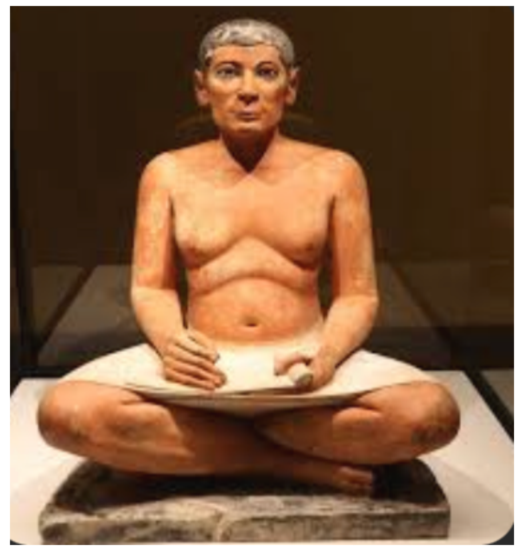
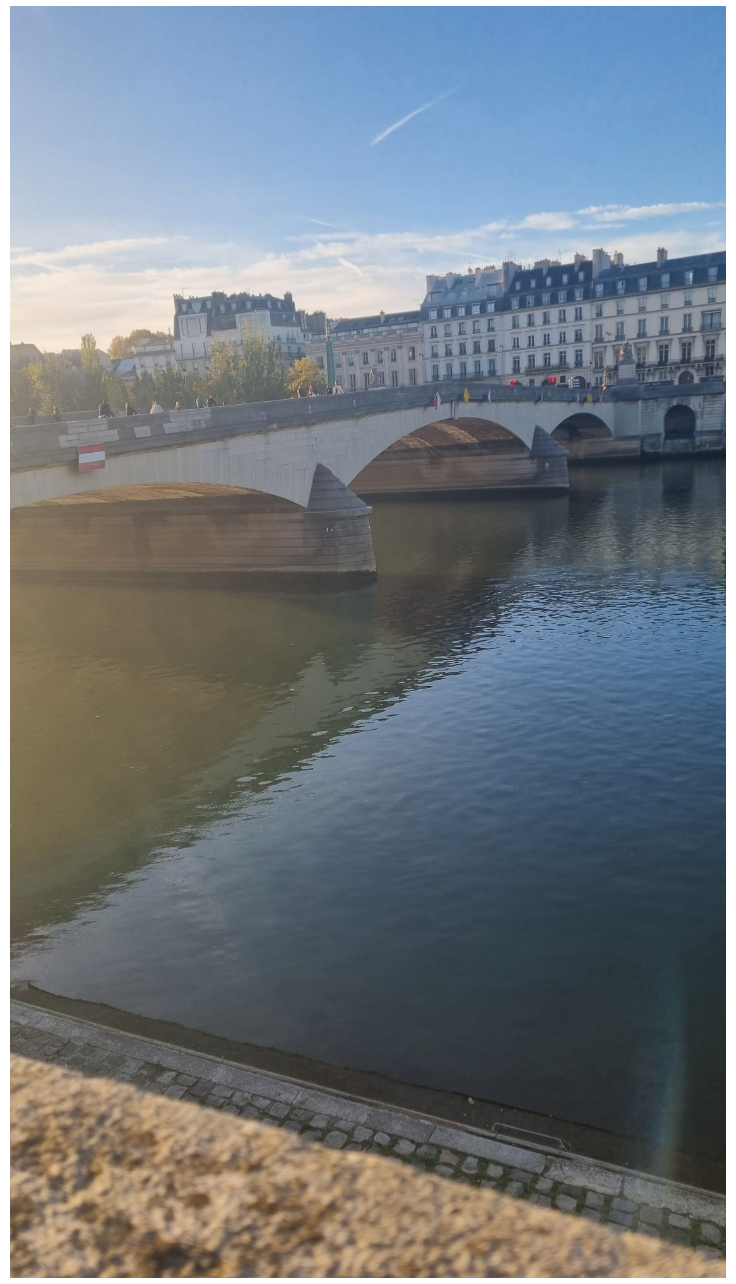
Istituto Nazionale di Fisica Nucleare

Taming hadronic uncertainties in and beyond Standard Model



Orsay, 22-24 October 2025



Hadronic uncertainties in Kaon decays

Giancarlo D'Ambrosio

INFN Sezione di Napoli

Predictions for the rare kaon decays
from QCD in the limit of a large number of colour
With M. Knecht and S.Neshatpour

e-Print: 2409.08568 [hep-ph] MDPI

THANKS to Collaborations Nazila Iyer Neshatpour
- 2311.04878 2404.03643

THANKS to NA48 HIKE LHCB KOTO
T. Kitahara

CP violation in $K \to \mu^+\mu^-$ with and without time dependence through a tagged analysis
Giancarlo D'Ambrosio (INFN, Naples), Avital Dery (CERN), Yuval Grossman (Cornell U., LEPP), Teppei Kitahara (Chiba U. and KMI, Nagoya and Nagoya U.), Radoslav Marchevski (EPFL, Lausanne, LPPC) et al. (Jul 17, 2025)
e-Print: 2507.13445 [hep-ph]

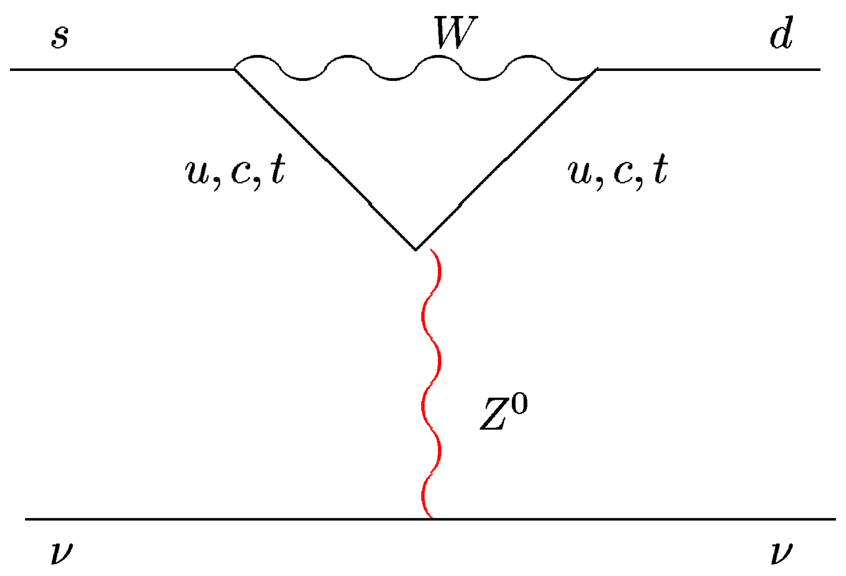
Outline

- Rare Kaon decays crucial in flavour physics
- Measuring $K_S\,K_L$ interference at LHCB
- Large Nc : may be useful to predict kaon decays $K^+ \to \pi^+ l^+ l^-$ $K_S \to \pi^0 l^+ l^-$

$K \to \pi\nu\overline{\nu}$

NEED KOTO/KOTO II AND LHCB

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \quad \times \left[\sum_{q=c,t} V^*_{qs}V_{qd}\; m_q^2\right]$$



$$\sim \qquad [A^2\lambda^5\;(1-\rho-i\eta)m_t^2 + \lambda m_c^2]$$

SM $\quad \underbrace{V-A\otimes V-A}_{}$ $\quad$ Littenberg

$$\Downarrow$$

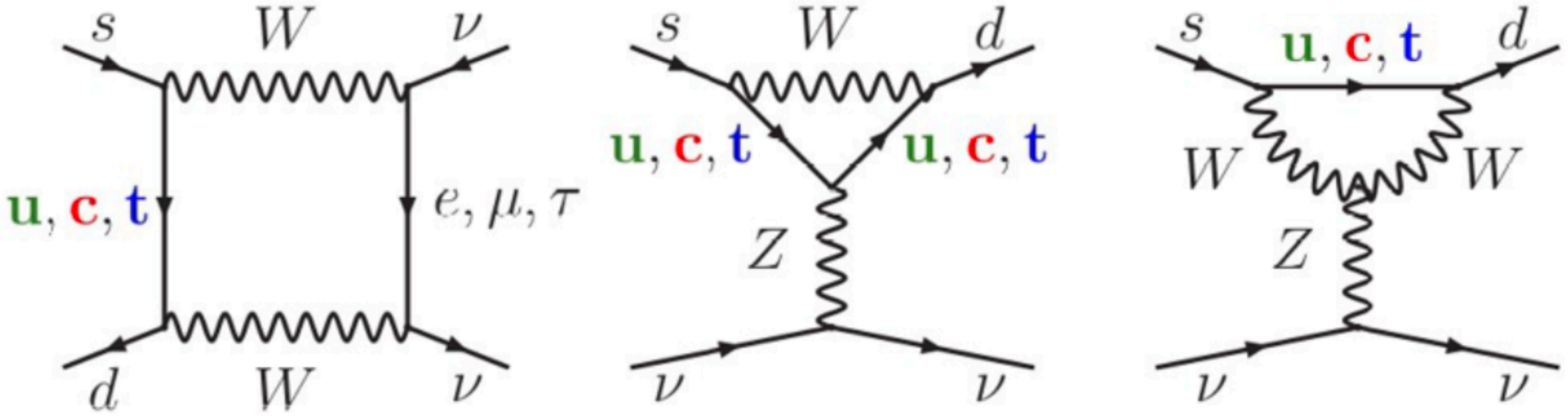
$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow\ J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

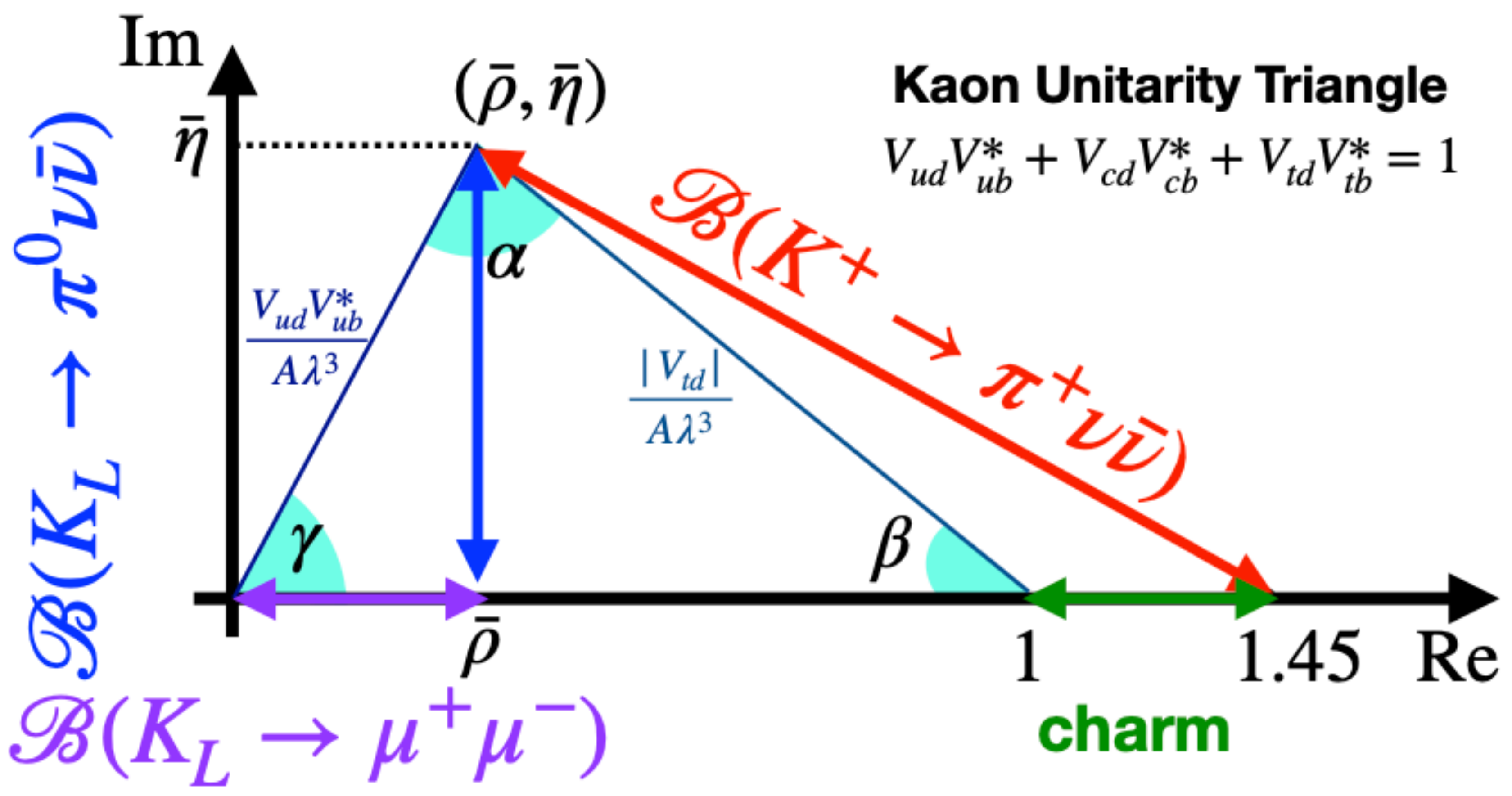
SM

Buchalla and Buras, hep-ph/9308272, Buras et al, 1503.02693.

$K \to \pi\nu\bar{\nu}$: Precision test of the Standard Model

SM: Z-penguin & box diagrams





- $\mathcal{B}(K \to \pi\nu\bar{\nu})$ **highly suppressed** in SM
  - GIM mechanism & maximum CKM suppression $s \to d$ transition: $\sim \frac{m_t}{m_W} \left| V_{ts}^* V_{td} \right|$
- Theoretically clean ⇒ high precision SM predictions
  - Dominated by short distance contributions.
  - Hadronic matrix element extracted from $\mathcal{B}(K \to \pi^0 \ell^+ \nu_\ell)$ decays via isospin rotation.

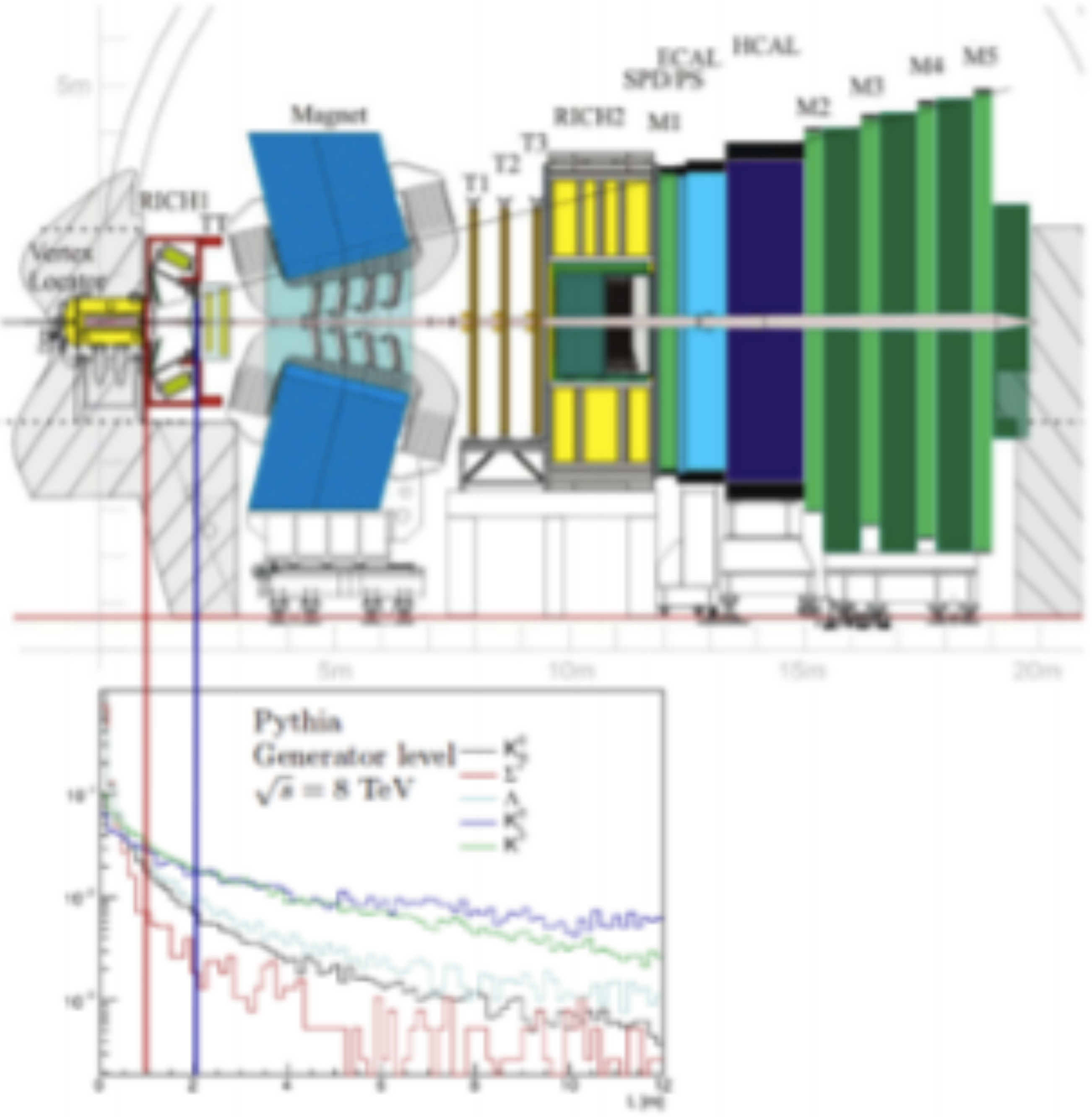
Mode	SM Branching Ratio [1]	SM Branching Ratio [2]	Experimental Status
$K^+ \to \pi^+\nu\bar{\nu}$	$(8.60 \pm 0.42) \times 10^{-11}$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6 \pm 4.0) \times 10^{-11}$ NA62 16−18
$K_L \to \pi^0\nu\bar{\nu}$	$(2.94 \pm 0.15) \times 10^{-11}$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 2 \times 10^{-9}$ KOTO (2021 data)

