IJCLab | Pôle Théorie

JOURNÉE DES NOUVEAUX ENTRANTS

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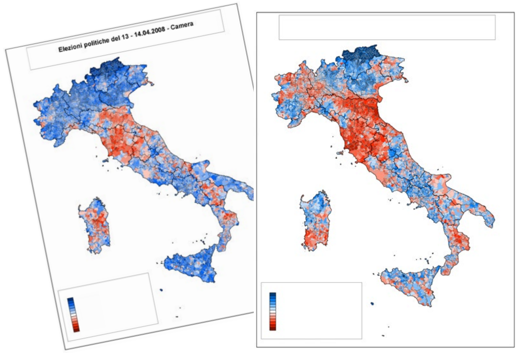
Orsay, 26 March 2025

Curriculum vitae et studiorum

- Born in Padova, 1990.
- B.Sc. in Physics, *Università degli Studi di Padova*, 2009-2012. Thesis title:
  «Il modello collettivo: soluzioni dell'Hamiltoniana di Bohr nel caso γ-instabile dell'oscillatore quartico »
- M.Sc. in Physics, *Università degli Studi di Padova*, 2012-2015. Thesis title:
  «Simmetrie e rotovibrazioni di nuclei α-coniugati »
- Ph.D. in Physics, HISKP, *Universität Bonn*, 2016-2020. Thesis title:
  «Nuclear Physics in a finite volume: Investigation of two-particle and α-cluster systems »
- Post-Doc, HISKP, Universität Bonn, 2020-2021.
- Post-Doc, CEA Paris-Saclay, 2022-2025.
- Post-Doc, IJCLab, CNRS, *Université Paris-Saclay*, since 2025.

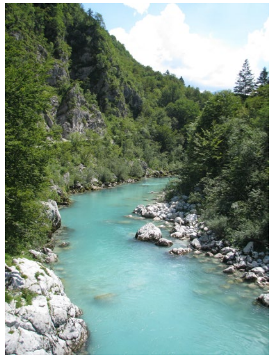
Memberships: SOCIETÀ ITALIANA DI FISICA

Hobbies and spare time activities



Mapping

Contributing to **Wikipedia**



Hiking

Travelling

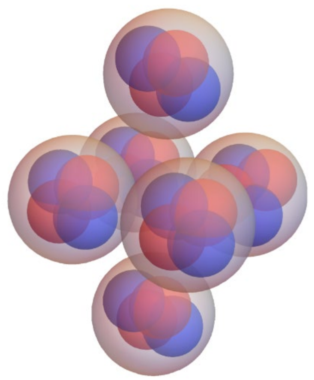
Drawing

Reading, doing sport (not much) **& more ...**

Clustering and intrinsic shape of ^{24}Mg

- The ***geometric alpha-cluster model*** (**GαCM**) is a macroscopic approach describing α-conjugate nuclei in terms of N α-partcles rotating and vibrating about their equilibrium positions, sitting at the vertices of a *polyhedral structure*. Equilibrium configurations are characterized by a *point symmetry group* $\mathcal{G}$.

Inspiration: R. Bijker and F. Iachello, *Nucl. Phys. A* **1006**, 122077 (2021) and G. S., L. Fortunato and A. Vitturi, *J. Phys. G.* **43**, 085104 (2016), *ArXiv:* **1512.025123** (2015).

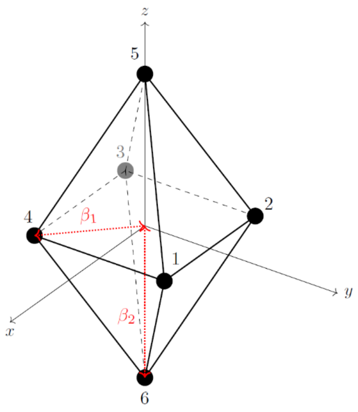


The system is described the quantum vibration-rotation Hamiltonian, with N = 6 α-clusters:

$$H = \frac{1}{2}\sum_{\alpha\beta}(J_\alpha - p_\alpha)\mu_{\alpha\beta}(J_\beta - p_\beta) + \frac{1}{2}\sum_{j=1}^{3N-6} P_j^2 + \frac{1}{2}\sum_{j=1}^{3N-6}\lambda_j Q_j^2 - \frac{\hbar^2}{8}\sum_{\alpha}\mu_{\alpha\alpha}$$

orbital angular momentum; vibrational angular momentum; **normal coordinates**; effective reciprocal inertia tensor; conjugate momenta

J.K.G. Watson, *Mol. Phys.* **15**, 479-490 (1968)



The structure parameters specifying the equilibrium α-configuration are β_1 and β_2.

