Short- vs Long-Distance Dynamics in $b \to s\bar{\ell}\ell$ Decays

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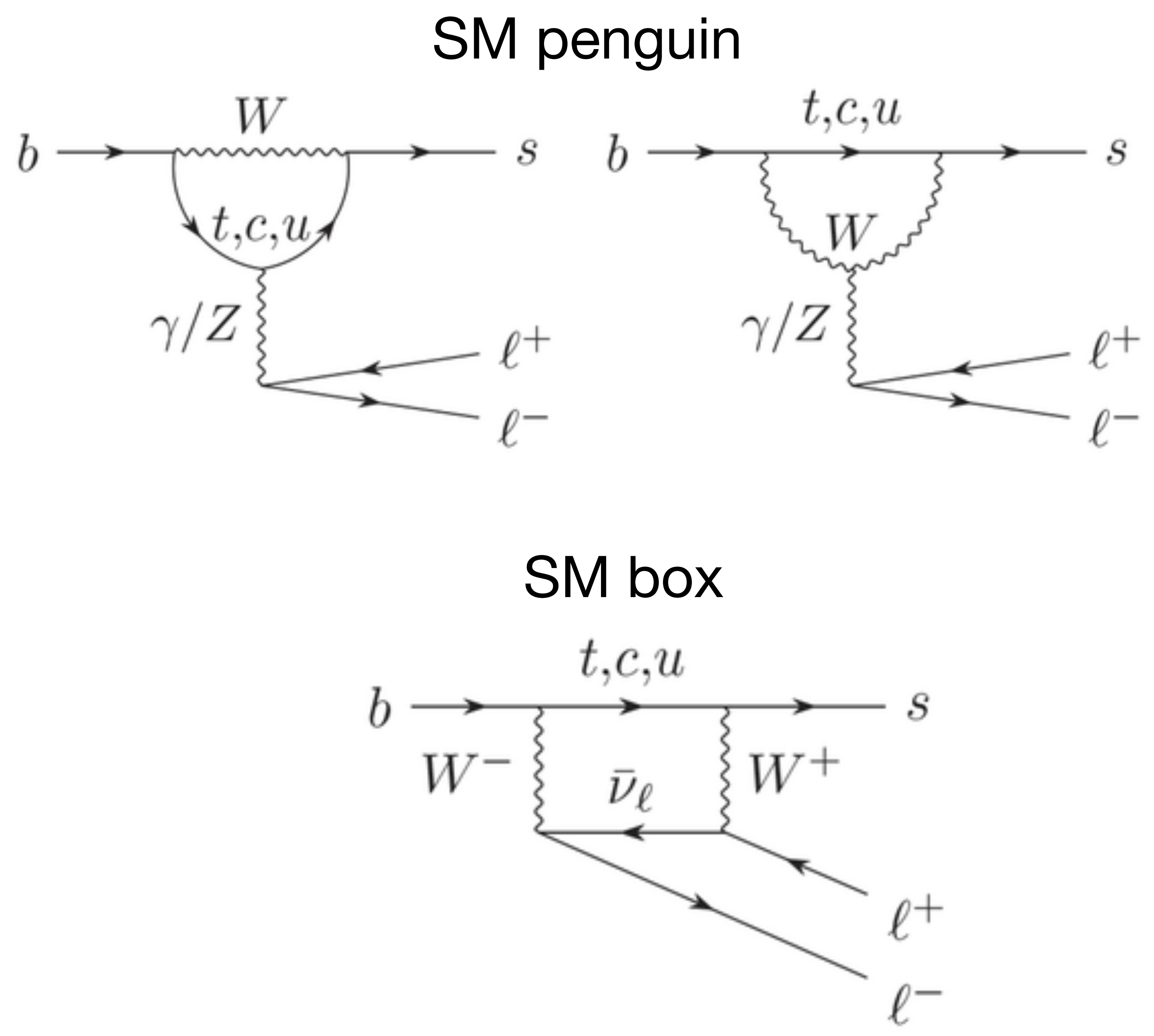
Based on 2507.17824, 2405.17551, 2401.18007
(with Gino Isidori, Zachary Polonsky, Marzia Bordone, Sandro Maechler)

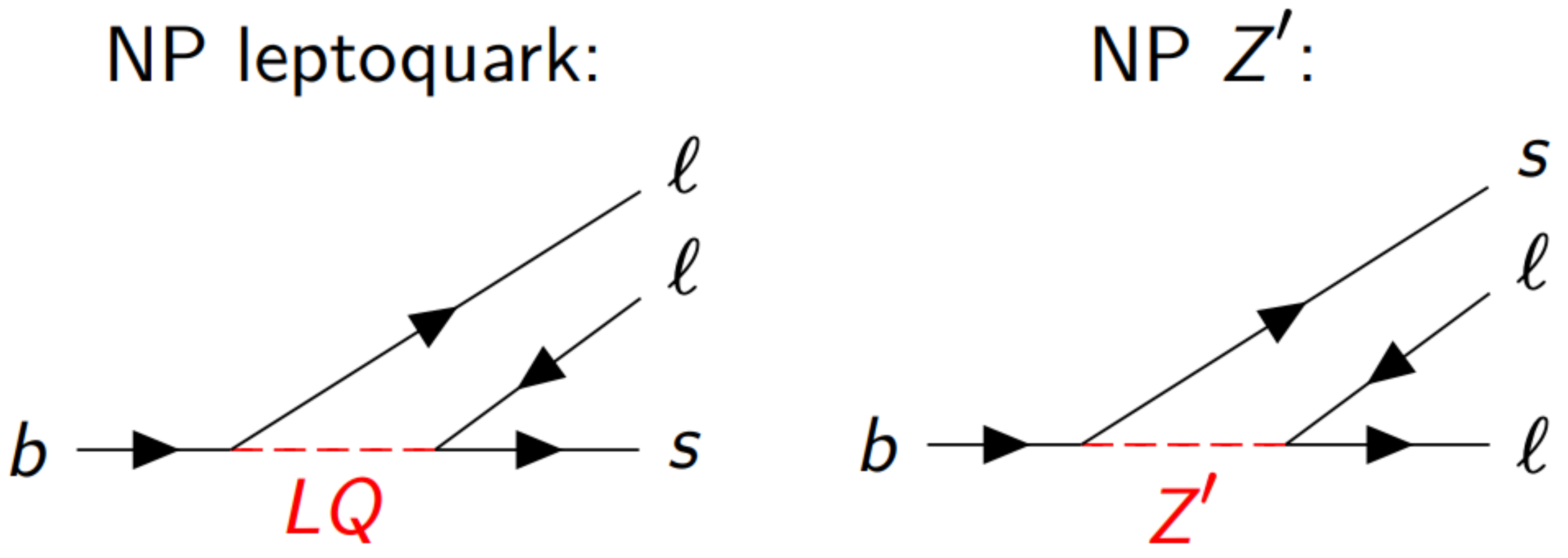
IJCLab, Orsay, 23.10.2025

Universität Zürich[UZH]

Introduction

- The **absence of direct signs** of new physics at the LHC strengthens the importance of **indirect searches** via observables that are suppressed in the Standard Model.
- **Flavor-changing neutral currents** (FCNCs), such as $b \to s\bar{\ell}\ell$, are sensitive indirect probes of BSM physics.

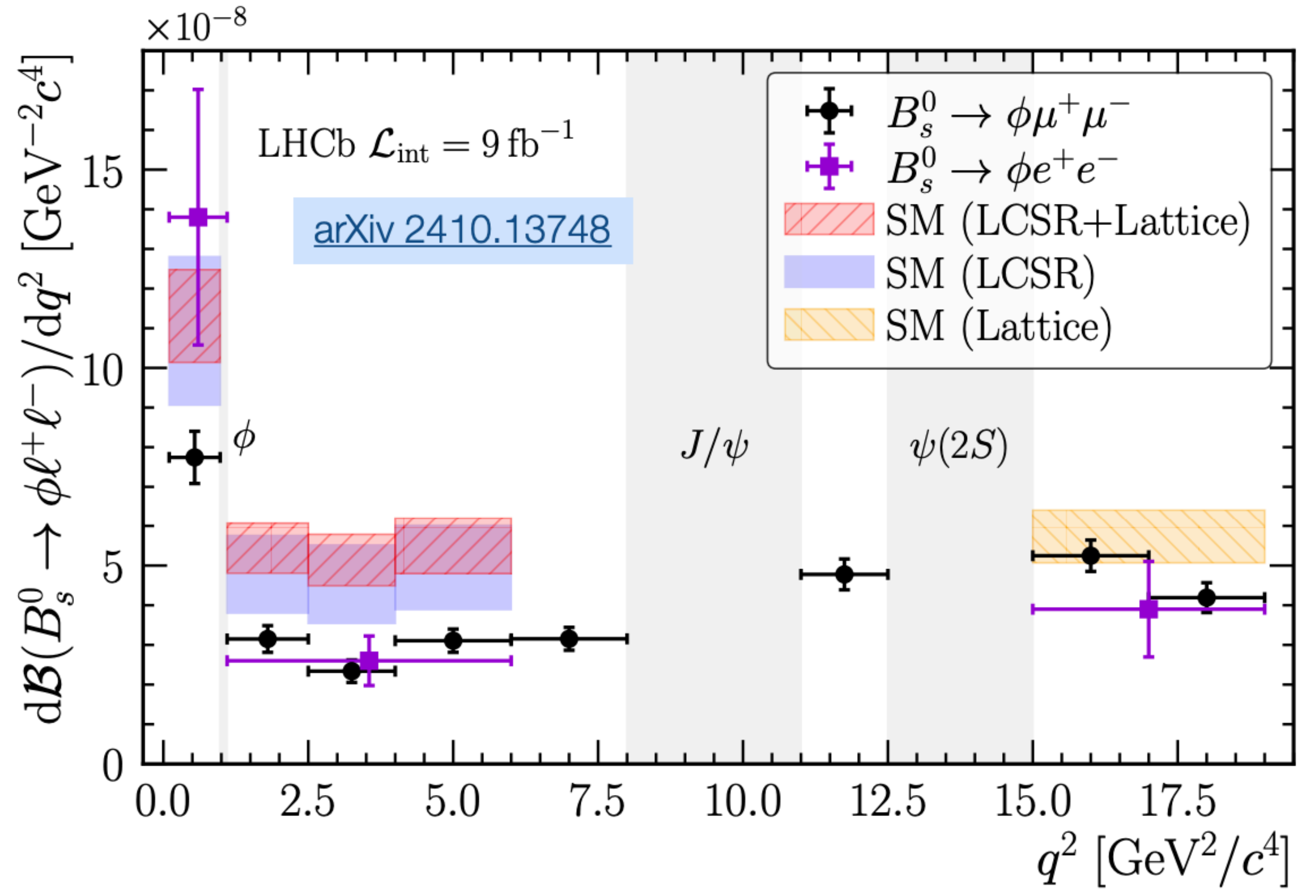




Experimental status

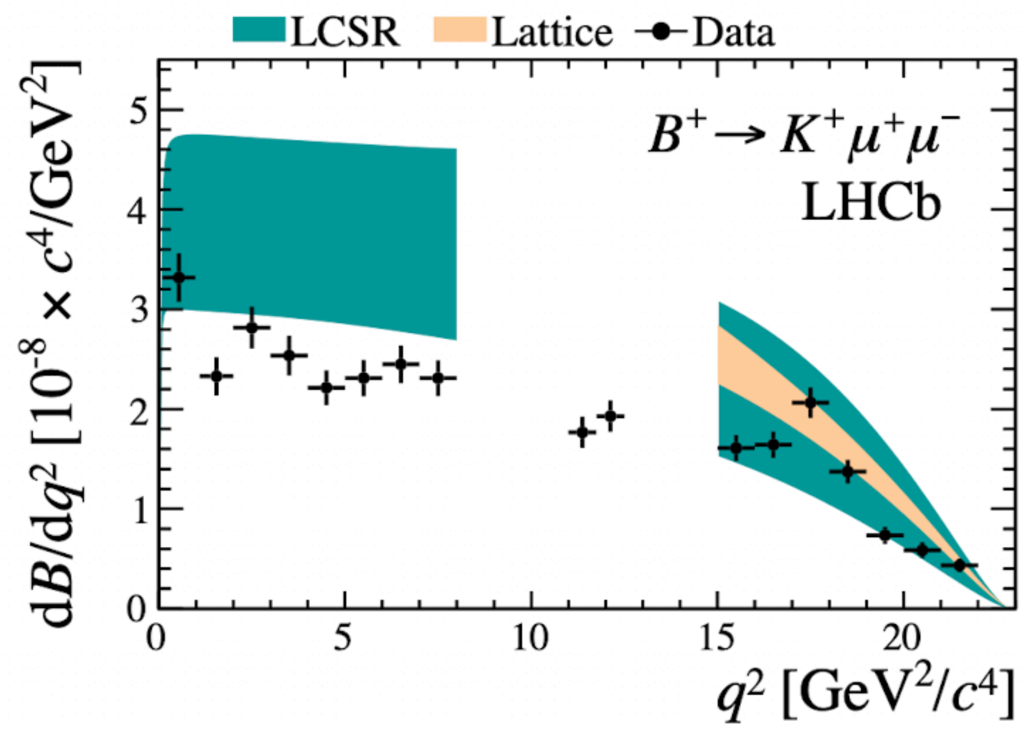
- Long-standing **tension** in measurements of **branching ratios** and **angular observables** in decays mediated by $b \to s\bar{\ell}\ell$:

