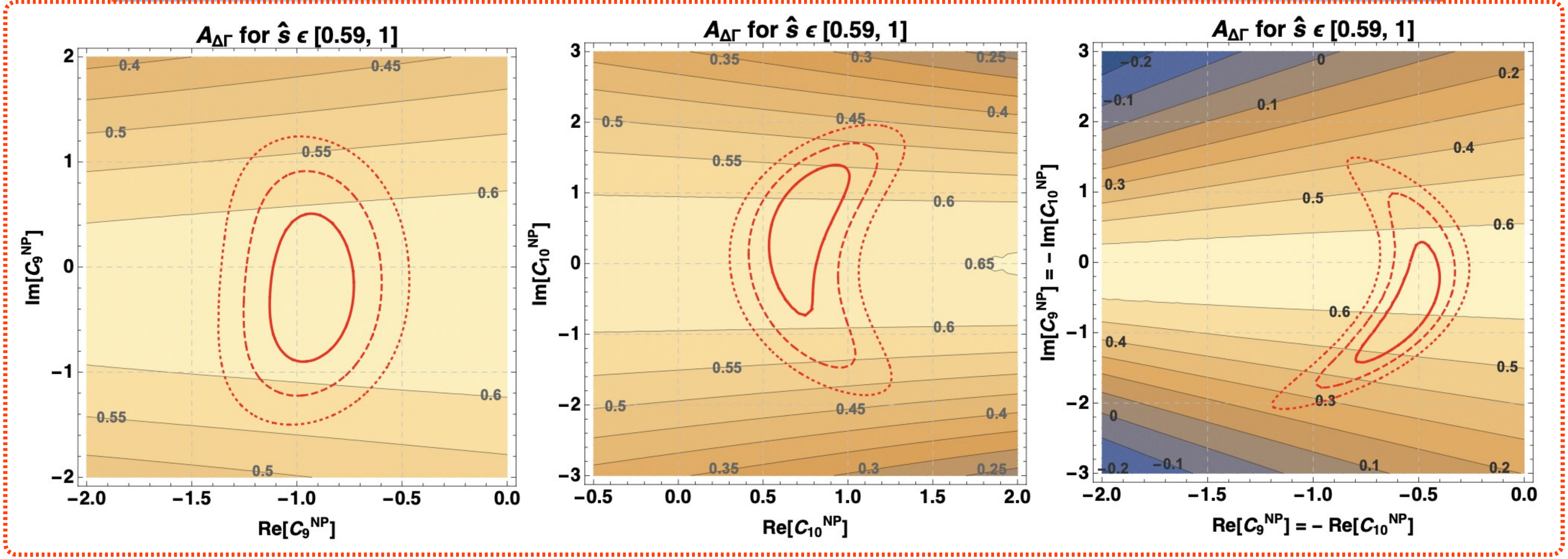

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- *Yields several observables – also in the ee channel*

Spares

Impact of broad $c\bar{c}$

[*Carvunis et al., '21*]