The model for ^{24}Mg predicts 9 normal modes of vibration, labeled by the irreducible representations of $\mathcal{D}_{4h}$, the symmetry group associated with the intrinsic structure of the nucleus at equilibrium, a **square bipyramid**:

$$\begin{array}{ccccccccc} \omega_1 & \omega_2 & \omega_3 & \omega_4 & \omega_5 & \omega_6 & \omega_7 & \omega_8 & \omega_9 \\ Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & (Q_7,Q_8) & (Q_9,Q_{10}) & (Q_{11},Q_{12}) \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ A_{1g} & A_{1g} & A_{2u} & B_{1g} & B_{2g} & B_{2u} & E_g & E_u & E_u \end{array} \in \mathcal{D}_{4h}$$

Moments of inertia

pointlike charge distribution:

$$I_{xx}^{\text{stat}} = I_{yy}^{\text{stat}} = 2m(\beta_1^2 + \beta_2^2)$$

$$I_{zz}^{\text{stat}} = 4m\beta_1^2$$

Gaussian charge distribution:

$$I_{xx}^{\text{stat}} = I_{yy}^{\text{stat}} = 2m(\beta_1^2 + \beta_2^2) + \frac{6m}{\alpha_1}$$

$$I_{zz}^{\text{stat}} = 4m\beta_1^2 + \frac{6m}{\alpha_1}$$

Adopted values

$(\beta_1, \beta_2) = (2.38, 3.72)$ fm

- The candidates for the 9 rotational bands (singly-excited normal modes) have been *identified* in the measured spectrum!

The exact Hamiltonian can be systematically approximated, by introducing higher-order coupling between rotations an vibrations. At LO rotations and vibrations are decoupled. Thus, the LO Hamiltonian is invariant under *parity*, *time reversal*, and the *equilibrium point group*, $\mathcal{D}_{4h}$.

$$E_{LO}(J,K,[\mathfrak{n}]) = \frac{\hbar^2}{2I_{xx}^{\text{stat}}}[J(J+1)-K^2] + \frac{\hbar^2}{2I_{zz}^{\text{stat}}}K^2 + \sum_{i=1}^{6}\hbar\omega_i(\mathfrak{n}_i + \frac{1}{2}) + \sum_{i=7}^{9}\hbar\omega_i(\mathfrak{n}_i + 1) - \frac{\hbar^2}{8}\sum_{\alpha} I_{\alpha\alpha}^{\text{stat}\,-1}$$

where $(\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6,\omega_7,\omega_7,\omega_8,\omega_8,\omega_9,\omega_9) \equiv (\lambda_1,\ \lambda_2\ \ldots\ \lambda_{12})$ $\rightsquigarrow$ frequencies of the normal modes

and $\mathfrak{n}_i$ $\rightsquigarrow$ number of vibrational quanta of the mode ω_i, vectorized as $[\mathfrak{n}]$

More details:
G.S. et al: ArXiv: **2412.17782** (2024)

The first excited rotational band B_{2g}

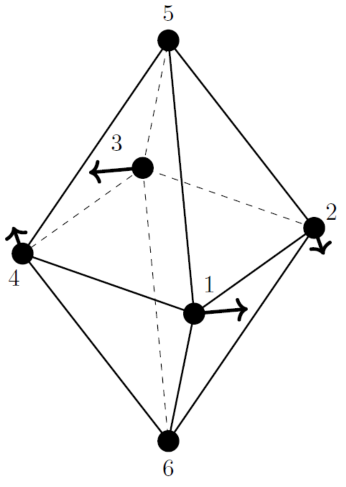
EXAMPLE

Excitation quantum: $\hbar\omega_5 = 2.997(29)$ MeV

associated with the normal coordinate:

$$Q_5 = \sqrt{m}\left(-\frac{\Delta x_1}{2\sqrt{2}} + \frac{\Delta x_2}{2\sqrt{2}} + \frac{\Delta x_3}{2\sqrt{3}} - \frac{\Delta x_4}{2\sqrt{3}} + \frac{\Delta y_1}{2\sqrt{2}} + \frac{\Delta y_2}{2\sqrt{2}} - \frac{\Delta y_3}{2\sqrt{2}} - \frac{\Delta y_4}{2\sqrt{2}}\right)$$