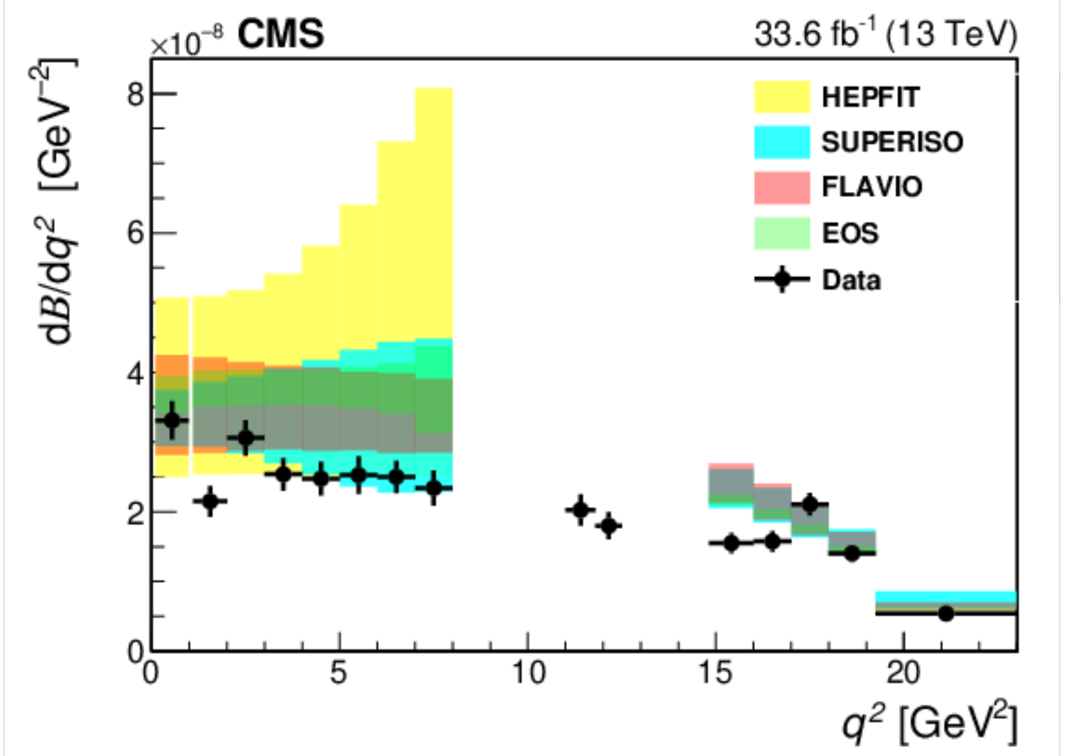
$B_s \to \phi\bar{\mu}\mu$



[LHCb, 2410.13748]

$B \to K\bar{\mu}\mu$

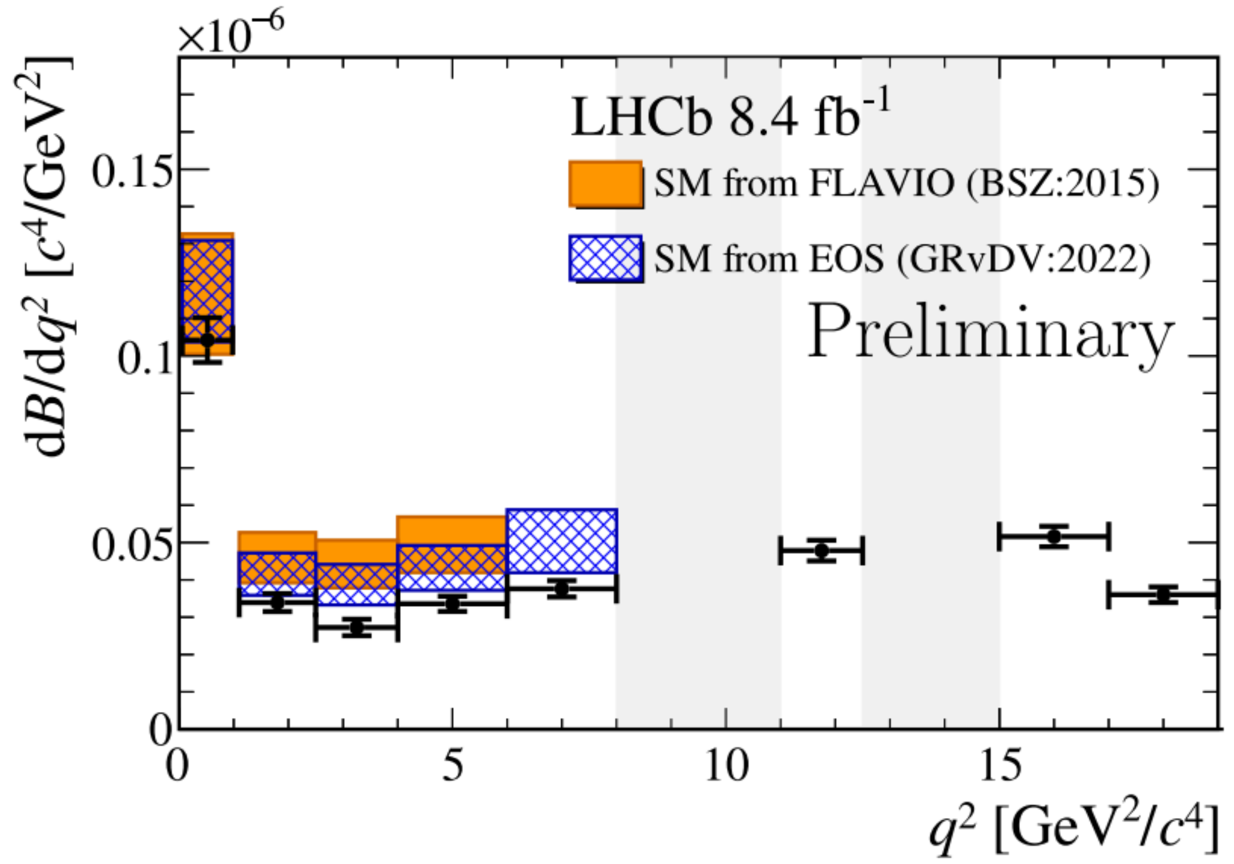


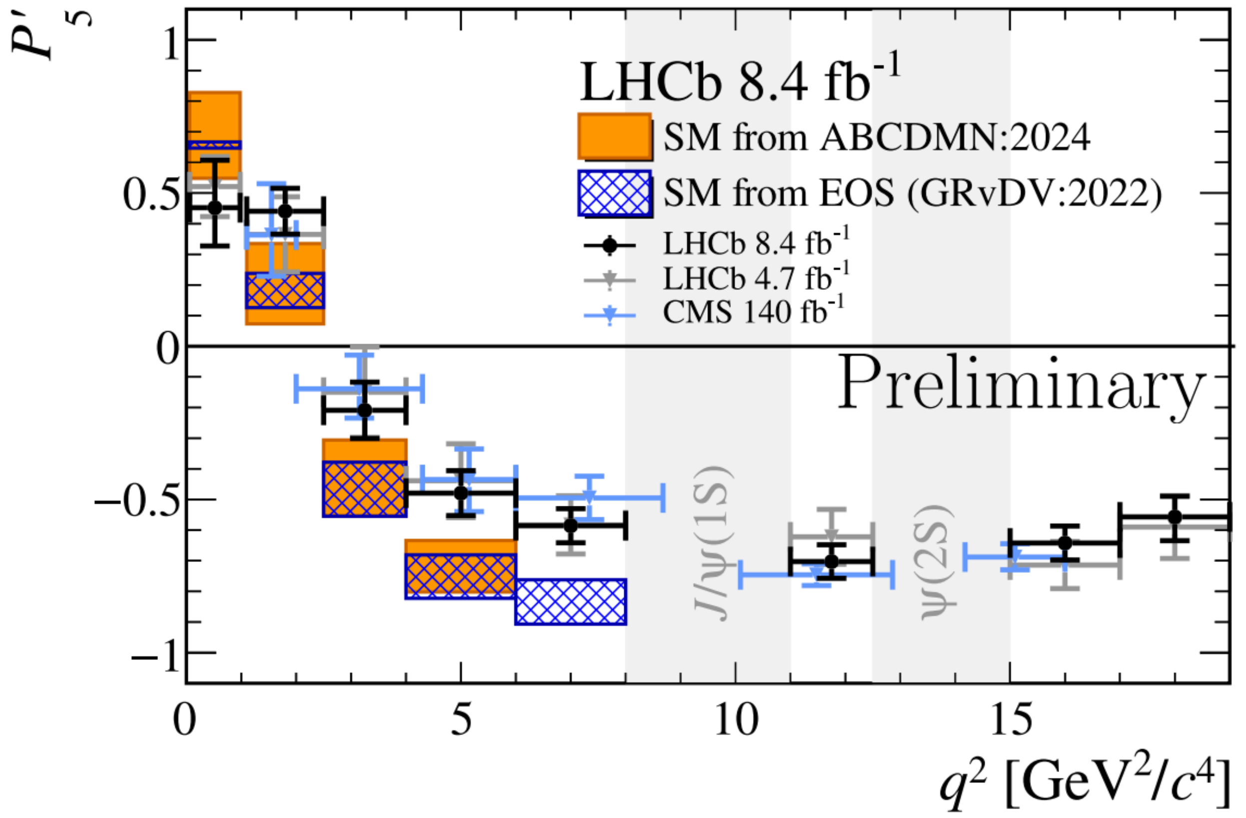


[JHEP 06 (2014) 133] [CMS: 2401.07090]

NEW!

$B \to K^*\bar{\mu}\mu$





https://indico.cern.ch/event/1584446/

Form factors

- **Effective description of $b \to s\bar{\ell}\ell$ decays below the EW scale:**

$$\mathscr{L} = \mathscr{L}^{[N_f=5]}_{\text{QCD+QED}} + \frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts} \sum_{i=1}^{10} C_i \mathcal{O}_i$$

$$\mathcal{O}_1 = \frac{4\pi}{\alpha_e}(\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L) \qquad \mathcal{O}_2 = \frac{4\pi}{\alpha_e}(\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

$$\mathcal{O}_3 = \frac{4\pi}{\alpha_e}(\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q}_L \gamma^\mu q_L) \qquad \mathcal{O}_4 = \frac{4\pi}{\alpha_e}(\bar{s}^a_L \gamma^\mu T^a b^b_L) \sum_q (\bar{q}_L{}^b \gamma^\mu T^a q^a_L)$$

$$\mathcal{O}_5 = \frac{4\pi}{\alpha_e}(\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_q (\bar{q}_L \gamma^\mu \gamma^\nu \gamma^\rho q_R) \qquad \mathcal{O}_6 = \frac{4\pi}{\alpha_e}(\bar{s}^a_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b^b_L) \sum_q (\bar{q}_L{}^b \gamma^\mu \gamma^\nu \gamma^\rho T^a q^a_R)$$

$$\mathcal{O}_7 = \frac{m_b}{e}(\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{e^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

$$\mathcal{O}_9 = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell) \qquad \mathcal{O}_{10} = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

four-quark operators

(Tree-level SM contribution)

semileptonic and electromagnetic operators

(Very sensitive to new physics!)

Global fits

- The tensions are explainable with a **shift in C_9 of around** $25\ \%$ relative to the SM value*

From $B \to K^* \bar{\mu}\mu$ by LHCb:

Wilson Coefficient results	
$\mathcal{C}_9$	$3.56 \pm 0.28 \pm 0.18$
$\mathcal{C}_{10}$	$-4.02 \pm 0.18 \pm 0.16$
$\mathcal{C}_9'$	$0.28 \pm 0.41 \pm 0.12$
$\mathcal{C}_{10}'$	$-0.09 \pm 0.21 \pm 0.06$
$\mathcal{C}_{9\tau}$	$(-1.0 \pm 2.6 \pm 1.0) \times 10^2$

[*JHEP* 09 (2024) 026]

$\Delta\mathcal{C}_9^{\rm NP} = -0.71 \pm 0.33$

Wilson coefficient	$b \to s\mu\mu$ best fit	$b \to s\mu\mu$ pull	LFU, $B_s \to \mu\mu$ best fit	LFU, $B_s \to \mu\mu$ pull	all rare B decays best fit	all rare B decays pull
$C_9^{bs\mu\mu}$	$-0.77^{+0.21}_{-0.21}$	3.6σ	$-0.21^{+0.17}_{-0.19}$	1.2σ	$-0.42^{+0.13}_{-0.14}$	3.2σ
$C_9'^{bs\mu\mu}$	$+0.29^{+0.25}_{-0.25}$	1.2σ	$-0.22^{+0.17}_{-0.18}$	1.3σ	$-0.04^{+0.13}_{-0.13}$	0.3σ
$C_{10}^{bs\mu\mu}$	$+0.33^{+0.24}_{-0.24}$	1.3σ	$+0.16^{+0.12}_{-0.11}$	1.4σ	$+0.17^{+0.10}_{-0.10}$	1.8σ
$C_{10}'^{bs\mu\mu}$	$-0.05^{+0.16}_{-0.15}$	0.3σ	$+0.04^{+0.11}_{-0.12}$	0.3σ	$+0.02^{+0.09}_{-0.09}$	0.2σ
$C_9^{bs\mu\mu} = C_{10}^{bs\mu\mu}$	$-0.27^{+0.15}_{-0.15}$	1.7σ	$+0.17^{+0.18}_{-0.18}$	1.0σ	$-0.08^{+0.11}_{-0.11}$	0.7σ
$C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$	$-0.53^{+0.13}_{-0.13}$	3.6σ	$-0.10^{+0.07}_{-0.07}$	1.4σ	$-0.17^{+0.06}_{-0.06}$	2.7σ
$C_9^{bs\ell\ell}$	$-0.77^{+0.21}_{-0.21}$	3.6σ			$-0.78^{+0.21}_{-0.21}$	3.7σ
$C_9'^{bs\ell\ell}$	$+0.29^{+0.25}_{-0.25}$	1.2σ			$+0.30^{+0.25}_{-0.25}$	1.2σ

[*JHEP* 05 (2023) 087, Greljo, Salko, Smolkovic, Stangl]

flavor-universal shifts in C_9
(after $R_K, R_{K^*}\ldots$)

2D Hyp.	All: Best fit	All: Pull$_{\rm SM}$	All: p-value
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{10\mu}^{\rm NP})$	$(-0.82, -0.17)$	4.4	21.9%
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{7'})$	$(-0.68, +0.01)$	4.2	19.4%
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{9'\mu})$	$(-0.78, +0.21)$	4.3	20.7%
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{10'\mu})$	$(-0.76, -0.12)$	4.3	20.5%
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{9e}^{\rm NP})$	$(-1.17, -0.97)$	5.6	40.3%

[*Eur.Phys.J.C* 83 (2023) 7, 648
Algueró, Biswas, Capdevila, Descotes-Genon, Matias]

$$({\rm Re}\, C_9{}^{\rm BSM}, {\rm Re}\, C_{10}{}^{\rm BSM}) \simeq (-1.0, +0.4)$$

[Gubernari, Reboud, van Dyk, Virto, 2206.03797]

Other fits: Hurth, Mahmoudi et al (1705.06274), Geng, Grinstein et al (1704.05446), Capdevila, Crivellin et al (1704.05340), Ciuchini et al (2110.10126, 2212.10516), Recent: Hurth et al, 2508.09986

* this assumes we have good theoretical control over the long-distance contributions in the SM

Form factors

- **Matrix element for exclusive modes:**

$$\mathcal{A}(B \to M\ell^+\ell^-) = \frac{G_F \alpha V_{ts}^* V_{tb}}{\sqrt{2}\pi}\Big[(C_9\ \ell\gamma^\mu\ell + C_{10}\ \ell\gamma^\mu\gamma_5\ell)\langle M|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle\ - \frac{1}{q^2}\ell\gamma^\mu\ell\ (2im_b C_7\langle M|\bar{s}\sigma_{\mu\nu}q^\nu P_R b|B\rangle + \mathcal{H}_\mu)\Big]$$

Local form factors

- **Lattice QCD:** at high q^2 and more recently, also at low q^2. Uncertainties are small (few % for $B \to K$) and reducible;

Horgan et al, 1310.3722, 1501.00367
Bailey et al, 1509.06235
Bouchard et al, 1306.2384
Bharucha, Straub, Zwicky, 1503.05534
Parrott et al, 2207.12468

- **Light-cone sum rules**: $10 - 20\ \%$ errors, not reducible below a certain threshold.