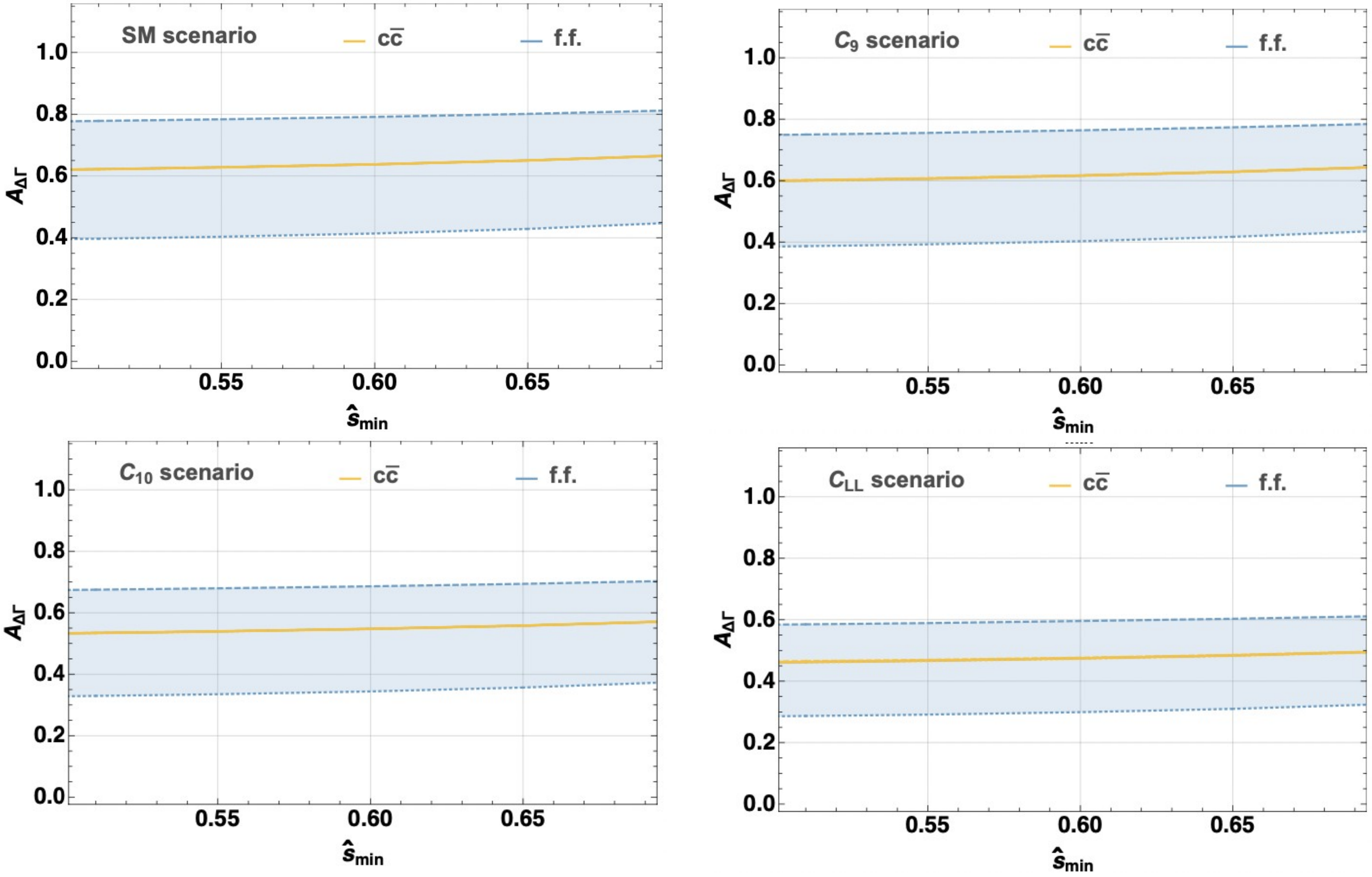
- Parameterize the effect most generally (e.g. discussion in [Lyon, Zwicky, '*14*])

$$C_9 \;\to\; C_9 \;-\; \frac{9\pi}{\alpha^2}\,\bar{C}\sum_V |\eta_V|e^{i\delta_V}\frac{\hat{m}_V\,\mathcal{B}(V\to\mu^+\mu^-)\,\hat{\Gamma}^V_{\text{tot}}}{\hat{q}^2-\hat{m}_V^2+i\hat{m}_V\hat{\Gamma}^V_{\text{tot}}}$$

  - $|\eta_V| \in [1, 3]$ & $\delta_V \in [0, 2\pi)$ (uniformly and independently for the 5 resonances)
  - for $s_{min} \in [0.5, 0.7]\ m_{Bs}^2$
  - for all TH scenarios

$s_{\psi(2S),\ \psi(3770),\ \psi(4040),\ \psi(4160),\ \psi(4415)} = \{0.47, 0.49, 0.57, 0.61, 0.68\}$

Impact of broad $c\bar{c}$



Impact of broad $c\bar{c}$

[Carvunis et al., ‘21]

- Bottom line: broad $c\bar{c}$ has surprisingly small impact on $A_{\Delta\Gamma}$

  But broad-$c\bar{c}$ shift to C_9 typically O(5%) – and with random phase

  ⮡ Far from obvious why such a small impact on $A_{\Delta\Gamma}$

- Closer look (App. D for an analytic understanding)

  Cancellation is a conspiracy between

  - Complete dominance of contributions quadratic in C_9 and C_{10}
  - Multiplying f.f.’s $F_V, F_A \in \mathbb{R}$
  - Broad $c\bar{c}$ can be treated as small modif. of (numerically large) C_9

  ⮡ Ease cancellations between num & den in $A_{\Delta\Gamma}$

Radiative leptonic FFs in LQCD

Large E_γ

- *The required correlator (weak & e.m. current insertion between a B and the vac) has always the desired large-Euclidean-t behavior*