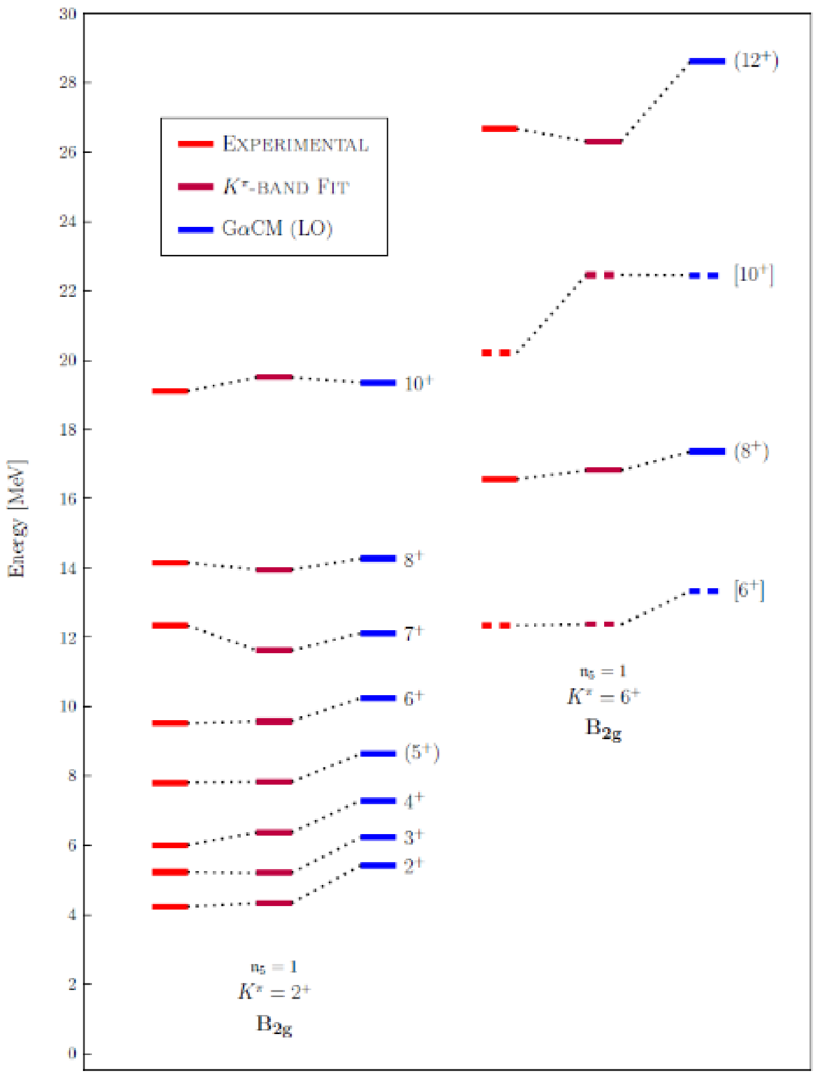
The $K^\pi = 2^+$ bandhead is a 2^+ at 4.123 MeV. The composition of the band reflects the literature assignments, corroborated by the NNDC. It is the most consolidated singly-excited band.



Q_5
ω_5

The 6^+ band is a new assignment and rather uncertain.

Description: Symmetric *scissoring mode* of pairs of adjacent α-clusters in the xy plane. The basis of the bipyramid becomes rectangular. The apical α-clusters do not move.



Intraband B_{2g}	Experimental		GαCM LO
	W.U.	e^2 fm^4	e^2 fm^4
$B[E2; 2^+ (4.123) \to 3^+ (5.235)]$	n.a.	n.a.	196.85
$B[E2; 2^+ (4.238) \to 4^+ (6.010)]$	26.82^{+216}_{-216}	110.30^{+888}_{-888}	84.36
$B[E2; 4^+ (6.010) \to 6^+ (9.528)]$	36.1^{+317}_{-130}	148.5^{+1307}_{-535}	133.62
$B[E2; 5^+ (7.812) \to 7^+ (12.340)]$	n.a.	n.a.	141.59
$B[E2; 6^+ (9.527) \to 8^+ (14.150)]$	n.a.	n.a.	146.02
$B[E2; 8^+ (14.150) \to 10^+ (19.110)]$	n.a.	n.a.	150.17

- The calculated values of the reduced transition probabilities at LO in the GαCM agree with the experimental values within one standard deviation.

Nucleon mass-specific moments of inertia:

$\mathscr{I}_x = 140.3(15)$ fm^2

$\mathscr{I}_z = 87.7(24)$ fm^2

More details:

G.S. et al: ArXiv: **2412.17782** (2024)

Work in progress: M1, M2 transitions and calculations at NLO

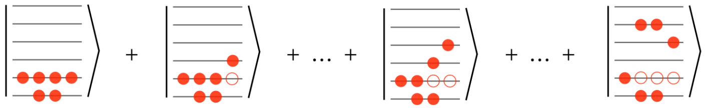
Light Nuclei Workshop CEA Paris-Saclay 2024

The **ESNT** workshop coorganized by G.S. last year, together with V. Somà and T. Duguet.

Self consistent Gorkov-Green's function theory

- Ab-initio approach, extending the self-consistent Green's function theory to *semimagic* nuclei. The Z ***protons*** and N ***neutrons*** interact through *Chiral EFT* potentials.

SCGGF is a **correlation-expansion** method, in which the exact eigenstate of the A-body Schrodinger equation is constructed by means of the particle-hole correlator Ω :

$|\Psi_0^A\rangle =$  $+ \dots$ $\rightsquigarrow$ $|\Psi_0^A\rangle = \Omega|\Phi_0^A\rangle = |\Phi_0^A\rangle + |\Phi_0^{A\,1p1h}\rangle + \ldots + |\Phi_0^{A\,2p2h}\rangle + \ldots + |\Phi_0^{A\,3p3h}\rangle + \ldots$

and the **reference state** Φ_0^A is the ground state of H_0, a solvable Hamiltonian, splitting the original one into $H = H_0 + H_I$ where H_I contains the 2-, 3-, ...-body interactions.

Since for **open-shell nuclei** the ground state is *almost degenerate* with respect to the excitation of pairs of nucleons in the lowest-lying single-particle (s.p.) energy levels, it is necessary to reopen the s.p. energy gaps. The problem is fixed by means of ***particle-number-symmetry breaking*** in the reference state (Bogoliubov vacuum).

