QED Effects in Exclusive B Decays

Philipp Böer

based on arXiv: 2008.10615, 2107.03819, 2108.05589, 2204.09091
(with M. Beneke, G. Finauri, J. Toelstede and K. Vos)

review article arXiv: 2312.12885 (with T. Feldmann)

Taming hadronic uncertainties in and beyond the SM
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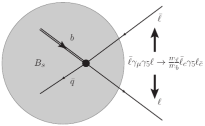
Outline

1. Introduction
2. QED Factorization in non-leptonic B decays
3. Leptonic and semi-leptonic decays
4. Conclusions

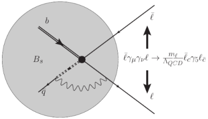
Introduction

Motivation

- Precision: Traditionally focus on hadronic uncertainties. Time to look at QED.
  - $\rightarrow$ QED effects can cause large logarithms $\ln m_\ell, \ln m_\pi, \ldots$ and $\ln \Delta E$
- Qualitatively new effects
  - $\rightarrow$ power-enhancement in $B_s \rightarrow \mu^+\mu^-$
  - $\rightarrow$ violation of isospin symmetry ($Q_u \neq Q_d$)
  - $\rightarrow$ requires careful definition of an observable (theory vs. experiment)
  - $\rightarrow$ . . .
- Photons couple weakly to strongly interacting quarks
  - $\rightarrow$ probe of hadronic physics, requires factorization theorems
- Theoretically interesting: Photons have long-range interactions with charged particles in the initial/final state
  - $\rightarrow$ QED factorization is more complicated than QCDF



$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle$



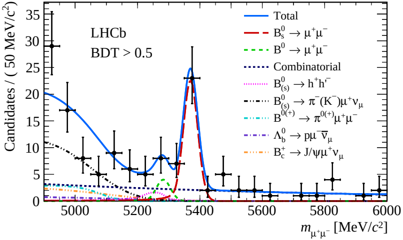
$\langle 0| \int d^4x \, T\{j_{\rm QED}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$

QED effects in B decays

IR finite observable:

- $\rightarrow$ must include ultrasoft photon radiation
- $\rightarrow$ soft-photon inclusive width

$$\Gamma(\Delta E) \equiv \Gamma[\bar{B} \to M_1 M_2 + X_s]\big|_{E_{X_s} \leq \Delta E}$$



[LHCb, $B_s \to \mu^+\mu^-$, 1703.05747]

factorizes in non-radiative amplitude and ultrasoft function for $\Delta E \ll m_M \sim \Lambda_{\rm QCD}$ (electrons ↯)

$$\Gamma(\Delta E) = |\mathcal{A}(\bar{B} \to M_1 M_2)|^2 \times \sum_{X_s} |\langle X_s|(\bar{S}_v^{(Q_B)} S_{v_1}^{\dagger(Q_1)} S_{v_2}^{\dagger(Q_2)})|0\rangle|^2 \, \theta(\Delta E - E_{X_s})$$

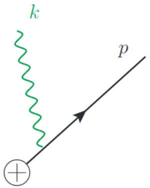
Simple classification:

- ultrasoft photons with energy $\ll \Lambda_{\rm QCD}$ see pointlike mesons (“universal”)
- (virtual) photons with energy $\gtrsim \Lambda_{\rm QCD}$ probe partonic sub-structure (structure dependent)

Ultrasoft Photons

- Eikonal approximation for point-like coupling

$$\epsilon_\mu(k)\bar{u}(p)\gamma^\mu \frac{\not{p}+\not{k}+m}{(k+p)^2-m^2} \to \epsilon_\mu(k)\frac{p^\mu}{p\cdot k}\bar{u}(p)$$



- Double-log's in ultrasoft corrections exponentiate and dress the non-radiative amplitude

$$\sum_{X_s} |\langle X_s|(\bar{S}_v^{(Q_B)} S_{v_1}^{\dagger(Q_1)} S_{v_2}^{\dagger(Q_2)})|0\rangle|^2 \sim \left(\frac{\Delta E}{\Lambda}\right)^{A(\alpha\to\beta)}$$

- **What is the cut-off Λ?**
  - $\to$ traditional treatment: pointlike coupling up to scales $\Lambda = m_B$ e.g. [Isidori et al.]
  - $\to$ However: theory requires $\Lambda \ll \Lambda_{\rm QCD}$

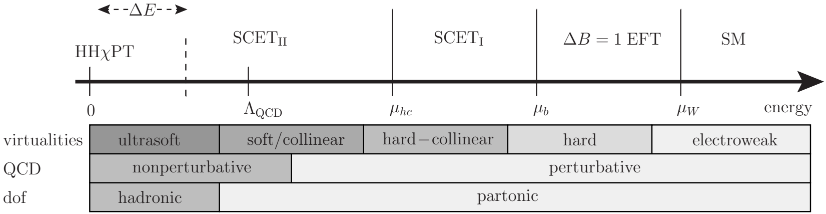
Photons with energy $\gtrsim \Lambda_{\rm QCD}$ probe the partonic structure of the mesons!