  [Kane, Lehner, Meinel, Soni, *'19*]

  Note that this is non-trivial – e.g. it doesn't seem to hold if there are hadronic final states

- *However, the low-q^2 spectrum is dominated by resonant contributions (~98% of the BR), that LQCD is unable to capture*

Amplitude structure

[Beneke-Bobeth-Wang, *'20*]

- *Take the weak operators as* $O_i \equiv J_i^{(l)} \cdot J_i^{(q)}$

  and i = 9,10 for definiteness (and simplicity)

$$\overline{A} \;\propto\; \epsilon^*_\mu \left\{ \sum_i C_i \left[T_i^{\mu\nu} \; \langle \ell\bar{\ell} | \; J^{(l)}_{i\,\nu}(0) \; |0\rangle \right.\right.$$
$$\left.\left. + S_\nu^{(i)} \; \mathrm{FT}_x \langle \ell\bar{\ell} | \; T\{J^\mu_{\mathrm{em}}(x), J_i^{(l)\,\nu}(0)\}|0\rangle \right]\right\}$$

FSR: only $S_\nu^{(10)} \neq 0$ $(\propto m_\ell)$ ⇨ *tiny*

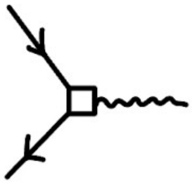
↳ *Main object to calculate*

$$T_i^{\mu\nu} \;\propto\; \mathrm{FT}_x \langle 0| \; T\{J^\mu_{\mathrm{em}}(x), J_i^{(q)\,\nu}(0)\}|B\rangle$$

Notes on structure

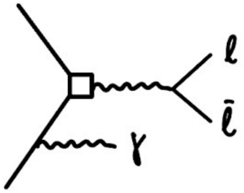
[Beneke-Bobeth-Wang, *'20*]

- *O_7* :



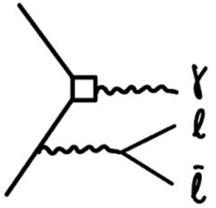
⇒

$T_{7A}^{\mu\nu}$:



but also

$T_{7B}^{\mu\nu}$:



- $$T_i^{\mu\nu} = T_i^{\mu\nu}(k,q) \propto (g^{\mu\nu}k\cdot q - q^\mu k^\nu)\overbrace{(F_L^{(i)} - F_R^{(i)})}^{=\,F_A^{(i)}} + i\varepsilon^{\mu\nu qk}\underbrace{(F_L^{(i)} + F_R^{(i)})}_{=\,F_V^{(i)}}$$

- For $E_\gamma \gg \Lambda_{\mathrm{QCD}}$ $\quad F_R^{(i)} \sim \frac{\Lambda_{\mathrm{QCD}}}{E_\gamma} F_L^{(i)}$ ⇒ $F_A^{(i)} \approx F_V^{(i)}$

Two-step matching onto SCET

[Beneke-Bobeth-Wang, *'20*]

- *Decoupling of h modes $O(m_b^2)$ in QCD → SCET_I matching*

$$\sum_i^9 \eta_i C_i\, T_i^{\mu\nu} = \sum_i^9 C_i H_i(q^2) \cdot$$

$$\cdot\, \mathrm{FT}_x \langle 0|\, T\{J^\mu_{\mathrm{em,SCET_I}} \underbrace{(x),\, [\bar q_{\mathrm{hc}} \gamma_L^{\nu\perp} h_v](0)}\}|B\rangle$$

separation $x \sim 1/\sqrt{E_\gamma \Lambda_{\mathrm{QCD}}}$

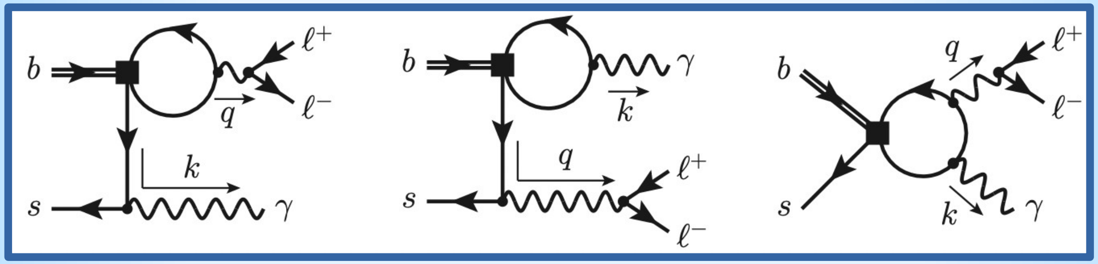
i.e. intermediate propagator is hc

- *Decoupling of hc modes $O(E_\gamma\, \Lambda_{QCD}\, ;\, m_b\, \Lambda_{QCD})$ in SCET_I → SCET_{II}*