Bharucha, Straub, Zwicky, 1503.05534
Gubernari et al, 1811.00983

- Combination in the whole q^2 range:

Gubernari, Reboud, van Dyk, Virto, 2011.09813, 2305.06301

Non-local form factors

$$\mathcal{A}(B \to M\ell^+\ell^-) = \frac{G_F \alpha V_{ts}^* V_{tb}}{\sqrt{2}\pi}\Big[(C_9\ \ell\gamma^\mu\ell + C_{10}\ \ell\gamma^\mu\gamma_5\ell)\langle M|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle\ - \frac{1}{q^2}\ell\gamma^\mu\ell\ (2im_b C_7\langle M|\bar{s}\sigma_{\mu\nu}q^\nu P_R b|B\rangle + \mathcal{H}_\mu)\Big]$$

Non-local form factors

matrix elements of the four-quark operators

$$\mathcal{M}(B \to H_\lambda \ell\ell)|_{C_{1-6}} = -i\frac{32\pi^2\mathcal{N}}{q^2}\bar{\ell}\gamma^\mu\ell\int d^4x e^{iqx}\langle H_\lambda|T\{j_\mu^{\text{em}}(x), \sum_{i=1,6} C_i\mathcal{O}_i(0)\}|B\rangle$$

only $\mathcal{O}_1, \mathcal{O}_2$ give a significant contribution

$$\mathcal{O}_1 = (\bar{s}_L^\alpha\gamma_\mu c_L^\beta)(\bar{c}_L^\beta\gamma^\mu b_L^\alpha) \qquad \mathcal{O}_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$

- The (regular for $q^2 \to 0$) contributions of the non-local matrix elements of the four-quark operators can be effectively taken into account by a **shift in C_9**:

$$\mathcal{M}(B \to H_\lambda \ell\ell)|_{C_{1-6}} = \left(\Delta_9^\lambda(q^2) + \frac{m_B^2}{q^2}\Delta_7^\lambda\right)\langle H_\lambda\ \ell^+\ell^-|\mathcal{O}_9|B\rangle$$

$$C_9 \to C_9^\lambda(q^2) = C_9^{\text{SM}} + \Delta_9^\lambda(q^2) + C_9^{\text{NP}}$$

Long-Distance or New Physics?

Non-local form factors

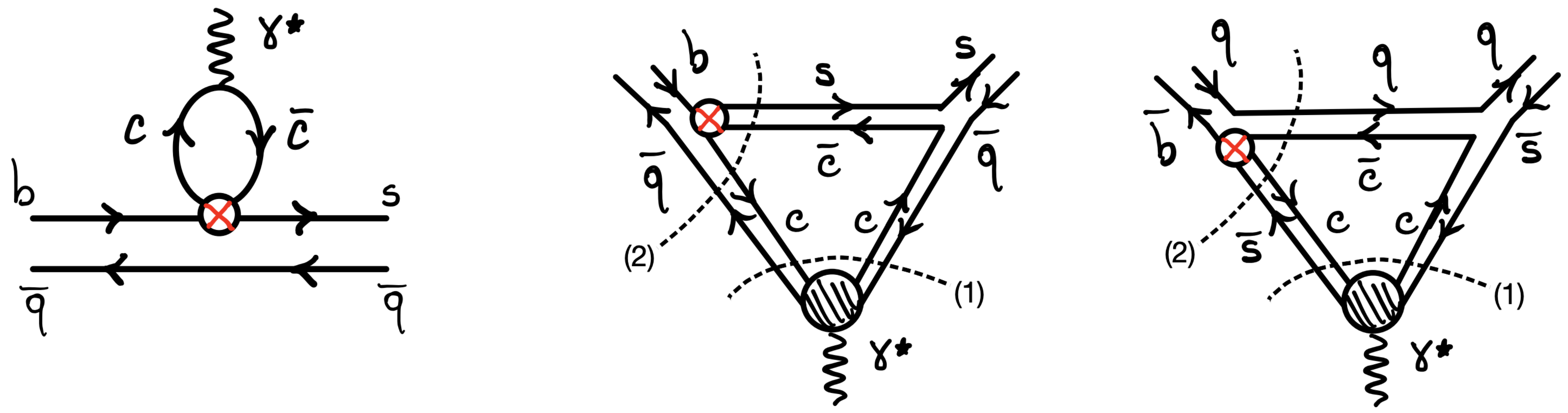
There is no doubt that the tension with the data could be well described by a shift in C_9 of $\mathcal{O}(25\%)$ with respect to the SM value

BUT

this shift could come from an **inaccurate description of the non-local matrix elements**

Non-local form factors

The correlator in $\int d^4x\ e^{iqx}\langle H_\lambda | T\{j_\mu^{\text{em}}(x), \sum_{i=1,6} C_i \mathcal{O}_i(0)\} | B\rangle$ receives **two kinds of contributions**:

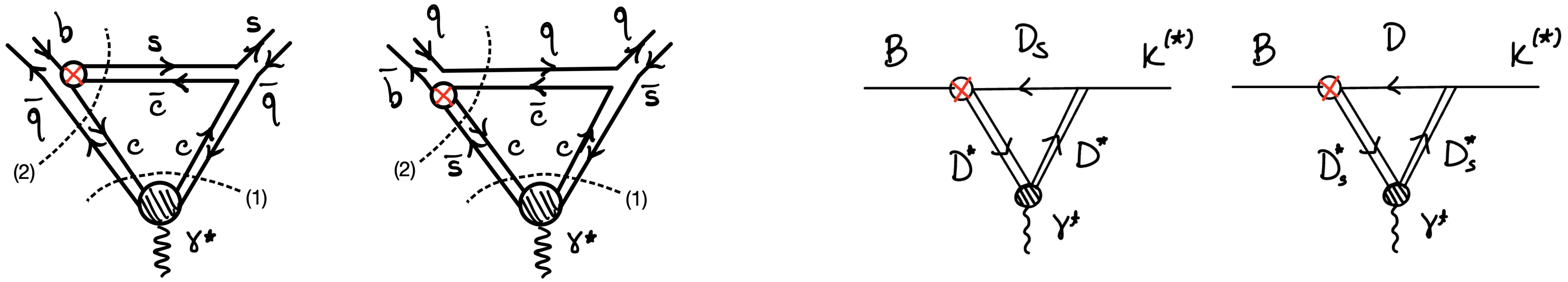


(a) (b)

Pictures from [Ciuchini, Fedele, Franco, Paul, Silvestrini, Valli, 2212.10516]

- Studied with **light-cone sum rules** for $q^2 \ll 4m_c^2$ + **dispersion relations** to extend to larger values of q^2 **Khodjamirian, Mannel, Pivovarov, Wang, 1006.4945, 1211.0234**
- Also using negative q^2 region to further constrain **Bobeth, Chrzaszcz, van Dyk, Virto, 1707.07305, Chrzaszcz et al, 1805.06378**
- Unitarity bounds **Gubernari, van Dyk, Virto, 2011.09813**
- **Small effect** in the large-recoil region **Gubernari, Reboud, van Dyk, Virto, 2206.03797, Mahajan, Mishra 2409.00181**

Rescattering contributions: triangle topology



[Ciuchini, Fedele, Franco, Paul, Silvestrini, Valli, 2212.10516]

- **Rescattering** of a pair of charmed and charmed-strange mesons.
- Doubts about the use of dispersive methods (anomalous thresholds, see Mutke, Hoferichter, Kubis *JHEP* 07 (2024) 276, Gopal, Gubernari *Phys.Rev.D* 111 (2025) 3).

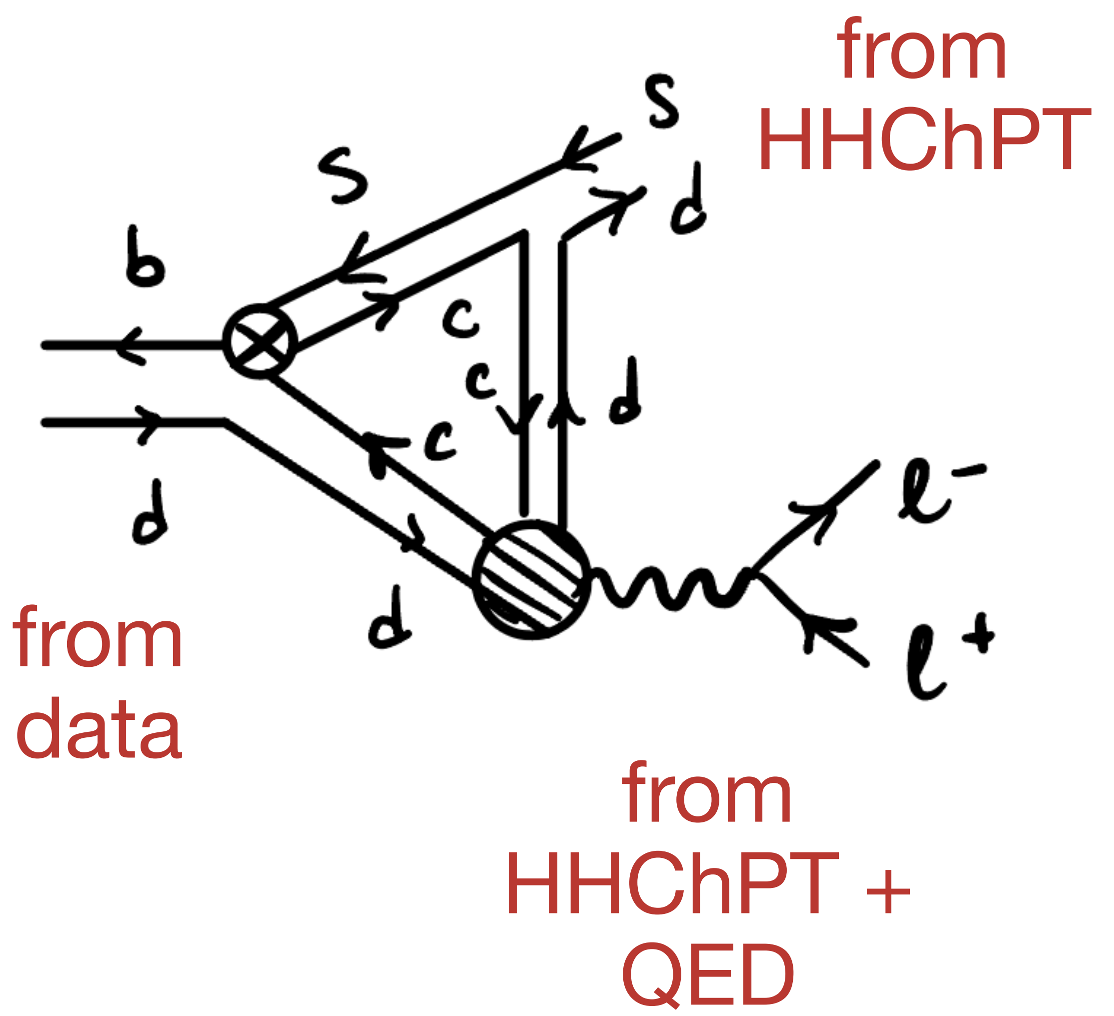
See talk by Simon!