(Pointlike coupling requires wavelenght $\ll$ typical size of the meson $\sim 1/\Lambda_{\rm QCD}$)

Scales and EFTs

Hierarchy of energy scales:

$M_W \sim 80\,\text{GeV} \gg m_b \sim 4.2\,\text{GeV} \gg \text{few times}\ \Lambda_{\text{QCD}} \gg \Delta E \sim 60\,\text{MeV}$



(figure from [Beneke, Bobeth, Szafron '19])

- ✓ short-distance QED at $\mu \gtrsim m_b \rightarrow$ Wilson coefficients of weak eff. Lagrangian
- ✓ Far IR (ultrasoft) region $\mu_{\text{us}} \ll \Lambda_{\text{QCD}}$ described by point-like hadrons

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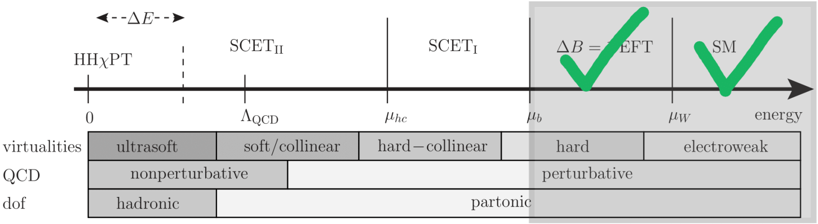

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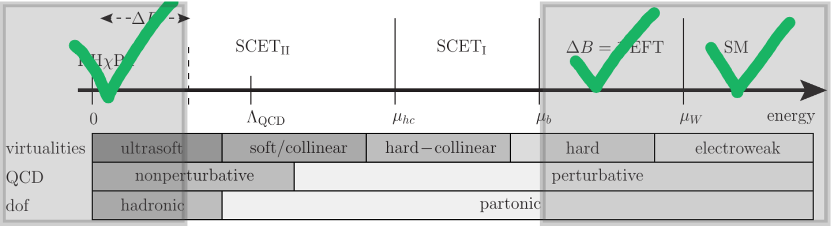

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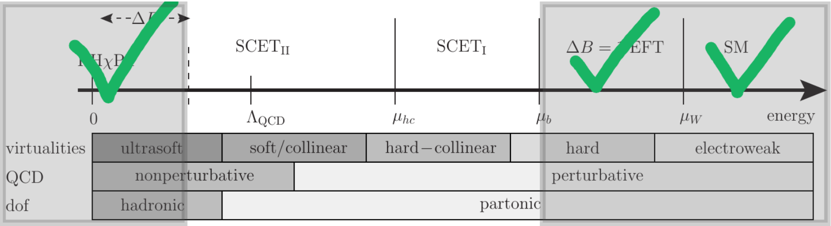

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Goal: theory for QED corrections between m_b and $\Lambda_{\rm QCD}$ ("structure dependent effects")

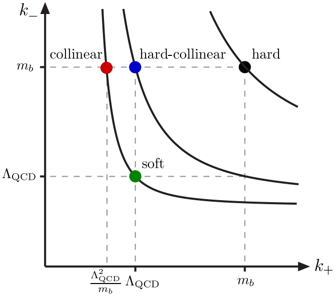
$\to$ Soft-collinear effective field theory (SCET) for light and energetic particles

Soft-Collinear Effective Theory

- Soft-Collinear Effective Theory (SCET) is designed to describe the long-distance physics in processes with energetic particles (jets)

$$h \sim (1,1,1)m_b$$
$$hc \sim (1,\sqrt{\lambda},\lambda)m_b$$
$$c \sim (1,\lambda,\lambda^2)m_b$$
$$s \sim (\lambda,\lambda,\lambda)m_b$$
$$(\lambda = \Lambda/m_b)$$

- What are the relevant degrees of freedom (momentum regions)?
  - $\rightarrow$ "**hard**" scale: $m_b = 4.2$GeV
  - $\rightarrow$ "soft"/"collinear" scale: $\Lambda \sim 0.5$GeV
  - $\rightarrow$ "hard-collinear" scale: $\sqrt{m_b\Lambda} \sim 1.5$GeV