^Recent SM calculations [1:Buras et al. EPJC 82 (2022) 7, 615][2:D'Ambrosio et al. JHEP 09 (2022) 148]
(Differences in SM calculations from choice of CKM parameters: see [Eur.Phys.J.C 84 (2024) 4, 377])

NA62 16-22 $(13.0)^{+3.3}_{-3.0} \times 10^{-11}$ 18^{+3}_{-2}

Bkg

Kaon in LHCb



- LHCb experiment has been designed for efficient reconstructions of ***b*** and ***c***
- Huge production of strangeness [$O(10^{13})$/fb^{-1} K^0_S] is suppressed by its trigger efficiency [ε~1-2%@LHC Run-I, ε~18%@LHC Run-II]
- LHCb upgrade (LS2=Phase I upgrade, LS4=Phase II upgrade) could realize high efficiency for K^0_S [ε~90%@LHC Run-III] [M. R. Pernas, HL/HE LHC meeting, Fermilab, 2018]
- In LHC Run-III and HL-LHC, we could probe the *ultra* rare decay Br~$O(10^{-11\sim-12})$

Rare Kaon decay program at LHCB

	PDG	Prospects
$K_S \to \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	$(\mathrm{LD})(5.0 \pm 1.5) \cdot 10^{-12}$ $\quad$ NP $< 10^{-11}$
$K_S \to \mu\mu\mu\mu$	–	SM LD $\sim 2 \times 10^{-14}$
$K_S \to ee\mu\mu$	–	$\sim 10^{-11}$
$K_S \to eeee$	–	$\sim 10^{-10}$
$K_S \to \pi^0\mu\mu$	$(2.9 \pm 1.3) \cdot 10^{-9}$	$\sim 10^{-9}$
$K_S \to \pi^+\pi^-e^+e^-$	$(4.79 \pm 0.15) \cdot 10^{-5}$	SM LD $\sim 10^{-5}$
$K_S \to \pi^+\pi^-\mu^+\mu^-$	–	SM LD $\sim 10^{-14}$

Prospects for Measurements with Strange Hadrons at LHCb

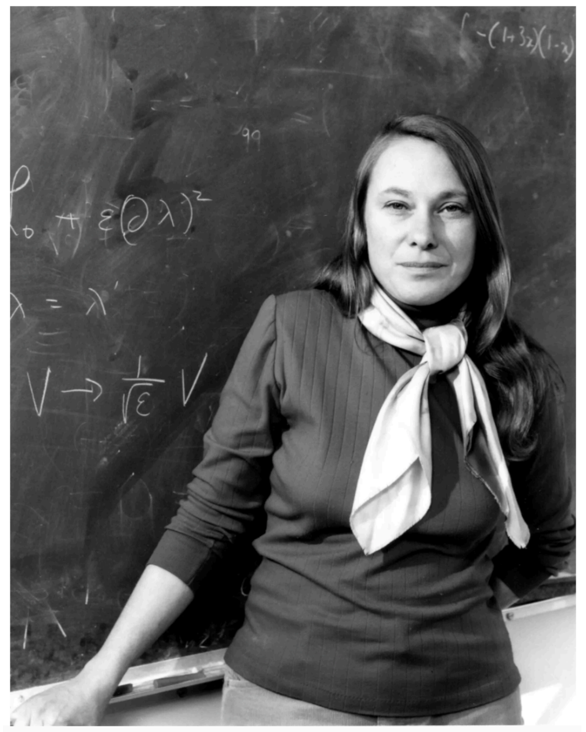
A.A. Alves Junior (Santiago de Compostela U., IGFAE), M.O. Bettler (Cambridge U.), A. Brea Rodríguez (Santiago de Compostela U., IGFAE), A. Casais Vidal (Santiago de Compostela U., IGFAE), V. Chobanova (Santiago de Compostela U., IGFAE) et al. (Aug 10, 2018)

Published in: *JHEP* 05 (2019) 048 • e-Print: 1808.03477 [hep-ex]

Rare n Strange 2017: strange physics at LHCb

$$K_{L,S} \to \mu\mu$$

$K_L \to \mu\mu$



Mary K. Gaillard in 1985, four years after she became the first woman hired by the physics department at the University of California, Berkeley. University of California, Berkeley

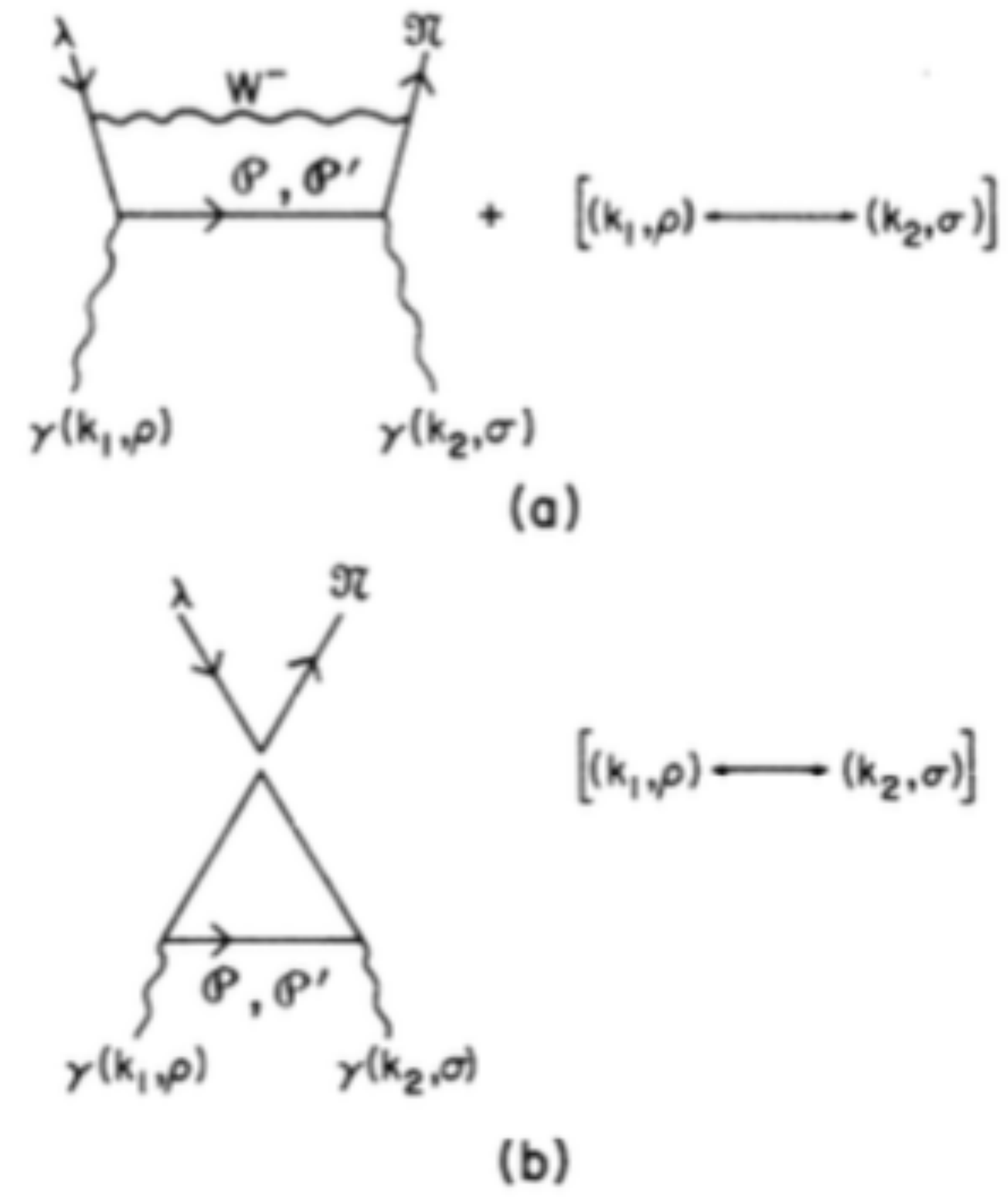


FIG. 7. Leading contributions to $\lambda + \bar{\mathfrak{N}} \to \gamma + \gamma$. To leading order in M_W^{-2}, the diagrams in (a) reduce to those of (b).

Gaillard Lee

$\Gamma(K_L^0 \to \mu^+\mu^-)/\Gamma(K_L^0 \to \pi^+\pi^-)$

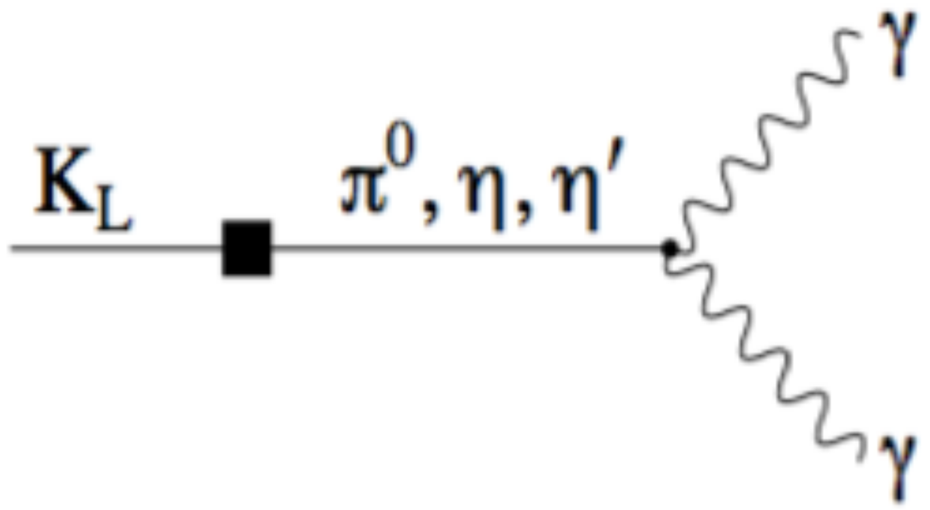
VALUE (10^{-6})	EVTS	DOCUMENT ID		TECN	COMMENT
3.48 ± 0.05	**OUR AVERAGE**				
3.474 ±0.057	6210	AMBROSE	2000	B871	
3.87 ±0.30	179	[1] AKAGI	1995	SPEC	
3.38 ±0.17	707	HEINSON	1995	B791	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
3.9 ±0.3 ±0.1	178	[2] AKAGI	1991B	SPEC	In AKAGI 1995

$$\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm exp} = (6.84 \pm 0.11) \times 10^{-9}$$

$K_L \to \gamma\gamma\,|_{\rm exp}$ known

Dispersive calculation: Re A, Im A

We do not know the sign of $A(K_L \to \gamma\gamma)$



$$
\begin{aligned}
A(K_L \to 2\gamma_\perp)_{O(p^4)} &= A(K_L \to \pi^0 \to 2\gamma_\perp) + A(K_L \to \eta_8 \to 2\gamma_\perp) \\
&= A(K_L \to \pi^0) A(\pi^0 \to 2\gamma_\perp) \left[\frac{1}{M_K^2 - M_\pi^2} + \frac{1}{3} \cdot \frac{1}{M_K^2 - M_8^2} \right] \simeq 0
\end{aligned}
$$

Kaon Decays in the Standard Model
Vincenzo Cirigliano (Los Alamos), Gerhard Ecker, Helmut Neufeld (Vienna U.), Antonio Pich, Jorge Portoles, refs therein

Low energy form factor + matching with VMD to QCD

We use a QCD motivated form factor G.D. Isidori, Portoles;Isidori, Unterdorf

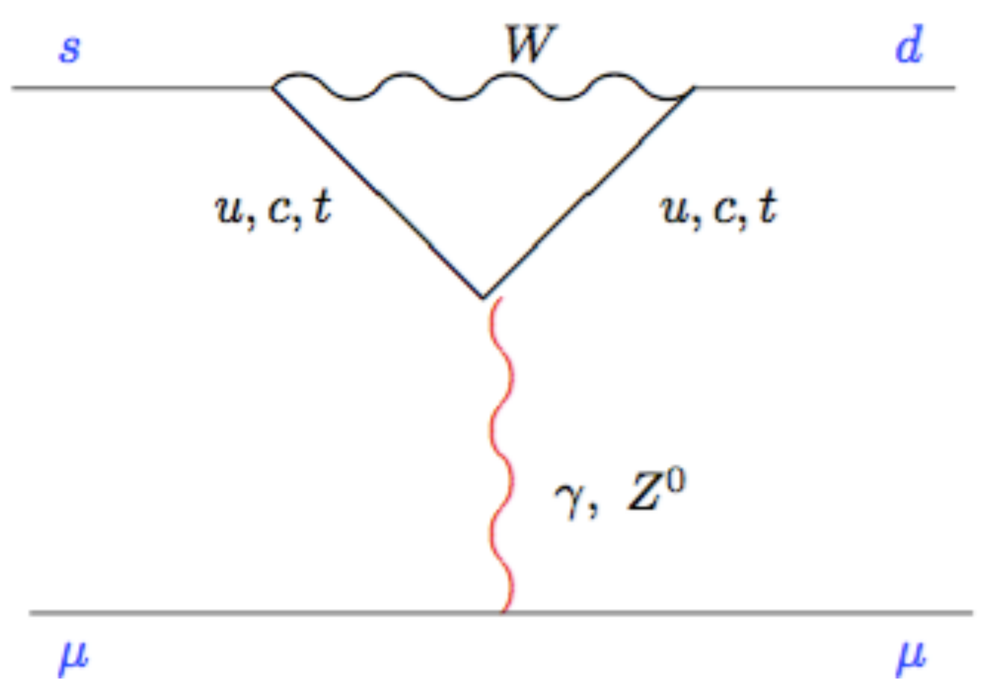
$$A(K_L \to \gamma^*(q_1)\gamma^*(q_2)) = A_{\gamma\gamma}^{\exp}\left[1 + \alpha\left(\frac{q_1^2}{q_1^2 - M_V^2} + \frac{q_2^2}{q_2^2 - M_V^2}\right)\right.$$
$$\left. + \beta\frac{q_1^2 q_2^2}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)}\right]$$

α fixed from $K_L \to \gamma\gamma^*$ β from $K_L \to e^+e^-\mu\bar{\mu}$ (not yet)

Matching with QCD $q_1^2, q_2^2 \to \infty$ imposes $|1 + 2\alpha + \beta| \simeq 0.3$