- Suggestion that these effects could mimic short-distance physics.
- Parametrization that includes anomalous thresholds: **Gopal, Gubernari 2412.04388**
- **Some progress in lattice!** **Frezzotti et al, 2508.03655**

Model for charm rescattering

2405.17551 Isidori, Polonsky, AT
2507.17824 Isidori, Polonsky, AT

- We look at the simplest decay mode, $B^0 \to K^0 \bar{\ell}\ell$.
- Model in terms of **hadronic degrees of freedom**.
- We use **data** for the B vertex, heavy-hadron chiral perturbation theory (**HHChPT**) combined with QED for the remaining vertices.
- We obtain an accurate description in the low recoil (or **high** q^2) limit; we extrapolate to the whole kinematical region introducing appropriate form factors.
- Considering the largest B decays, we classify all the possible intermediate states that allow a parity-conserving strong interaction with the kaon.
- Goal: **estimate the size of these contributions**



Charm rescattering contributions

* **Dynamics of $D_{(s)}^{(*)}$ mesons close to their mass shell, determined by:**
  * Lorentz + Gauge invariance under QED
  * $SU(3)$ light-flavor symmetry
  * Heavy-quark spin symmetry

$$\mathcal{L}_{D,\text{free}} = -\frac{1}{2}\left(\Phi^{\mu\nu}_{D^*}\right)^\dagger \Phi_{D^*\,\mu\nu} - \frac{1}{2}\left(\Phi^{\mu\nu}_{D_s^*}\right)^\dagger \Phi_{D_s^*\,\mu\nu} + \left(D_\mu \Phi_D\right)^\dagger D^\mu \Phi_D + \left(D_\mu \Phi_{D_s}\right)^\dagger D^\mu \Phi_{D_s} + m_D^2\left[\left(\Phi^\mu_{D^*}\right)^\dagger \Phi_{D^*\,\mu} + \left(\Phi^\mu_{D_s^*}\right)^\dagger \Phi_{D_s^*\,\mu}\right] - m_D^2\left[\Phi^\dagger_D \Phi_D + \Phi^\dagger_{D_s}\Phi_{D_s}\right] + \text{h.c.}\,.$$

minimally coupled photon:

$$D_\mu \Phi = (\partial_\mu + ieQ_\Phi A_\mu)\Phi$$

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- **Weak $B \to D^{(*)}D^{(*)}$ transitions (using heavy-quark spin symmetry):**

$$\mathcal{L}_{BDD} = -g_{DD}\Phi_B \Phi^\dagger_{D_s}\Phi_D + \text{h.c.}$$

$$\mathcal{L}_{BD} = g_{DD^*}\left(\Phi^{\mu\dagger}_{D_s^*}\,\Phi_D \partial_\mu \Phi_B + \Phi^\dagger_{D_s}\,\Phi^\mu_{D^*}\partial_\mu \Phi_B\right) + \text{h.c.}$$

$$\mathcal{L}_{BD*D*} = -g'_{1^*}\Phi_B\left(\Phi^\mu_{D_s^*}\right)^\dagger \Phi_{D^*\mu} - \frac{g_{2^*}}{2m_D^2}\Phi_B\left(\Phi^{\mu\nu}_{D_s^*}\right)^\dagger \Phi_{D^*\mu\nu} - \frac{g_{3^*}}{2m_D^2}\Phi_B\left(\tilde{\Phi}^{\mu\nu}_{D_s^*}\right)^\dagger \Phi_{D^*\mu\nu} + \text{h.c.}$$

Extract from data up to a phase

Charm rescattering contributions

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Extract from data up to a phase

* **From HHChPT** (valid close to endpoint $q^2 \approx m_B^2$):

$$\mathcal{L}_{DK,0} = \frac{2g_\pi}{f}\left(im_{D^{*+}D_s}\Phi^{\dagger\mu}_{D^{*+}}\Phi_{D_s}\partial_\mu\Phi^\dagger_{K^0} - im_{D_s^*D^+}\Phi^\dagger_{D^+}\Phi^\mu_{D_s^*}\partial_\mu\Phi^\dagger_{K^0} + \sqrt{\frac{m_{D^{*+}}}{m_{D_s^*}}}\epsilon_{\alpha\beta\mu\nu}\partial^\alpha\Phi^{\dagger\mu}_{D^{*+}}\partial^\beta\Phi^\nu_{D_s^*}\Phi^\dagger_{K^0}\right) + \text{h.c.}$$

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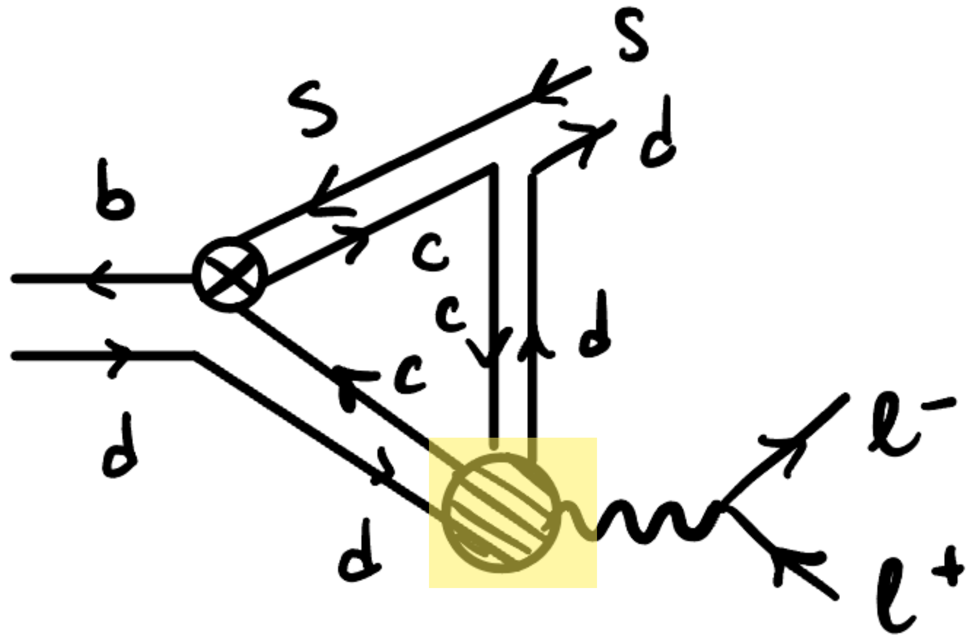
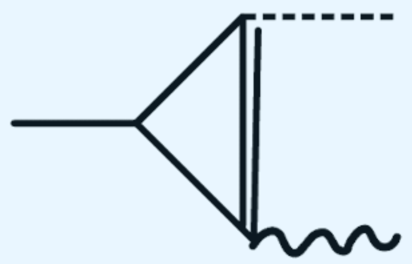
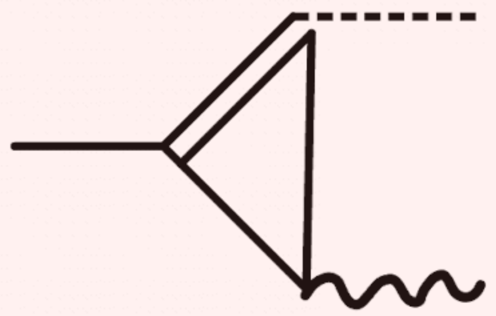
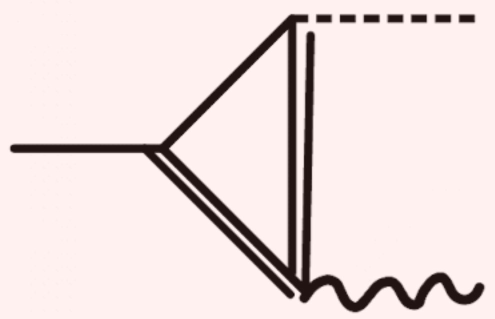
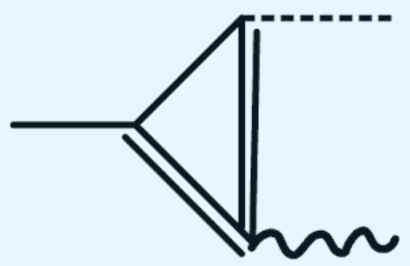
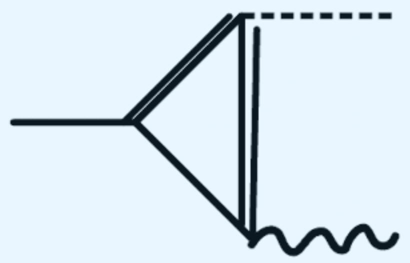
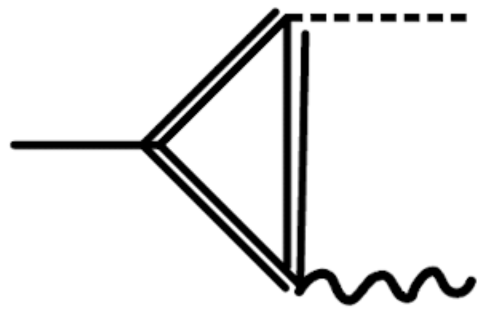
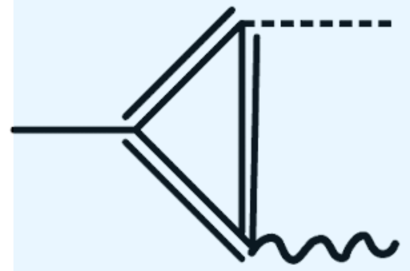
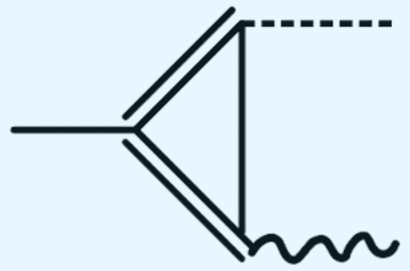
* **Dipole coupling**

$$\text{Tr}\left[\bar{H}_v\sigma_{\mu\nu}F^{\mu\nu}H_v\right] \to 4g_{\text{dip}}(q^2)\left[\frac{1}{m_D}\left(\Phi^\dagger_D\Phi_{D^*\mu\nu}\tilde{F}^{\mu\nu} + \text{h.c.}\right) + i\Phi^\dagger_{D^*\mu}\Phi_{D^*\nu}F^{\mu\nu}\right]$$

Charm rescattering contributions

- Considering **two possible interactions** at the photon vertex, these are the possible topologies:



	Monopole	Dipole
$B \to DD_s$	Not possible	
$B \to D^*D_s$ $B \to D_s^*D$	  2405.17551 Isidori, Polonsky, AT	 
$B \to D^*D_s^*$	 harder…	 

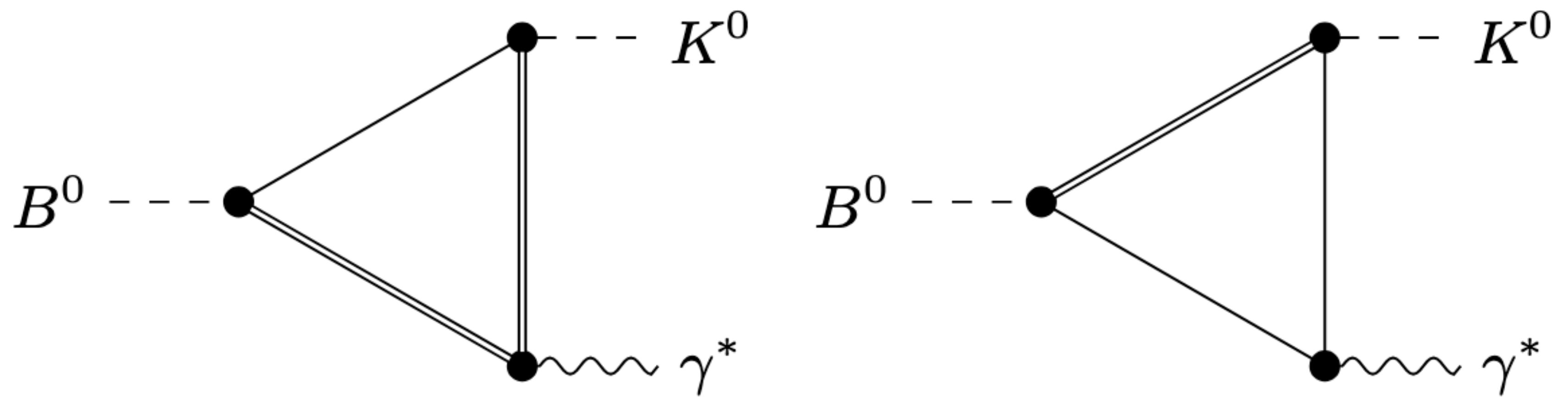
2507.17824 Isidori, Polonsky, AT

Single line is a D, double line is a D^*

Monopole contributions

2405.17551 Isidori, Polonsky, AT

- We compute the **one-loop diagrams** generated by the $B \to DD^*$ transition.
- In the $SU(3)$-symmetric limit, the diagrams obtained by swapping $D_s^{(*)} \leftrightarrow D^{(*)}$ are symmetric.
- Sum of diagrams shows an **ultraviolet divergence**; we use an $\overline{MS}$-like renormalization scheme to discard it and use the scale dependence to estimate the uncertainty.