- light-cone vectors: $n^\mu_\pm = (1{,}0{,}0, \pm 1)$

$$k^\mu = \frac{k_-}{2}n^\mu_- + k^\mu_\perp + \frac{k_+}{2}n^\mu_+ = (k_-, k_\perp, k_+)$$

- two-step matching

$$\mathrm{QCD} \xrightarrow{m_b \to \infty} \mathrm{SCET_I} \xrightarrow{\sqrt{m_b\Lambda} \to \infty} \mathrm{SCET_{II}}$$

$\rightarrow$ Derive **Factorization Theorems!**

Soft-Collinear Effective Theory

- Soft-Collinear Effective Theory (SCET) is designed to describe the long-distance physics in processe

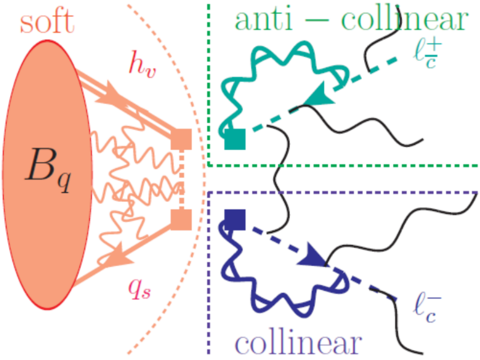
- What are th
  (momentun
  → "**harc**
  → "soft"
  → "hard

- light-cone v

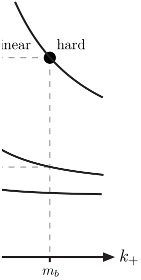
$$k^\mu = \frac{k}{2}$$

- two-step m

$$\text{QCD} \xrightarrow{m_b \to}$$

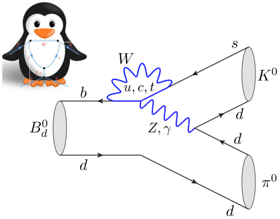


$(1, 1, 1)m_b$
$(1, \sqrt{\lambda}, \lambda)m_b$
$(1, \lambda, \lambda^2)m_b$
$(\lambda, \lambda, \lambda)m_b$
$= \Lambda/m_b)$



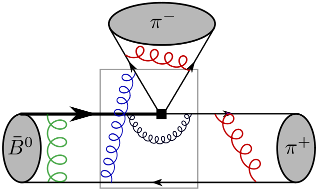
$\frac{\Lambda_{\text{QCD}}}{m_b}$ Λ_{QCD}

QED Factorization in non-leptonic B decays



QCD Factorization

[BBNS '99-'01]



$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \sim F^{B\to M_1}(q^2=0) \int_0^1 \mathrm{d}u \, \mathbf{T}_i^{\mathrm{I}}(u) \, \phi_{M_2}(u)$$
$$+ \int_0^\infty \mathrm{d}\omega \int_0^1 \mathrm{d}u \, \mathrm{d}v \; \mathbf{T}_i^{\mathrm{II}}(u,v,\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u) \, \phi_B^+(\omega) \; + \; \mathcal{O}(\Lambda/m_b)$$

Scale separation for $m_b \to \infty$

- $\to$ perturbative hard-scattering kernels $T_i^{\mathrm{I,II}}$
- $\to$ non-perturbative (but universal) Light-cone distribution amplitudes (LCDAs)

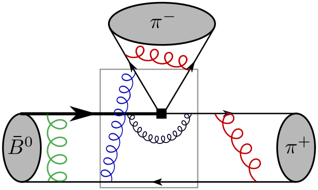
Field theoretic basis in Soft-Collinear Effective Theory

- $\to$ holds to all orders in α_s
- $\to$ power-corrections $\sim \mathcal{O}(\Lambda/m_b)$

Second term absent for heavy-light final states

QCD Factorization

[BBNS '99-'01]



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$$+ \int_0^\infty \mathrm{d}\omega \int_0^1 \mathrm{d}u \, \mathrm{d}v \; \mathbf{T}_i^{\mathrm{II}}(u,v,\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u) \, \phi_B^+(\omega) \; + \; \mathcal{O}(\Lambda/m_b)$$

Perturbative corrections:

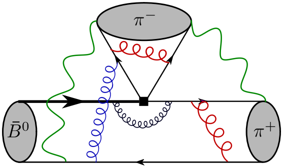
- $\rightarrow$ scattering kernels partially known to NNLO — e.g. [Bell, Beneke, Huber, Li]
- $\rightarrow$ 3-loop (2-loop) anomalous dimension for ϕ_M (ϕ_B^+) — [Braun et al.]
- $\rightarrow$ QED may compete

Non-perturbative input:

- $\rightarrow$ Decay constants and first Gegenbauer moments from lattice — e.g. [FLAG, Braun et al.]
- $\rightarrow$ parameters of $\phi_B^+(\omega)$ poorly constraint

QED Factorization

[Beneke, PB, Toelstede, Vos '20-'22]



$\otimes = (Q_1, Q_2)$

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \Big|_{\text{non-rad.}} = \mathcal{F}_{Q_2}^{B \to M_1}(q^2 = 0) \int_0^1 \mathrm{d}u \, \mathbf{T}^{\mathrm{I}}_{i,Q_2}(u) \, \Phi_{M_2}(u) + \int \mathrm{d}\omega \int_0^1 \mathrm{d}u \, \mathrm{d}v \; \mathbf{T}^{\mathrm{II}}_{i,\otimes}(u,v,\omega) \, \Phi_{M_1}(v) \, \Phi_{M_2}(u) \, \Phi_{B,\otimes}(\omega)$$

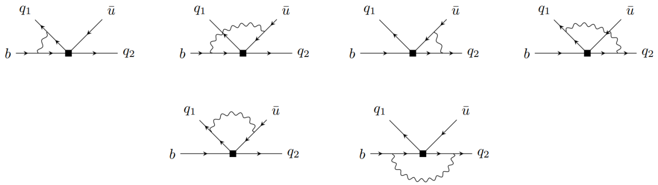
→ looks like the QCDF formula, but new (non-perturbative) hadronic matrix elements
(photons couple weakly to strongly interacting quarks)

→ form factor $F^{B\to\pi}$ → semi-leptonic amplitude $A^{B\to\pi\ell\bar{\nu}_\ell}$ for charged M_2

→ **Soft physics qualitatively different** from standard hard-scattering picture.
Process-dependence & rescattering phases due to soft Wilson lines from charged mesons

$$S_{n_+}^{(q)}(x) = \exp\left\{ -iQ_q e \int_0^\infty \mathrm{d}s \, n_+ \cdot A_s(x + s n_+) \right\}$$

Perturbative Input: Hard-Scattering Kernels



- finite $\mathcal{O}(\alpha_{\rm em})$ radiative corrections to short-distance kernels, e.g. for $B \to$ light-light:

$$
\begin{aligned}
H_{2,-}^{\rm I(1)}(u) = {} & Q_{q_1} Q_{q_2} \left(L^2 - 4L_\nu + L\,(4 + 2i\pi - 2\ln u) + \ln^2 u - 2i\pi \ln u - \frac{7\pi^2}{6} + 1 \right) \\
& - Q_u Q_{q_2} \left(L^2 - L_\nu + L\,(4 + 2i\pi - 2\ln \bar{u}) - \ln \bar{u}\,(3 + 2i\pi - \ln \bar{u}) - \frac{7\pi^2}{6} + 3i\pi + 6 \right) \\
& + Q_u Q_d \left(\frac{1}{2} L^2 - 4L_\nu - 2L\,(-1 + \ln \bar{u}) + 2\ln^2 \bar{u} - \frac{2}{u} \ln \bar{u} + 2\mathrm{Li}_2\,(u) + \frac{\pi^2}{12} - 3 \right) \\
& - Q_d Q_{q_1} \left(\frac{1}{2} L^2 - L_\nu + L(2 - 2\ln u) + 2\ln^2 u - 3\ln u + \frac{\ln u}{\bar{u}} + 2\mathrm{Li}_2(\bar{u}) + \frac{\pi^2}{12} + 2 \right) \\
& - 3 \left(Q_{q_1} + Q_u \right) \left(Q_{q_2} + Q_d \right) \\
& - Q_{q_2} Q_d \left(\frac{1}{2} L^2 - L_\nu + 2L + \frac{\pi^2}{12} + 4 \right) - Q_d^2 \left(\frac{1}{2} L_\nu + L + 2 \right) - \frac{1}{2} Q_{q_2}^2 \left(L_\nu - L \right) \\
& - \frac{1}{2} \left(Q_{q_1}^2 + Q_u^2 - 2 Q_u Q_{q_1} \right) \left(L_\nu - L \right) \,,
\end{aligned}
$$

Non-perturbative input: LCDA for charged pions

$$R_{\bar{c}}\langle\pi^-|\bar{\chi}_{\bar{c}}^{(d)}(tn_-)\frac{\not{n}_-}{2}\gamma_5\chi_{\bar{c}}^{(u)}(0)|0\rangle = -iE\int_0^1 du\; e^{iu\hat{t}} f_\pi \Phi_{\pi^-}(u)$$