Isidori Unterdorfer
+many refs…
Hoferichter et al `23

$K_L \to \mu\mu$



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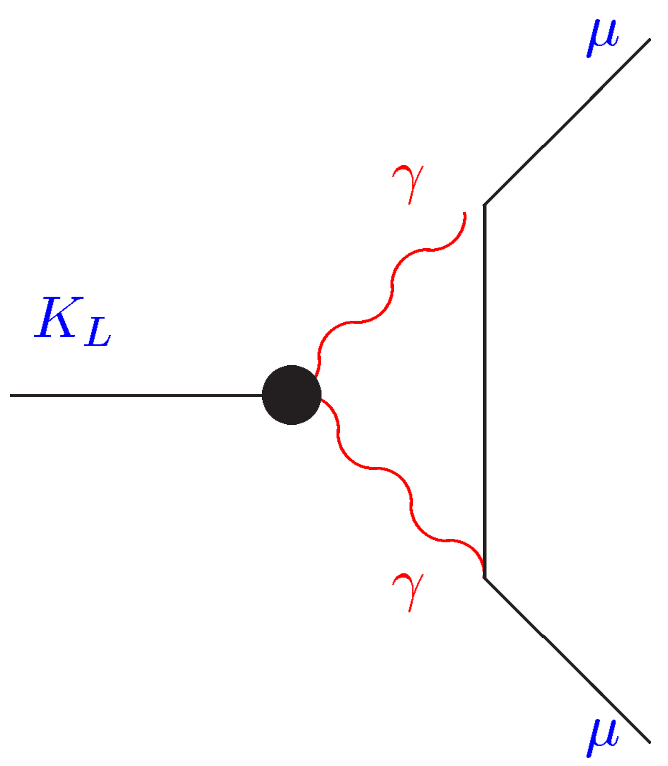


$$\frac{\Gamma(K_L \to \mu\bar{\mu})}{\Gamma(K_L \to \gamma\gamma)} \sim \quad |ReA|^2 + |ImA|^2$$

Absorptive calculation
model independent

$$27.14$$

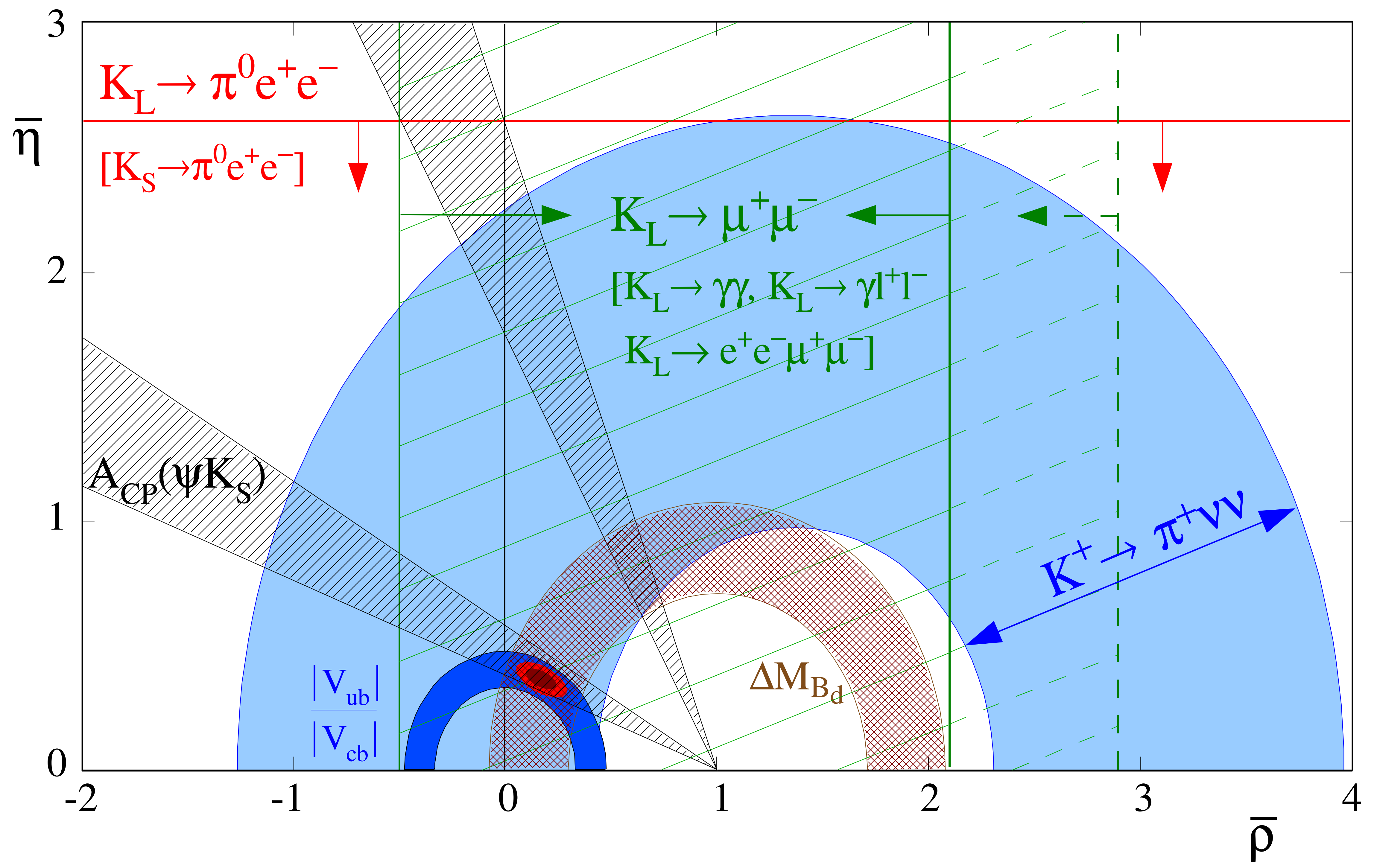
Subtracting from expt. the Absorptive contribution

$$0.98 \pm 0.55 = |ReA|^2 = (\chi_{\gamma\gamma}(M_\rho) + \chi_{\rm short} - 5.12)^2$$

$$|\chi_{\rm short}^{\rm SM}| = 1.96(1.11 - 0.92\bar{\rho})$$

Isidori Unterdorfer

$K_L \to \mu\mu$: our sign ignorance



We do not know the sign

We know the sign

Magically comes LHCB

measuring $K_S \to \mu\mu$

Mostly K_L decays outside fiducial volume

$K_S \to \mu\mu$

PHYSICAL REVIEW D VOLUME 10, NUMBER 3 1 AUGUST 1974

Rare decay modes of the K mesons in gauge theories

M. K. Gaillard* and Benjamin W. Lee†
National Accelerator Laboratory, Batavia, Illinois 60510‡

(Received 4 March 1974)

Rare decay modes of the kaons such as $K \to \mu\bar{\mu}$, $K \to \pi\nu\bar{\nu}$, $K \to \gamma\gamma$, $K \to \pi\gamma\gamma$, and $K \to \pi e\bar{e}$ are of theoretical interest since here we are observing higher-order weak and electromagnetic interactions. Recent advances in unified gauge theories of weak and electromagnetic interactions allow in principle unambiguous and finite predictions for these processes. The above processes, which are "induced" $|\Delta S| = 1$ transitions, are a good testing ground for the cancellation mechanism first invented by Glashow, Iliopoulos, and Maiani (GIM) in order to banish $|\Delta S| = 1$ neutral currents. The experimental suppression of $K_L \to \mu\bar{\mu}$ and nonsuppression of $K_L \to \gamma\gamma$ must find a natural explanation in the GIM mechanism which makes use of extra quark(s). The procedure we follow is the following: We deduce the effective interaction Lagrangian for $\lambda + \mathfrak{N} \to l + \bar{l}$ and $\lambda + \bar{\mathfrak{N}} \to \gamma + \gamma$ in the free-quark model; then the appropriate matrix elements of these operators between hadronic states are evaluated with the aid of the principles of conserved vector current and partially conserved axial-vector current. We focus our attention on the Weinberg-Salam model. In this model, $K \to \mu\bar{\mu}$ is suppressed due to a fortuitous cancellation. To explain the small K_L-K_S mass difference and nonsuppression of $K_L \to \gamma\gamma$, it is found necessary to assume $m_{\mathcal{P}}/m_{\mathcal{P}'} \ll 1$, where $m_{\mathcal{P}}$ is the mass of the proton quark and $m_{\mathcal{P}'}$ the mass of the charmed quark, and $m_{\mathcal{P}'} < 5$ GeV. We present a phenomenological argument which indicates that the average mass of charmed pseudoscalar
sti-
sup-
very
has the
l upper
tion

Dr. Gaillard at Berkeley in the early 1980s. AIP Emilio Segrè Visual Archives, Physics Today Collection

$K^0_S \to \mu^+\mu^-$

INSPIRE search

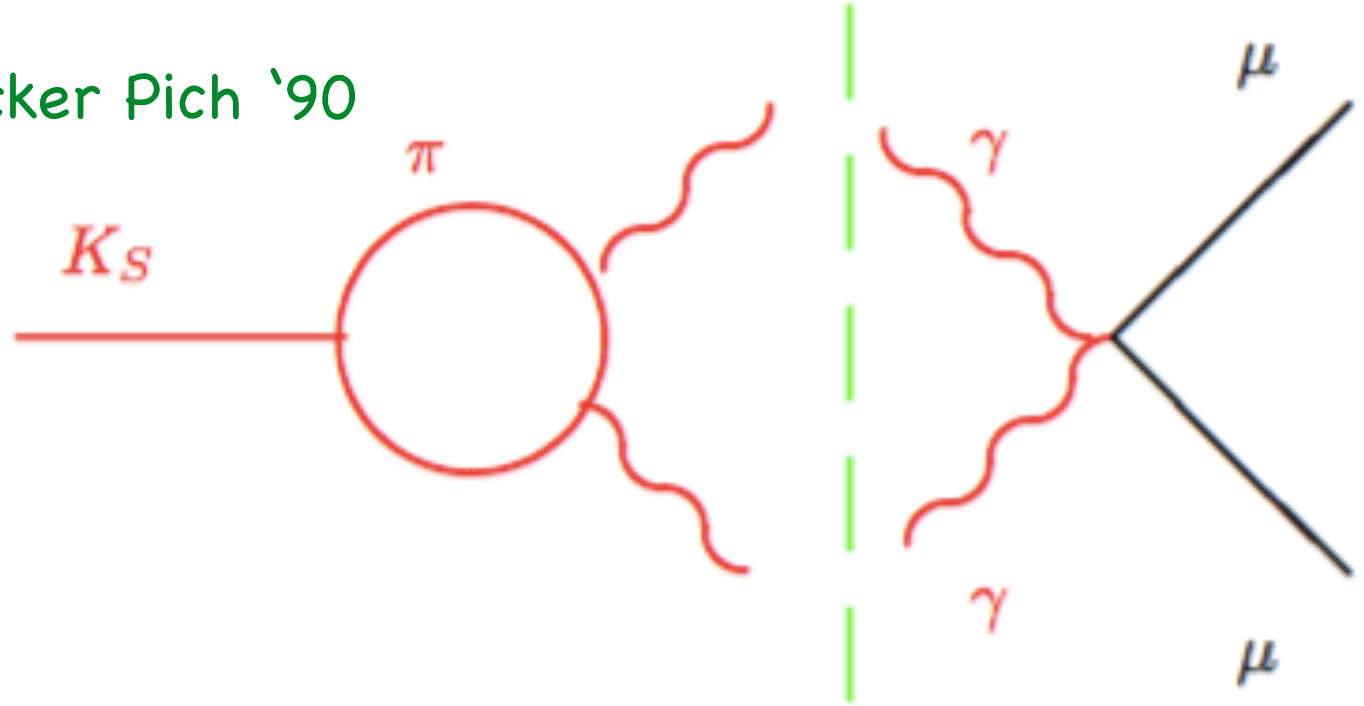
$\Gamma(K^0_S \to \mu^+\mu^-)/\Gamma_{\text{total}}$ Γ_{11}/Γ

Test for $\Delta S = 1$ weak neutral current. Allowed by first-order weak interaction combined with electromagnetic interaction.

VALUE	*CL%*	*DOCUMENT ID*		*TECN*
$< 2.1 \times 10^{-10}$	90	[1] AAIJ	2020AE	LHCB
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$< 8 \times 10^{-10}$	90	[2] AAIJ	2017BQ	LHCB
$< 9 \times 10^{-9}$	90	[3] AAIJ	2013G	LHCB
$< 3.2 \times 10^{-7}$	90	GJESDAL	1973	ASPK

$K_S \to \mu\mu$

Ecker Pich '90



No CP conserving Short Distance due to Furry Theorem

Gaillard Lee

LD $\quad 5 \times 10^{-12}$ 20% TH err

Dispersive treatment of $K_S \to \gamma\gamma$ and $K_S \to \gamma l^+ l^-$

Gilberto Colangelo, Ramon Stucki, and Lewis C. Tunstall

Short Distance

SM $\quad 10^{-5} |\Im(V_{ts}^* V_{td})|^2 \sim 10^{-13}$

NP $\quad$ few 10^{-11} allowed

Summarizing

$$\mathcal{B}\left(K_S^0 \to \mu^+\mu^-\right)_{\mathrm{SM}} = (5.18 \pm 1.50_{\mathrm{LD}} \pm 0.02_{\mathrm{SD}}) \times 10^{-12}$$

$$\parallel$$

$$(4.99_{\mathrm{LD}} + 0.19_{\mathrm{SD}})$$

[Ecker, Pich '91; Isidori, Unterdorfer '04; Chobanova, D'Ambrosio, TK, Martínez, Santos, Fernández, Yamamoto '18]

SD??

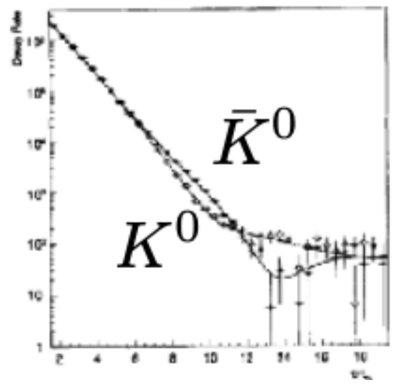
{

CPLEAR Flavor tagging

Such an interference has been discussed from '67 [Sehgal and Wolfenstein], and has been observed/utilized in many processes: e.g., $K^0 \to \pi\pi$, $K^0 \to 3\pi^0$, $K^0 \to \pi^+\pi^-\pi^0$, and $K^0 \to \pi^0 e^+ e^-$

cf. CPLEAR experiment (1990-99@CERN)

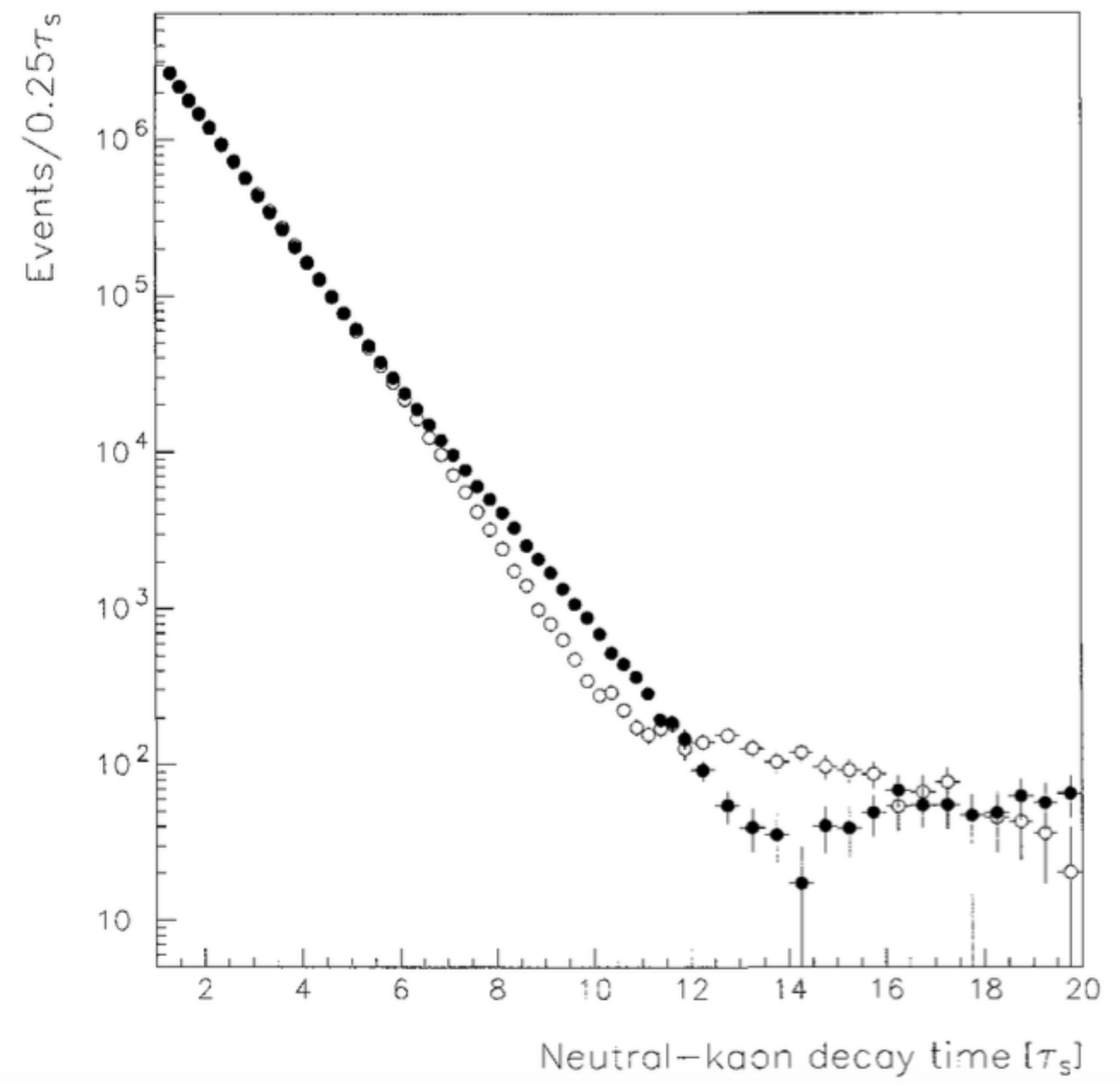
$$p\bar{p} \to \left[\begin{array}{l} K^0 K^- \pi^+ \\ \bar{K}^0 K^+ \pi^- \end{array} \right.$$