With reference to the size M of the basis of single-particle eigenstates, the $U_Z(1) \times U_N(1)$ - symmetry breaking mechanism gives:

✓ **Advantage:** polynomial scaling (M^α) ✗ **Disadvantage:** M increases and symmetries must be restored

Tool: the present implementations of SCGGF are based on the one-body propagator, which in time representation is defined as:

$$i\mathbf{G}_{ab}(t,t') \equiv \langle \Psi_0 | T\{\mathbf{A}_a(t) \odot \mathbf{A}_b^*(t')\} | \Psi_0 \rangle$$

where $\mathbf{A}_a^\dagger = \begin{pmatrix} a_a^\dagger & a_{\bar{a}} \end{pmatrix}$ $\quad \mathbf{A}_a \equiv \begin{pmatrix} a_a \\ a_{\bar{a}}^\dagger \end{pmatrix}$ are rank-one Nambu tensors.

single-particle index $a \equiv (n, \ell, j, m, q)$

involuted single-particle index $\tilde{a} \equiv (n, \ell, j, -m, q)$

- The Fourier transform of time delivers the ***Lehmann representation:***

$$G_{ab}^{gg'}(\omega) = \sum_k \frac{{}^k\chi_a^g \; {}^k\chi_b^{g'*}}{\omega - (\Omega_k - \Omega_0)/\hbar + i\eta} + \sum_k \frac{{}^k\Upsilon_a^g \; {}^k\Upsilon_b^{g'*}}{\omega + (\Omega_k - \Omega_0)/\hbar - i\eta}$$

where

$${}^k\chi_b^1 \equiv \langle \Psi_0 | A_b^1 | \Psi_k \rangle = \langle \Psi_0 | a_b | \Psi_k \rangle$$
$${}^k\chi_b^2 \equiv \langle \Psi_0 | A_b^2 | \Psi_k \rangle = \langle \Psi_0 | a_{\bar{b}}^\dagger | \Psi_k \rangle$$

involuted Nambu index $\bar{2} = 1 \quad \bar{1} = 2$

and $(-1)^g \, [{}^k\chi_a^g]^* = {}^k\Upsilon_{\bar{a}}^{\bar{g}}$

are the **spectroscopic amplitudes**

$\Omega_k - \Omega_0 \rightsquigarrow$ approx. ***separation energies*** betw. the g.s. of the *A*-body system and the energy eigenstate *k* of the A±1-body system

Observables: binding energies, charge radii, one-nucleon separation energies, spectrum of even-odd nuclei ...

Phys. Rev. C **84**, 064317 (2011) *Eur. Phys. J A* **57**, 135 (2021)
Phys. Rev. Lett. **128**, 022502 (2022)

Self consistent Gorkov-Green's function theory

- **New tool**: we introduce Gorkov's ***polarization propagator***, whose definition reflects Dyson's case:

$$\Pi^{gg''g'''g'}_{acdb}(t,t') \equiv \lim_{\substack{t''\to t^+ \\ t'''\to t'^+}} iG^{gg'g''g'''}_{abcd}(t,t',t'',t''') - iG^{gg''}_{ac}(t,t'')G^{g'g'''}_{bd}(t',t''') \quad \text{where} \quad i^2\mathbf{G}_{abcd}(t,t',t'',t''') \equiv \langle\Psi_0|T\{\mathbf{A}_a(t)\odot\mathbf{A}_b(t')\odot\mathbf{A}^*_d(t''')\odot\mathbf{A}^*_c(t'')\}|\Psi_0\rangle$$

The Fourier transform of time delivers the ***Lehmann representation***,

$$\Pi^{gg''g'''g}_{acdb}(\omega) \equiv \Pi^{+\,gg''g'''g}_{acdb}(\omega) + \Pi^{-\,gg''g'''g}_{acdb}(\omega) \quad \text{where} \quad \Pi^{+\,gg''g'''g'}_{acdb}(\omega) = \sum_{k\neq 0}\frac{{}^k\chi^{gg''}_{ac}\;{}^k\chi^{*g'''g'}_{db}}{\omega-(\Omega_k-\Omega_0)/\hbar+i\eta} \quad \text{and} \quad \Pi^{-\,gg''g'''g'}_{acdb}(\omega) = -\sum_{k\neq 0}\frac{{}^k\Upsilon^{gg''}_{ac}\;{}^k\Upsilon^{*g'''g'}_{db}}{\omega+(\Omega_k-\Omega_0)/\hbar-i\eta}$$

where the + (-) part is analytical in the upper (lower) part of the complex plane for ω .

- The poles approx. coincide with the **energies of the excited states** of the A-body system with respect to the g.s. energy $E_k \approx \Omega_k - \Omega_0$. At the numerator

$${}^k\chi^{11}_{bc} \equiv \langle\Psi_0|A^1_bA^{\dagger\,1}_c|\Psi_k\rangle = \langle\Psi_0|a_ba^\dagger_c|\Psi_k\rangle \qquad {}^k\chi^{12}_{bc} \equiv \langle\Psi_0|A^1_bA^{\dagger\,2}_c|\Psi_k\rangle = \langle\Psi_0|a_ba_{\bar{c}}|\Psi_k\rangle \qquad {}^k\chi^{21}_{bc} \equiv \langle\Psi_0|A^2_bA^{\dagger\,1}_c|\Psi_k\rangle = \langle\Psi_0|a^\dagger_{\bar{b}}a^\dagger_c|\Psi_k\rangle \qquad {}^k\chi^{22}_{bc} \equiv \langle\Psi_0|A^2_bA^{\dagger\,2}_c|\Psi_k\rangle = \langle\Psi_0|a^\dagger_{\bar{b}}a_{\bar{c}}|\Psi_k\rangle$$

are the **transition amplitudes** , fulfilling $-(-1)^{g+g'}\;{}^k\Upsilon^{gg'}_{ab} = [{}^k\chi^{\bar{g}\bar{g}'}_{\bar{a}\bar{b}}]^*$ hence $\Pi^{+\,gg''g'''g'}_{acdb}(\omega) = (-1)^{\bar{g}+\bar{g}'+\bar{g}''+\bar{g}'''}[\Pi^{-\,\bar{g}\bar{g}''\bar{g}'''\bar{g}'}_{\bar{a}\bar{c}\bar{d}\bar{b}}(-\omega)]^*$

Observables: reduced *electric* $(R=E)$ and *magnetic* $(R=M)$ multipole transition probabilities between states with angular momentum J and J_p:

$$B(J_0\to J_p, R\ell) \equiv \frac{1}{2J_0+1}\sum_{M_0}\sum_{M_p}\sum_m |\langle\Psi_p|\mathfrak{Q}_{\ell m}(R)|\Psi_0\rangle|^2$$

where $\mathfrak{Q}_{\ell m}(R)$ are the transition operators with angular momentum ℓ and projection m: $\langle\Psi_p|\mathfrak{Q}_{\ell m}(R)|\Psi_0\rangle = \sum_{ab}(a|\mathfrak{Q}_{\ell m}(R)|b)\langle\Psi_p|[A^{1\,\dagger}_a\otimes A^1_b]^\ell_m|\Psi_0\rangle$

which are expressed in terms of the angular-momentum-coupled transition matrix elements: $[A^{\dagger\,1}_a\otimes A^1_b]^\ell_m = [a^\dagger_a\otimes a_b]^\ell_m = \sum_{m_am_b}(j_aj_b\ell|m_a-m_bm)(-1)^{-m_b}a^\dagger_aa_b$

and the matrix elements between the s.p. states and the EM mult. transition oper. are given by: $(a|\mathfrak{Q}_{\ell m}(E)|b) = \int d^3r\;(a|r^\ell Y^m_\ell(\theta,\phi)\rho(\mathbf{r})|b)$ and $(a|\mathfrak{Q}_{\ell m}(M)|b) = \int d^3r\;(a|\mathbf{j}(\mathbf{r})\cdot\mathbf{L}r^\ell Y^m_\ell(\theta,\phi)|b)$

where $\rho(\mathbf{r}) = e\sum_{i=1}^Z\delta(\mathbf{r}-\mathbf{r}_i)$ and $\mathbf{j}(\mathbf{r}) = \frac{e\hbar}{2mi}\sum_{i=1}^Z[\delta(\mathbf{r}-\mathbf{r}_i)\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\delta(\mathbf{r}-\mathbf{r}_i)]$ (pointlike charge distribution)

The ADC method for Gorkov's polarization propagator

- **Strategy**: By means of the automated implementation of Wick's theorem (AIWT) codes (on **GitHub**), the Nambu comp. of the polarization propagator are expanded in *perturbation theory* up to third order. The expression of each time-ordered or Goldstone contribution in energy representation is also provided.

  - **Method**: application of the *algebraic diagrammatic construction* (ADC) scheme. The starting-point is general one-body *transition operator*

$$\mathcal{D} = \sum_{g_1 g_2} \sum_{rs} D^{g_1 g_2}_{rs} A^{g_1\,\dagger}_r A^{g_2}_s$$

Defining the transition function as $T(\omega) \equiv \sum_{g_1 g_2 g_3 g_4} \sum_{abcd} D^{g_1 g_3 *}_{ac} \Pi^{+\, g_1 g_3 g_4 g_2}_{acdb}(\omega) D^{g_4 g_2}_{db}$ consistently with J. Schirmer, *Phys. Rev. A* **26**, 5, 2395-2416 (1982)

For the polarization propagator, the ADC ansatz recalls the Lehmann representation: $T(\omega) \equiv \mathbf{F}^\dagger(\omega \mathbb{1} - \mathbf{K} - \mathbf{C})^{-1}\mathbf{F}$

where $\mathbf{K} \Rightarrow$ 'matrix' depending on the eigenvalues associated with the unperturbed Hamiltonian (grand-canonical potential).

and $\mathbf{C} \Rightarrow$ modified interaction 'matrix' $\mathbf{C} \equiv \mathbf{C}^{(1)} + \mathbf{C}^{(2)} + \dots$ and $\mathbf{F} \Rightarrow$ modified transition moments $\mathbf{F} \equiv \mathbf{F}^{(0)} + \mathbf{F}^{(1)} + \mathbf{F}^{(2)} + \dots$

The geometric series gives: $T(\omega) \overset{\text{ADC}}{=} \mathbf{F}^\dagger(\omega\mathbb{1} - \mathbf{K})^{-1} \sum_{n=0}^{+\infty} \left\{ \mathbf{C}(\omega\mathbb{1} - \mathbf{K})^{-1} \right\}^n \mathbf{F} \quad \rightsquigarrow \quad T(\omega) \equiv T^{(0)}(\omega) + T^{(1)}(\omega) + T^{(2)}(\omega) + \dots$

  - A *matching* procedure with the perturbative expansion of the polarization propagator yields the expressions for $\mathbf{F}$, $\mathbf{C}$ and $\mathbf{K}$.

Approx. expressions for the poles of Lehmann's repr. of the polarization prop. are obtained by means of the diagonalization of the 'matrix', $\mathbf{K} + \mathbf{C}$.

- **Example:** first order

$$T^{(1)}(\omega) = \mathbf{F}^{\dagger(1)}[\omega\mathbb{1} - \mathbf{K}]^{-1}\mathbf{F}^{(0)} + \mathbf{F}^{\dagger(0)}[\omega\mathbb{1} - \mathbf{K}]^{-1}\mathbf{F}^{(1)} + \mathbf{F}^{\dagger(0)}[\omega\mathbb{1} - \mathbf{K}]^{-1}\mathbf{C}^{(1)}[\omega\mathbb{1} - \mathbf{K}]^{-1}\mathbf{F}^{(0)}$$

The matching procedure for the modified transition moments yields:

$$^{k_3 k_2}F^{(1)} \equiv \sum_{k_2 k_3} \sum_{db} \sum_{g_5 g_6} D^{g_6 g_5}_{db} \frac{^{k_2}\Upsilon^{(0) g_5}_{b}\, {}^{k_3}\chi^{*(0) g_6}_{d} - {}^{k_3}\chi^{*(0)\bar{g}_5}_{\bar{b}}\, {}^{k_2}\Upsilon^{(0)\bar{g}_6}_{\bar{d}}}{\sqrt{2}} \sum_{\substack{g_1 g_2 \\ g_3 g_4}} \sum_{pqrs} \left({}^{k_1 k_4 k_3 k_2} f^{(1,d)\ g_1 g_4 g_3 g_2}_{prsq} + {}^{k_1 k_4 k_3 k_2} f^{(1,c)\ g_1 g_4 g_3 g_2}_{prsq} \right)$$

$$^{k_1 k_4 k_3 k_2} f^{*(1,\mathrm{d})\ 1111}_{\bar{p}\bar{r}\bar{s}\bar{q}} \equiv \frac{1}{2\hbar} \sum_{k_5} \frac{^{k_5}\Upsilon^{(0)2}_{p}\, {}^{k_4}\chi^{*(0)2}_{r}\, \bar{v}_{\bar{s}\bar{r}\bar{q}\bar{p}}\, {}^{k_5}\Upsilon^{*(0)2}_{s}\, {}^{k_2}\Upsilon^{(0)2}_{q}\, \delta_{k_1 k_3}}{-\left(\frac{\Omega_{k_2} - \Omega_0}{\hbar}\right) - \left(\frac{\Omega_{k_4} - \Omega_0}{\hbar}\right)} - \frac{1}{2\hbar} \sum_{k_5} \frac{^{k_1}\chi^{(0)2}_{p}\, {}^{k_5}\Upsilon^{*(0)2}_{r}\, \bar{v}_{\bar{s}\bar{r}\bar{q}\bar{p}}\, {}^{k_3}\Upsilon^{*(0)2}_{s}\, {}^{k_5}\Upsilon^{(0)2}_{q}\, \delta_{k_2 k_4}}{-\left(\frac{\Omega_{k_1} - \Omega_0}{\hbar}\right) - \left(\frac{\Omega_{k_3} - \Omega_0}{\hbar}\right)} + \dots$$

$$^{k_1 k_4 k_3 k_2} f^{*(1,\mathrm{d})\ 1122}_{\bar{p}\bar{r}\bar{s}\bar{q}} \equiv -\frac{1}{2\hbar} \sum_{k_5} \frac{^{k_5}\chi^{*(0)2}_{p}\, {}^{k_4}\Upsilon^{(0)2}_{r}\, \bar{v}_{\bar{r}q\bar{p}s}\, {}^{k_5}\chi^{(0)2}_{s}\, {}^{k_2}\chi^{*(0)1}_{q}\, \delta_{k_1 k_3}}{-\left(\frac{\Omega_{k_2} - \Omega_0}{\hbar}\right) - \left(\frac{\Omega_{k_4} - \Omega_0}{\hbar}\right)} + \dots$$

Outlook: complete the procedure, at least, up to second order, i.e. ADC[2].

Properties of dilute neutron matter with 3-body forces

Purpose: Investigation of the properties of the upper layers of the inner crust of neutron stars, which could explain the phenomenon of the *glitches*. Neutron matter is in superfluid state, with a density close to the neutron-drip density $n \approx 2.5 \times 10^{-4}\ \mathrm{fm}^{-3}$. Primary targets are the **equation of state** (EoS) and the **pairing gaps**.

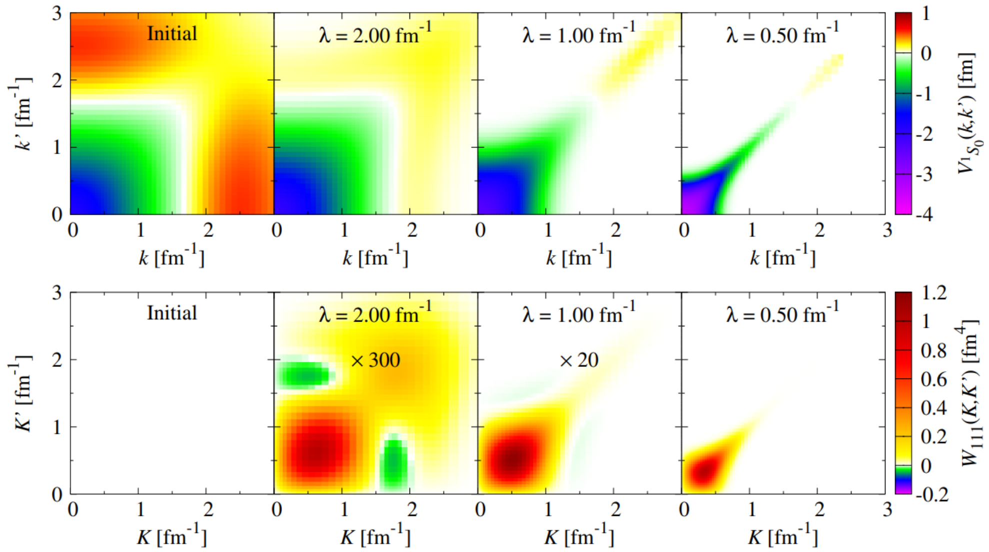
Inspiration : *V. Palanappian, S. Ramanan and M. Urban: ArXiv:* **2412.00137** (2024)

▶ **Tools:** As in ab-initio methods, nuclear interactions are drawn from Chiral Effective Field Theory (e.g. at N²LO)

$$H = T + V_{nn}^{N^2LO} + W_{nnn}^{N^2LO} \qquad \text{where} \qquad W_{nnn}^{N^2LO} = \frac{g_A^2}{4f_\pi^4}\mathcal{A}_{123}\sum_{i\neq j\neq k}\frac{(\boldsymbol{\sigma}_i\cdot\mathbf{q}_i)(\boldsymbol{\sigma}_j\cdot\mathbf{q}_j)}{(q_i^2+m_\pi^2)(q_j^2+m_\pi^2)}[-2c_1m_\pi^2 + c_3\mathbf{q}_i\cdot\mathbf{q}_j]$$

New!

▶ Preprocessing of nuclear interactions with **similarity renormalization group** (SRG) techniques:



▶ SRG-induced 3-body interactions, from S-wave V_{nn}
[*ArXiv:* **2412.00137**]

I. addition of SRG-induced 3-body forces into the second-quantized Hamiltonian $\hat{H}$, together with the dependence of the 2- and 3-body matrix elements on the evolution parameter s

II. Derivation of the *SRG flow equations* for the matrix elements of the antisymmetrized 2- and 3-body interaction from the flow eq. for $\hat{H}$

$$\frac{d\hat{H}_s}{ds} = [\eta_s, \hat{H}_s] \quad \text{with generator} \quad \eta_s \equiv [\hat{T}, \hat{H}_s]$$

III. Numerical solution of SRG flow equations for the matrix elements of the 2- and 3-body forces, in the *hyperspherical plane wave basis.*

Typical values: $\lambda \equiv s^{-1/4} \lesssim 0.50\ \mathrm{fm}^{-1}$

also $\lambda \approx 2k_F$ where $k_F \equiv (3\pi^2 n)^{1/3}$ is the *Fermi momentum*

Properties of dilute neutron matter with 3-body forces

- Next, comes the derivation of the **equation of state** for dilute neutron matter with three-body forces.

It consists in calculating the g.s. energy per particle, E_{HFB} for the Hamiltonian $\hat{H}$.
Mean field *approximation*: Hartree-Fock (HF) or Hartree-Fock-Bogoliubov (HFB):

$$E_{\text{HFB}} = \frac{\mathcal{E}_{\text{HFB}}}{n_{\text{HFB}}} \quad \text{and the ratio} \quad \mathcal{E}_{\text{HFB}}/\mathcal{E}_{\text{FG}} \quad \text{where} \quad \mathcal{E}_{\text{FG}} = \frac{k_F^5}{10\pi^2 m_n}$$

HFB energy density

HFB particle density

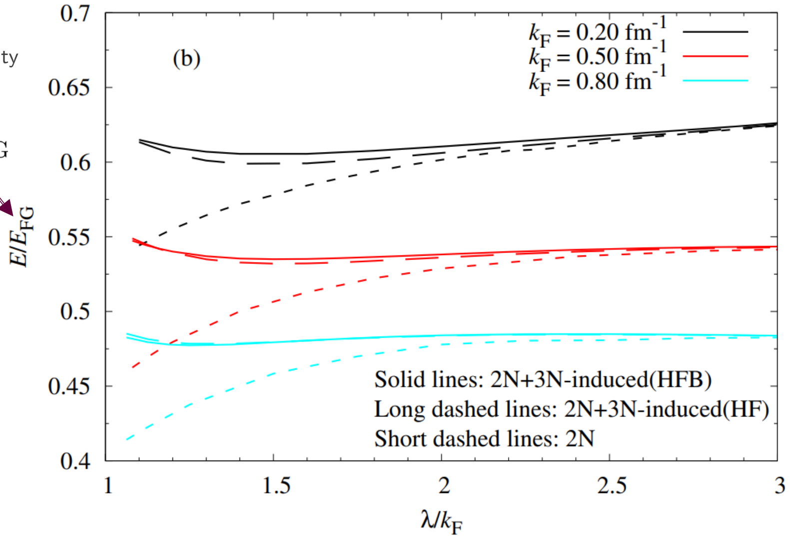
energy density of the non-interacting neutron Fermi gas

- Improvements are obtained by applying *Bogoliubov many-body perturbation theory* (BMBPT).

Further reading: M. Urban et al., *Phys. Rev. A* **103**, 063306 (2021) V. Palaniappan et al., *Phys. Rev. C* **107**, 025804 (2023)

Fermi gas particle density

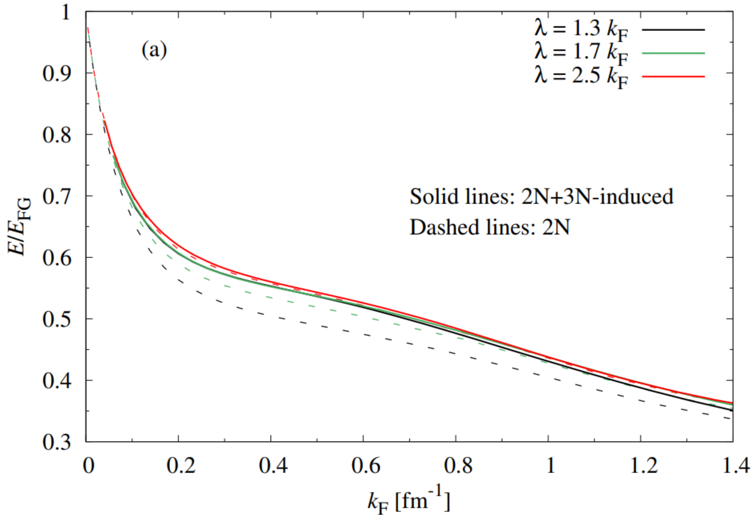
$\mathcal{E}_{\text{FG}}/n_{\text{FG}}$

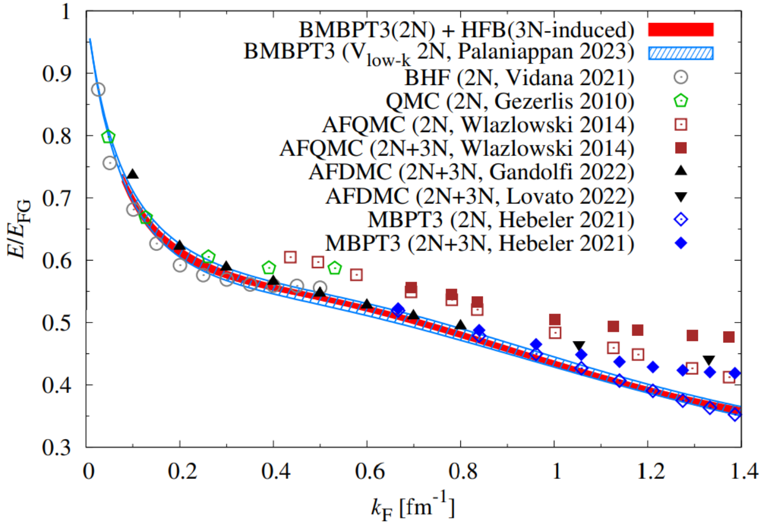


[*ArXiv:* **2412.00137**]

- EoS as a function of the SRG evolution parameter λ (left), divided by the Fermi momentum. Effects of the induced 3-body forces in the EoS with S-wave 2-body forces (right).

- EoS with different approaches, including quantum Monte Carlo with (AFMC) and without (QMC) auxiliary fields as a function of the Fermi momentum. The addition of induced 3-body forces increases the ratio of energies per particle.





[*ArXiv:* **2412.00137**]

THANK YOU *for your attention!*