- UV-scale evolution IR-divergent $\rightarrow$ well-defined after soft rearrangement
- choose decay constant in QCD

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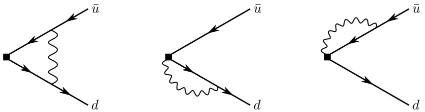

$$\gamma(u,v) = -\frac{\alpha_{\rm em}}{\pi}\delta(u-v)Q_M\left(Q_M \ln\frac{\mu}{2E} - Q_d \ln u + Q_u \ln(1-u) + \frac{3}{4}\right)$$
$$-\left(\frac{\alpha_s C_F}{\pi} + \frac{\alpha_{\rm em}}{\pi}Q_u Q_d\right)\left[\left(1+\frac{1}{v-u}\right)\frac{u}{v}\theta(v-u) + \left(1+\frac{1}{u-v}\right)\frac{1-u}{1-v}\theta(u-v)\right]_+$$

- LCDA $\Phi_{\pi^-}(u;\mu)$ is a non-perturbative function, but μ-dependence perturbative
  - $\rightarrow$ resum log-enhanced contributions perturbatively through renormalization group

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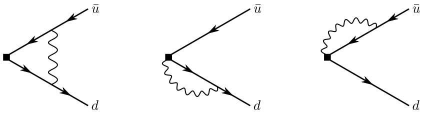

Numerical results for inverse moments: ($a_2^\pi = 0.116$ @ 2GeV)

$$\langle\bar{u}^{-1}\rangle_{\pi^-}(5.3\,\text{GeV}) = 0.9997|^{\text{QED}}_{\text{point charge}}(3.285^{+0.05}_{-0.05}|_{\text{LL}} - 0.020|_{\text{NLL}} + 0.017|^{\text{QED}}_{\text{partonic}})$$
$$\langle\bar{u}^{-1}\rangle_{\pi^-}(80.4\,\text{GeV}) = 0.985|^{\text{QED}}_{\text{point charge}}(3.197^{+0.03}_{-0.03}|_{\text{LL}} - 0.022|_{\text{NLL}} + 0.042|^{\text{QED}}_{\text{partonic}})$$

$\rightarrow$ QED small, but as important as QCD NLL resummation

Non-perturbative Input: B-meson LCDA (soft function)

$$\frac{1}{R_c R_{\bar{c}}} \langle 0| \bar{q}_s^{(d)}(tn_-)[tn_-,0]_{n_-}^{(d)} \frac{\not{n}_-}{2} h_v \, (S_{n_+}^{\dagger,Q_2} S_{n_-}^{Q_2}) \, |\bar{B}^0\rangle = i m_B \int \mathrm{d}\omega e^{-i\omega t} f_B \Phi_{B,+-}(\omega)$$

- Process dependent: soft photons sensitive to charge and direction of final-state particles
- “Soft functions” instead of LCDA: rescattering phases, support for negative ω, ...

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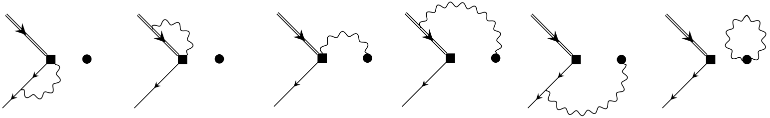

Anomalous dimension:

$$\gamma_{\otimes}(\omega,\omega') = \frac{\alpha_s C_F}{\pi} \left[\left(\ln \frac{\mu}{\omega - i0} - \frac{1}{2} \right) \delta(\omega - \omega') - H_+(\omega,\omega') \right]$$
$$+ \frac{\alpha_{\rm em}}{\pi} \left[\left((Q_{\rm sp}^2 + 2Q_{\rm sp} Q_{M_1}) \ln \frac{\mu}{\omega - i0} - \frac{3}{4} Q_{\rm sp}^2 - \frac{1}{2} Q_d^2 \right.\right.$$
$$\left.\left. + i\pi (Q_{\rm sp} + Q_{M_1}) Q_{M_2} \right) \delta(\omega - \omega') - Q_{\rm sp} Q_d H_+(\omega,\omega') + Q_{\rm sp} Q_{M_2} H_-(\omega,\omega') \right]$$

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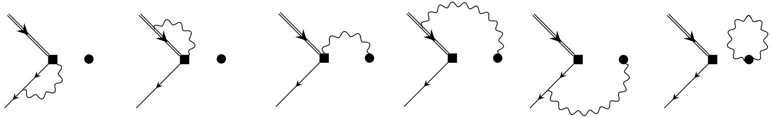