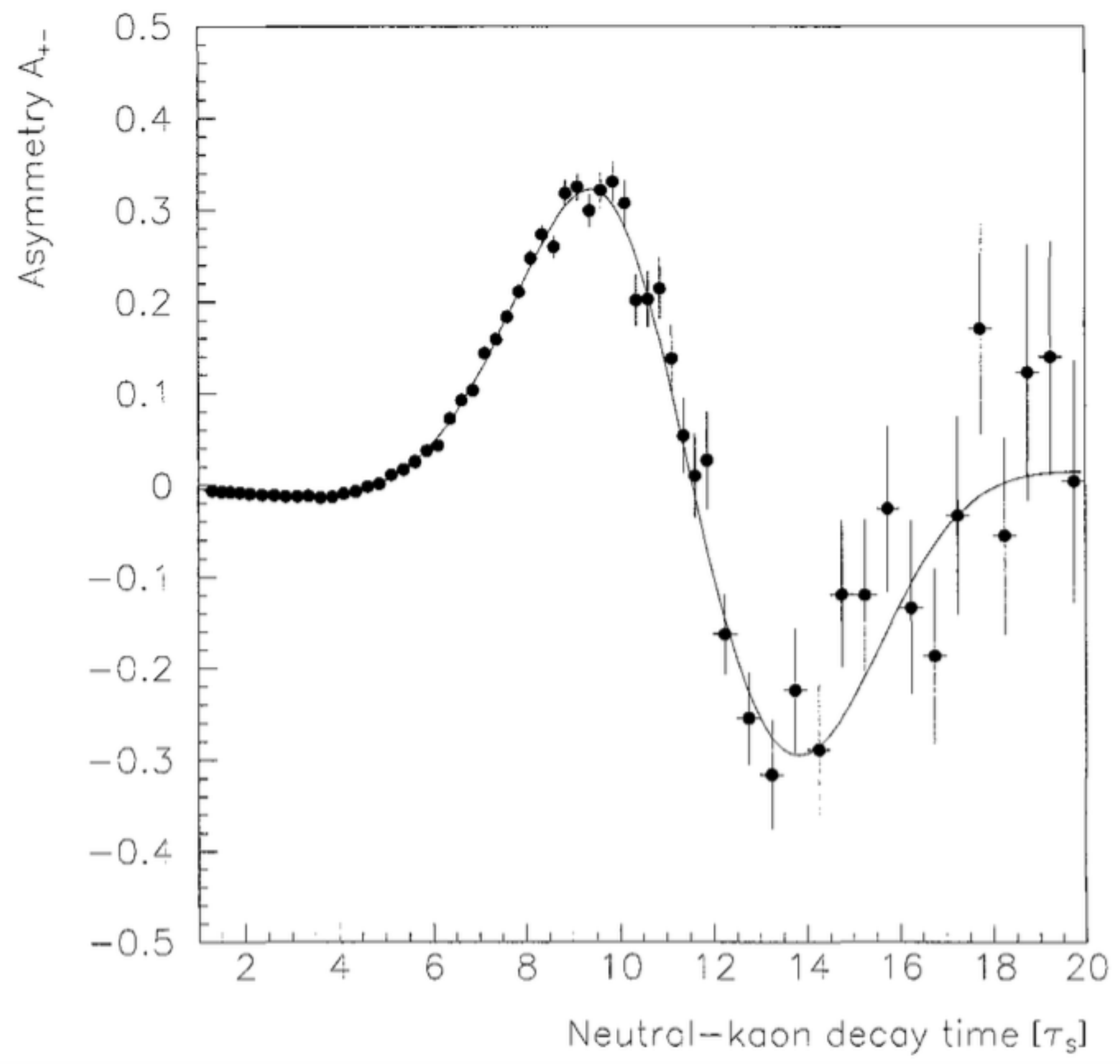


$\{K_S, K_L\} \to \pi^+\pi^-$

measured the interference between K_L and K_S

[CPLEAR collaboration '95]





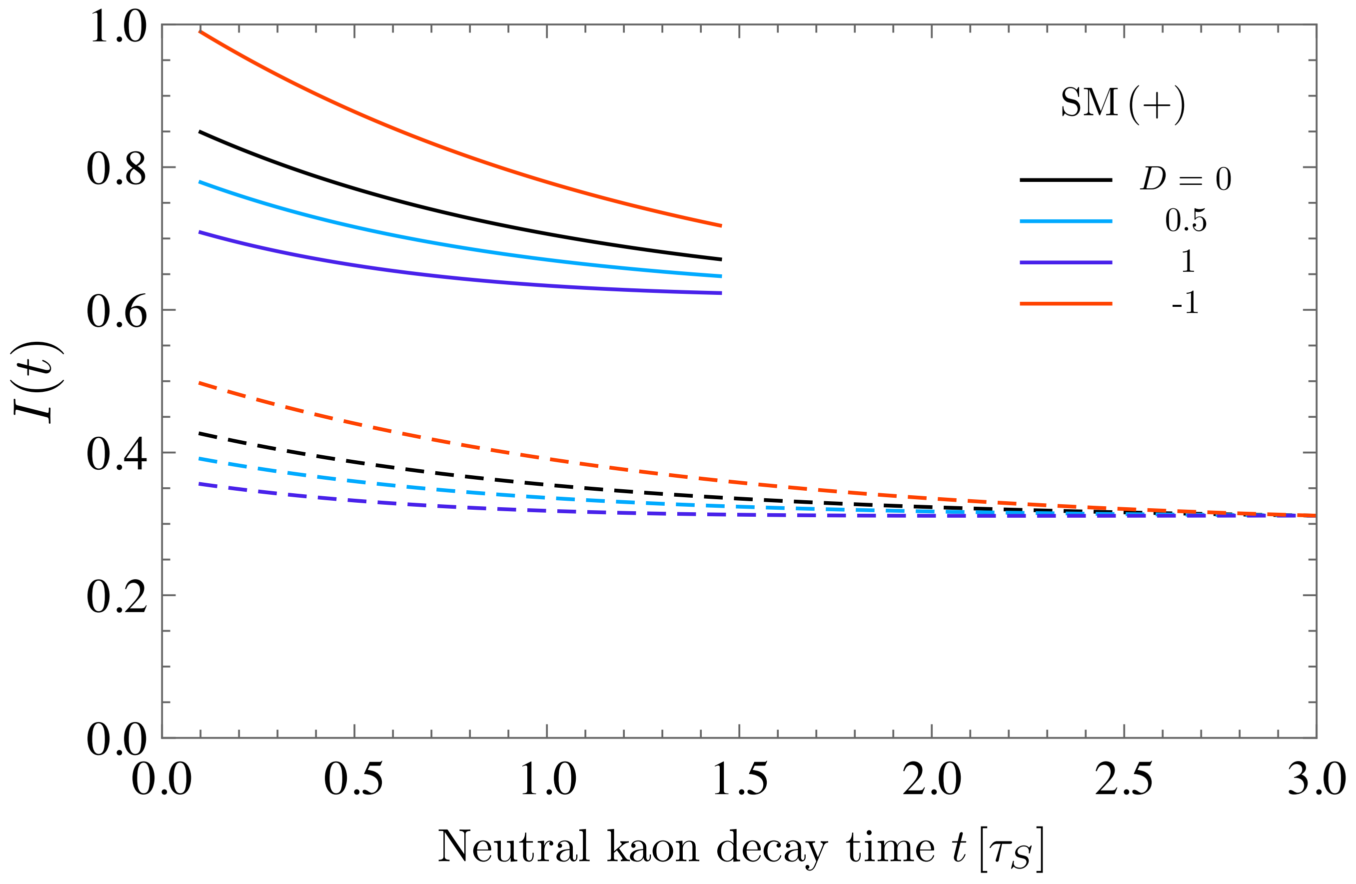
}

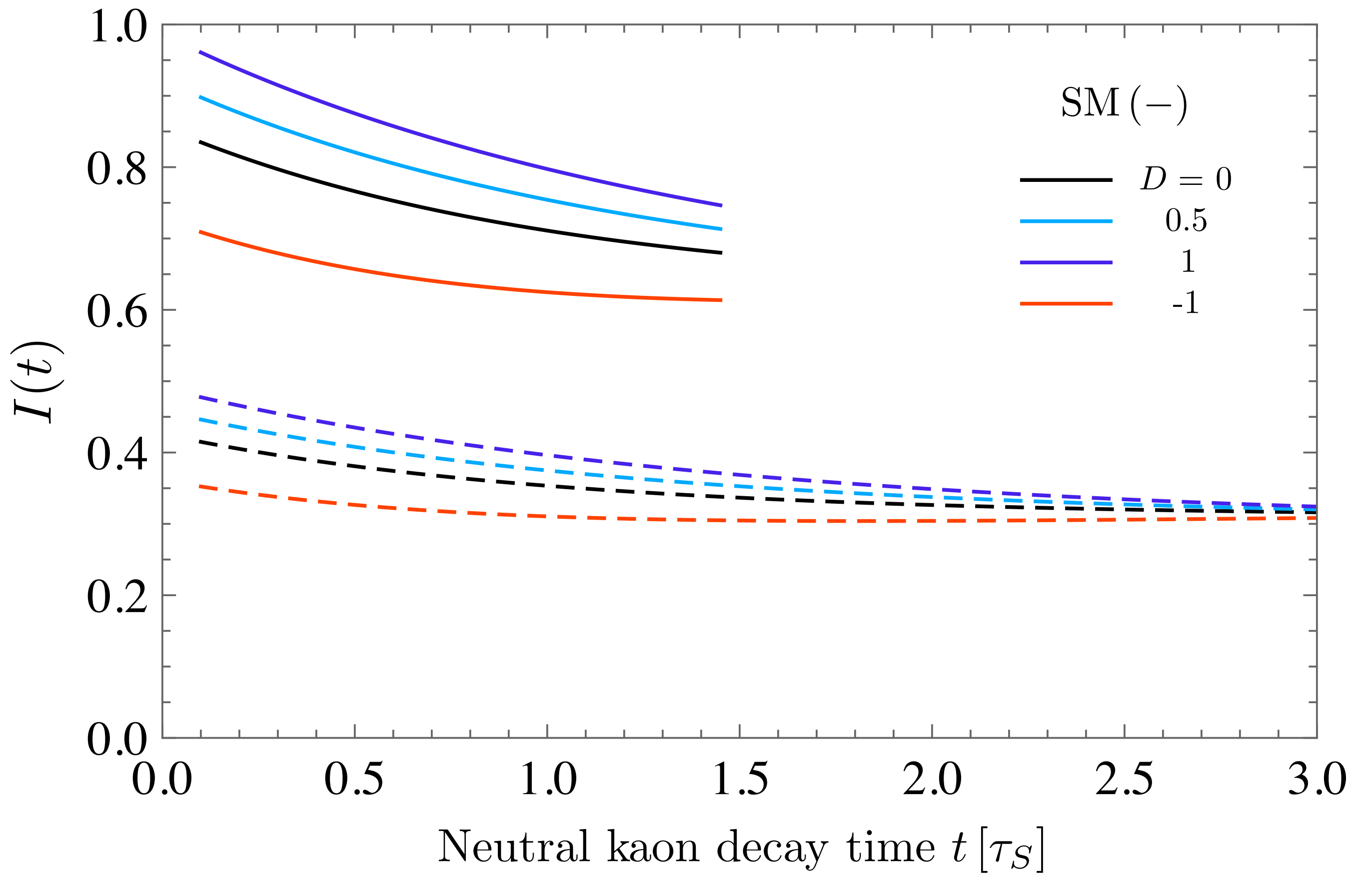
Can we study $K^0(t)$?

GD , Kitahara
1707.06999 PRL

$$pp \to K^0 K^- X$$

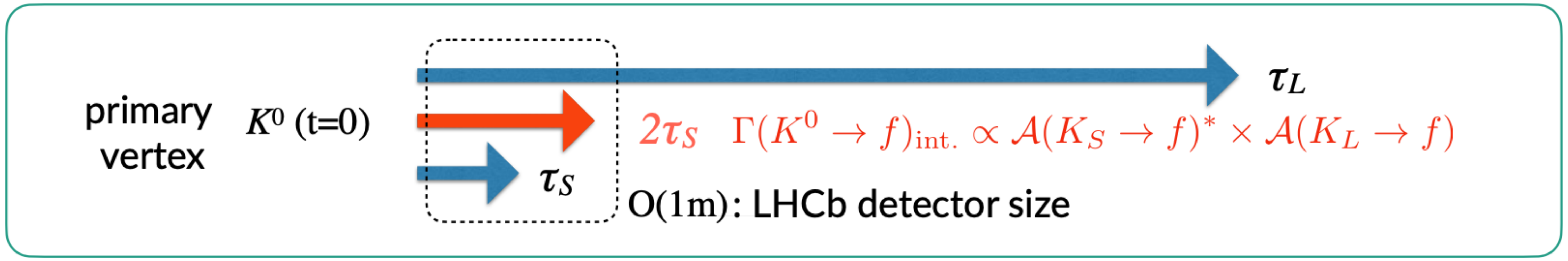
$$pp \to K^{*+} X \to K^0 \pi^+ X$$



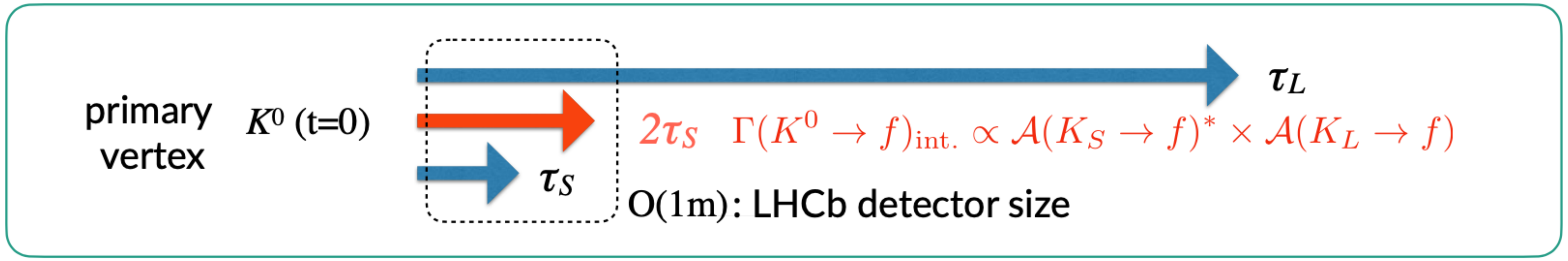


$$|\overset{(-)}{K}{}^0(t)\rangle = \frac{1}{\sqrt{2}(1\pm\bar{\epsilon})}\left[e^{-iH_S t}\left(|K_1\rangle + \bar{\epsilon}|K_2\rangle\right) \pm e^{-iH_L t}\left(|K_2\rangle + \bar{\epsilon}|K_1\rangle\right)\right]$$

$$D = \frac{K^0 - \overline{K}^0}{K^0 + \overline{K}^0}$$



$$D = \frac{K^0 - \overline{K}^0}{K^0 + \overline{K}^0}$$



Interference $\qquad \tau \sim 2\tau_S$

$$I(t) = \frac{1}{2}\left|\mathcal{A}\left(K_S\right)\right|^2 e^{-\Gamma_S t} + \frac{1}{2}\left|\mathcal{A}\left(K_L\right)\right|^2 e^{-\Gamma_L t} + D\,\mathrm{Re}\left[e^{-i\Delta M_K t}\mathcal{A}\left(K_S\right)^* \mathcal{A}\left(K_L\right)\right] e^{-\frac{\Gamma_B+\Gamma_L}{2}t} + \mathcal{O}(\bar{\epsilon})$$

$$\mathcal{H}_{\rm eff} = \frac{G_F \alpha}{\sqrt{2}} \lambda_t y'_{7A} \left(\bar{s}\gamma_\mu \gamma_5 d\right)\left(\bar{\mu}\gamma^\mu \gamma_5 \mu\right) + \text{h.c.}$$

$$\sum_{spin} \mathcal{A}\left(K_1 \to \mu^+\mu^-\right)^* \mathcal{A}\left(K_2 \to \mu^+\mu^-\right)$$

$$= \frac{16 i G_F^4 M_W^4 F_K^2 M_K^2 m_\mu^2 \sin^2\theta_W}{\pi^3} \mathrm{Im}\left[\lambda_t\right] y'_{7A} \Big\{ \underbrace{A^\mu_{L\gamma\gamma}}_{\gamma\gamma \text{ loop}} - 2\pi \sin^2\theta_W \left(\mathrm{Re}\left[\lambda_t\right] y'_{7A} + \mathrm{Re}\left[\lambda_c\right] y_c\right)\Big\}$$