- To obtain a reliable estimate **over the entire kinematical range,** we introduce a correction for the DD^*K **vertex**:

$$\frac{1}{f_K} \to \frac{1}{f_K} G_K(q^2)\,,$$

$$G_K(q^2) = \frac{1}{1 + E_K(q^2)/f_K} = \frac{2m_B f_K}{2m_B f_K + m_B^2 - q^2}$$

(Kaon emission amplitude growing as E_K/f_K is only correct in the soft-kaon limit)

Monopole form factor

- Correction for **QED vertex:** the point-like description must be improved away from $q^2 \to 0$
- Dominant contribution: vector charmonium states that mix with the photon
- Model by a **tower of resonances**:

$$e \to eF_V(q^2)\,, \qquad F_V(q^2) = \sum_V M_V^2 \frac{y_V}{q^2 - M_V^2 + i\sqrt{q^2}\Gamma_V} e^{i\varphi_V}$$

- Form factor fit to BESIII data on $e^+e^- \to D_s^+ D_s^-$ + normalization at $q^2 = 0$.

$$\sigma(e^+e^- \to D^+D^-) = \frac{\pi\alpha^2}{3s}|F_V(s)|^2\left(\frac{s - 4m_D^2}{s}\right)^{3/2}$$

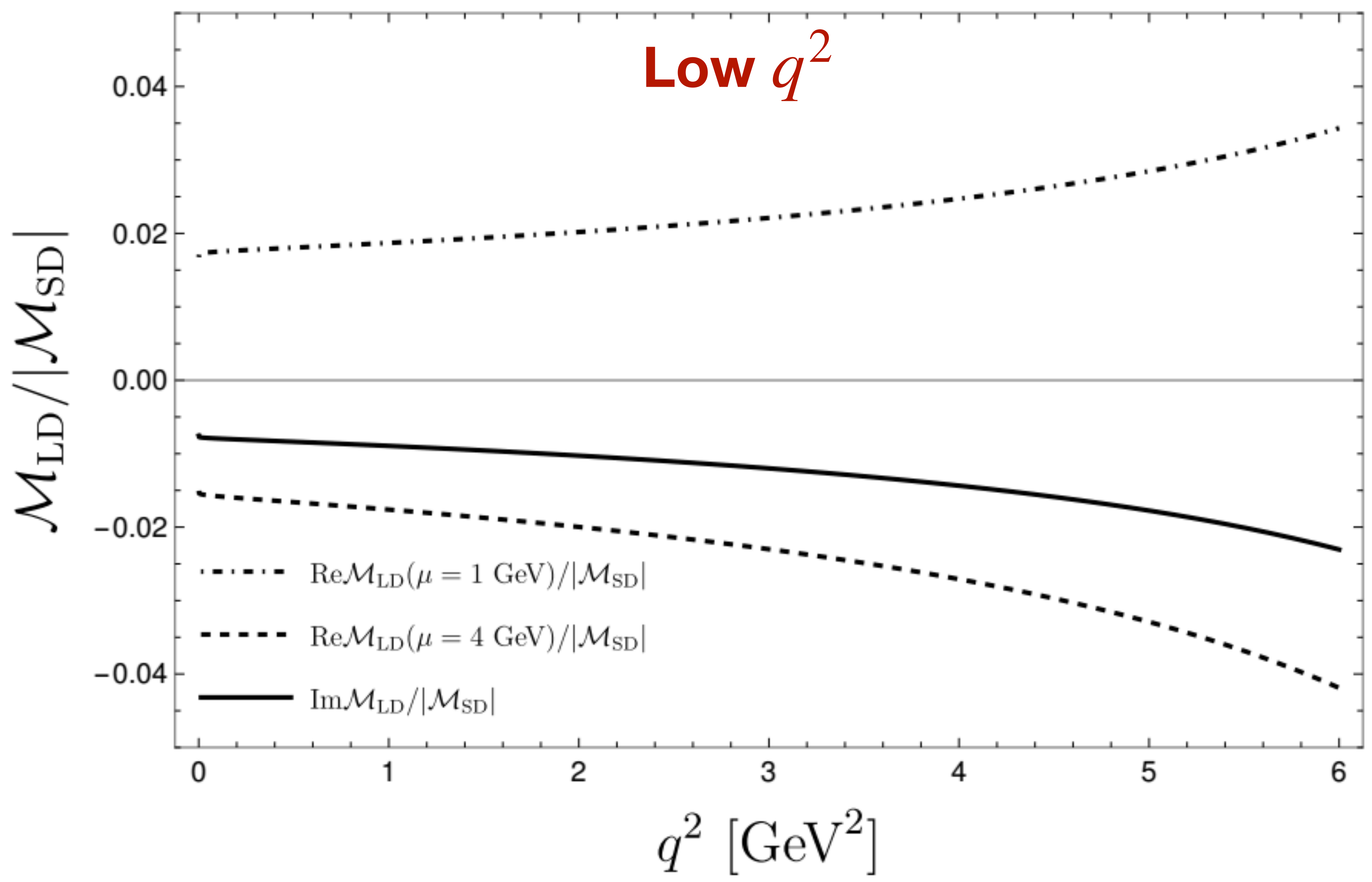
$$F_V(q^2 \to 0) \to 1 \Rightarrow \sum_V y_V e^{i\varphi_V} = -1$$

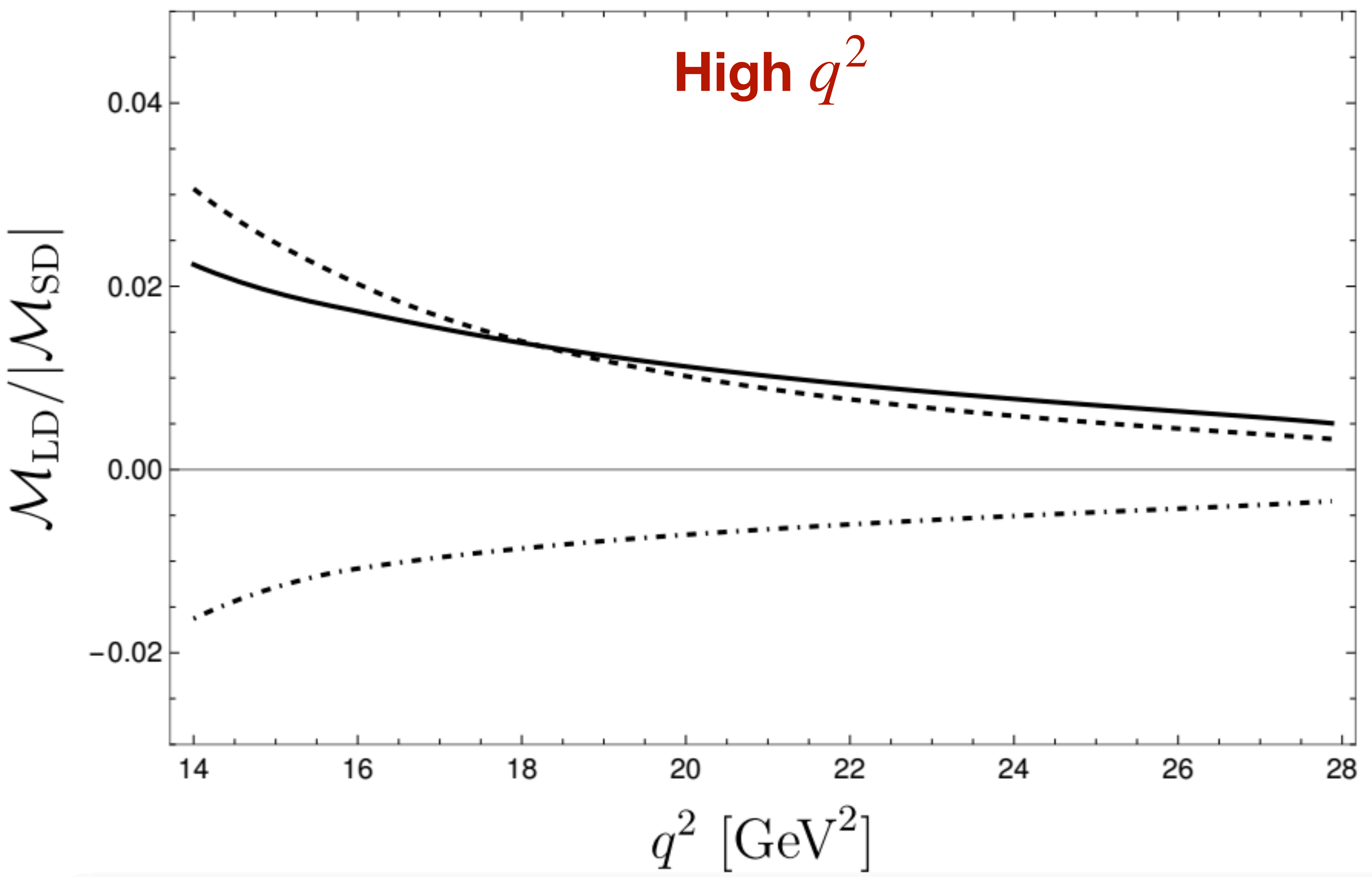
$$V = \{J/\psi, \psi(2S), \psi(3770)\}$$

→ 2 free parameters to fit:

	J/Ψ	$\Psi(2S)$	$\Psi(3770)$
y_V	1.50	1.01	0.63
φ_V	π	5.75	2.19

Monopole contributions





- Monopole LD contributions do not exceed a few percent relative to the SD one: $\mathcal{M}_{\rm SD} = \frac{4G_F}{\sqrt{2}}\frac{e}{16\pi^2}V_{tb}^*V_{ts}(p_B \cdot j_{\rm em})f_+(q^2)(2C_9)$
- The **absorptive part** is independent of the renormalization scheme used, and corresponds to the analytic discontinuity of the amplitude corresponding to the kinematical regions where the internal mesons go on-shell.

~MODEL INDEPENDENT!

- Encoding this effect in a shift in C_9 averaged over the q^2 range and varying μ in the range $[1,4]$ GeV: $\frac{\delta C_9}{C_9^{SM}} \approx 2.5\,\%$

Sign of shift in C_9

- δC_9^{LD} has an **opposite sign** in the low- and high-q^2 regions;
- This is a consequence of the structure of the **vector form factor**.
- It is not a consequence of vector meson dominance, it is a **general feature** of imposing the normalization at $q^2 = 0$ on the general parametrization:

$$F_V(q^2 \to 0) \to 1 \Rightarrow \sum_V y_V e^{i\varphi_V} = -1\,.$$

$$F_V(q^2) = \sum_V M_V^2 \frac{y_V}{q^2 - M_V^2 + i\sqrt{q^2}\Gamma_V} e^{i\varphi_V}$$

VMD:

$$F_V(q^2) = \frac{m_{J/\psi}^2}{m_{J/\psi}^2 - q^2}$$

$$\epsilon_V = \frac{M_V^2}{\overline{M}^2} - 1 \ll 1\,.$$

$$F_V(q^2) = \frac{\overline{M}^2}{q^2 - \overline{M}^2} \times \times \left\{ -1 + \frac{q^2}{q^2 - \overline{M}^2} \sum_V y_V e^{i\varphi_V} \epsilon_V + O(\epsilon^2) \right\}$$

In the low-q^2 region, where $q^2 \sim \overline{M}^2/2$

$$F_V(q^2_{\mathrm{low}}) \sim 2 + 2\sum_V y_V e^{i\varphi_V} \epsilon_V + O(\epsilon^2)$$

while in the high-q^2 region with $q^2 \sim 2\overline{M}^2$, this becomes

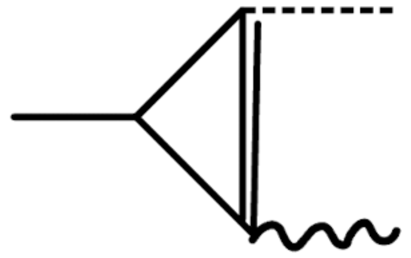
$$F_V(q^2_{\mathrm{high}}) \sim -1 + 2\sum_V y_V e^{i\varphi_V} \epsilon_V + O(\epsilon^2)$$

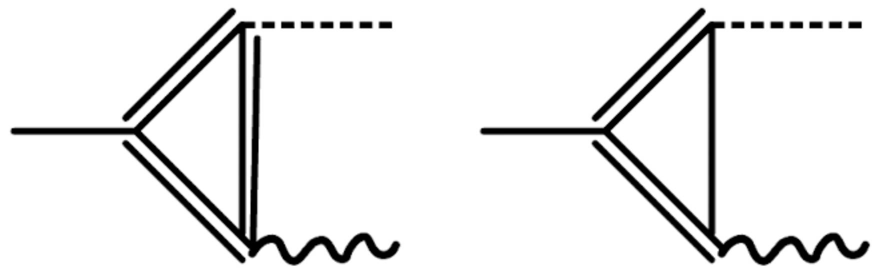
- **Comparing the extraction of C_9 at low- and high-q^2 provides a useful data-driven check!**

Dipole contributions

2507.17824 Isidori, Polonsky, AT

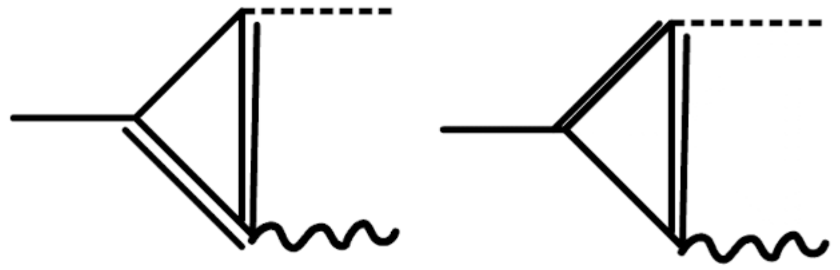
- We evaluate all the topologies appearing with a dipole interaction:

$B \to D D_s :$  $= 0$ because of the Lorentz structure

$B \to D^* D_s^* :$  $= 0$ when adding together the different diagrams obtained by $D_s^{(*)} \leftrightarrow D^{(*)}$

$B \to D^* D_s :$

$B \to D_s^* D :$



only non-vanishing topologies, corresponding to 4 diagrams to evaluate

it turns out we don't need other B couplings…

Dipole form factor

- The dipole interaction has the following form: $\mathscr{L} \ni 4g_{\text{dip}}(q^2)\left[\frac{1}{m_D}\left(\Phi_D^\dagger \Phi_{D^*\mu\nu}\tilde{F}^{\mu\nu} + \text{h.c.}\right) + i\Phi^\dagger_{D^*\mu}\Phi_{D^*\nu}F^{\mu\nu}\right].$
- We assuming the following parametrization of the form factor: $g_{\gamma^* D^* D}(s) = \sum_V M_V^2 \frac{\eta_V}{s - M_V^2 + i\sqrt{s}\,\Gamma_V} e^{i\phi_V}$ $\quad V = J/\psi, \psi(2s), \psi(3770)$
- We extract the dipole form factor from **Belle data** on $e^+e^- \to DD^*$ and the lattice QCD results for $D_s^* \to D_s\gamma$ (that gives access to $g_{dip}(0)$).