For exp. model $\phi(\omega,\mu_0) = \omega/\omega_0^2 e^{-\omega/\omega_0}$ at $\mu_0 = 1$ GeV with $\omega_0 = \lambda_B(\mu_0) = 0.3$:

	QCD	$(0,0)$	$(-,0)$	$(0,-)$	$(+,-)$
$\lambda_B^{-1}(2\text{GeV})$	2.792	2.792	2.802	$2.790 + 0.010i$	$2.798 + 0.010i$
$\sigma_1(2\text{GeV})$	-0.213	-0.213	-0.210	-0.214	-0.211

$\rightarrow$ no unexpected large QED effects from soft functions

$\rightarrow$ (recent interest in "LCDAs on two light-cones": QED, power-corrections, charm-loop)

Numerical Estimates for πK Final States

Universal contributions: ("Bloch-Nordsieck factors")

- $\rightarrow$ from ultra-soft radiation and universal part of virtual corrections
- $\rightarrow$ resums large soft $\sim \ln \Delta E/m_B$ and collinear $\sim \ln m_i/m_B$ logarithms

$$U(M_1 M_2) = \left(\frac{2\Delta E}{m_B}\right)^{-\frac{\alpha_{\rm em}}{\pi}\left(Q_B^2 + Q_1^2\left[1+\ln\frac{m_1^2}{m_B^2}\right] + Q_2^2\left[1+\ln\frac{m_2^2}{m_B^2}\right]\right)}$$

For $\Delta E = 60$ MeV: ($\Delta E = \pi K$ invariant mass window around m_B)

$$U(\pi^+ K^-) = 0.914 \qquad U(\pi^0 K^-) = 0.976$$

$$U(\pi^- \bar{K}^0) = 0.954 \qquad U(\bar{K}^0 \pi^0) = 1$$

Structure-dependent (virtual) corrections in non-rad. amplitude partly included

- ✓ Electroweak scale to m_B: QED corrections to Wilson coefficients
- ✓ m_B to $\Lambda_{\rm QCD}$: $\mathcal{O}(\alpha_{\rm em})$ corrections to short-distance kernels
- ↯ QED effects in LCDAs/form factors, but log-enhanced effects $\sim \ln \Lambda/m_B$ under control.

Ratios and Isospin Sum Rules

1. Consider ratios where QCD uncertainties drop out:

$$R_L = \frac{2\mathrm{Br}(\pi^0 K^0) + 2\mathrm{Br}(\pi^0 K^-)}{\mathrm{Br}(\pi^- K^0) + \mathrm{Br}(\pi^+ K^-)} = R_L^{\mathrm{QCD}} + \cos\gamma \mathrm{Re}\, \delta_{\mathrm{E}} + \delta_U$$

[Beneke, Neubert '03]

→ Probe of new physics because of sensitivity to electroweak penguin contributions.
E.m. effects are isospin violating and may fake penguin amplitudes.

$$R_L^{\mathrm{QCD}} - 1 \approx (1 \pm 2)\% \qquad \delta_E \approx 0.1\% \qquad \delta_U \approx 5.8\%$$

→ QED corrections larger than QCD and QCD uncertainty, but short-distance QED negligible

2. Isospin sumrule

$$\begin{aligned}\Delta(\pi K) &\equiv A_{\mathrm{CP}}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 K^-) - \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 \bar{K}^0) \\ &\equiv \Delta(\pi K)^{\mathrm{QCD}} + \delta\Delta(\pi K)\end{aligned}$$

[Gronau, Rosner '06]

$$\Delta(\pi K)^{\mathrm{QCD}} = (0.5 \pm 1.1)\% \qquad \delta_\Delta(\pi K) \approx -0.4\%$$

→ Isospin sumrule robust against QED effects (QED of similar size but small)

Ratios with semi-leptonic decays

$$R_L^{(0),(*)}(\Delta E) \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+} L^-)(\Delta E)}{d\Gamma^{(0)}(\bar{B}_d \to D^{(*)+} \mu^- \bar{\nu}_\ell)/dq^2|_{q^2=m_L^2}}$$

$$R_L^{(*)}(\Delta E) \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+} L^-)(\Delta E)}{d\Gamma(\bar{B}_d \to D^{(*)+} \mu^- \bar{\nu}_\ell)(\Delta E)/dq^2|_{q^2=m_L^2}}$$

- short-distance QED $\approx -1\%$, ultrasoft up to $\approx -7\%$ (depending on the semi-leptonic normalization)
- not large enough to explain the -15% amplitude deficit [Bordone et al., 2020], but highlights the importance of proper treatment of ultrasoft radiation effects.