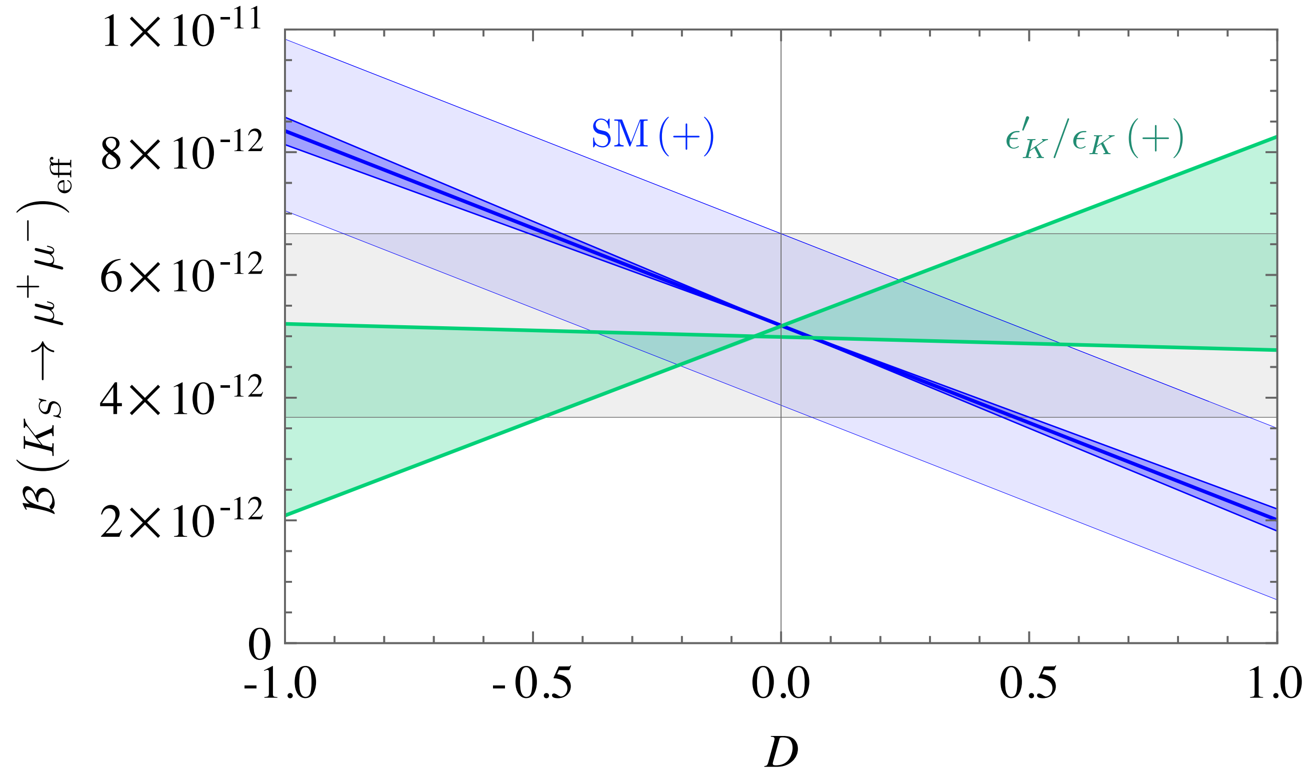
Coefficient is pure imaginary

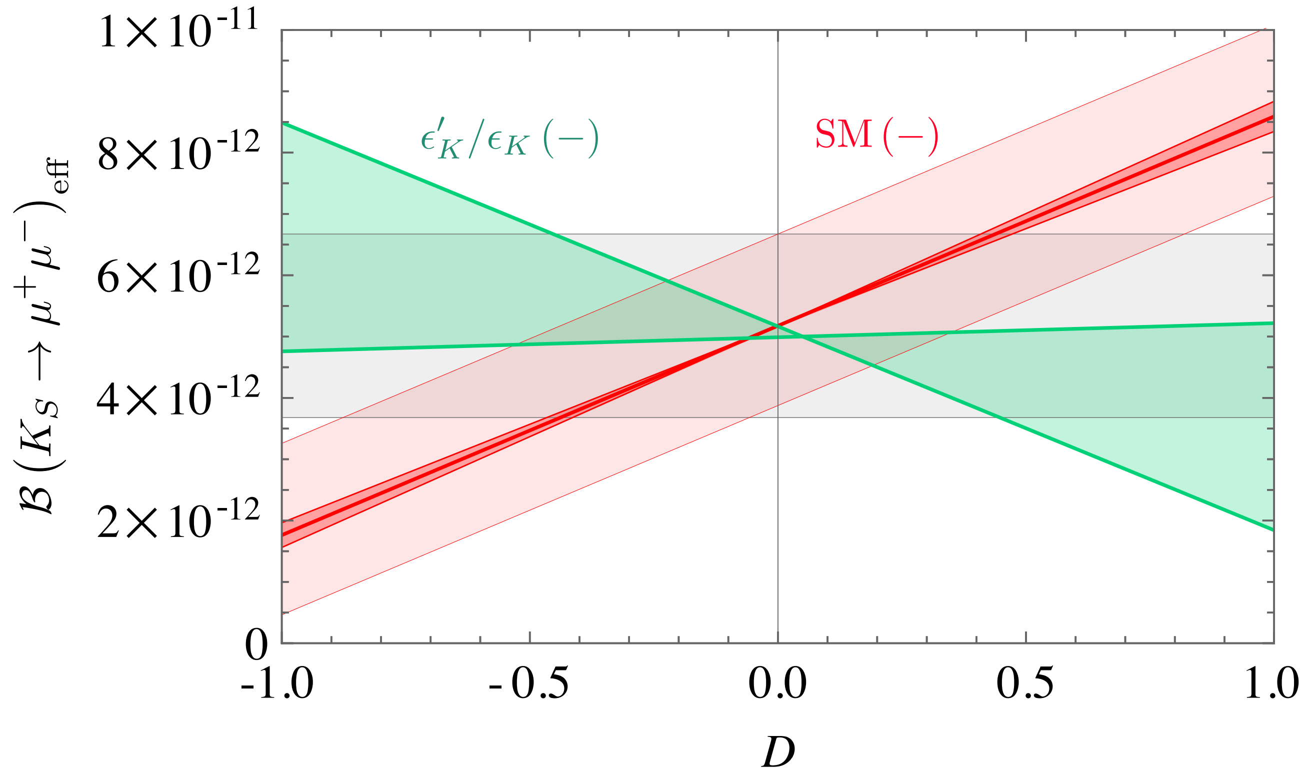
Leading contribution is proportional to $D \times \mathrm{Im}[\lambda_t] \times \mathrm{Im}[A(K_L \to \gamma\gamma \to \mu\mu)]$

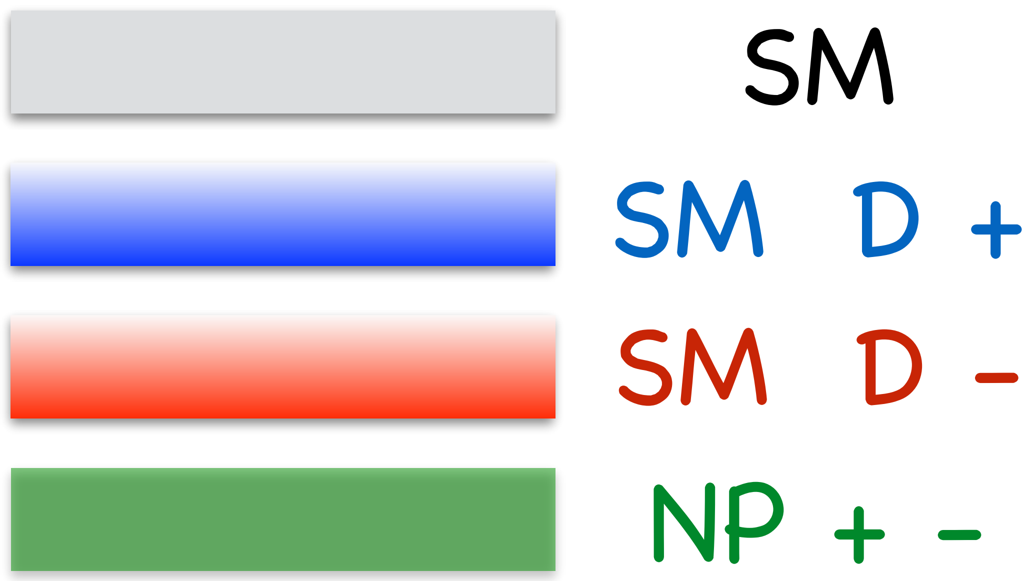
It looks like [weak phase] × [strong phase], namely **the direct CPV in meson decay**

Short distance window

GD , Kitahara
1707.06999 PRL

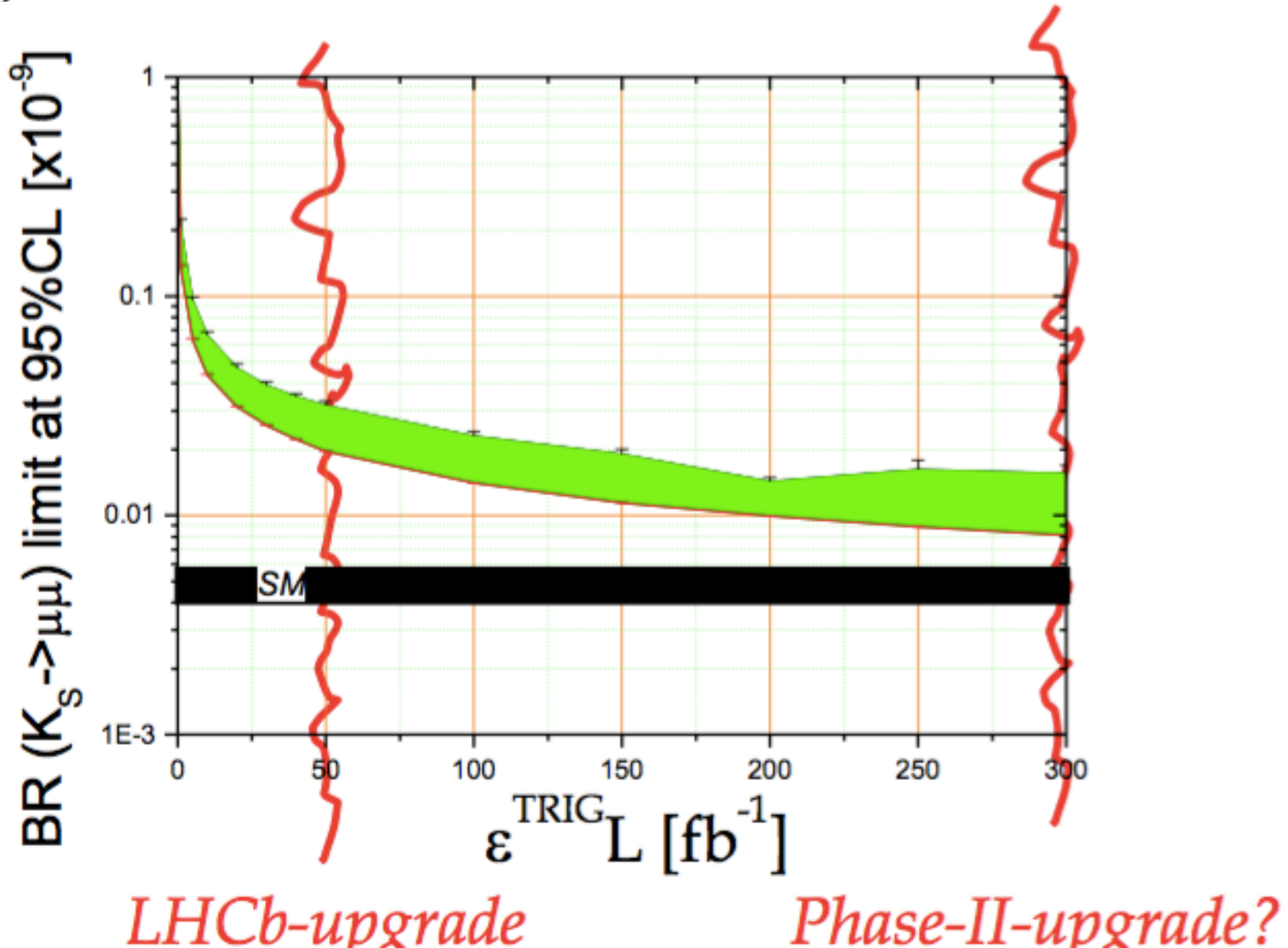






modified Z-coupling model. ϵ'_K/ϵ

$$\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm eff}$$
$$= \tau_S \left[\int_{t_{min}}^{t_{max}} dt \left(\Gamma(K_1) e^{-\Gamma_S t} + \frac{D}{8\pi M_K} \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} \sum \mathrm{Re} \left[e^{-i\Delta M_K t} \mathcal{A}(K_1)^* \mathcal{A}(K_2) \right] e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \right) \varepsilon(t) \right]$$
$$\times \left(\int_{t_{min}}^{t_{max}} dt\, e^{-\Gamma_S t} \varepsilon(t) \right)^{-1},$$



KAONS: RICH POTENTIAL

Rare Kaon decay program at LHCB

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$K_S \to \mu\mu\mu\mu$	–	SM LD $\sim 2 \times 10^{-14}$
$K_S \to ee\mu\mu$	–	$\sim 10^{-11}$
$K_S \to eeee$	–	$\sim 10^{-10}$
$K_S \to \pi^0\mu\mu$	$(2.9 \pm 1.3) \cdot 10^{-9}$	$\sim 10^{-9}$
$K_S \to \pi^+\pi^- e^+e^-$	$(4.79 \pm 0.15) \cdot 10^{-5}$	SM LD $\sim 10^{-5}$
$K_S \to \pi^+\pi^-\mu^+\mu^-$	–	SM LD $\sim 10^{-14}$