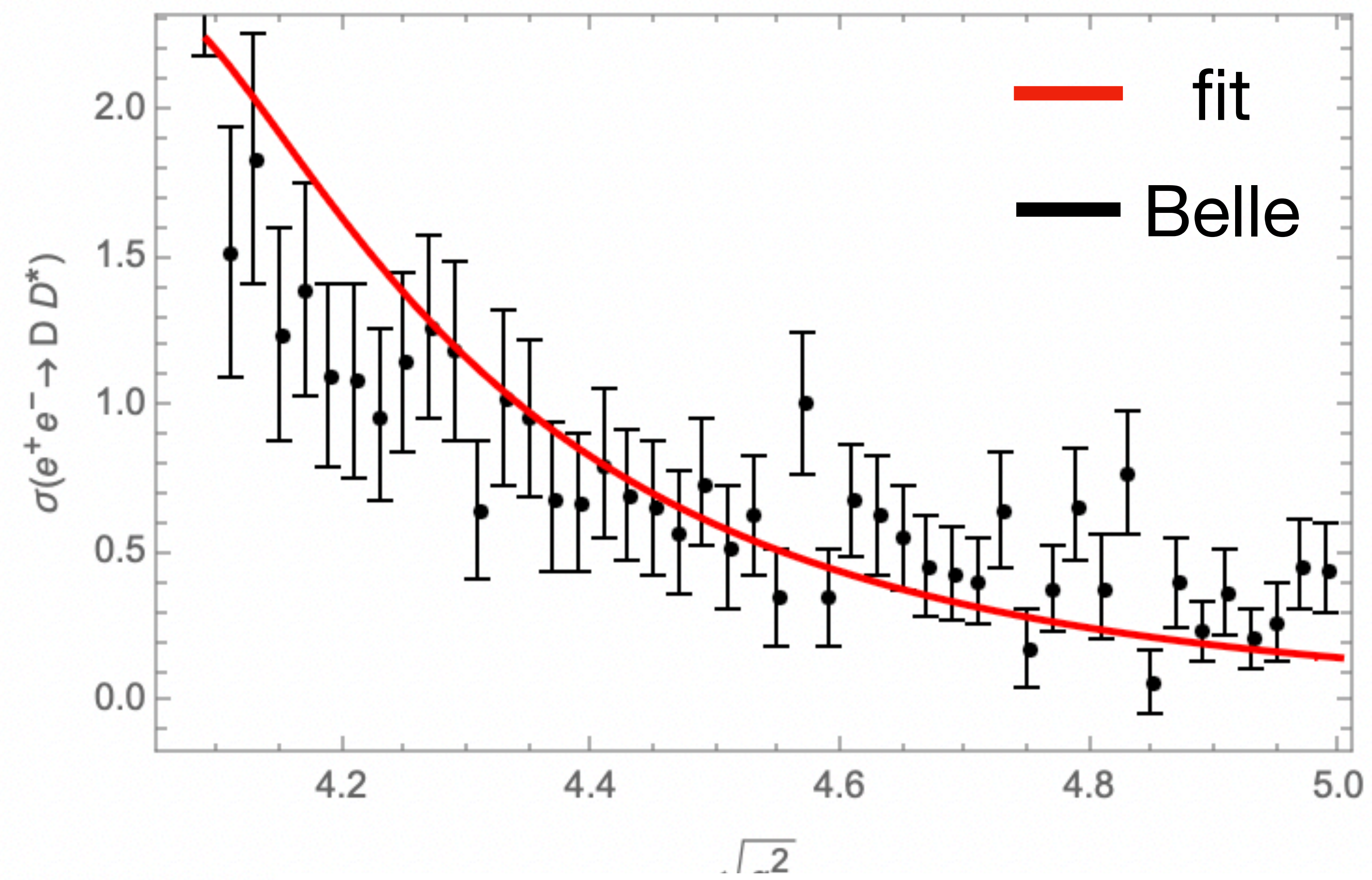
$$\sigma(e^+e^- \to DD^*) = \frac{e^2|g_{\text{dip}}(s)|^2}{6\pi m_D^2}\left(\frac{s - 4m_D^2}{s}\right)^{3/2}$$

- We impose the following conditions:

$$\sum_V M_V^2 \eta_V e^{i\phi_V} = 0$$

$$g_{\gamma^* D^* D}(0) \text{ from } D^* \to D\gamma$$

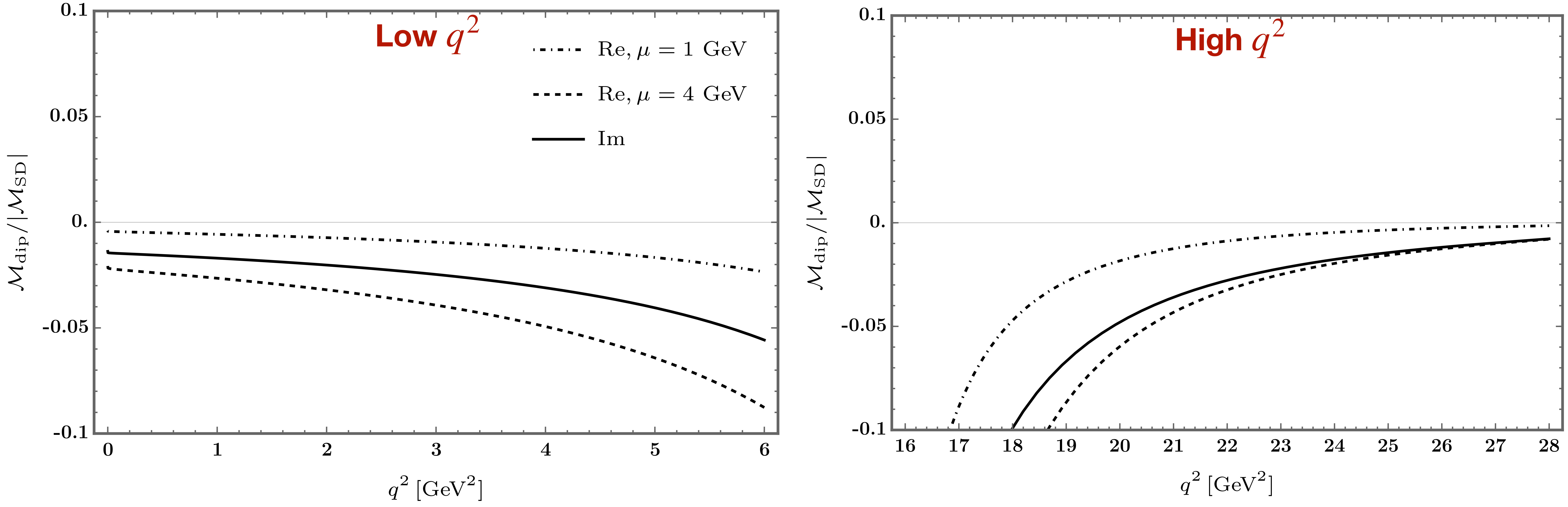
$$\phi_{J/\psi} = 0$$



Belle Collaboration, Phys.Rev.Lett. 98 (2007), 092001

- $\underline{D^* \to D\gamma}$:
  - CLEO $\Gamma(D^* \to D\gamma) = (1.33 \pm 0.33)$ keV (hep-ex/9711011)
  - Lattice $\Gamma(D_s^* \to D_s\gamma) = 0.0549(54)$ keV (2401.13475)
  - Lattice $\Gamma(D_s^* \to D_s\gamma) = = 0.066(26)$ keV (HPQCD 1312.5264)

Dipole contributions



- Dipole long-distance contributions are of the order of a **few percent** relative to the short-distance matrix element.
- Bigger effects in the resonance regions.

Multiplicity factors

- In the monopole case, we focused on the D^*D_s or D_s^*D intermediate states, but in principle there are **other states** with $\bar{c}c\bar{s}d$ valence structure.
- Consider all intermediate states the allow parity-conserving strong interactions with the kaon:

B^0 Decay	$\mathcal{B}(B^0 \to X) \times 10^3$
D^*D_s	8.0 ± 1.1
DD_s^*	7.4 ± 1.6
$D^*D_s^*$	17.7 ± 1.4
$DD_{s0}(2317)$	1.06 ± 1.6
$D^*D_{s1}(2457)$	9.3 ± 2.2
$D^*D_{s1}(2536)$	0.50 ± 0.14
$DD_{s2}(2573)$	$(3.4 \pm 1.8) \times 10^{-2}$
$D^*D_{s2}(2573)$	< 0.2
$DD_{s1}(2700)$	0.71 ± 0.12

$$\mathcal{N} = \frac{\sum_X \mathcal{M}(B^0 \to X)}{\mathcal{M}(B^0 \to D^*D_s) + \mathcal{M}(B^0 \to DD_s^*)} \approx \frac{1}{2} \sum_X \sqrt{\frac{\mathcal{B}(B^0 \to X)}{\mathcal{B}(B^0 \to DD_s^*)}} \approx 3$$

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- We estimate the multiplicity factor in the dipole case in the following way:
- We take the largest B decays that we haven't included so far, and then consider the possible intermediate states connecting the kaon to the photon.

$$B \to DD_s : (7.2 \pm 0.8) \times 10^{-3}$$
$$B \to D_{s1}(2457)D^* : (9.3 \pm 2.2) \times 10^{-3}$$

- Two diagrams each: $\mathcal{N} \approx 2$

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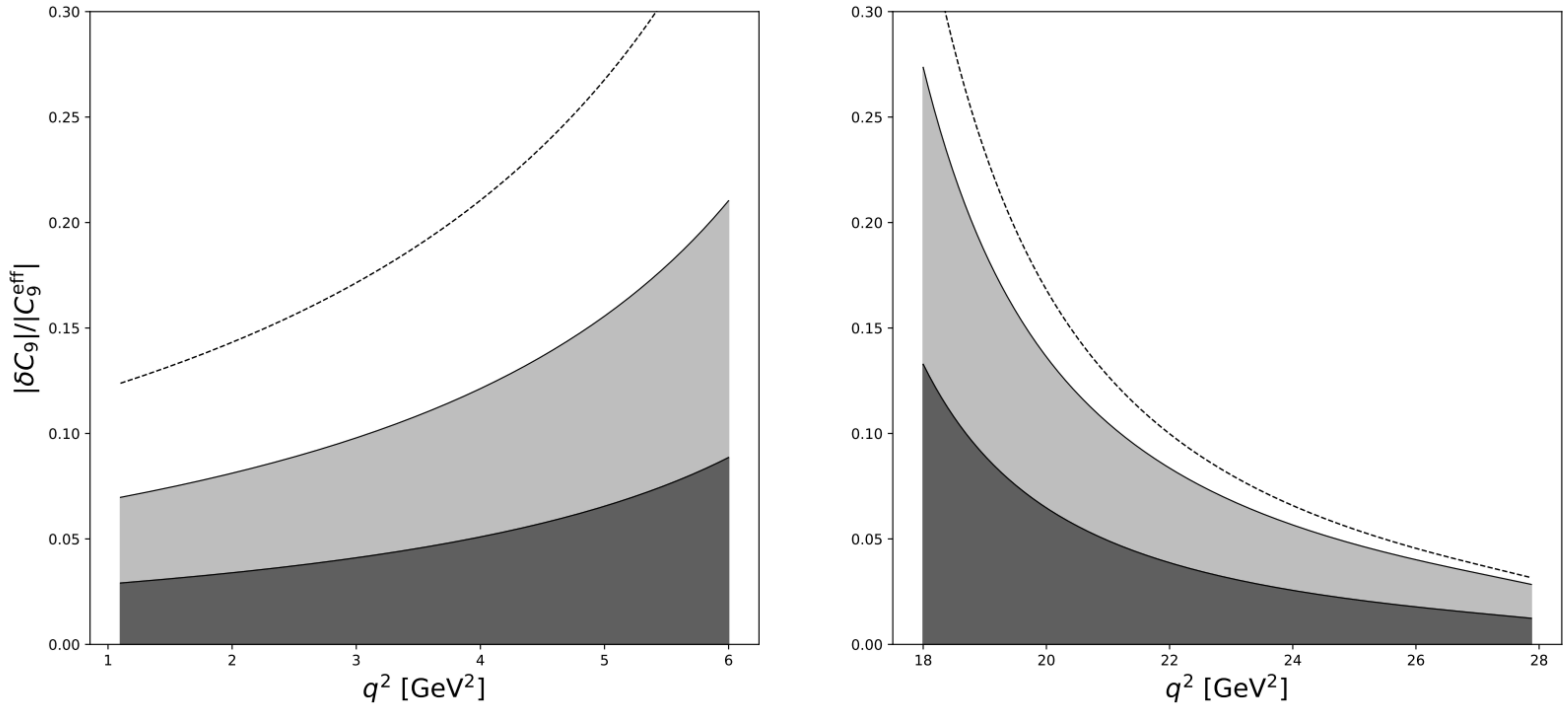
- Two diagrams each: $\mathcal{N} \approx 2$

- We don't know the **relative sign** between the monopole and the dipole contributions, because $g_{\rm dip}(q^2)$ is extracted from data up to a phase → we can see what happens if we **maximize** over this phase…

Long-distance rescattering contributions

- Let’s sum all the long-distance contributions we estimated and compare it to the SD contribution:

- **Black region**: "natural size” of the effect (adding constructively absorptive parts of monopole and dipole).