$R_L^{(*)}$	LO	QCD NNLO	$+\delta_{\text{QED}}$	$+\delta_{\text{U}}$ $(\delta_{\text{U}}^{(0)})$
R_π	0.969 ± 0.021	$1.078^{+0.045}_{-0.042}$	$1.069^{+0.045}_{-0.041}$	$1.074^{+0.046}_{-0.043}\,(1.003^{+0.042}_{-0.039})$
R_π^*	0.962 ± 0.021	$1.069^{+0.045}_{-0.041}$	$1.059^{+0.045}_{-0.041}$	$1.065^{+0.047}_{-0.042}\,(0.996^{+0.043}_{-0.039})$
$R_K \cdot 10^2$	7.47 ± 0.07	$8.28^{+0.27}_{-0.26}$	$8.21^{+0.27}_{-0.26}$	$8.44^{+0.29}_{-0.28}\,(7.88^{+0.26}_{-0.25})$
$R_K^* \cdot 10^2$	6.81 ± 0.16	$7.54^{+0.31}_{-0.29}$	$7.47^{+0.30}_{-0.29}$	$7.68^{+0.32}_{-0.30}\,(7.19^{+0.29}_{-0.28})$

Table 3: Theoretical predictions for $R_L^{(*)}$ expressed in GeV^2 at LO, NNLO QCD and subsequently adding δ_{QED} given in (82) and the ultrasoft effects δ_{U} (or in brackets $\delta_{\text{U}}^{(0)}$). The last column presents our final results.

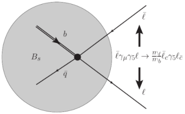
Leptonic and semi-leptonic decays

Leptonic $\bar{B}_s \to \mu^+\mu^-$ Decays

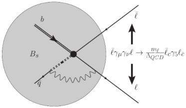
Helicity suppressed: $\mathcal{A}(\bar{B}_s \to \mu\mu) \sim m_\mu$

In QCD $\sim$ decay constant

$$\langle 0|\, \bar{s}\gamma^\mu \gamma_5 b \,|\bar{B}_s(p)\rangle = i f_{B_s} p^\mu$$



$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle$



$\langle 0| \int d^4x\, T\{j_{\rm QED}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$

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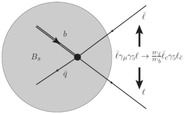
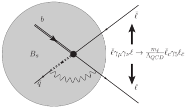


$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle$



$\langle 0| \int d^4x\, T\{j_{\rm QED}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$

Structure-dependent QED: [Beneke et al. '17+'19; Feldmann et al. '22]

- $\to$ hadronic and leptonic tensor no longer factorize
- $\to$ non-local interaction: $f_{B_s} \to \phi_B^+(\omega)$
- $\to$ power-enhancement: $m_\mu/m_B \to m_\mu/\Lambda_{\rm QCD}$ (enhanced by double-logs $\ln^2 \Lambda_{\rm QCD}/m_B$)

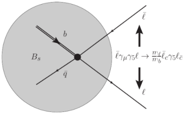
$$\begin{aligned} i\mathcal{A} \quad \sim \quad & f_{B_s} m_\mu \Big\{ \frac{\alpha}{4\pi}\, Q_\mu Q_s\, M\, [\bar{\ell}\,(1+\gamma_5)\,\ell] \\ & \times \left(\int_0^1 du\,(1-u) \int_0^\infty \frac{d\omega}{\omega}\, \phi_B^+(\omega) \left(\ln\frac{m_B\omega}{m^2} + \ln\frac{u}{1-u}\right) C_9^{\rm eff}(u m_b^2) \right. \\ & \left. - Q_\mu \int_0^\infty \frac{d\omega}{\omega}\, \phi_B^+(\omega) \left(\ln^2\frac{m_B\omega}{m^2} - 2\ln\frac{m_B\omega}{m^2} + \frac{2\pi^2}{3}\right) C_7^{\rm eff} \right) \Big\} \end{aligned}$$

Leptonic $\bar{B}_s \to \mu^+\mu^-$ Decays

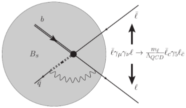
Helicity suppressed: $\mathcal{A}(\bar{B}_s \to \mu\mu) \sim m_\mu$

In QCD $\sim$ decay constant

$$\langle 0|\, \bar{s}\gamma^\mu\gamma_5 b\,|\bar{B}_s(p)\rangle = i f_{B_s} p^\mu$$



$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle$



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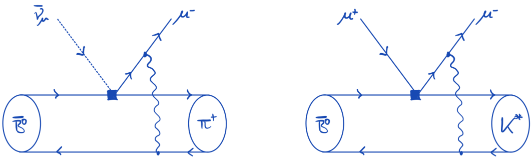
- $\to$ power-enhancement: $m_\mu/m_B \to m_\mu/\Lambda_{\rm QCD}$ (enhanced by double-logs $\ln^2 \Lambda_{\rm QCD}/m_B$)

