# Journée des nouveaux entrants 2025 : Mathematical Physics

Robin Zegers

IJCLab, CNRS/IN2P3 & Université Paris-Saclay, Orsay, France

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## Definition (from J. of Math. Phys.)

"…the application of mathematics to problems in physics, the development of mathematical methods suitable for such applications and for the formulation of physical theories."

## Another (more symbolic) way to put it

$$M\Phi : M \overset{\longrightarrow}{\underset{\longleftarrow}{}} \Phi$$

where

- $M \longrightarrow \Phi$: application of existing mathematical framework/methods to theoretical Physics;
- $M \longleftarrow \Phi$: construction of the mathematical framework/methods needed to make sense of physical theories/solve problems form theoretical Physics.

N.B.: could apply to any part of $\Phi$

$\Rightarrow$ $M\Phi$ more a method (theorems, proofs etc as in Maths) than a particular subject!

# Members of the Mathematical Physics team

- Michel Dubois-Violette
- Samuel Friot
- Valentine Maris
- Parham Radpay
- Vincent Rivasseau
- Jean-Christophe Wallet
- Robin Zegers

Main research topics:

- Analytical methods for QFT
- Quantum Gravity and Random Tensors
- Quantum Gravity and QFT on Quantum space-times
- Quantum symmetries and Quantum integrability

## Special functions for Feynman integrals (FI)

Dimensionally-regulated FI $\rightarrow$ Multivariable hypergeometric functions ("too complicated" objects)

Simple subclass describing many FI: Multiple polylogarithms (well-understood)

Beyond one-loop: more intricate functions also appear (elliptic polylogarithms, iterated integrals of modular forms etc). These are not fully understood $\Rightarrow$ Numerical evaluation difficult

Recent progress for the Mellin-Barnes (MB) representation computational technique

## Aim

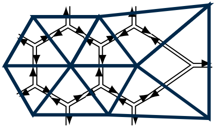
- Improve numerical evaluation by finding new analytic properties using tools such as MB technique
- Improvement of MB technique opens new research directions:
  - study of unexpected links with some aspects of computational geometry
  - direct application to multivariable hypergeometric functions
  - ...

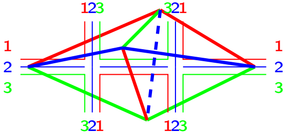
## The tensor track to Quantum Gravity

A theoretical framework for Quantum Gravity

### Random tensor theories

- a generalization of random matrices which describe 2d Quantum Gravity...

$d = 2$: 

$d = 3$: 


$d > 3\dots$

- ...to higher rank tensors $\Rightarrow$ to higher dimensional space-times

## Applications cover

- Random geometry and integrable models
- The "tensor track" to Quantum Gravity
- Signal/Image/Video analysis, Pattern recognition, Tensorial Principal Component Analysis
- Random tensors for AI and Complexity (Collaboration with CEA-LIST)

# Quantum Gravity and QFT on Quantum Space-times

Non-commutative Geometry provides a mathematical description of "Quantum spaces" and "Quantum space-times"

E.g. $\kappa$-Minkowski space-time $\simeq$ "Deformed" version of the Minkowski space-time of SR

## Motivation: an access to Quantum Gravity Phenomenology

(High-energy) effective regime of Quantum Gravity expected to by described by gauge field theories on quantum space-times

## Question: can one construct QFT on Quantum space-times?

- Construction of the 1st physically acceptable gauge theory on $\kappa$-Minkowski space-time
- Specific Phenomenology at Planck scale:
  - Modified causality
  - Modified dispersion relations
  - Modified symmetries (Lorentz, C,P,T)

# Quantum Symmetries and Quantum integrability

Non-commutative geometry also provides a mathematical description of "Quantum Symmetries"
They can be:

- quantum symmetries of quantum spaces
  E.g. $\kappa$-Poincaré Hopf-algebra $\simeq$ Deformation of the Poincaré (universal enveloping) algebra of SR
- hidden/dynamical symmetries of quantum integrable systems (algebraic Bethe ansatz)
  E.g. Quantum affine algebras $\simeq$ Deformations of Kac-Moody Lie algebras.

## Main questions

- Construct and understand the structure of these quantum algebras
- Classify their (irreducible) representations

## Quantum Toroidal Algebras and their representation theory

Essential for various integrable structures in: 2d CFT, SUSY gauge theories, string theory...
But still poorly understood!

New family of quantum algebras : Double Quantum Affinizations

- Construction of Quantum Toroidal Algebra from any simple Lie algebra
- Classification of all their (weight-finite) irreducible representations

Thanks for your attention!