NLP

[Beneke-Bobeth-Wang, *‘20*]

- *Three sources*
  - *coupling of γ to b quark*
  - *power corr’s to SCET$_I$ correlator at tree level*
  - *annihilation-type insertions of 4q operators* ⇨ *local*



- *Two soft FFs*
  - $\xi(E_\gamma)$: *computable as in* $B_u \to \ell\, \nu\, \gamma$ [Beneke-Rohrwild, *‘11*]
  - *For B-type contributions:* $\tilde{\xi}(E_\gamma)$

    Its Im develops resonances, thus escaping a factorization description

Resonances

[Beneke-Bobeth-Wang, *'20*]

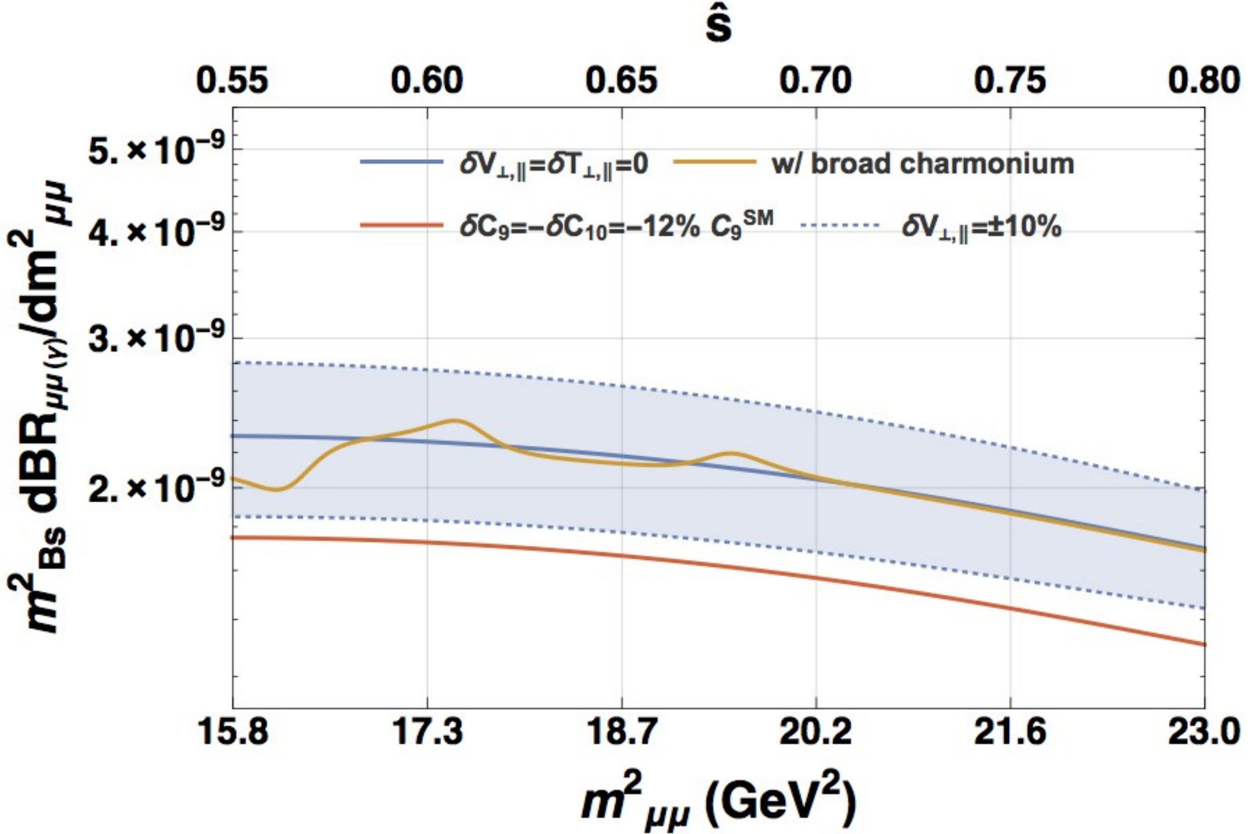
- $T_{7B}^{\mu\nu}$ *leads to* $\bar{A}_{res}$
  - *standard spectral repr. (à la BW)*
  - *formally power-suppressed*

    hence inclusion won't lead to double counting
    of some short-distance contributions

$B_s \to \mu\mu\gamma$ spectrum

- In [DG, Reboud, Zwicky, ‘17] resonant ansatz used to rewrite low-q^2 BR in terms of the measured BR($B_s \to \phi\gamma$)

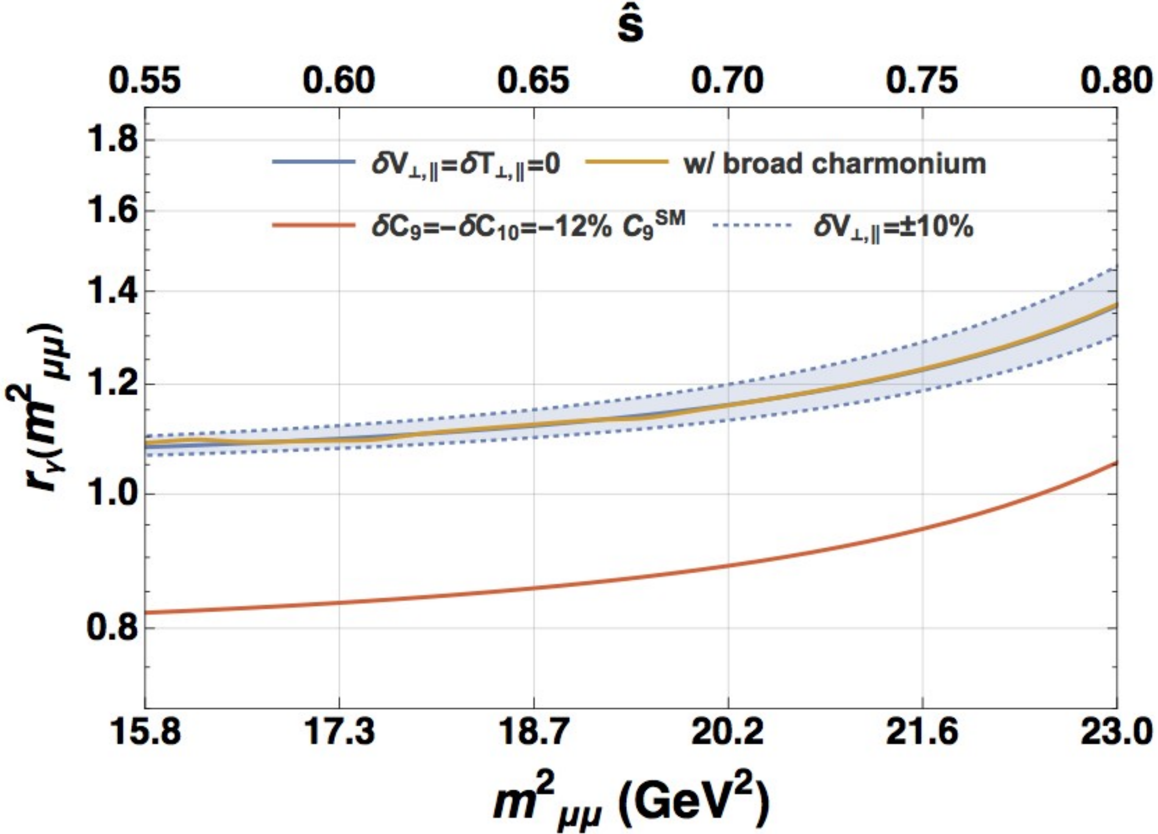
- Then main focus on large-q^2 region, above narrow charmonium. Broad-charmonium pollution estimated with similar resonant ansatz



$B_s \to \mu\mu\gamma$ spectrum

- *In [DG, Reboud, Zwicky, '17] resonant ansatz used to rewrite low-q^2 BR in terms of the measured BR($B_s \to \phi\gamma$)*

- *Then main focus on large-q^2 region, above narrow charmonium. Pollution substantially tamed in suitable ratio observable*



$$r_\gamma \equiv \frac{d\,BR(B_s \to \mu\mu\gamma)/dq^2}{d\,BR(B_s \to e\,e\,\gamma)/dq^2}$$