Leads to reduction of the (non-radiative) branching fraction by 0.5%.
(destructive interference between Q_7 and Q_9; individual terms at percent level)

No power-enhancement in $B^- \to \mu^-\bar{\nu}_\mu$ [Cornella et al. '22 + to appear]

Semi-leptonic Decays

Similar methods applicable to semi-leptonic decays in certain parts of phase-space.
(Back-to-back limit studied in [Beneke, PB, Finauri, Toelstede, Vos '20-'22]; similar to non-leptonic two-body decays.)



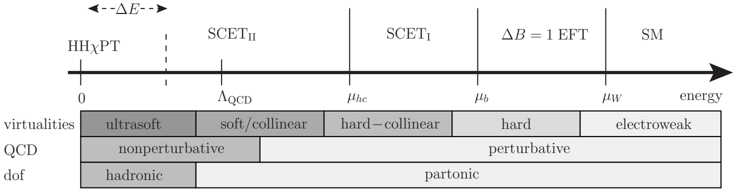
→ enhancements in certain parts of phase-space?

→ impact on angular distributions?

→ relevant for V_{ub} from $B \to \pi \ell \nu$ at %-level?

→ muons (exclusive particles) vs electrons ("jets" due to collinear radiation)

Conclusion



→ EFT methods allow calculation of structure-dependent QED effects between $m_b \gg \Lambda_{\rm QCD}$.

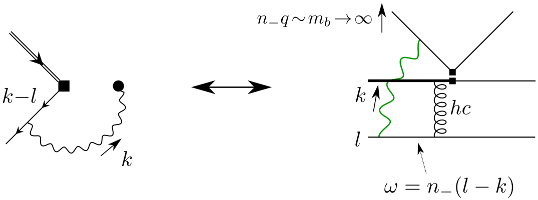
→ QED factorization more complicated than QCD due to charged external states. Description requires generalized non-perturbative hadronic matrix elements. Can sum log-enhanced corrections through renormalization group.

→ Structure dependent contributions small in non-leptonic decays, but can compete with QCD uncertainty. More relevant in leptonic decays. What about semi-leptonics??

→ Comparison between theory and experiment requires precise statement about how QED effects are treated in the exp. analysis (PHOTOS).

Backup-Slides

On the Support of QED B LCDAs



- even for on-shell massive partons with $\Phi^{(0)}(\omega) = \delta(\omega - m)$ the one-loop soft photon exchange with the anti-coll. π^- generates a support for $\omega < 0$
- diagram has e.g. the following contribution

$$\int \mathrm{d}^d k \, \frac{\delta(\omega - n_-\ell + n_-k)}{(k^2 + i0)[(k-\ell)^2 - m^2 + i0]\,(n_+k - i0)} \quad \Big| \text{pick up residues in } (n_+k)$$

$$\sim \Gamma(\epsilon) \int_{n_-\ell}^{\infty} \mathrm{d}(n_-k)\,(n_-k)^{-1-\epsilon}\delta(\omega - m + n_-k) = \Gamma(\epsilon)\,(m-\omega)^{-1-\epsilon}\theta(-\omega)$$

- QED B LCDA no longer linear in ω as $\omega \to 0$ but rather const.

  $\rightarrow$ no endpoint singularity in first inverse moment

$$\int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{\omega - i0} \Phi_{B,+-}(\omega)$$

Soft Rearrangement

- effective SCET operator composend of soft and (anti-)collinear fields
  $\rightarrow$ has well-defined UV-scale evolution ✓

- But: Factorization requires renormalization of each individual mode

↯ Problem: UV-scale evolution of individual pieces IR-divergent! ("factorization anomaly")

  $\rightarrow$ Anomalous dimension depends on IR-regulator (off-shellness, quark masses, ...)

  $\rightarrow$ can be cured by a "soft rearrangement" that removes the soft overlap

cf. [Beneke, Bobeth, Szafron]

$$\mathcal{O} = \mathcal{O}_{\bar{c}} \times \mathcal{O}_{s,C} \to (\mathcal{O}_{\bar{c}} R_{\bar{c}}) \times \left(\frac{\mathcal{O}_{s,C}}{R_{\bar{c}}}\right) \quad \text{with} \quad \left|\langle 0 | S_{n_+}^{\dagger (Q_2)} S_{n_-}^{(Q_2)} | 0 \rangle\right| \equiv R_{\bar{c}} R_c$$