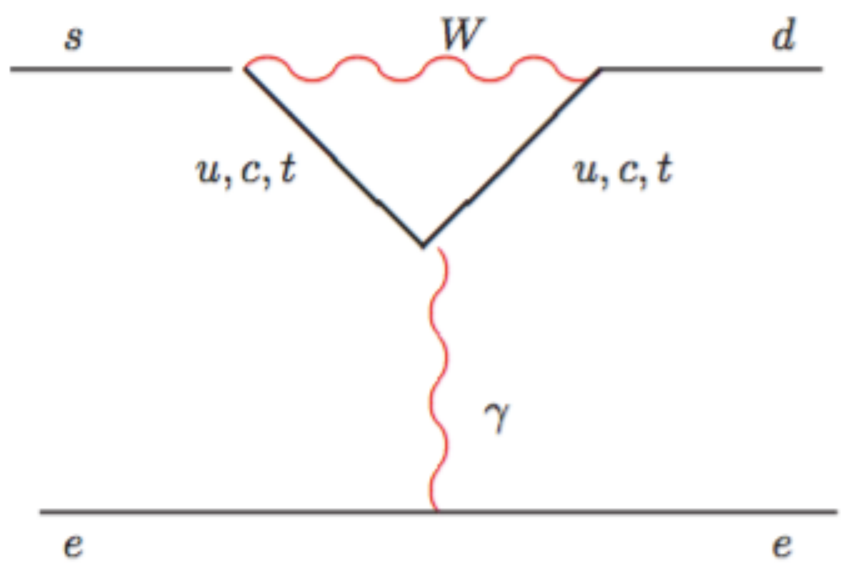
HIKE

$K^+ \to \pi^+\nu\bar{\nu}$	$\sigma_{\mathcal{B}}/\mathcal{B} \sim 5\%$	BSM physics, LFUV
$K^+ \to \pi^+\ell^+\ell^-$	Sub-% precision on form-factors	LFUV
$K^+ \to \pi^-\ell^+\ell^+$, $K^+ \to \pi\mu e$	Sensitivity $\mathcal{O}(10^{-13})$	LFV / LNV
Semileptonic K^+ decays	$\sigma_{\mathcal{B}}/\mathcal{B} \sim 0.1\%$	V_{us}, CKM unitarity
$R_K = \mathcal{B}(K^+ \to e^+\nu)/\mathcal{B}(K^+ \to \mu^+\nu)$	$\sigma(R_K)/R_K \sim \mathcal{O}(0.1\%)$	LFUV
Ancillary K^+ decays (e.g. $K^+ \to \pi^+\gamma\gamma$, $K^+ \to \pi^+\pi^0 e^+e^-$)	% – ‰	Chiral parameters (LECs)
$K_L \to \pi^0\ell^+\ell^-$	$\sigma_{\mathcal{B}}/\mathcal{B} < 20\%$	$\mathrm{Im}\lambda_t$ to 20% precision, BSM physics, LFUV
$K_L \to \mu^+\mu^-$	$\sigma_{\mathcal{B}}/\mathcal{B} \sim 1\%$	Ancillary for $K \to \mu\mu$ physics
$K_L \to \pi^0(\pi^0)\mu^\pm e^\mp$	Sensitivity $\mathcal{O}(10^{-12})$	LFV
Semileptonic K_L decays	$\sigma_{\mathcal{B}}/\mathcal{B} \sim 0.1\%$	V_{us}, CKM unitarity
Ancillary K_L decays (e.g. $K_L \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$)	% – ‰	Chiral parameters (LECs), SM $K_L \to \mu\mu$, $K_L \to \pi^0\ell^+\ell^-$ rates

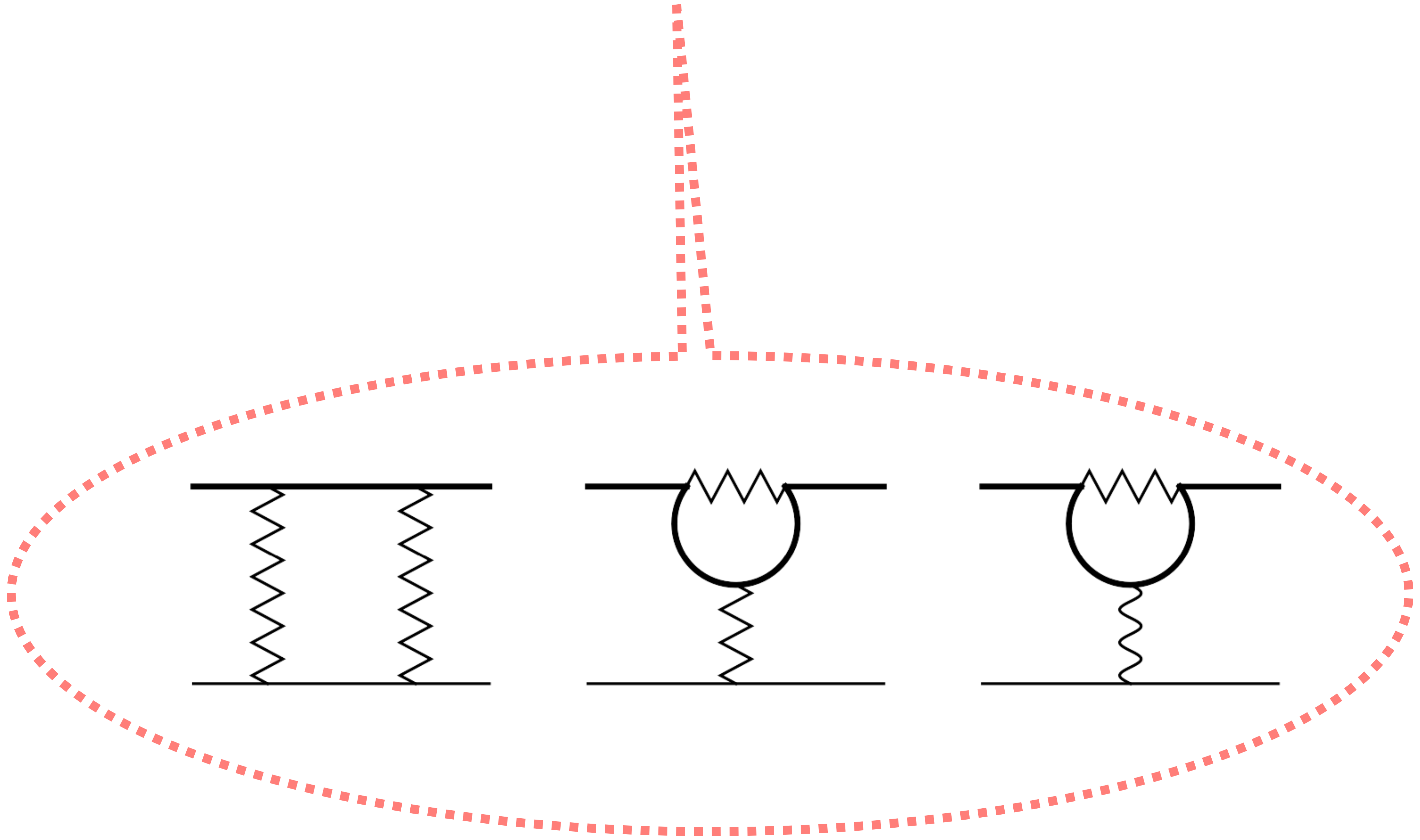
Long Distance enhancement

$$K^{\pm} \to \pi^{\pm} l^{+} l^{-} \qquad K_S \to \pi^0 l^+ l^-$$

Gilman Wise 1980



Short distance



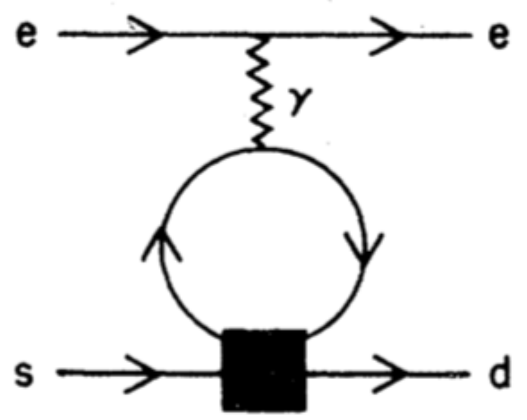


FIG. 1. Diagram contributing to C_7. The black box represents W exchange plus all strong-interaction corrections.

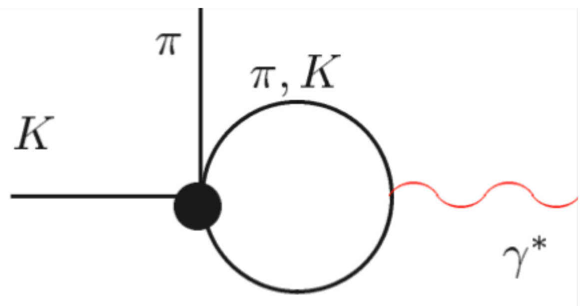
$$\begin{aligned}\mathcal{H}_{\text{eff}} = &-\frac{G_F}{\sqrt{2}} s_1 c_1 c_3 (\bar{s}_\alpha u_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A} \\ &-\frac{G_F}{\sqrt{2}}\frac{2}{9\pi}\frac{e^2}{4\pi}\left[A_c \ln\left(\frac{m_c^2}{\mu^2}\right) + A_t \ln\left(\frac{m_t^2}{\mu^2}\right)\right] \\ &\times (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{e}e)_V \\ &+ \text{H.c.}\end{aligned}$$

$\mathcal{O}(p^4)$ CHPT

$K^+ \to \pi^+ l^+ l^-$

'87 Ecker Pich de Rafael

$k^2 = M_K^2\,,\quad p^2 = M_\pi^2\,,\quad q = k - p\,,\quad \boxed{z = q^2/M_K^2}\,,\quad r_\pi = M_\pi/M_K$

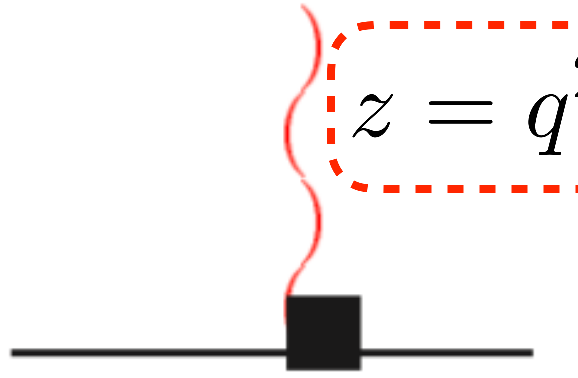


Chiral loop

$\boxed{W_{\pi\pi}(z)}$

Known from K->3pi

+



$\boxed{z = q^2/M_K^2}$

CT

$\boxed{a_+}$

Gauge and Lorentz invariance

$$\frac{\boxed{W(z)}}{(4\pi)^2}\left[z(k+p)^\mu - (1 - r_\pi^2)q^\mu\right]$$

$$\frac{d\Gamma}{dz} \sim \lambda^{3/2}(1, z, r_\pi^2)\sqrt{1 - 4\frac{r_\ell^2}{z}}\left(1 + 2\frac{r_\ell^2}{z}\right)\boxed{|W(z)|^2},$$

KS No pion loop

$\boxed{\mathcal{O}(p^4) \quad a_+}$

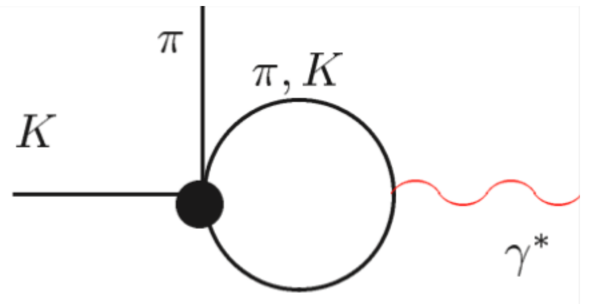
Data: the rate and spectrum not consistent with pheno

$\mathcal{O}(p^4)$ CHPT

'87 Ecker Pich de Rafael

$K_S \to \pi^0 l^+ l^-$

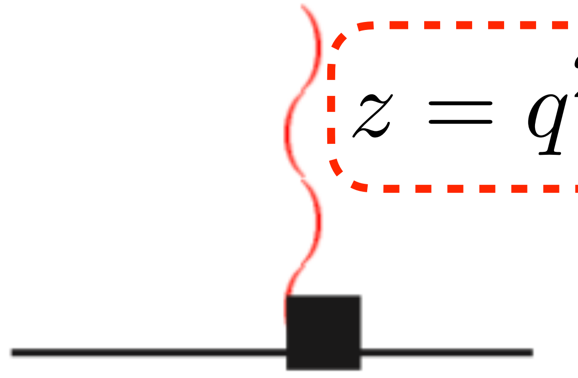
$k^2 = M_K^2\ , \quad p^2 = M_\pi^2\ , \quad q = k - p\ , \quad \boxed{z = q^2/M_K^2}\ , \quad r_\pi = M_\pi/M_K$



Chiral loop

No pion loop

+



CT

a_S

Gauge and Lorentz invariance

$$\frac{W(z)}{(4\pi)^2}\left[z(k+p)^\mu - (1-r_\pi^2)q^\mu\right]$$

$$\frac{d\Gamma}{dz} \sim \lambda^{3/2}(1, z, r_\pi^2)\sqrt{1 - 4\frac{r_\ell^2}{z}}\left(1 + 2\frac{r_\ell^2}{z}\right)|W(z)|^2\ ,$$

KS No pion loop for KS, CT

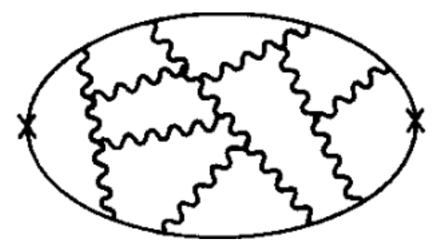
$\mathcal{O}(p^4)$ a_S

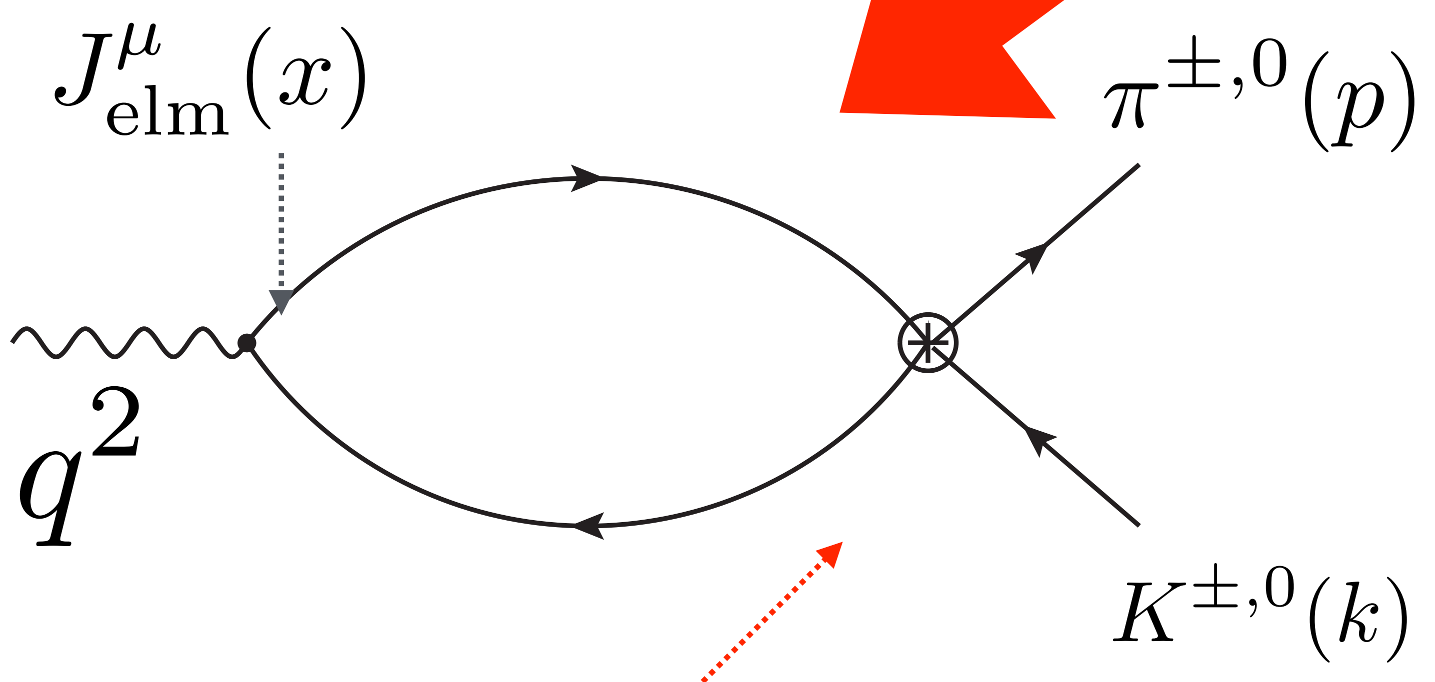
GD,Ecker,Isidori,Portoles ‘98

General consideration on the form factor

Lorentz and gauge invariance tell us on the structure of the amplitude and ff

QCD corrections





$$z = \frac{q^2}{M_K^2}$$

$$i\int d^4x\, e^{iqx}\langle\pi(p)|T\left\{J^\mu_{\rm elm}(x)\mathcal{L}_{\Delta S=1}(0)\right\}|K(k)\rangle = \frac{W(z)}{(4\pi)^2}\left[z(k+p)^\mu-(1-r_\pi^2)q^\mu\right]$$

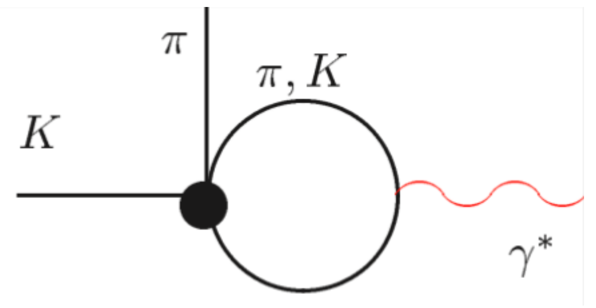
The integral at very short distances QCD quark loop

Pragmatic decision for O(p⁶) ff

GD,Ecker,Isidori,Portoles 98

$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)$$

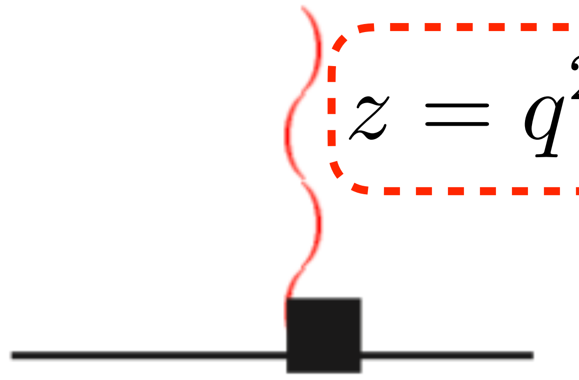
$$\boxed{W_i^{\rm pol}(z) = a_i + b_i z \qquad (i = +, S)}$$



Chiral loop

$$W_{\pi\pi}(z)$$

Known from K->3pi



$z = q^2/M_K^2$

CT

$$a_i + b_i z$$

$$a_i + b_i z$$ Determined expt.