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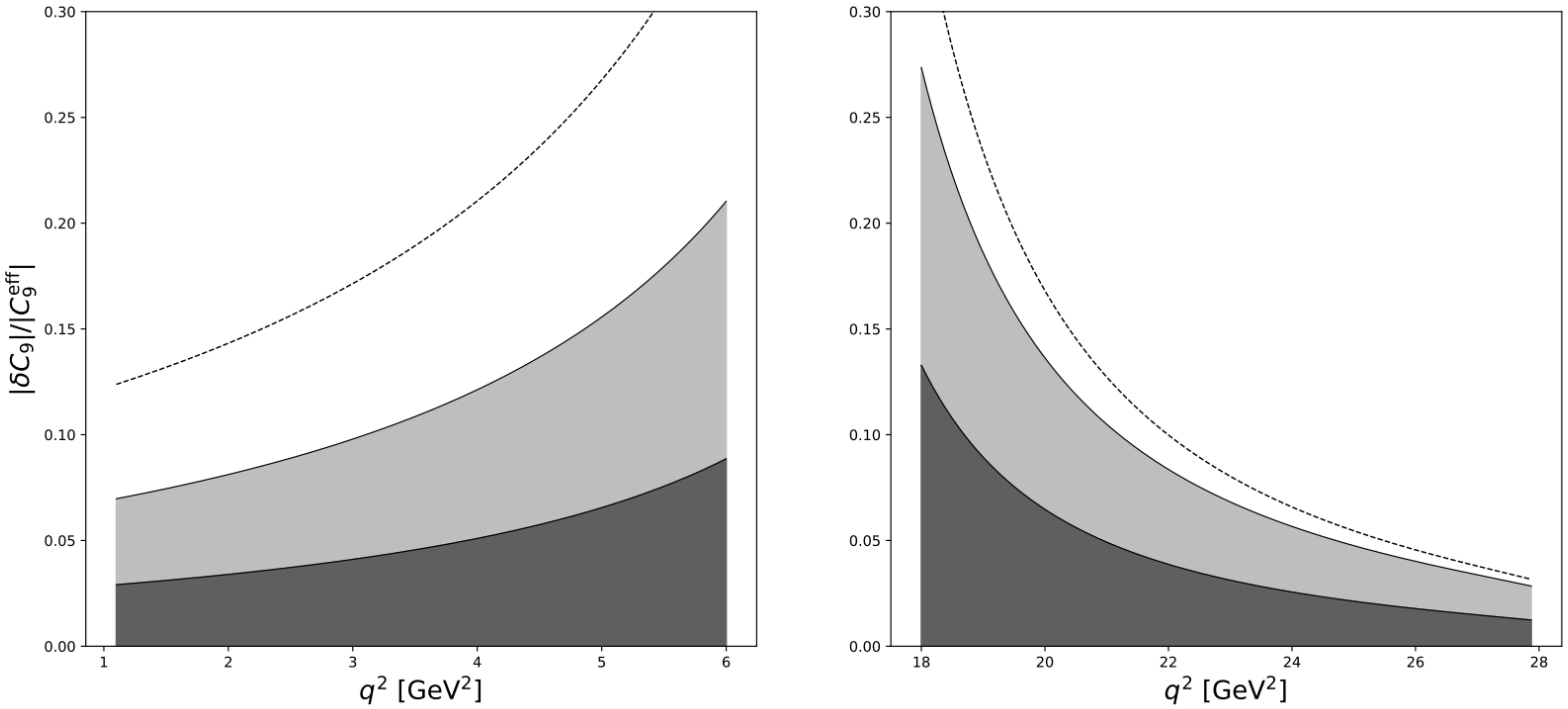

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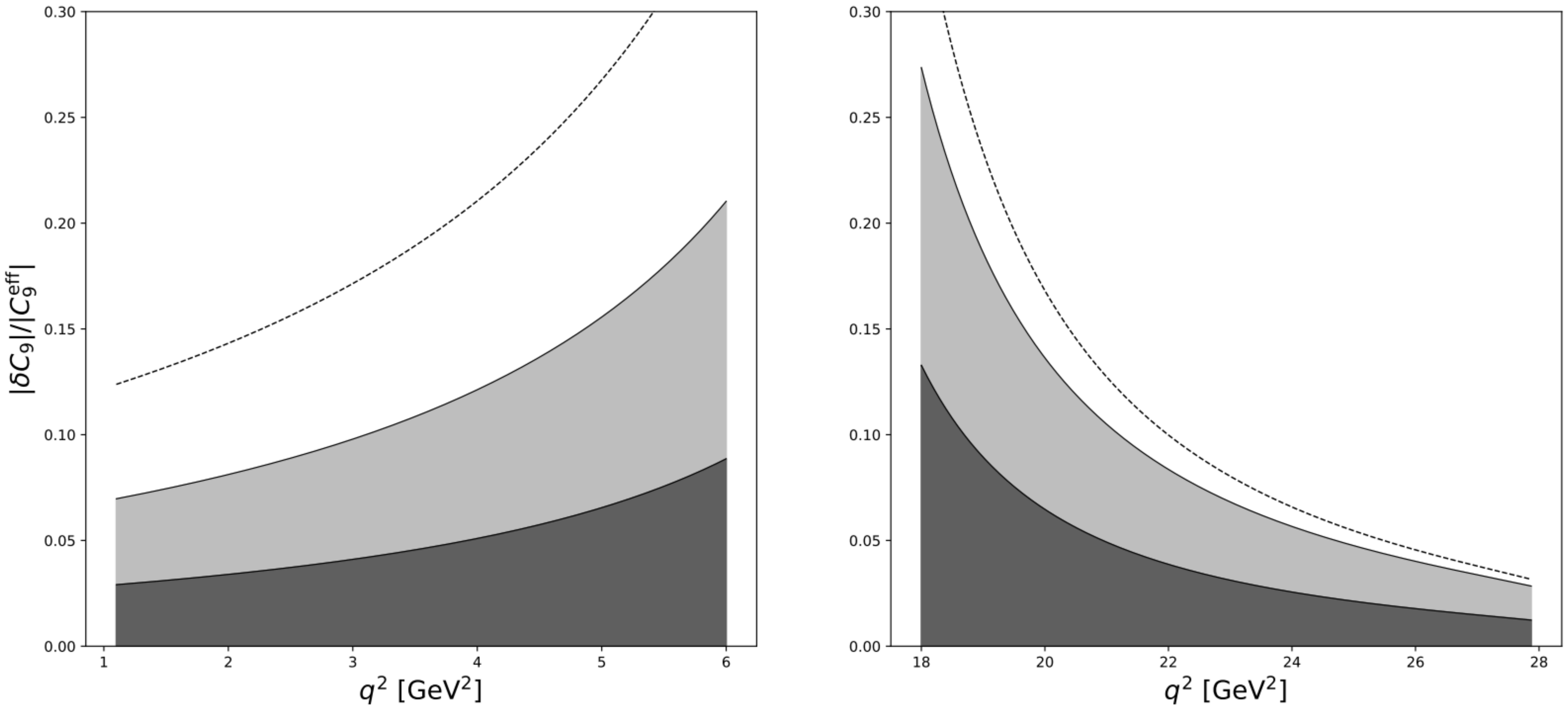
- Dashed line: **perfect conspiracy** of relative phases between SD, monopole, and dipole contributions, maximum multiplicity factors, and maximizing the dispersive part by setting $\mu = 4$ GeV (super fine tuned).



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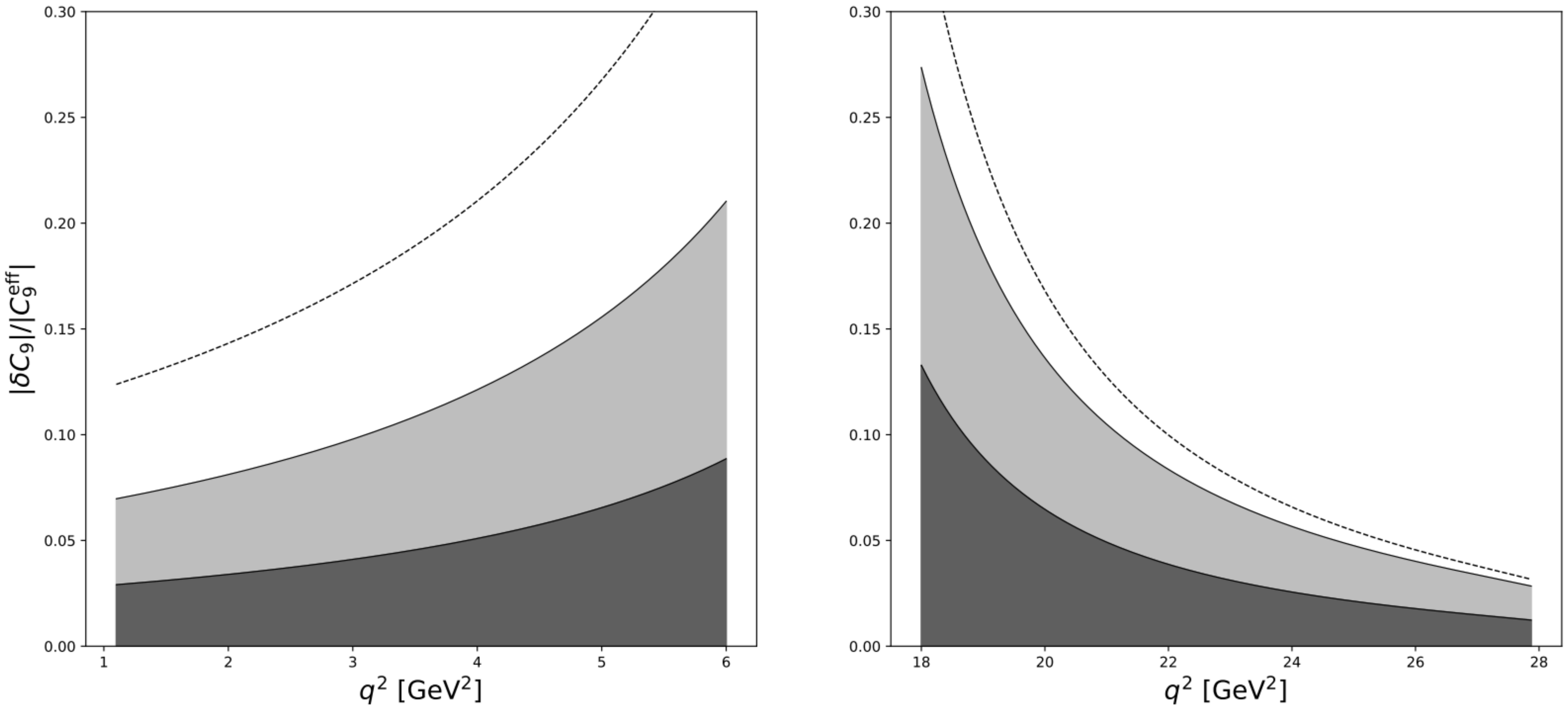


- **Hard to go above $10\,\%$. And in that case, there must be some visible q^2 dependence.**

Long-distance rescattering contributions

- With tuning, it is not unfeasible for rescattering effects to give a sizable, $O(20\%)$ contribution over a large q^2 region, **at the cost of a more pronounced q^2-dependence**, contrary to the hypothesis that these effects mimic short-distance physics.

- Testing the q^2 dependence is key $\rightarrow$ **Extraction of C_9 in as many q^2 bins as possible**.

- With current data, can we detect **any dependence on q^2** (and on the mode)?

Fit of C_9 bin by bin with data-driven approach

M. Bordone, G.Isidori, S. Mächler, AT, 2401.18007

$$C_9 \to C_9 + Y^\lambda(q^2)$$
$$\lambda = K, \perp, //, 0$$

More precisely, this shift includes:

$$C_9 \to C_9^\lambda(q^2) + Y_{q\bar{q}}^{[0]}(q^2) + Y_{b\bar{b}}^{[0]}(q^2) + Y_{c\bar{c}}^\lambda(q^2)$$

encodes (factorizable) **perturbative** contributions from 4-quark operators

encodes the **perturbative** charm-loop contributions and $c\bar{c}$ **resonances**

Dispersive relations:

$$Y_{c\bar{c}}^\lambda(q^2) = Y_{c\bar{c}}^\lambda(q_0^2) + \frac{16\pi^2}{\mathcal{F}_\lambda(q^2)}\Delta\mathcal{H}_{c\bar{c}}^\lambda(q^2),\ q_0^2 = 0$$

$$\Delta\mathcal{H}_{c\bar{c}}^{\lambda,1P} = \sum_V \eta_V^\lambda e^{i\delta_V^\lambda}\frac{q^2}{m_V^2}A_V^{\text{res}}(q^2)$$

$$A_V^{\text{res}}(q^2) = \frac{m_V\Gamma_V}{m_V^2 - q^2 - im_V\Gamma_V}$$

Extracting resonance parameters with inputs from data by LHCb (1612.06764, 2405.17347)

Fit of C_9 bin by bin with data-driven approach

M. Bordone, G.Isidori, S. Mächler, AT, 2401.18007

We extract the residual contribution to C_9:

$$C_9 \to C_9^\lambda(q^2) + Y_{q\bar{q}}^{[0]}(q^2) + Y_{b\bar{b}}^{[0]}(q^2) + Y_{c\bar{c}}^{\lambda}(q^2)$$

↓ extract from data

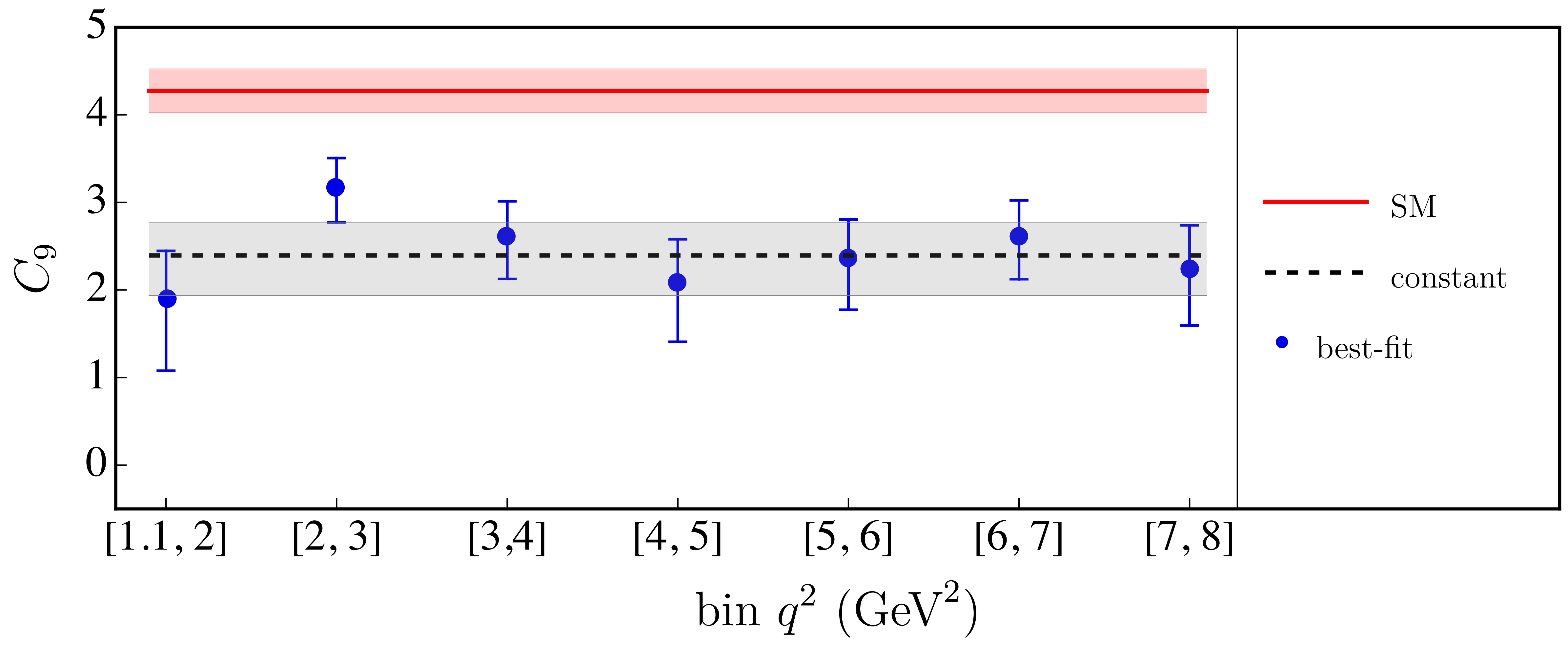
$$C_9^\lambda(q^2) = C_9^{\rm SM} + C_9^{{\rm LD},\lambda}(q^2) + C_9^{\rm SD}$$