Extremely good agreement with data few %

$$\text{LFUV test } a_+^{\mu\mu} - a_+^{ee}$$

Crivellin et al '16, Nazila et al '22

$K \to \pi\gamma^*$: Experimental situation

exp.	mode	number of events	a_+	b_+
BNL-E865	$K^+ \to \pi^+ e^+ e^-$	$10\,300$	$-0.587(10)$	$-0.655(44)$
NA48/2	$K^\pm \to \pi^\pm e^+ e^-$	$7\,253$	$-0.578(16)$	$-0.779(66)$
NA48/2	$K^\pm \to \pi^\pm \mu^+ \mu^-$	3120	$-0.575(39)$	$-0.813(145)$
NA62	$K^+ \to \pi^+ \mu^+ \mu^-$	27679	$-0.575(13)$	$-0.722(43)$

E. Cortina Gil et al. [NA62 Collaboration], JHEP 11, 011 (2022)

exp.	mode	number of events
NA48/1	$K_S \to \pi^0 e^+ e^-$	7
NA48/1	$K_S \to \pi^0 \mu^+ \mu^-$	6

$a_S = -1.29(3.15) \quad b_S = +17.8(10.6)$

or

$a_S = +1.28(3.16) \quad b_S = -17.6(10.6)$

G. D'Ambrosio, D. Greynat, MK, JHEP 02, 049 (2019)

- Neither the sign of a_S nor the sign of a_S/b_S are fixed by data

Matching long and short distances at order $\mathcal{O}(\alpha_s)$ in the form factors for $K \to \pi\ell^+\ell^-$ #1
Giancarlo D'Ambrosio (INFN, Naples), David Greynat (Unlisted), Marc Knecht (Marseille, CPT) (Jun 7, 2019)
Published in: *Phys.Lett.B* 797 (2019) 134891 • e-Print: 1906.03046 [hep-ph]
pdf DOI cite claim reference search 20 citations

On the amplitudes for the CP-conserving $K^\pm(K_S) \to \pi^\pm(\pi^0)\ell^+\ell^-$ rare decay modes #2
Giancarlo D'Ambrosio (INFN, Naples), David Greynat (Marseille, CPT), Marc Knecht (Marseille, CPT) (Dec 3, 2018)
Published in: *JHEP* 02 (2019) 049 • e-Print: 1812.00735 [hep-ph]
pdf DOI cite claim reference search 32 citations

arXiv:2510.17316 [pdf, ps, other] hep-ph

A dispersive approach to the CP conserving $K \to \pi\ell^+\ell^-$ radiative decays

Authors: Véronique Bernard, Sébastien Descotes-Genon, Marc Knecht, Bachir Moussallam

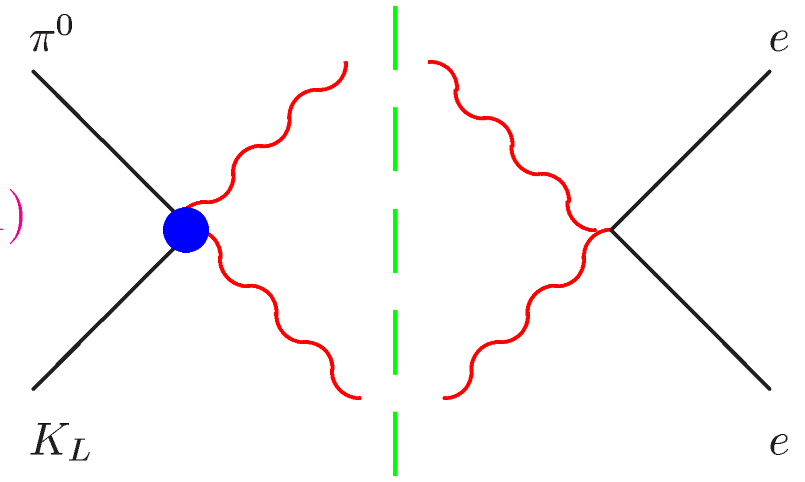
- Description of W_+, W_S based on analyticity/unitarity (2 dominant channels) +assumption of minimality

- $|W_+|^2$ energy dep. can be reproduced which constrains both a_+ and a_S

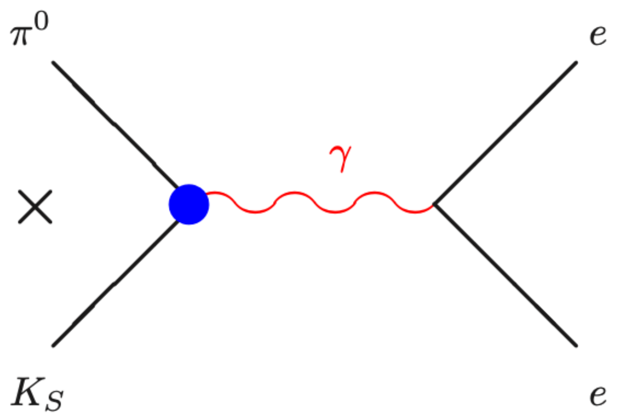
- a_+ must be negative, $a_S > 0$ favoured

Why interesting $K_L \to \pi^0 l^+ l^-$?

KOTO II , european strategy group

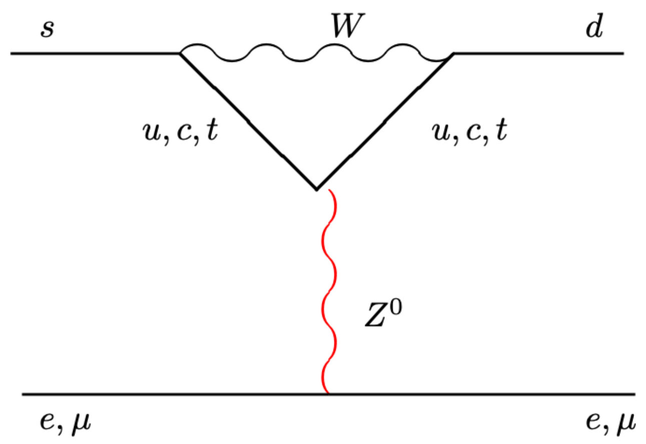
1) 

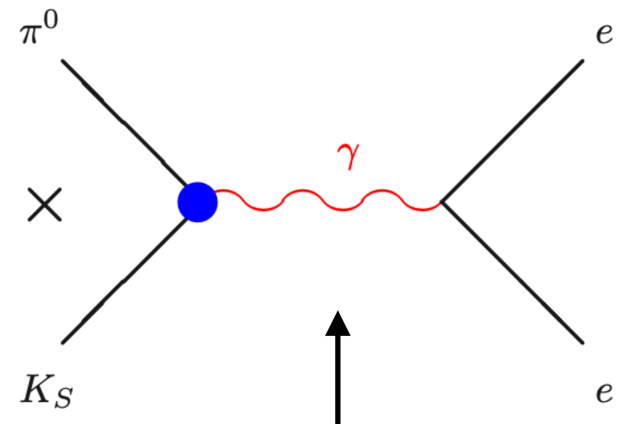
Negligible for e, calculable for μ

2) $\varepsilon\ \times$ 

$\sim \epsilon(a_S + b_S z)$

form factor

3) 

$\varepsilon\ \times$ 

$$\mathrm{BR}(K_L \to \pi^0 \ell\bar{\ell}) = \left(C^\ell_{\rm dir} \pm C^\ell_{\rm int}|a_S| + C^\ell_{\rm mix}|a_S|^2 + C^\ell_{\gamma\gamma}\right) \cdot 10^{-12}$$

$$|a_S| \;=\; 1.20 \pm 0.20$$

	$C^\ell_{\rm dir}$	$C^\ell_{\rm int}$	$C^\ell_{\rm mix}$	$C^\ell_{\gamma\gamma}$
$\ell = e$	$(4.62 \pm 0.24)(w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3) w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)(w_{7V}^2 + 2.32 w_{7A}^2)$	$(2.63 \pm 0.06) w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 e\bar{e}) = 3.46^{+0.92}_{-0.80}\,\left(1.55^{+0.60}_{-0.48}\right) \times 10^{-11}$$

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 \mu\bar{\mu}) = 1.38^{+0.27}_{-0.25}\,\left(0.94^{+0.21}_{-0.20}\right) \times 10^{-11}$$

$$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 e\bar{e}) < 28 \times 10^{-11} \qquad \text{at 90\% CL}$$

$$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 \mu\bar{\mu}) < 38 \times 10^{-11} \qquad \text{at 90\% CL}$$

Buchalla et al 03, Isidori et al 06

Nazila et al 22, Knecht GD 24 , Hoferichter et al.24

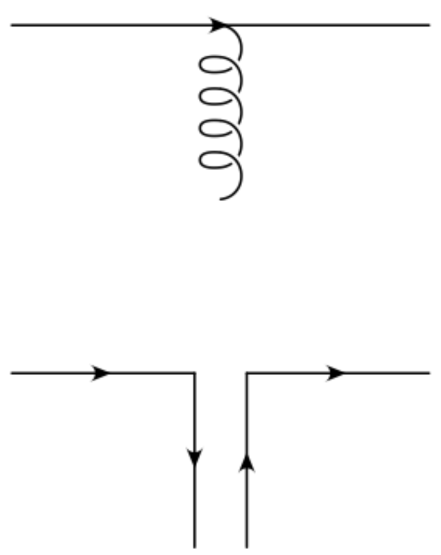
$$w_{7A,7V} = \mathrm{Im}\,(\lambda_t y_{7A,7V})\,/\,\mathrm{Im}\,\lambda_t$$

LARGE N QCD

't Hooft, Witten, Coleman

$$\widetilde{\mathcal{L}}_{QCD} = \sum_{q=u,d,s} \bar{q}\gamma^{\mu}\left(i\partial_{\mu} - g_s \frac{\lambda_a}{2} G^a_{\mu}\right) q - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

QCD properties OK: asympt. Freedom $\mu\frac{dg}{d\mu} = -b_0 \frac{g^3}{16\pi^2} + \mathcal{O}\left(g^5\right), \quad b_0 = \frac{11}{3}N - \frac{2}{3}N_F$ Confinement assumed



$$8 + 1 = 3 + \bar{3}$$

Large N => correct expansion parameter? Geometric interpretation of this expansion: planar diagrams

Question in Large N: The leading order term in N of your amplitude

't Hooft model QCD_2 — Exactly solved

Gross Neveu model — SSB+ mass Gap

Successes:

Zweig's rule (suppression of gluon exchange decays)

VMD, computability..

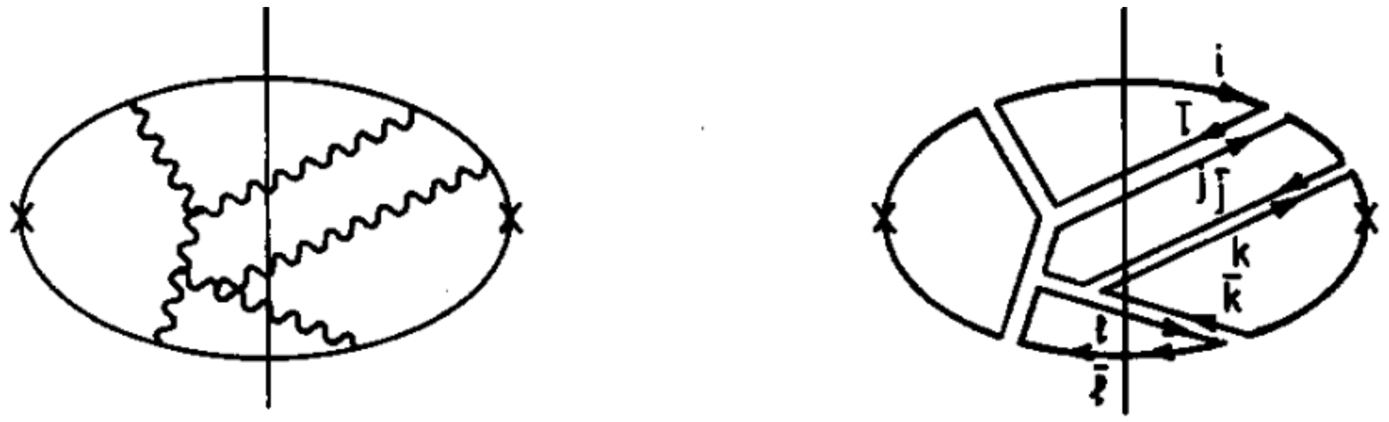
Witten '79 Large N QCD

Section 3

- Meson are free, stable, non-interacting, the number of states infinite

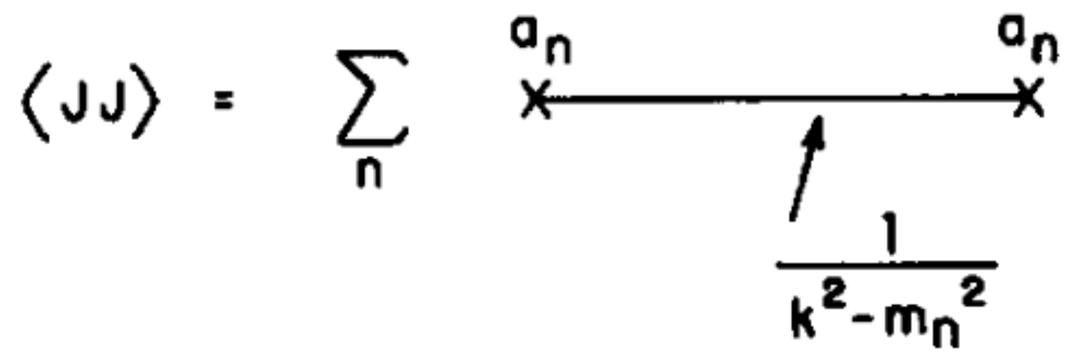
- Meson decay amplitudes $\mathcal{O}(1/\sqrt{N})$

- One meson exchange leading (see two point function with J bilinear)



$$\langle J(q)J(-q)\rangle = \sum_n \frac{a_n^2}{q^2 - m_n^2}\,.$$

$$\langle J(q)J(-q)\rangle \underset{q_2\to\infty}{\sim} \log q^2$$



ig. 21. $\langle JJ\rangle$ represented as a sum of one-meson poles.

$K_S \to \pi^0 l^+ l^-$: determination of the form factor

$$i\int d^4x e^{iqx}\langle \pi(p)|T\left\{J^\mu_{\rm elm}(x)\mathcal{L}_{\Delta S=1}(0)\right\}|K(k)\rangle = \frac{W(z)}{(4\pi)^2}\left[z(k+p)^\mu - (1-r_\pi^2)q^\mu\right]$$

$$k^2 = M_K^2\ , \quad p^2 = M_\pi^2\ , \quad q = k-p\ , \quad z = q^2/M_K^2\ , \quad r_\pi = M_\pi/M_K\ ,$$

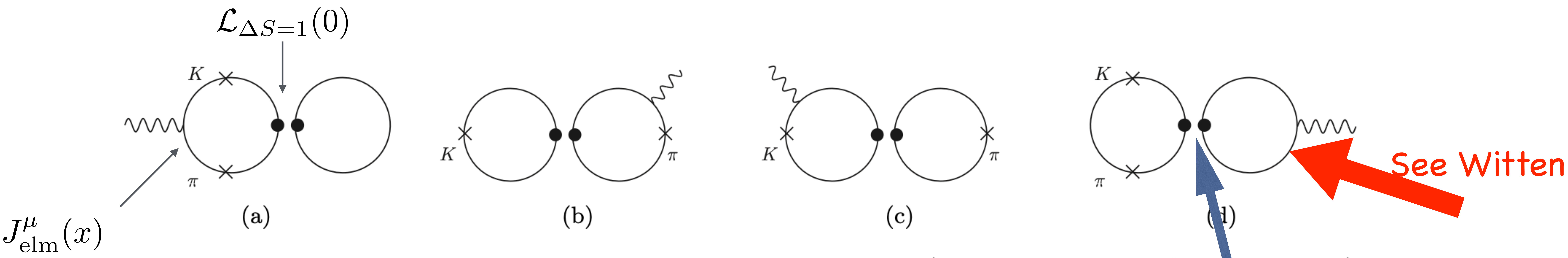


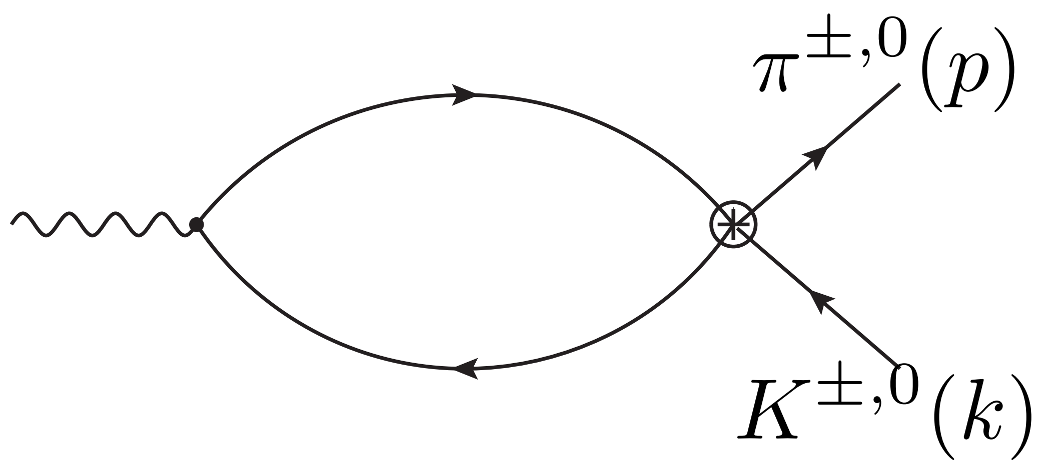
Figure 1: Potential diagrams contributing at leading order in large N_c (all at the order $(N_c)^2/(\sqrt{N_c})^2 = N_c$). The dots represent the effective $\Delta S = 1$ operators.

$$\boxed{\mathcal{L}^{|\Delta S|=1}_{\rm non-lept} = -\frac{G_{\rm F}}{\sqrt{2}}V_{us}V_{ud}\sum_{I=1}^{6} C_I(\nu)Q_I(\nu) + \text{H.c}}$$

$$\boxed{Q_1 = (\bar{s}^i u_j)_{V-A}(\bar{u}^j d_i)_{V-A},\ Q_2 = (\bar{s}^i u_i)_{V-A}(\bar{u}^j d_j)_{V-A}}$$

Large Nc results for $K_S \to \pi^0 l^+ l^-$

GD Knecht, Neshatpour '24



$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)$$

$$\boxed{W_i^{\rm pol}(z) = a_i + b_i z \qquad (i = +, S)}$$

$${\rm Br}(K_S \to \pi^0 e^+ e^-) = 5.1(1.7) \cdot 10^{-9}$$

TH $${\rm Br}(K_S \to \pi^0 e^+ e^-)|_{m_{ee} > 165 \ {\rm MeV}} = 2.9(1.0) \cdot 10^{-9}$$

NA48/1 $${\rm Br}(K_S \to \pi^0 e^+ e^-)|_{m_{ee} > 165 \ {\rm MeV}} = (3.0^{+1.5}_{-1.2} \pm 0.2) \cdot 10^{-9}$$

Large Nc predictions =>Wilson coefficients +hadr. parameters

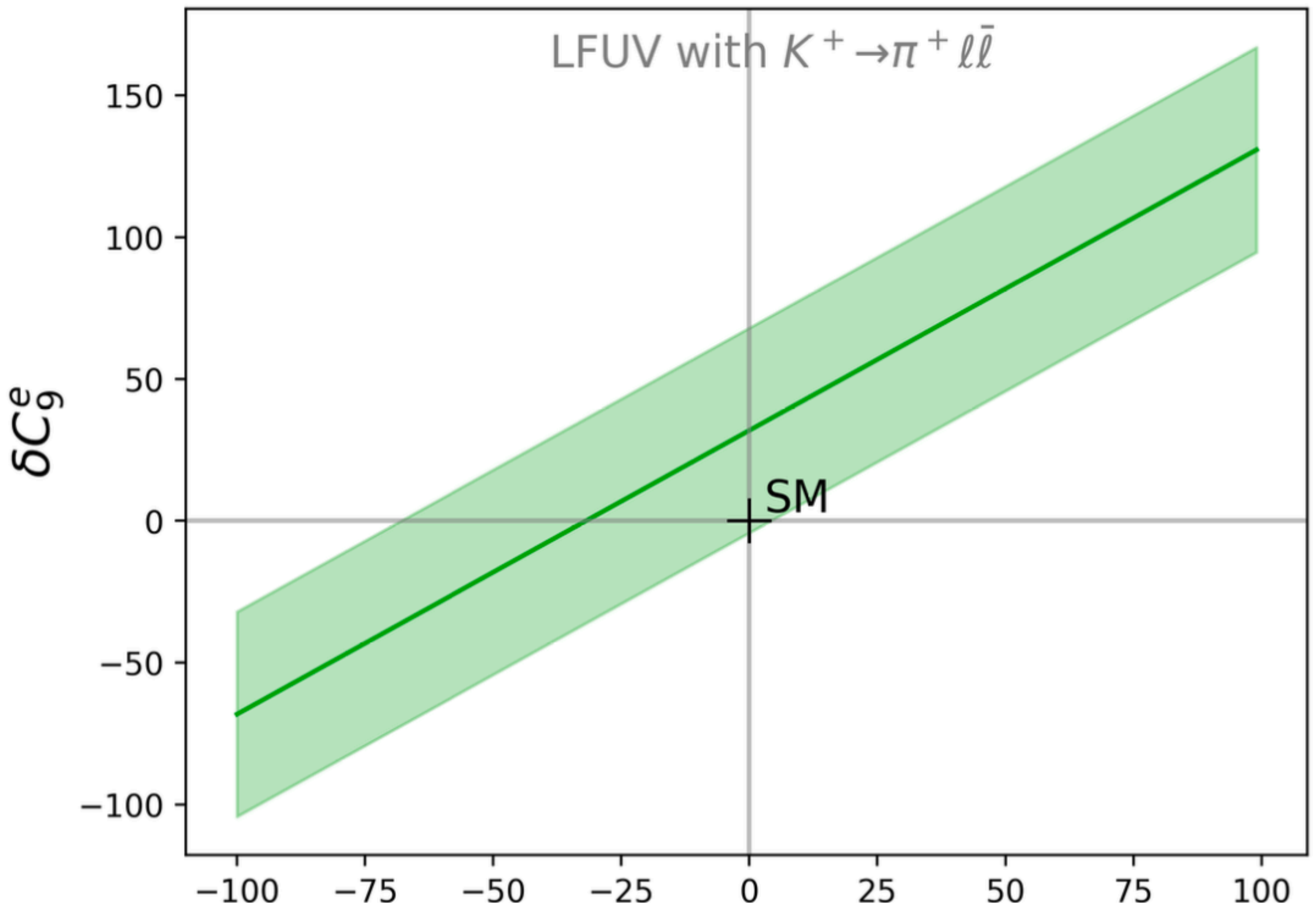
TH

$${\rm Br}(K_S \to \pi^0 \mu^+ \mu^-) = 1.3(0.4) \cdot 10^{-9}$$

$a^{ee} \neq a^{\mu\mu}$ indicates LFUV NP: $$a_+^{\mu\mu} - a_+^{ee} = -\sqrt{2}\,\mathrm{Re}\,[V_{td}V_{ts}^*(\boxed{C_9^\mu - C_9^e})]$$

[Crivellin, D'Ambrosio, Hoferichter, Tunstall '16

Channel	a_+	b_+	Reference
ee	-0.561 ± 0.009	-0.694 ± 0.040	E865 '99 and NA48/2 '09 comb. [D'Ambrosio, Greynat, Knecht '18]
$\mu\mu$	-0.575 ± 0.013	-0.722 ± 0.043	NA62 Coll. '22



Exploring scalar contributions with $K^+ \to \pi^+ \ell^+ \ell^-$

Phys. Lett. B 855 (2024) 138824

GD Nazila Iyer Neshatpour

Let's consider $K^+ \to \pi^+ \ell\ell$

$$\mathcal{M} = \frac{\alpha G_F}{4\pi} \underbrace{f_V}_{\text{SM}}(z) P^\mu \bar{\ell}\gamma_\mu \ell + G_F M_K \underbrace{f_S}_{\text{NP}} \bar{\ell}\ell$$

[Gao. '03, Chen et al. '03]

$$\frac{d^2\Gamma}{dz\, d\cos\theta} = \frac{G_F^2 M_K^5}{2^8\pi^3}\beta_\ell\, \lambda^{1/2}(z) \times \left\{ |f_V|^2 \frac{\alpha^2}{16\pi^2}\lambda(z)(1-\beta_\ell^2\cos^2\theta) + |f_S|^2\, z\beta_\ell^2 \right.$$

$$\left. + \mathrm{Re}(f_V^* f_S)\frac{\alpha\, r_\ell}{\pi}\beta_\ell \lambda^{1/2}(z)\cos\theta \right\},$$

$$r_\ell = m_\ell / M_K$$

$$A_{\mathrm{FB}}(z) = \frac{\alpha G_F^2 M_K^5}{2^8\pi^4} r_\ell \beta_\ell^2(z)\lambda(z)\mathrm{Re}\,(f_V^* f_S) \Big/ \left(\frac{d\Gamma(z)}{dz}\right)$$

Difficult to do this for the electron mode

Exploring scalar contributions with $K^+ \to \pi^+ \ell^+ \ell^-$

Phys. Lett. B 855 (2024) 138824

GD Nazila Iyer Neshatpour

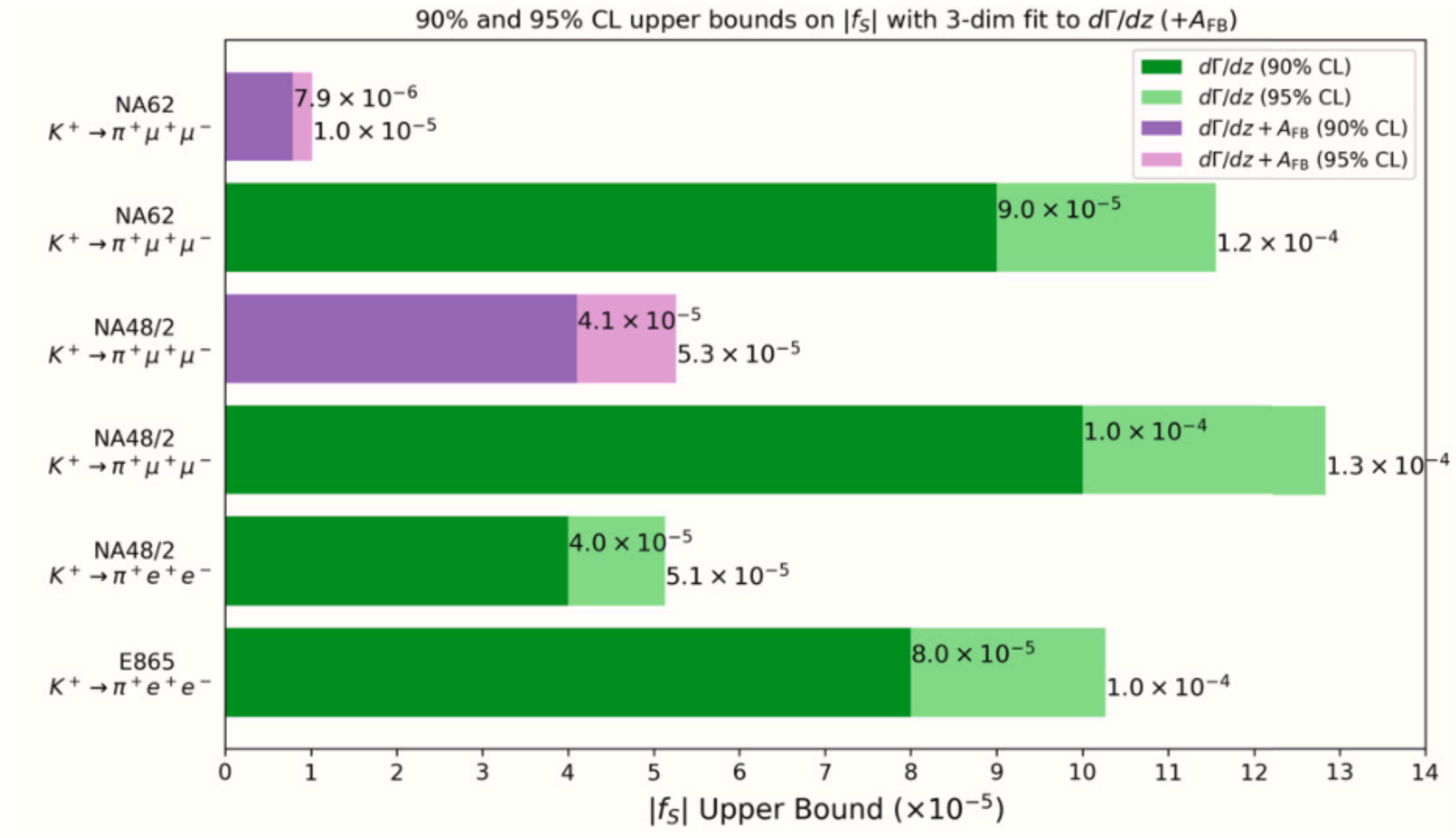


Table 2
Upper bound for $|f_S|$ at 90% CL, from the three parameter fit to a, b and f_S using various datasets. For the relevant inputs regarding the theoretical calculations we have considered PDG 2022 [42], and the external parameters α_+, β_+ are taken from [43], in agreement with NA62 [21].

	$d\Gamma/dz$	$d\Gamma/dz + A_{FB}$
$K \to \pi ee$	$\lvert f_S \rvert <$	$\lvert f_S \rvert <$
E865	8.0×10^{-5}	–
NA48/2	4.0×10^{-5}	–
$K \to \pi\mu\mu$	$\lvert f_S \rvert <$	$\lvert f_S \rvert <$
NA48/2	10.0×10^{-5}	4.1×10^{-5}
NA62	9.0×10^{-5}	7.9×10^{-6}

Conclusions

- We have focused maybe too much on

$$\Delta I = 1/2 \text{ rule}$$

$$K^+ \to \pi^+ l^+ l^-$$

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma, K\to\pi l^+ l^-} + \cdots$$

π	2π	3π	N_i
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N^r_{14} - N^r_{15}$ $K^+ \to \pi^+ l^+ l^-$
$\pi^0\gamma^*$ (S)	$\pi^0\pi^0\gamma^*$ (L)		$2N^r_{14} + N^r_{15}$ $K_S \to \pi^0 l^+ l^-$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$
	$\pi^+\pi^-\gamma\gamma$ (S)		"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
	$\pi^+\pi^-\gamma$ (S)	$\pi^+\pi^0\pi^0\gamma$	"
		$\pi^+\pi^-\pi^0\gamma$ (L)	"
		$\pi^+\pi^-\pi^0\gamma$ (S)	$7(N^r_{14} - N^r_{16}) + 5(N^r_{15} + N_{17})$
	$\pi^+\pi^-\gamma^*$ (L)		$N^r_{14} - N^r_{15} - 3(N^r_{16} - N_{17})$
	$\pi^+\pi^-\gamma^*$ (S)		$N^r_{14} - N^r_{15} - 3(N^r_{16} + N_{17})$
	$\pi^+\pi^0\gamma^*$		$N^r_{14} + 2N^r_{15} - 3(N^r_{16} - N_{17})$
	$\pi^+\pi^-\gamma$ (L)	$\pi^+\pi^-\pi^0\gamma$ (S)	$N_{29} + N_{31}$
		$\pi^+\pi^+\pi^-\gamma$	"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$
		$\pi^+\pi^-\pi^0\gamma$ (S)	$5N_{29} - N_{30} + 2N_{31}$
		$\pi^+\pi^-\pi^0\gamma$ (L)	$6N_{28} + 3N_{29} - 5N_{30}$

- $K_{S,L} \to \mu^+\mu^-(e^+e^-)$ 3-point function

LHCB