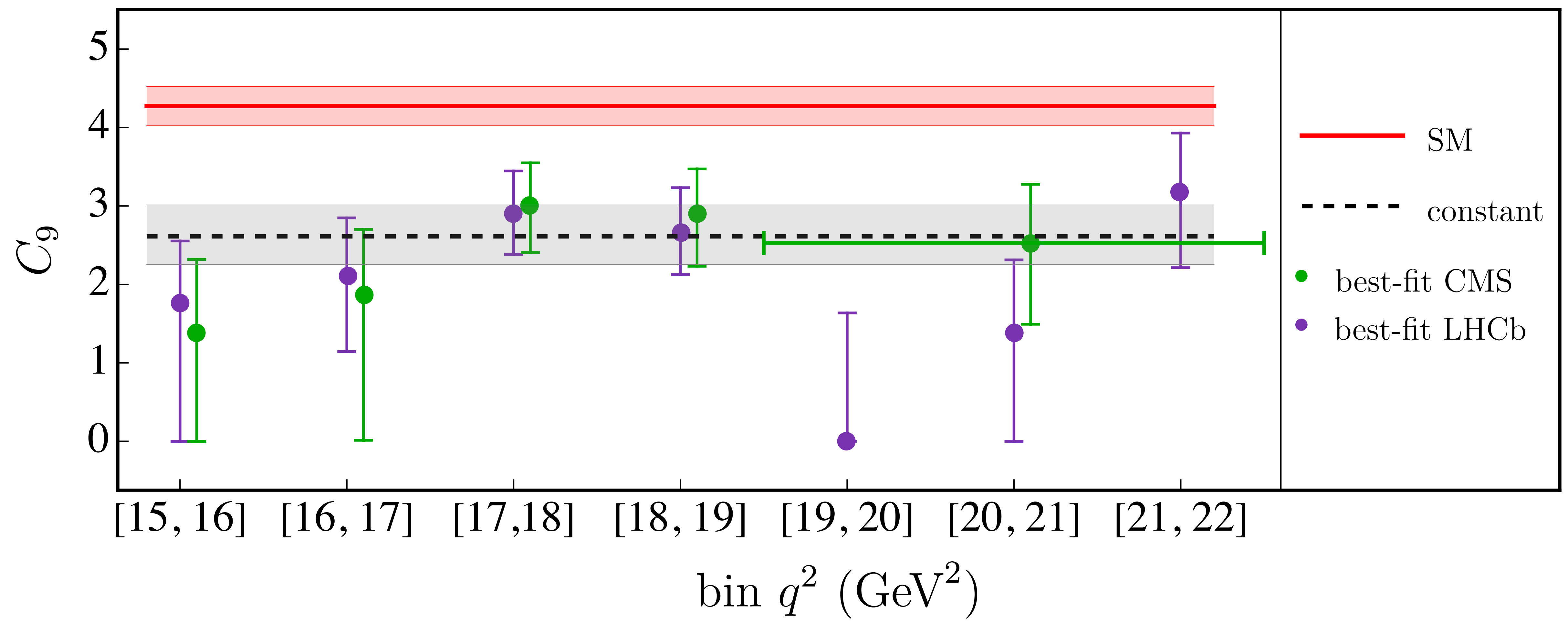
Long-distance, no reason to assume it is independent of λ or q^2

Short-distance, independent of λ and q^2

Can we find this contribution from data? → Fit from data for every bin in q^2 and every polarization

$B \to K$ results





q^2 (GeV^2)	C_9^K
[1.1, 2]	$1.9^{+0.5}_{-0.8}$
[2, 3]	$3.2^{+0.3}_{-0.4}$
[3, 4]	$2.6^{+0.4}_{-0.5}$
[4, 5]	$2.1^{+0.5}_{-0.7}$
[5, 6]	$2.4^{+0.4}_{-0.6}$
[6, 7]	$2.6^{+0.4}_{-0.5}$
[7, 8]	$2.3^{+0.5}_{-0.7}$
constant	$2.4^{+0.4}_{-0.5}$ ($\chi^2/\mathrm{dof} = 1.35$)

q^2 (GeV^2)	C_9^K (LHCb)	C_9^K (CMS)
[15, 16]	$1.8^{+0.8}_{-1.8}$	$1.4^{+0.9}_{-1.4}$
[16, 17]	$2.1^{+0.7}_{-1.0}$	$1.9^{+0.8}_{-1.9}$
[17, 18]	$2.9^{+0.5}_{-0.5}$	$3.0^{+0.5}_{-0.6}$
[18, 19]	$2.7^{+0.6}_{-0.5}$	
[18, 19.24]		$2.9^{+0.6}_{-0.7}$
[19, 20]	$0^{+1.6}_{-0}$	
[20, 21]	$1.4^{+0.9}_{-1.4}$	
[21, 22]	$3.2^{+0.8}_{-0.9}$	
[19.24, 22.9]		$2.5^{+0.7}_{-1.0}$
constant	2.6 ± 0.4 ($\chi^2/\mathrm{dof} = 1.06$)	

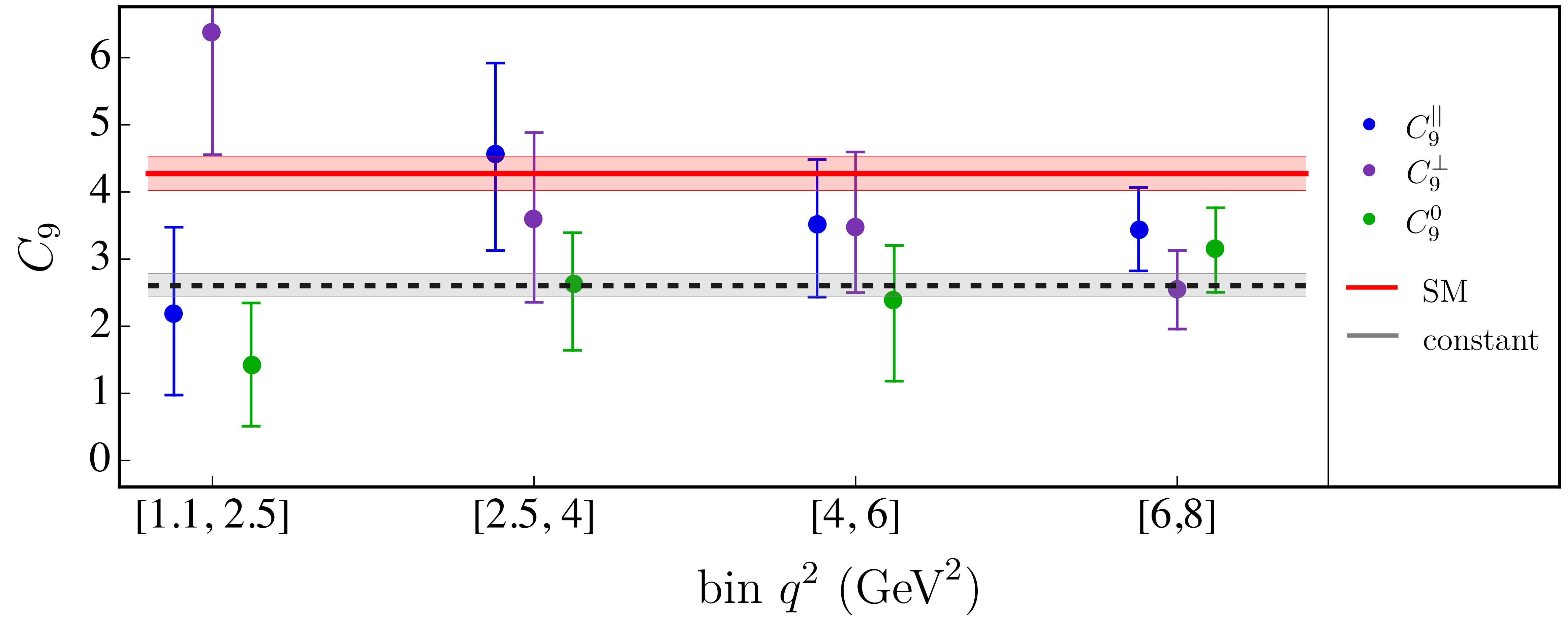
Table 3.3: Determinations of C_9 from $B \to K\mu^+\mu^-$ in the low-q^2 (left) and high-q^2 (right) regions. The p-values for the constant fits are 0.17 (low-q^2) and 0.39 (high-q^2).

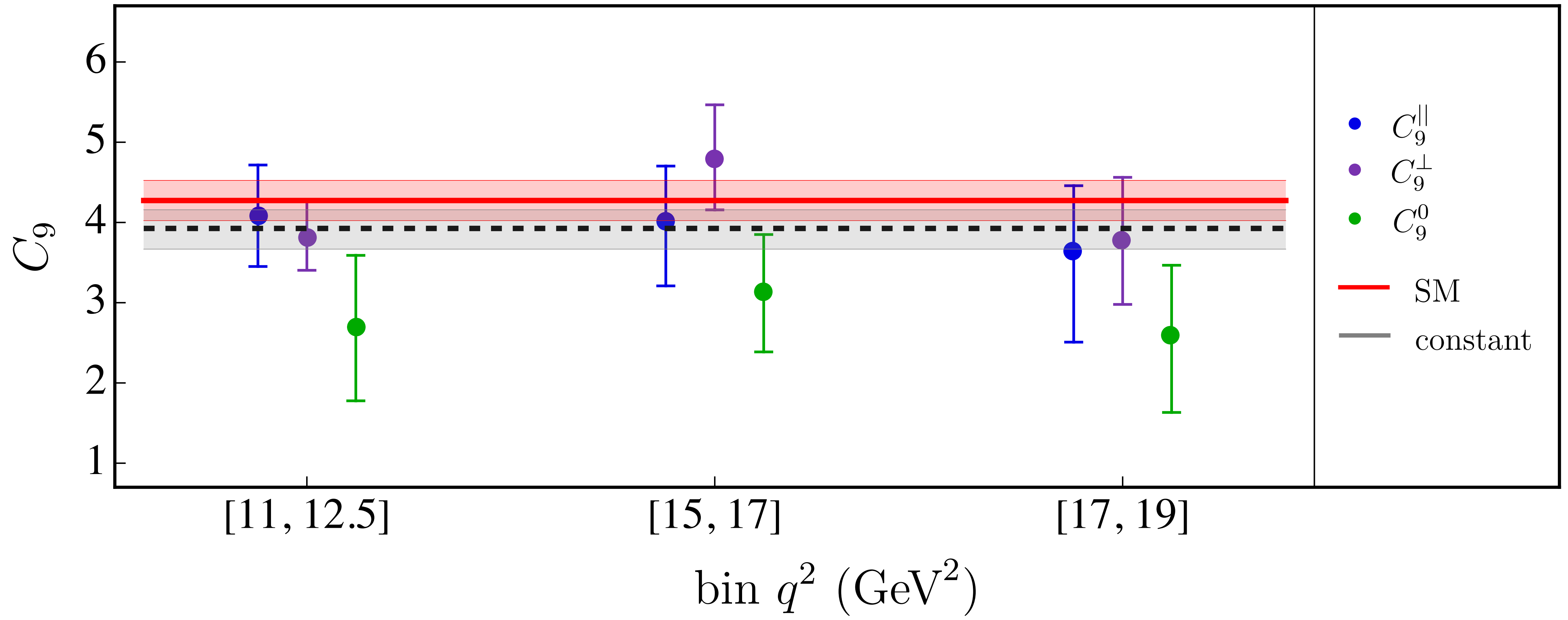
[M. Bordone, G.Isidori, S. Mächler, AT, 2401.18007]

$B \to K^*$ results

We're working on updating this with new LHCb data! (Half-sized bins!)

Using resonance parameters found by LHCb recently (2405.17347)





	constant C_9	$C_9^{\|\|}$	$C_9^{\perp}$	C_9^0
Low q^2	$2.60^{+0.18}_{-0.17}$	$2.4^{+0.6}_{-0.6}$	$2.6^{+0.7}_{-0.6}$	$2.8^{+0.7}_{-0.8}$
High q^2	$3.93^{+0.23}_{-0.26}$	$4.0^{+0.5}_{-0.5}$	$4.0^{+0.4}_{-0.4}$	$2.9^{+0.6}_{-0.6}$

$$C_9 = 3.40^{+0.16}_{-0.16} \quad (\chi^2/dof = 1.5)$$

Importance of extracting the value of C_9 at different values of q^2

Conclusions

- We have presented an estimate of the leading $B^0 \to K^0 \bar{\ell}\ell$ long-distance contributions induced by the rescattering of a charmed and a charmed-strange meson for both monopole and dipole photon couplings.
- From charm rescattering only, it seems hard to generate an effect large enough to explain the tensions while maintaining a **short-distance-like structure** favored by present data.
- A **high level of conspiracy** seems to be required to obtain an $\mathcal{O}(25\%)$ shift in C_9 in the whole q^2 range.
- Extracting C_9 experimentally in **different** q^2 **windows** helps.
- With more precise data the picture will become clearer: we'll be able to see if we are missing some long-distance contributions, or if the tension remains short-distance-like.

Thank you for your attention!

Backup

Conclusions

Independent determinations of C_9 assuming it to be constant: