

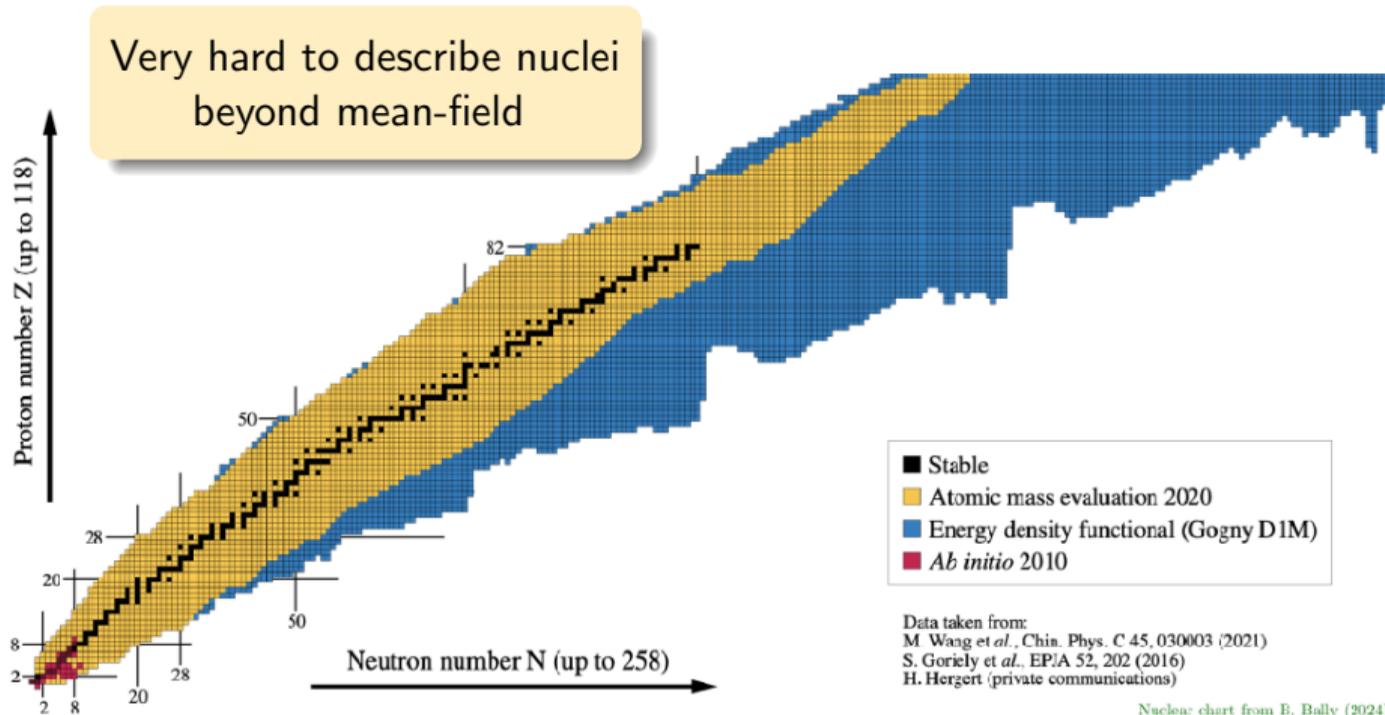
Dispersive Optical Models: Achievements and Perspectives

Mack C. Atkinson

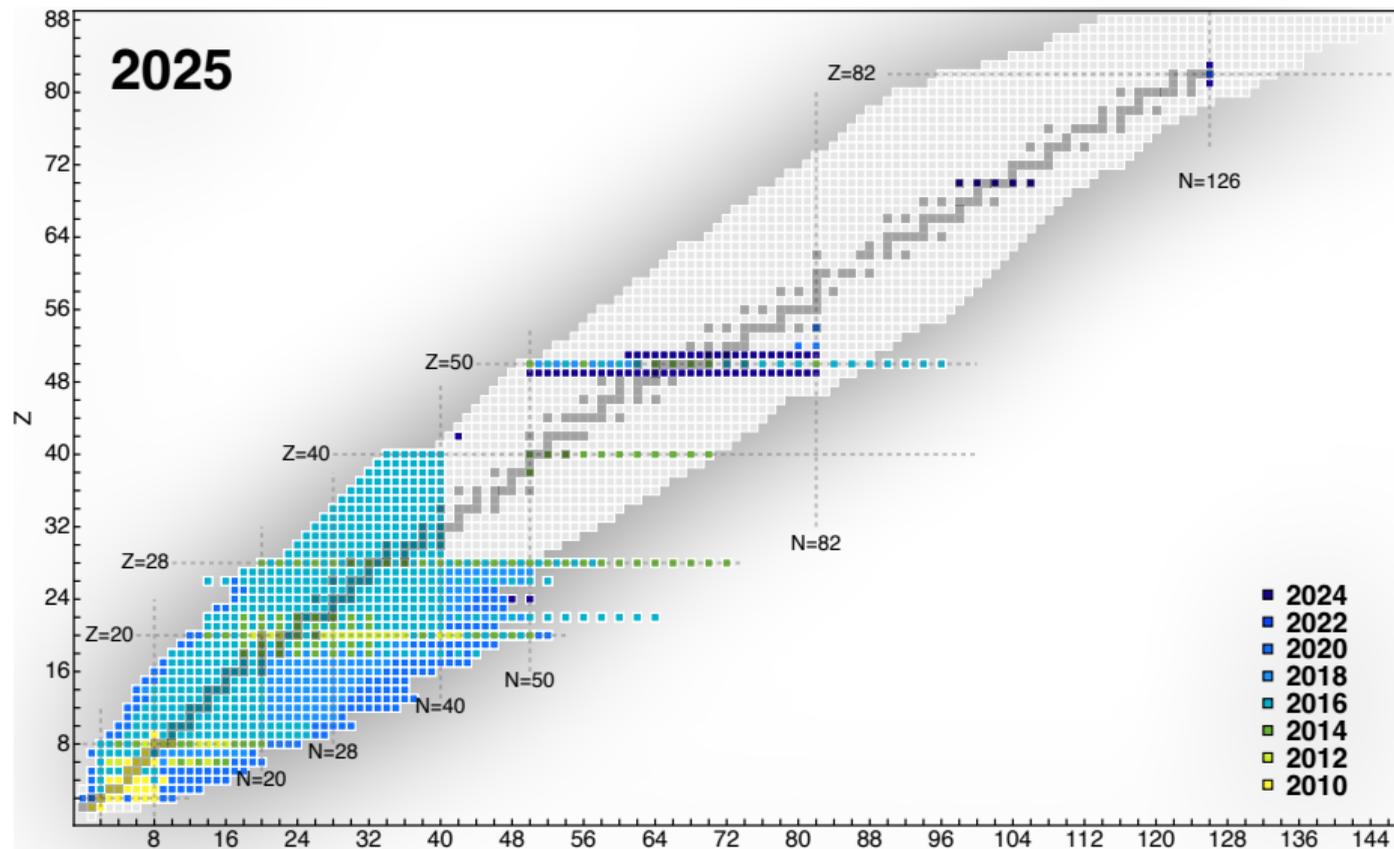
Reed College

Perspectives on Nuclear Data for the Next Decade P(ND)²-3

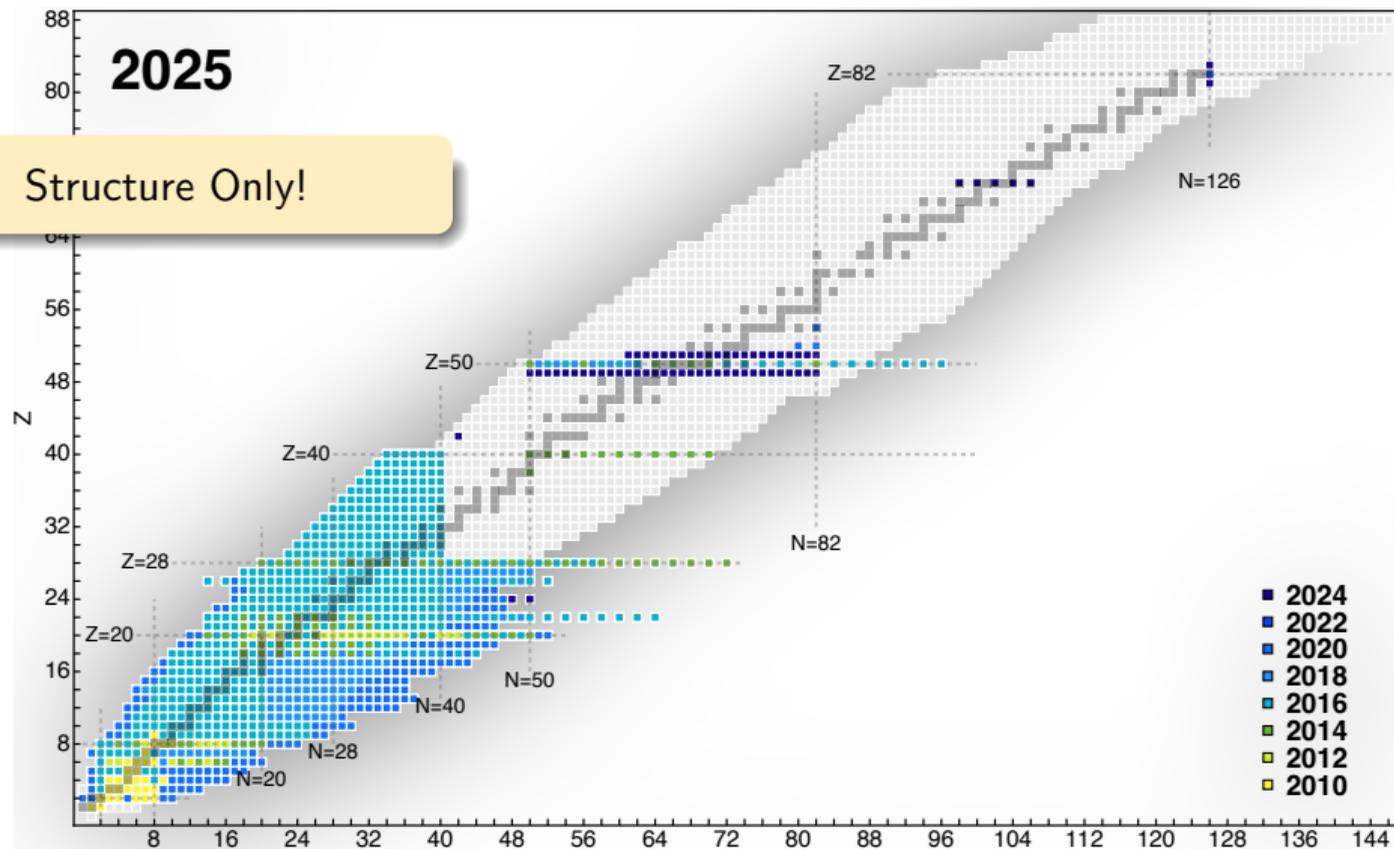
Progress in structure and reaction calculations across the nuclear chart



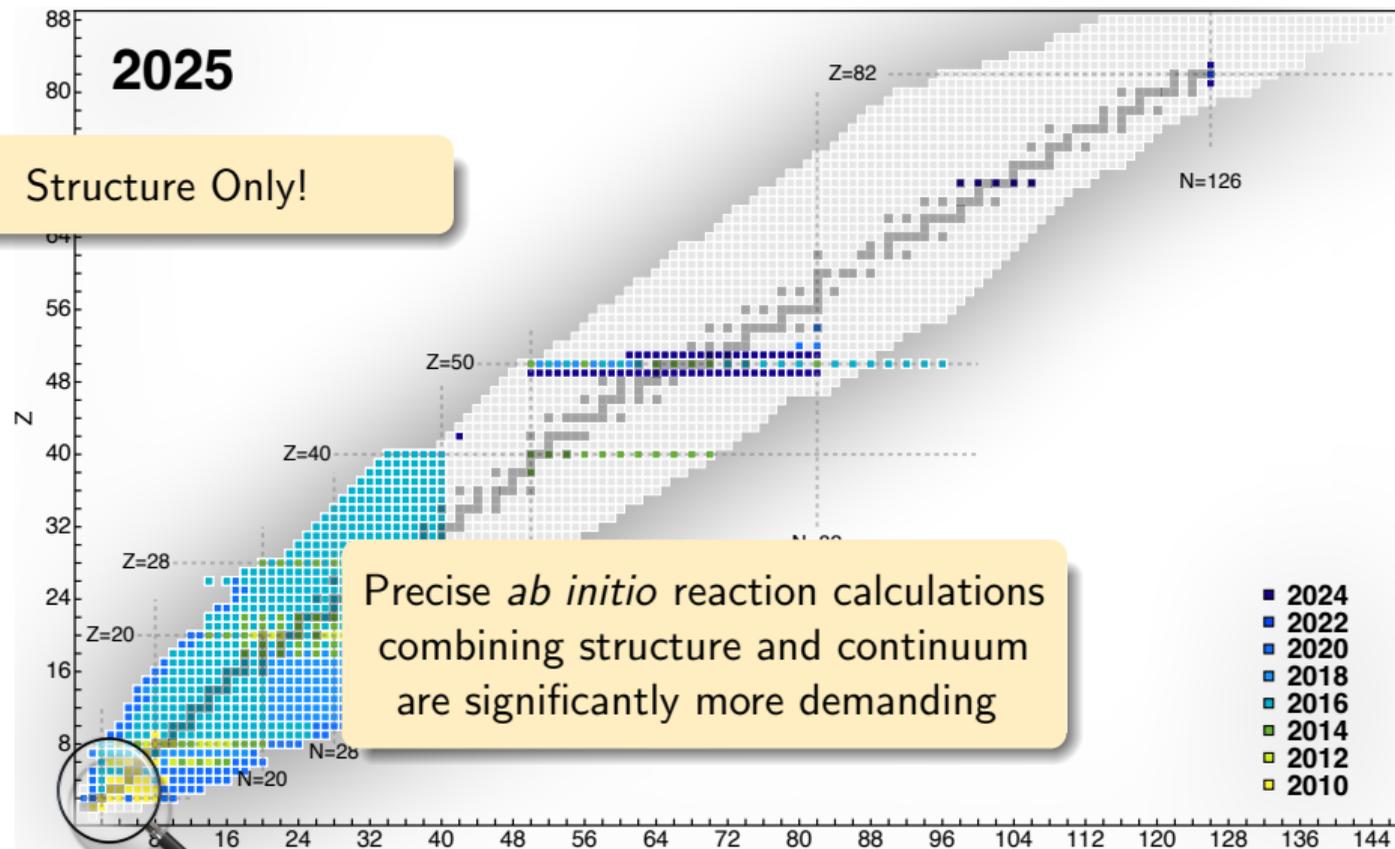
Progress in structure and reaction calculations across the nuclear chart



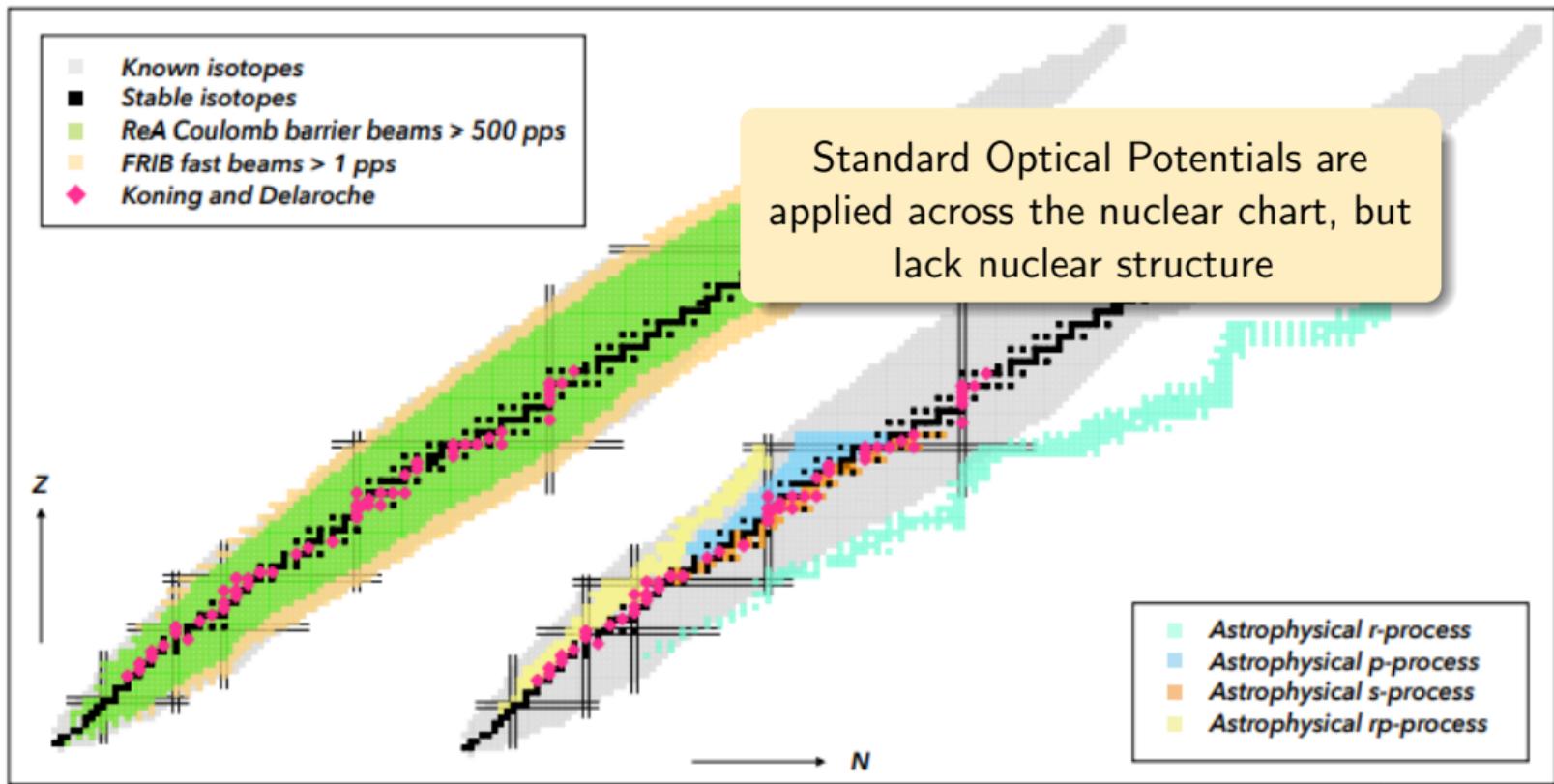
Progress in structure and reaction calculations across the nuclear chart



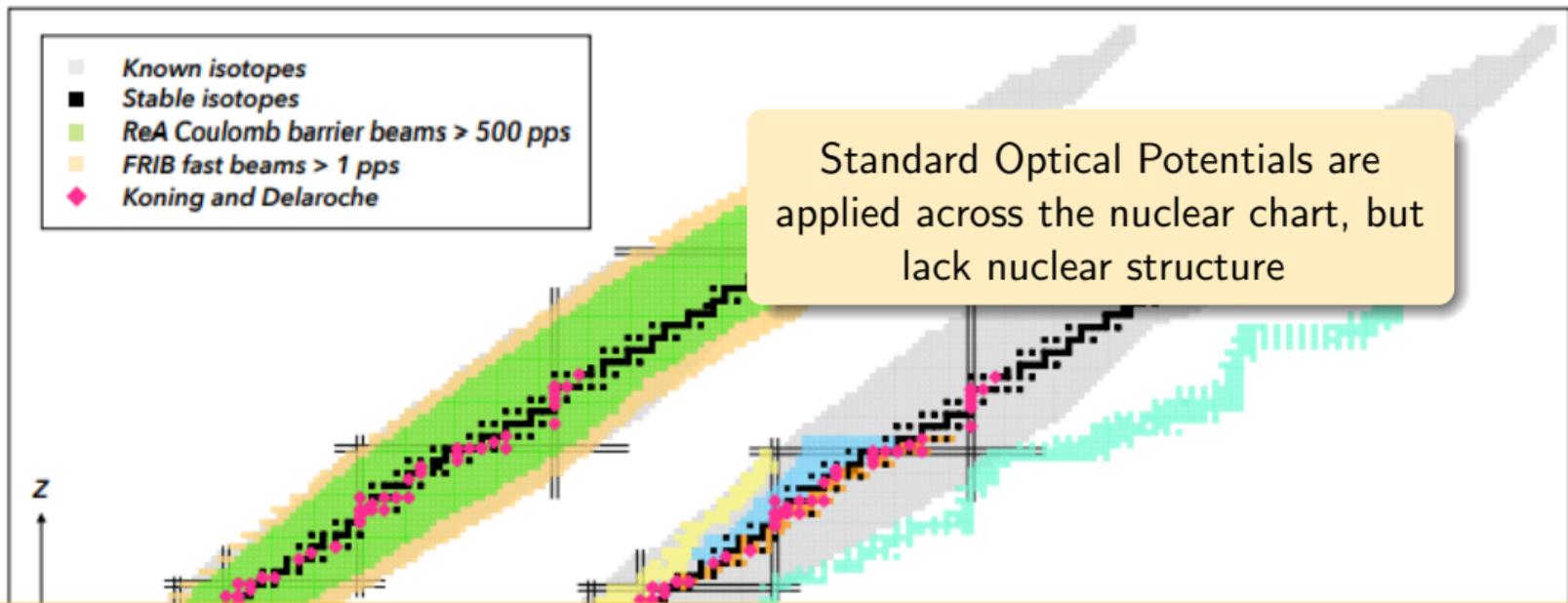
Progress in structure and reaction calculations across the nuclear chart



Structure and reaction calculations across the nuclear chart



Structure and reaction calculations across the nuclear chart

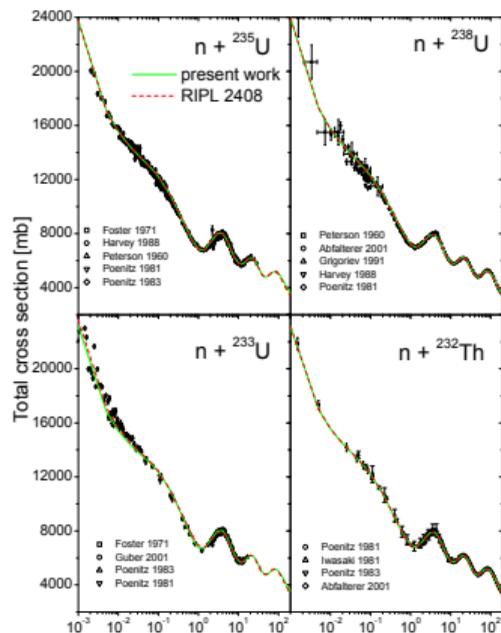


Dispersive Optical Model

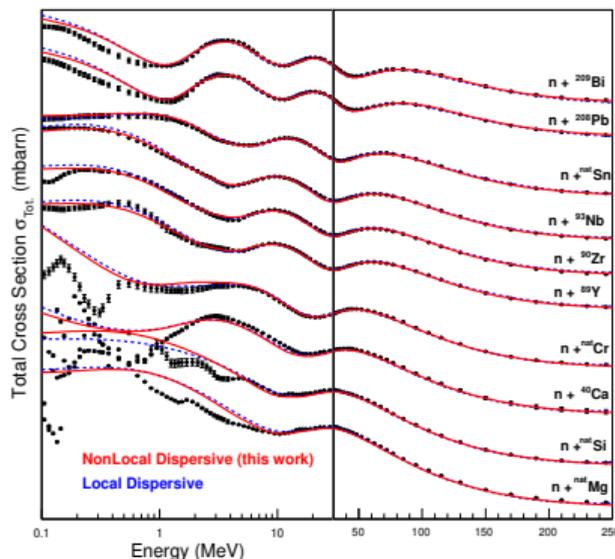
The DOM strikes a balance between computational cost and a consistent treatment of bound and continuum dynamics

Disclaimer

- There are several dispersive optical models
- This talk focuses on a particular DOM (Washington University in St. Louis) and its capabilities



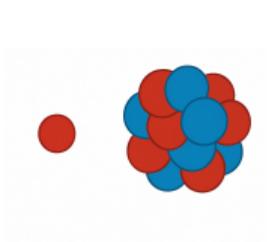
E. Sh. Soukhovitskii *et al.*, PRC **94**, 064605 (2016)



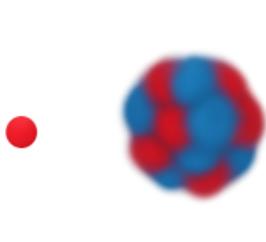
B. Morillon *et al.*, PRC **109**, 044611 (2024)

Dispersive Optical Model (DOM): uniting structure and continuum

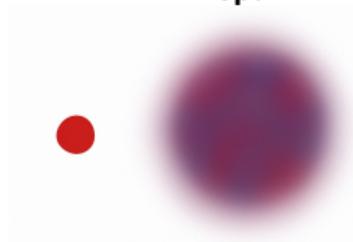
Exact



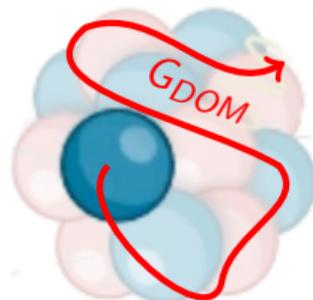
DOM



V_{opt}

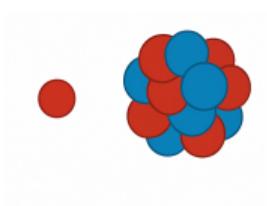


Dispersive Optical Model (DOM): uniting structure and continuum



G_{DOM} : Single-Particle Propagator

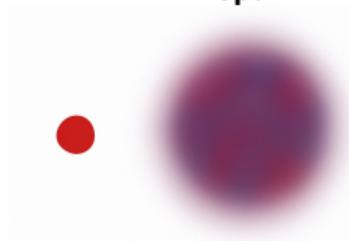
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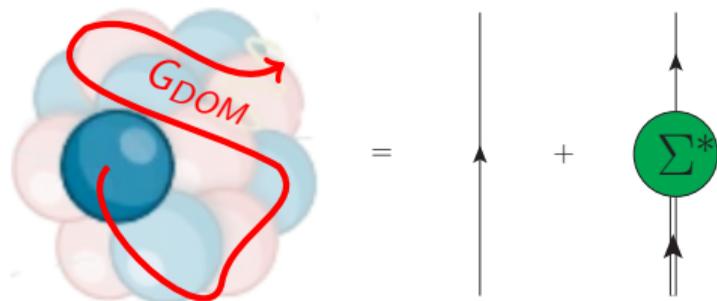
DOM



V_{opt}

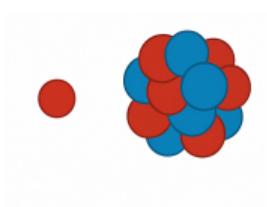


Dispersive Optical Model (DOM): uniting structure and continuum



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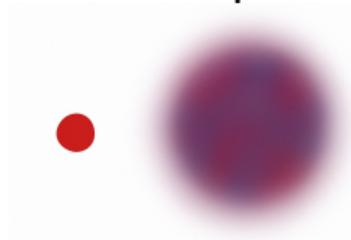
Exact



DOM

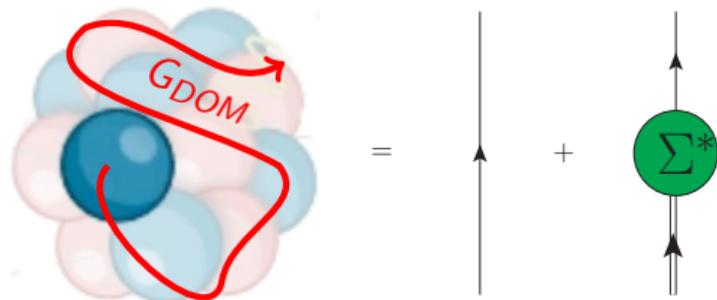


V_{opt}



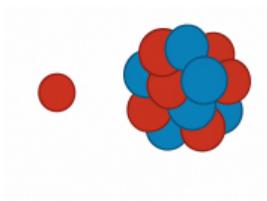
Dispersive Optical Model (DOM): uniting structure and continuum

- Self-energy: $\Sigma^*(r, r'; E) = V_{\text{opt}}(r, r'; E)$



G_{DOM} : Single-Particle Propagator

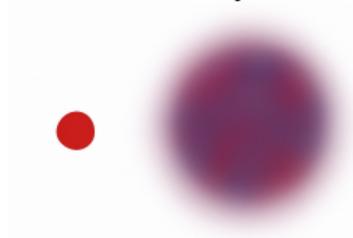
Exact



DOM



V_{opt}

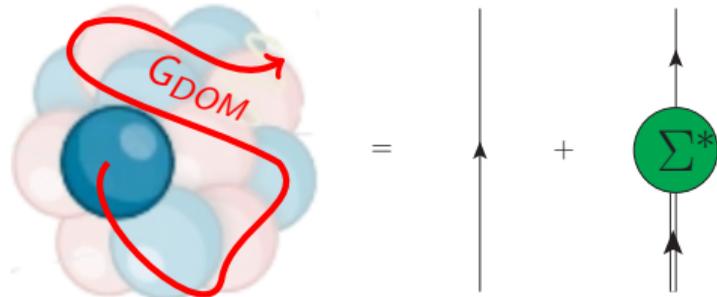


Dispersive Optical Model (DOM): uniting structure and continuum

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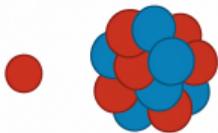
Dispersive Correction

$$\text{Re}\Sigma_{lj}(r, r'; E) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dE' \frac{\text{Im}\Sigma_{lj}(r, r'; E')}{E - E'}$$



G_{DOM} : Single-Particle Propagator

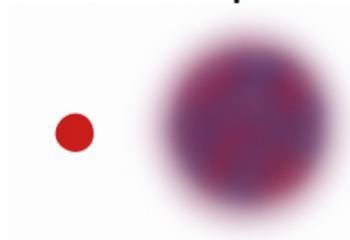
Exact



DOM



V_{opt}

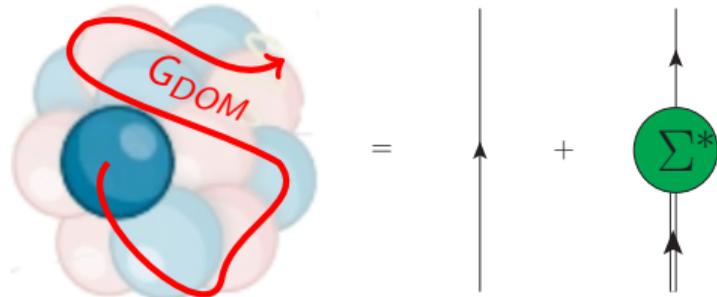


Dispersive Optical Model (DOM): uniting structure and continuum

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Dispersive Correction

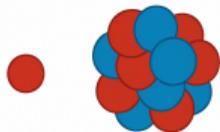
$$\text{Re}\Sigma_{\ell j}(r, r'; E) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dE' \frac{\text{Im}\Sigma_{\ell j}(r, r'; E')}{E - E'}$$



G_{DOM} : Single-Particle Propagator

- This constraint ensures bound and scattering quantities are simultaneously described

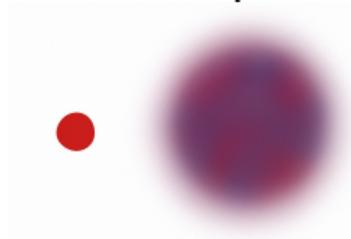
Exact



DOM



V_{opt}

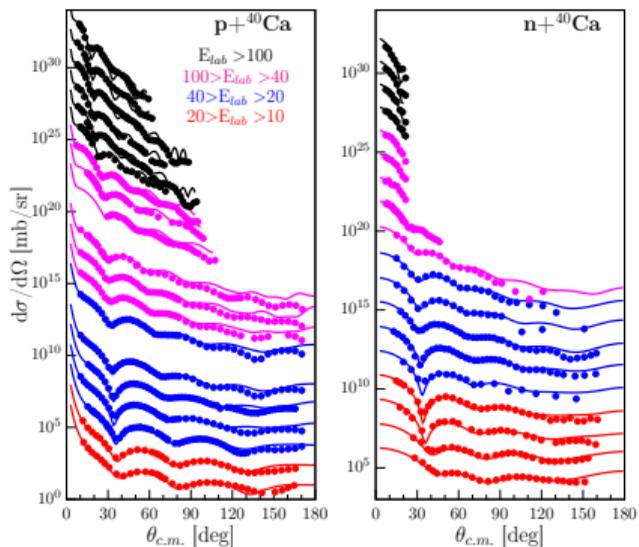


Fitting the self-energy (^{40}Ca)

- Parameters of self-energy varied to minimize χ^2

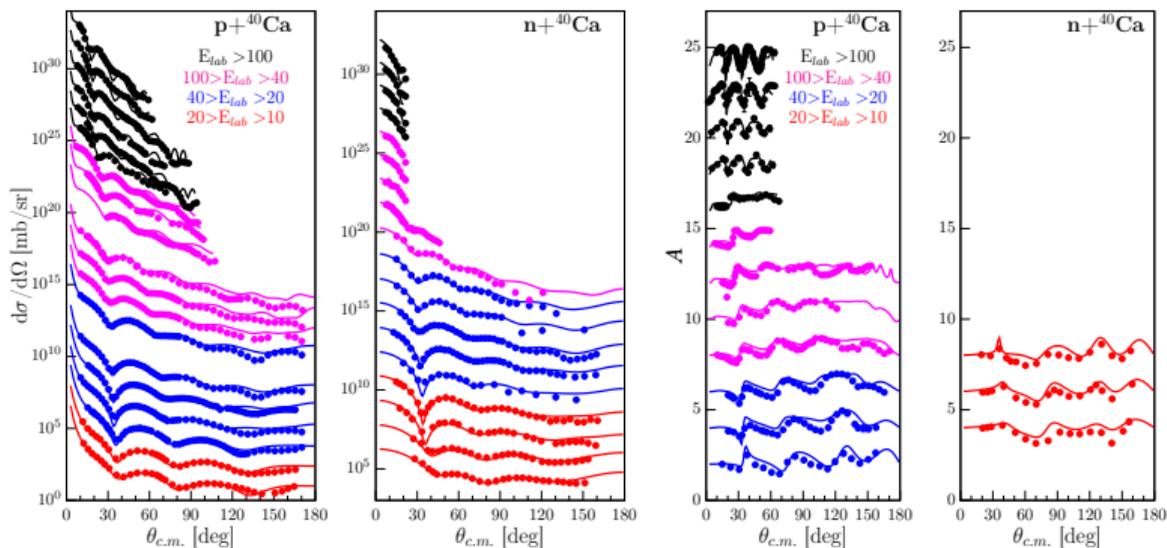
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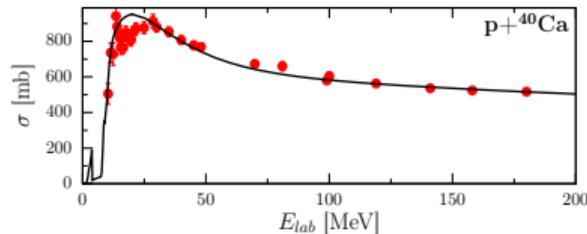
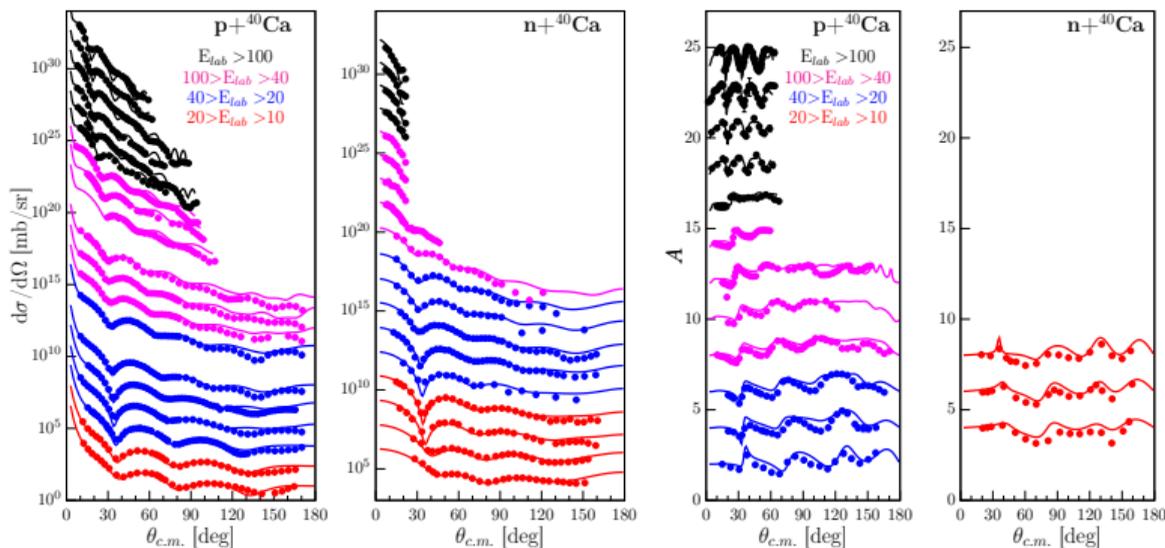
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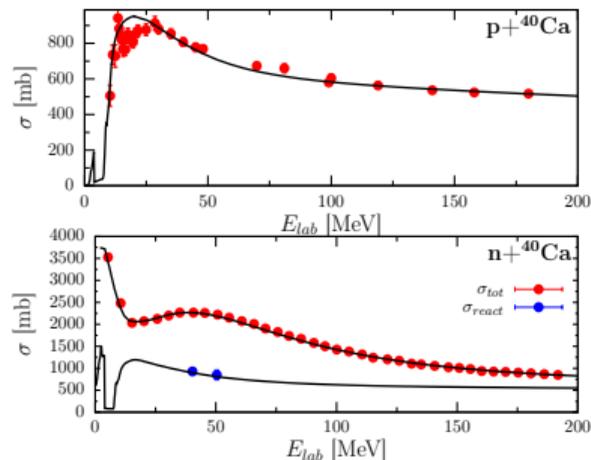
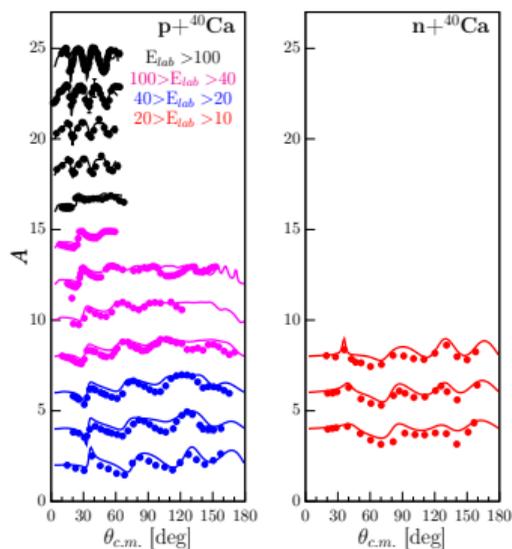
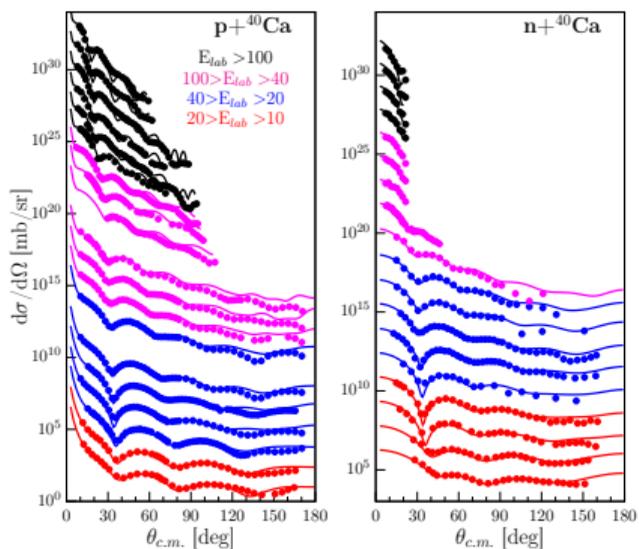
Fitting the self-energy (^{40}Ca)

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Fitting the self-energy (^{40}Ca)

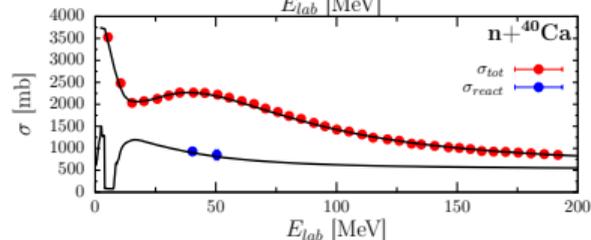
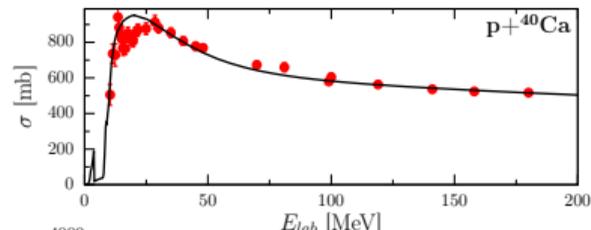
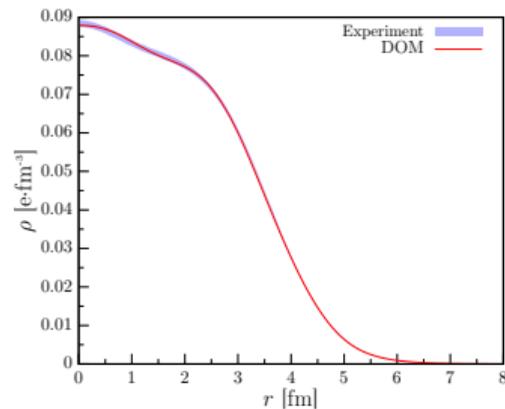
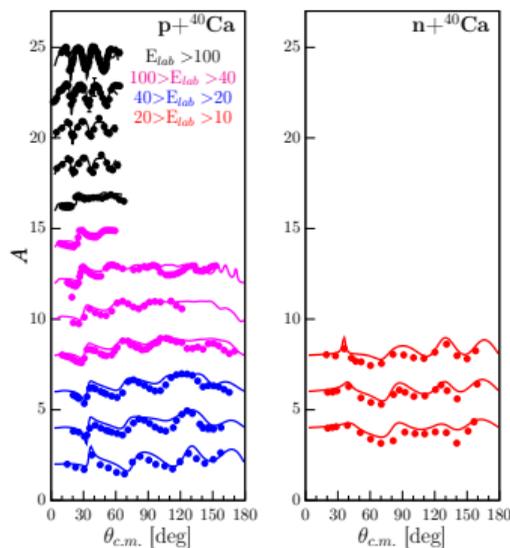
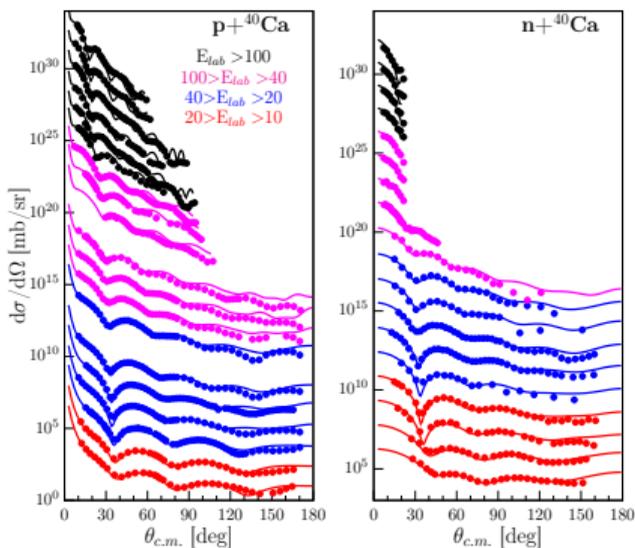
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M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

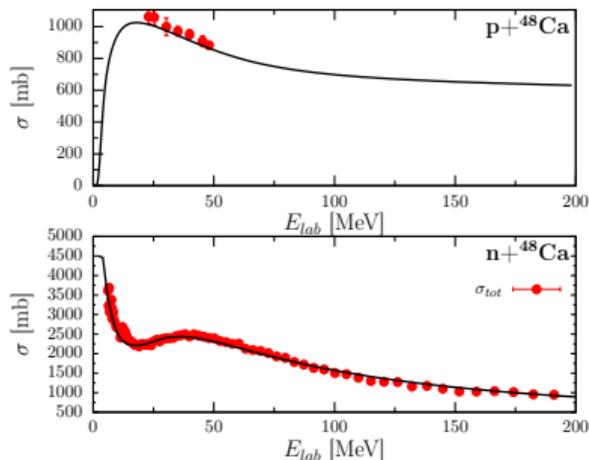
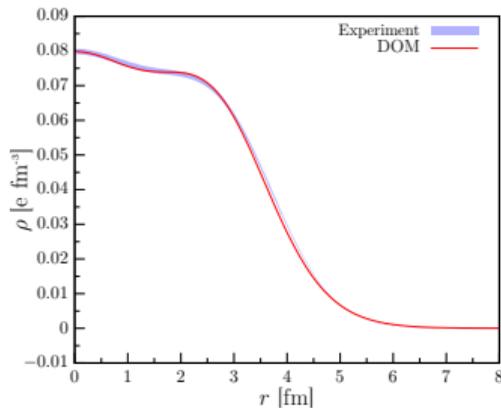
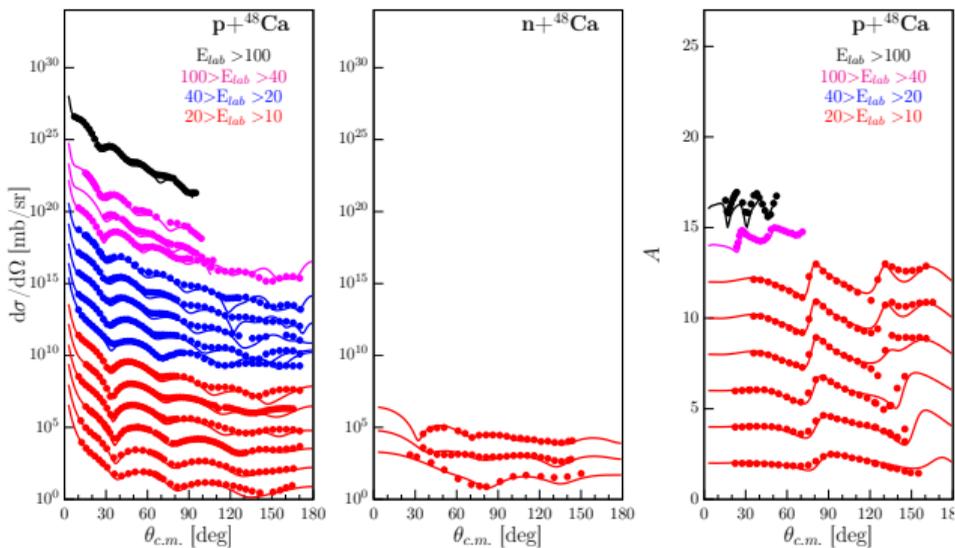
Fitting the self-energy (^{40}Ca)

- Parameters of self-energy varied to minimize χ^2
- Reproducing the data means self-energy is found



Fitting the self-energy (^{48}Ca)

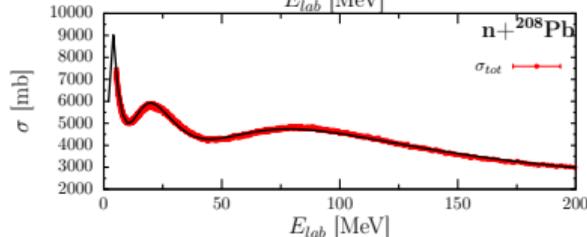
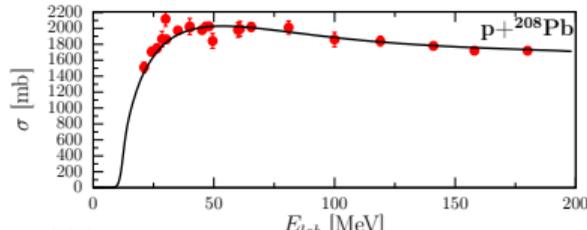
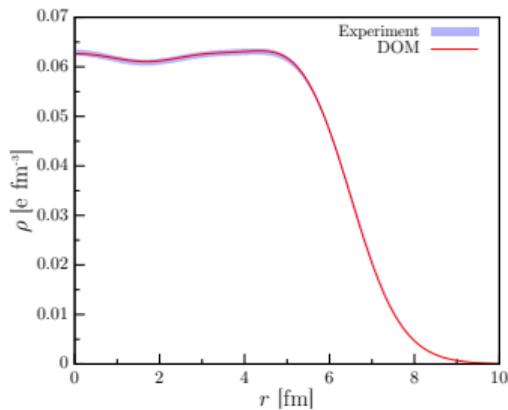
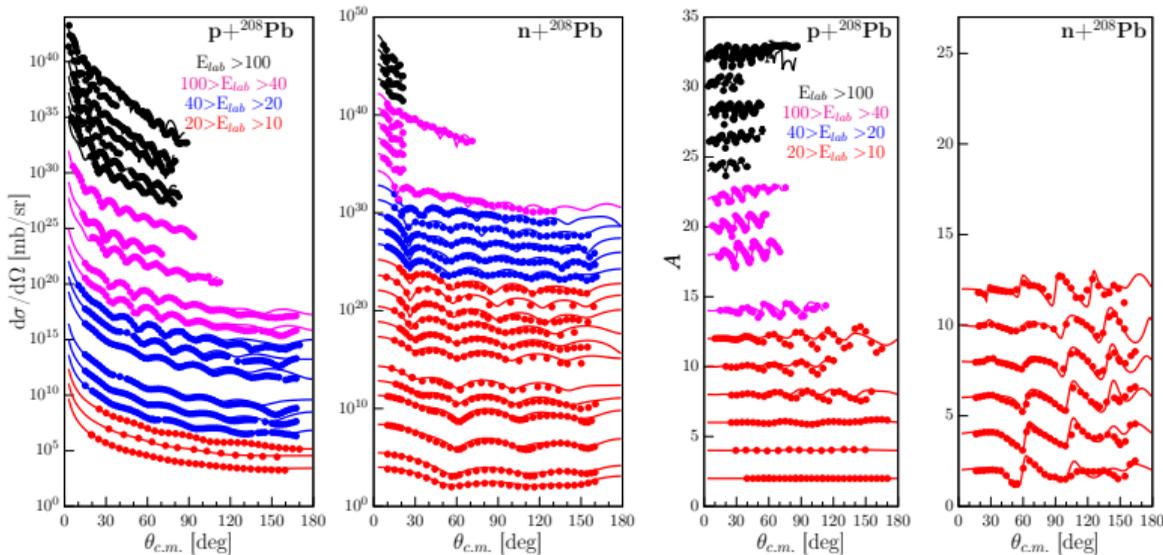
- Parameters of self-energy varied to minimize χ^2
- Reproducing the data means self-energy is found



M.C. Atkinson and W.H. Dickhoff, Phys. Lett. B, **798**, 135027 (2019)

Fitting the self-energy (^{208}Pb)

- Parameters of self-energy varied to minimize χ^2
- Reproducing the data means self-energy is found



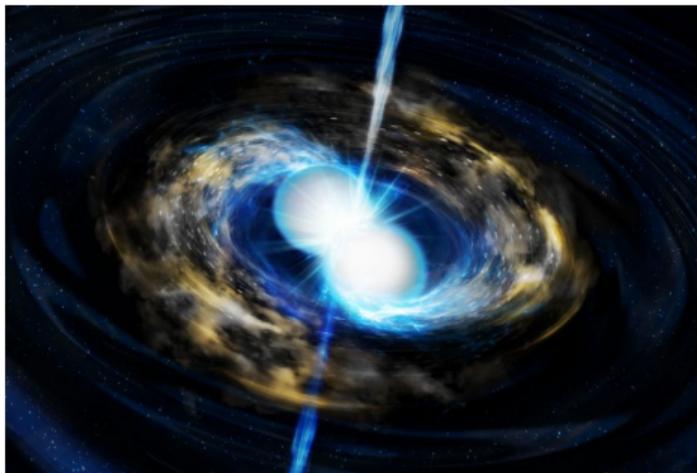
Neutron skin: $R_{\text{skin}} = R_n - R_p$

- r_n can be measured through parity-violating electron scattering (weak)



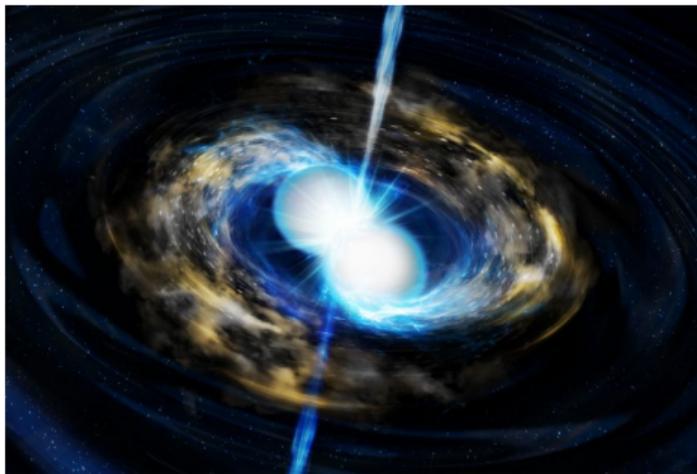
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- PREX-II at Jefferson Lab measured ^{208}Pb skin



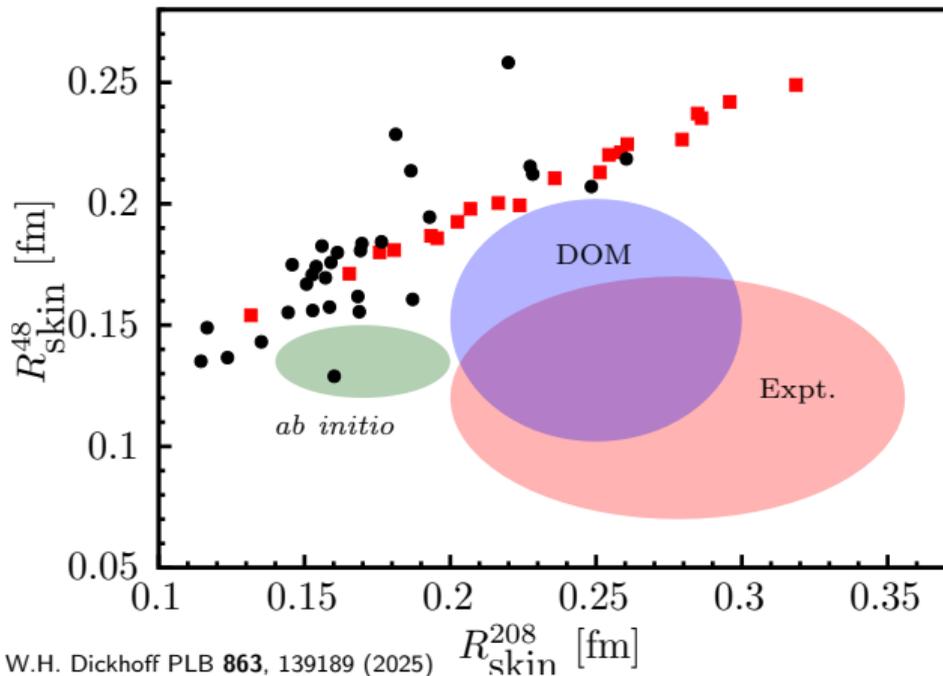
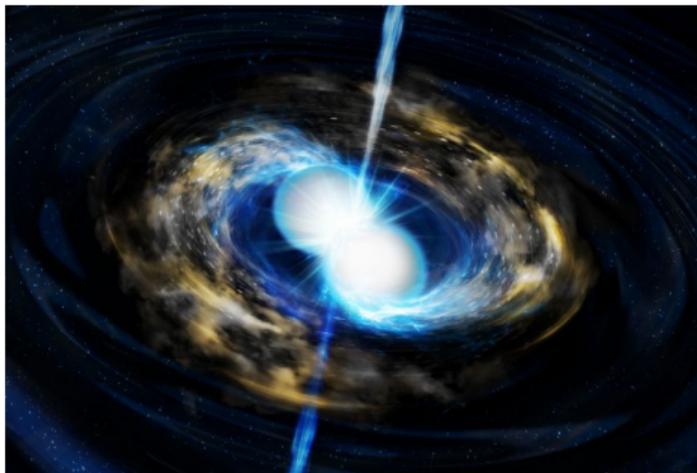
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Neutron skin: $R_{\text{skin}} = R_n - R_p$

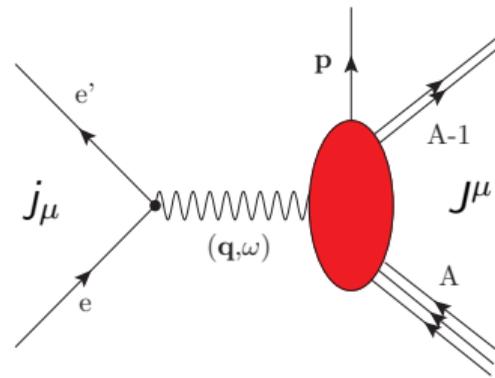
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- PREX-II at Jefferson Lab measured ^{208}Pb skin
 - Small skin from CREX very surprising
 - DOM can reproduce both skins!



N.L. Calleya, M.C. Atkinson, W.H. Dickhoff PLB **863**, 139189 (2025)

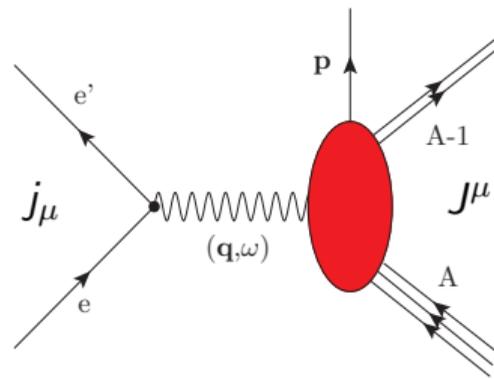
R_{skin}^{208} [fm]

The exclusive $(e, e'p)$ reaction



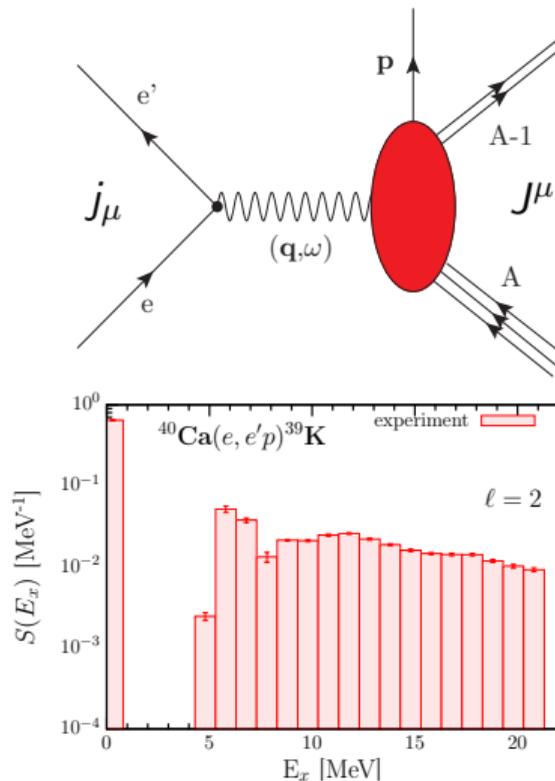
The exclusive $(e, e'p)$ reaction

- Excitation spectrum provides evidence of many-body correlations beyond mean-field



The exclusive $(e, e'p)$ reaction

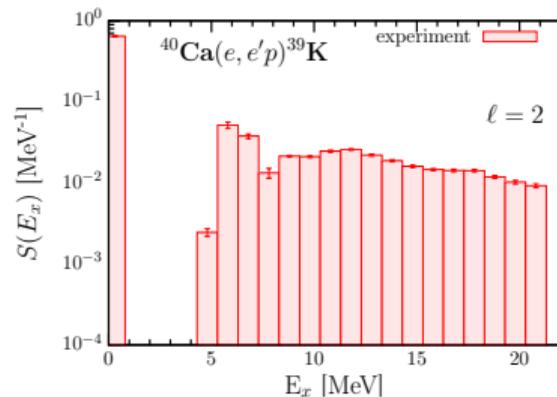
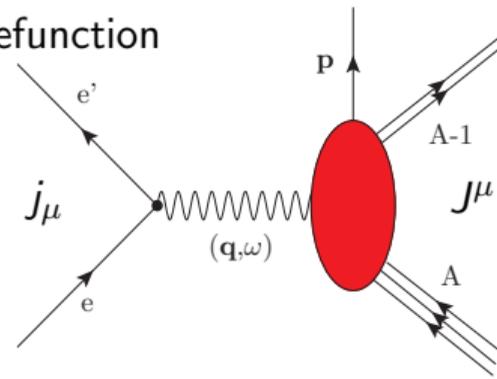
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M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

The exclusive $(e, e'p)$ reaction

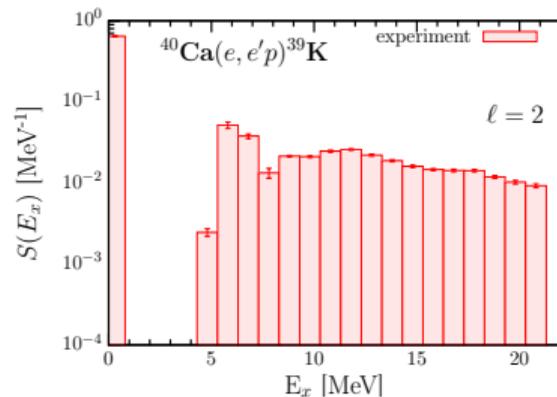
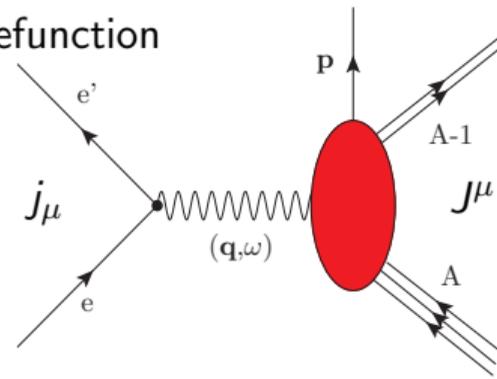
- Excitation spectrum provides evidence of many-body correlations beyond mean-field
- Momentum distribution is closely tied to the boundstate wavefunction



M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

The exclusive $(e, e'p)$ reaction

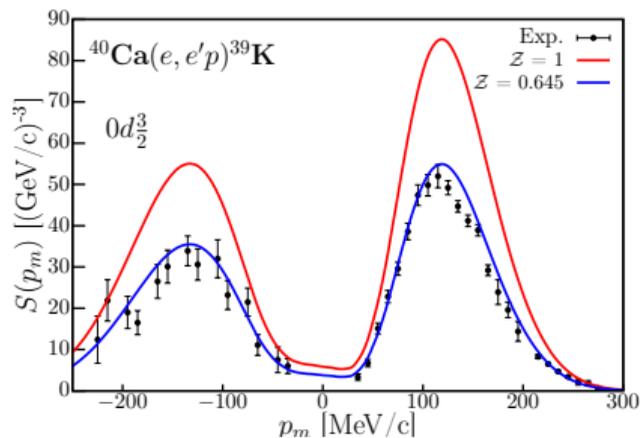
- Excitation spectrum provides evidence of many-body correlations beyond mean-field
- Momentum distribution is closely tied to the boundstate wavefunction
- Spectroscopic factor



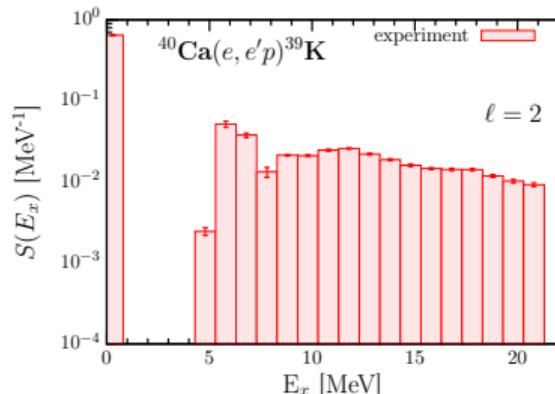
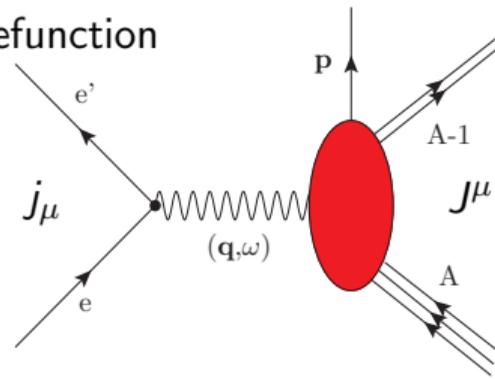
M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

The exclusive $(e, e'p)$ reaction

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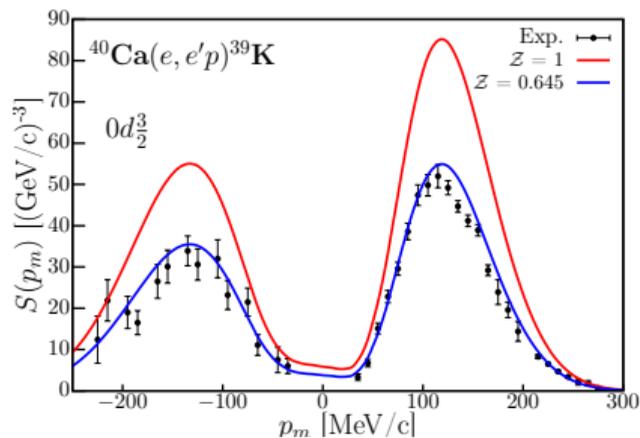


M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

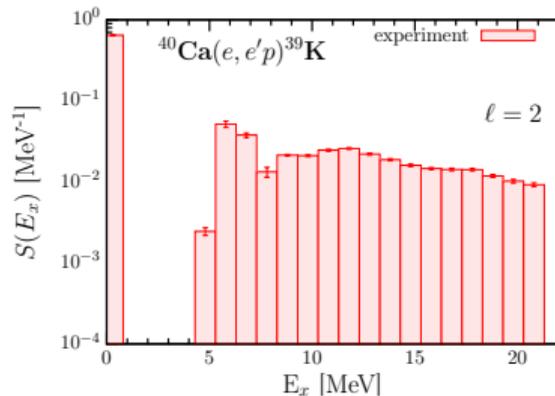
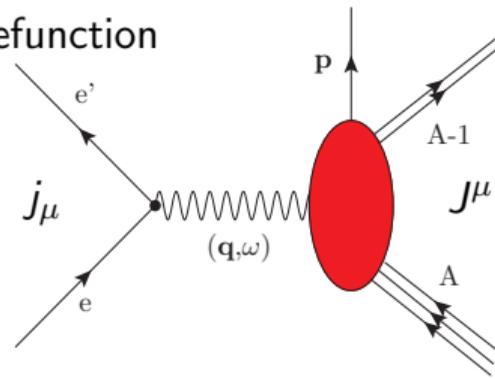


The exclusive $(e, e'p)$ reaction

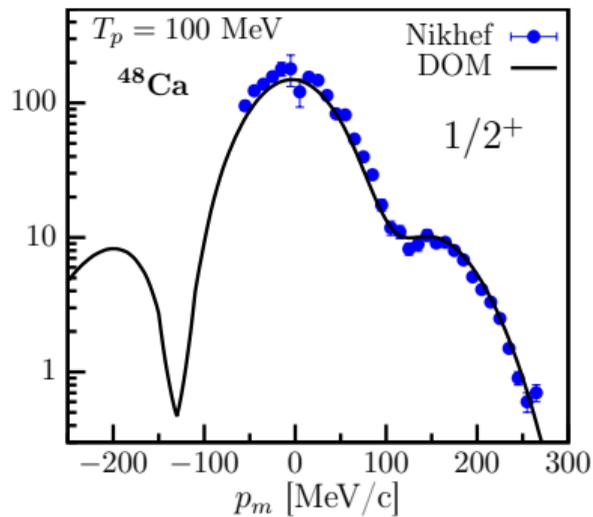
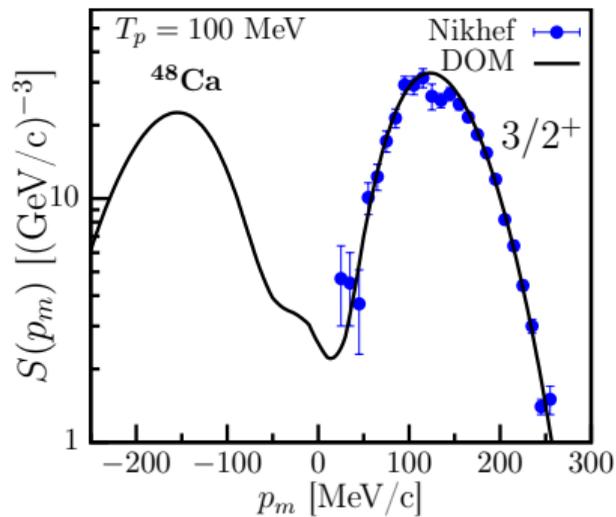
- Excitation spectrum provides evidence of many-body correlations beyond mean-field
- Momentum distribution is closely tied to the boundstate wavefunction
- Spectroscopic factor
- Electron interaction means clean knockout reactions



M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

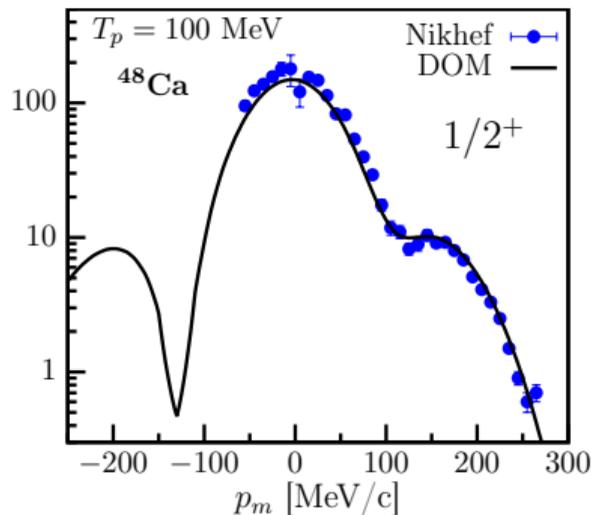
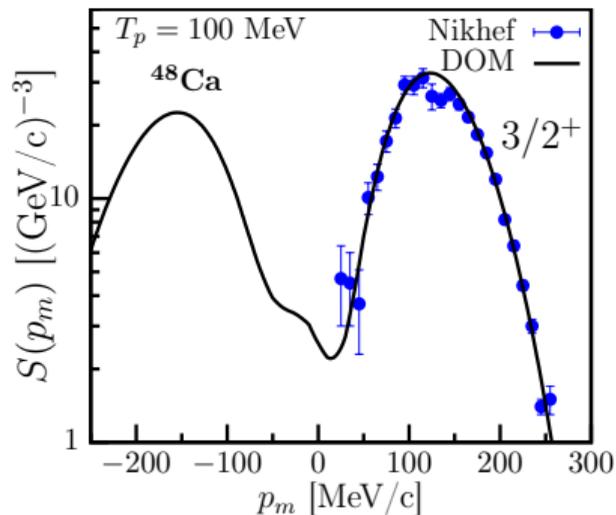


$^{48}\text{Ca}(e,e'p)^{47}\text{K}$ Momentum Distribution



Data: G. J. Kramer *et. al*, Nucl. Phys. A, **679**, 267 (2001)

$^{48}\text{Ca}(e,e'p)^{47}\text{K}$ Momentum Distribution

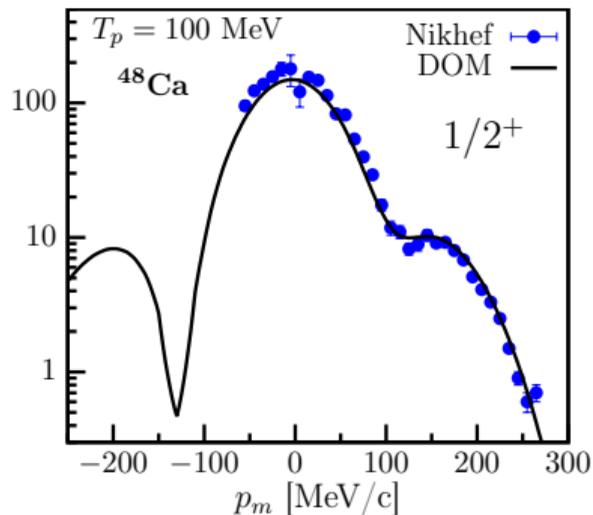
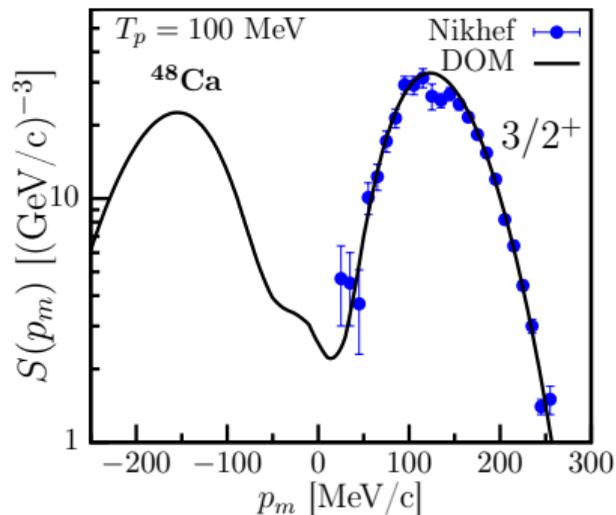


Data: G. J. Kramer *et. al*, Nucl. Phys. A, **679**, 267 (2001)

$S_{F_{lj}}^n$	$0d_{\frac{3}{2}}$	$1s_{\frac{1}{2}}$
^{40}Ca	0.71 ± 0.04	0.60 ± 0.03
^{48}Ca	0.58 ± 0.03	0.55 ± 0.03

M.C. Atkinson and W.H. Dickhoff, Phys. Lett. B, **798**, 135027 (2019)

$^{48}\text{Ca}(e,e'p)^{47}\text{K}$ Momentum Distribution

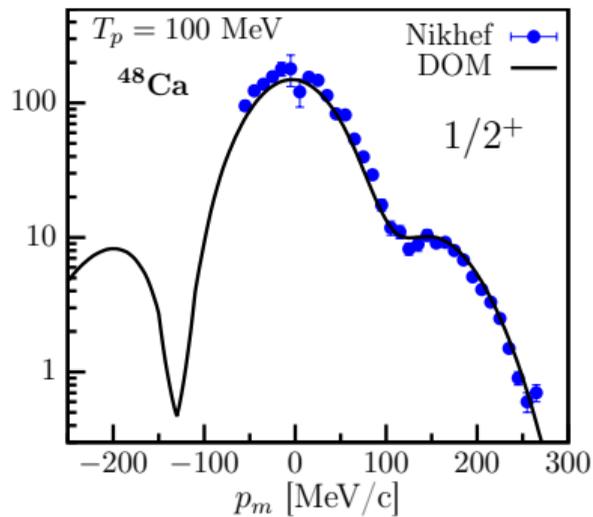
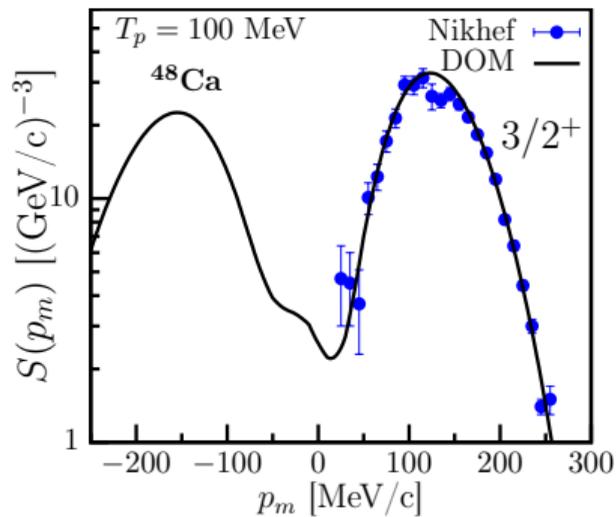


Data: G. J. Kramer *et. al*, Nucl. Phys. A, **679**, 267 (2001)

$S_{F_{lj}}^n$	$0d_{\frac{3}{2}}$	$1s_{\frac{1}{2}}$
^{40}Ca	0.71 ± 0.04	0.60 ± 0.03
^{48}Ca	0.58 ± 0.03	0.55 ± 0.03

M.C. Atkinson and W.H. Dickhoff, Phys. Lett. B, **798**, 135027 (2019)

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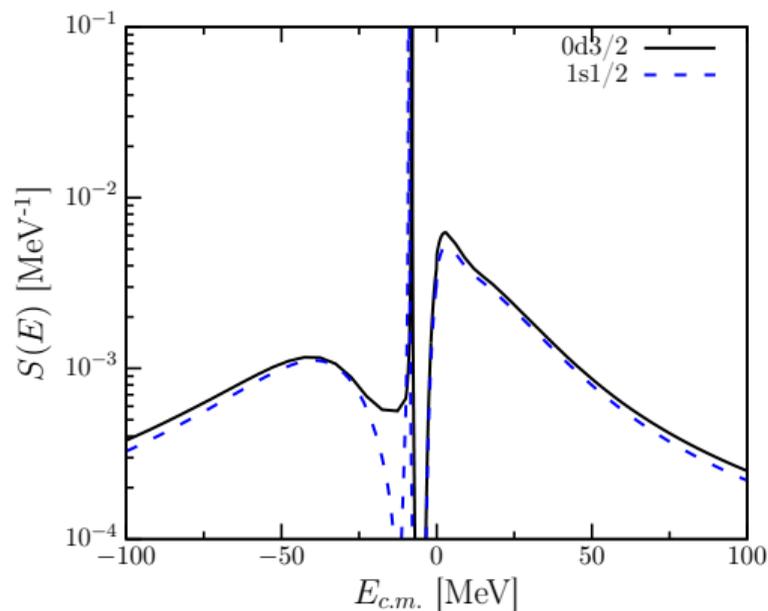
Protons are more correlated in ^{48}Ca !

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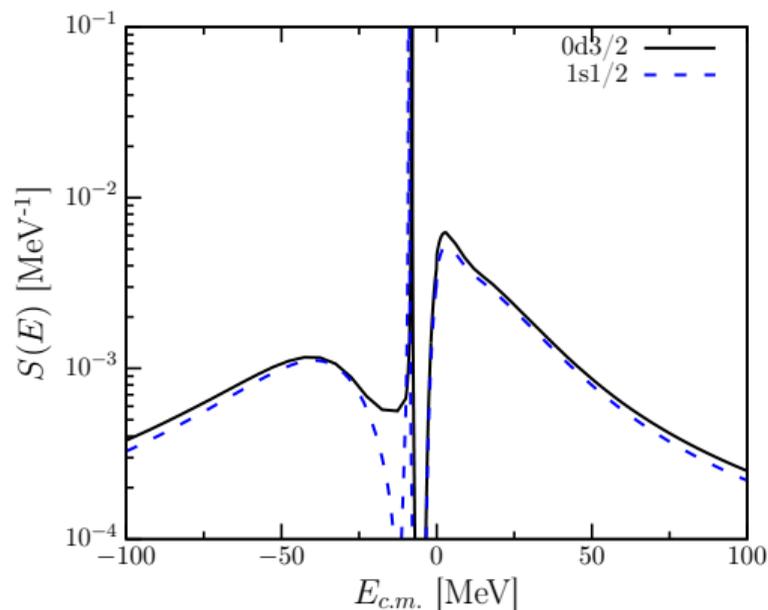
Constraints for S_F

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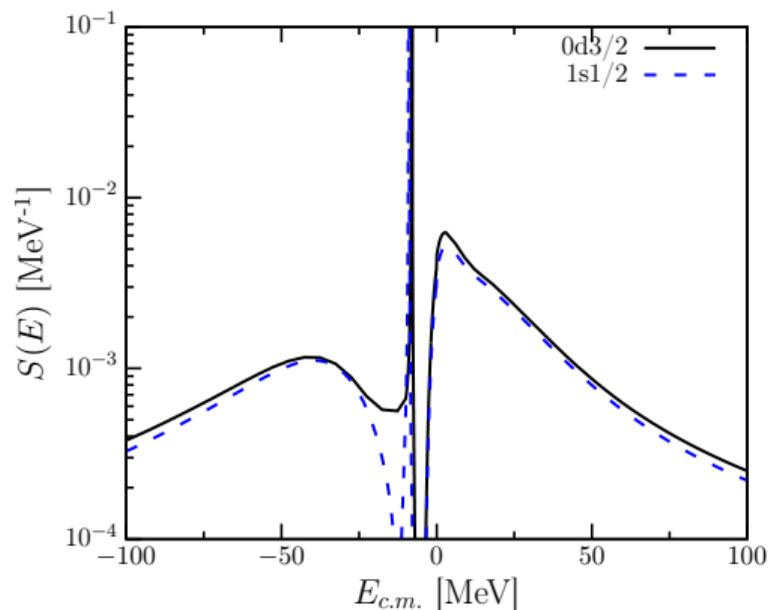
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- More strength above ϵ_f depletes the strength below



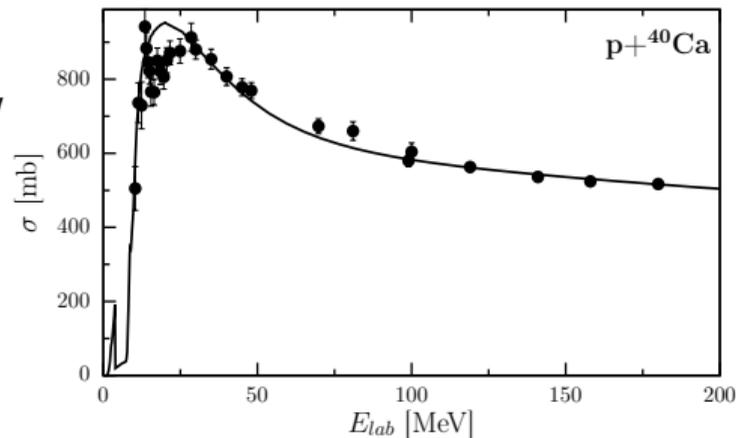
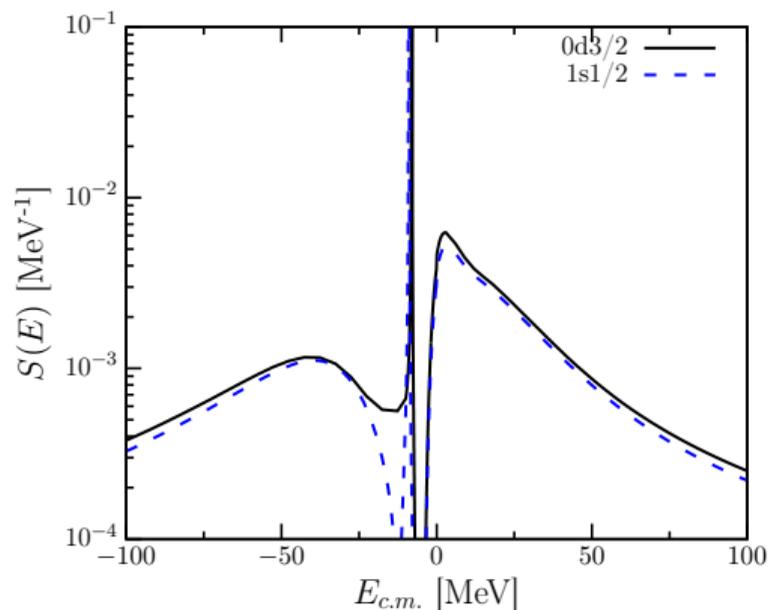
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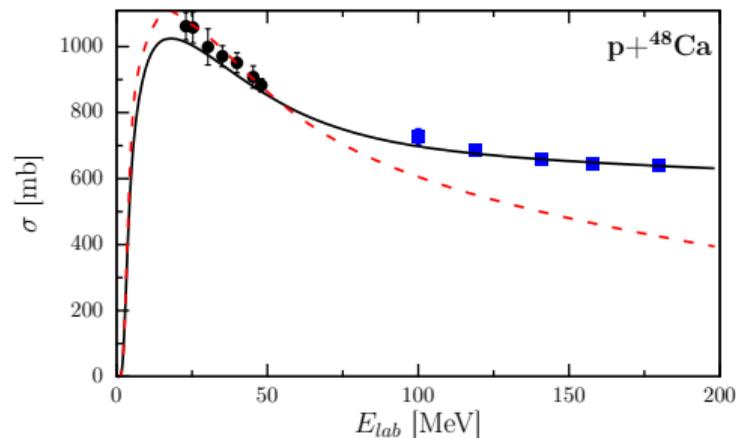
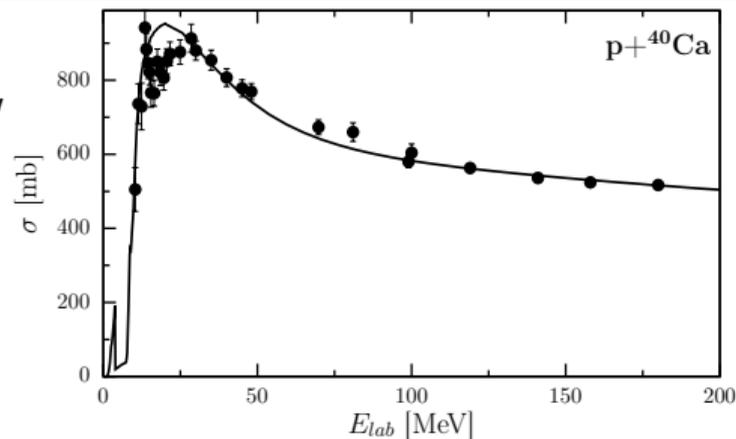
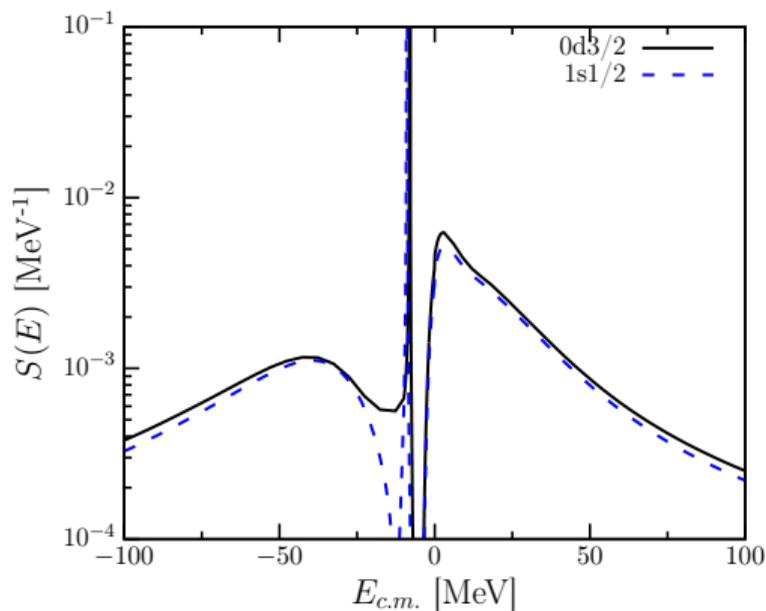
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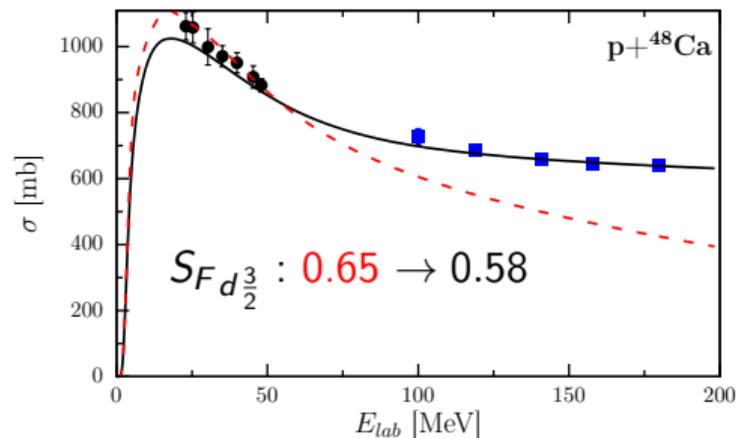
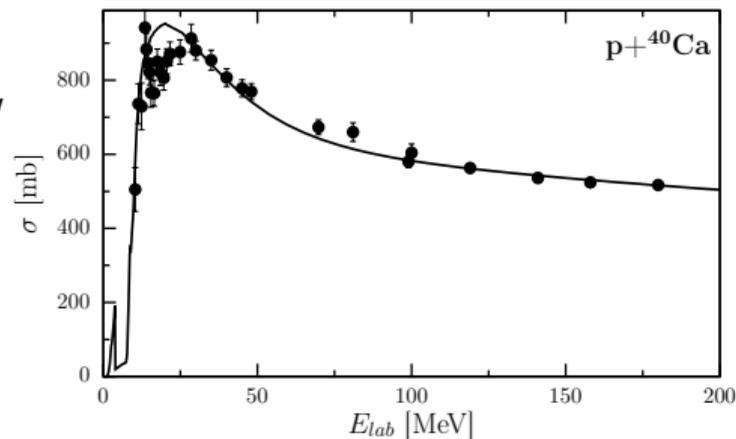
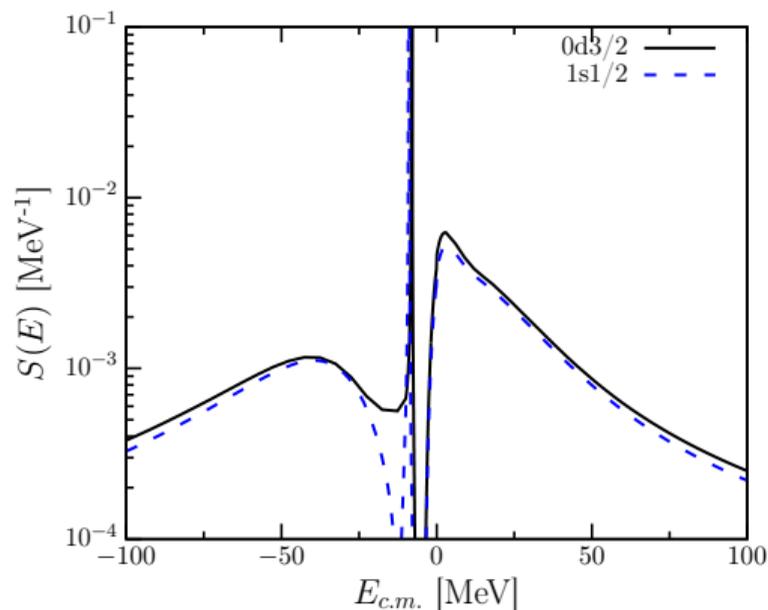
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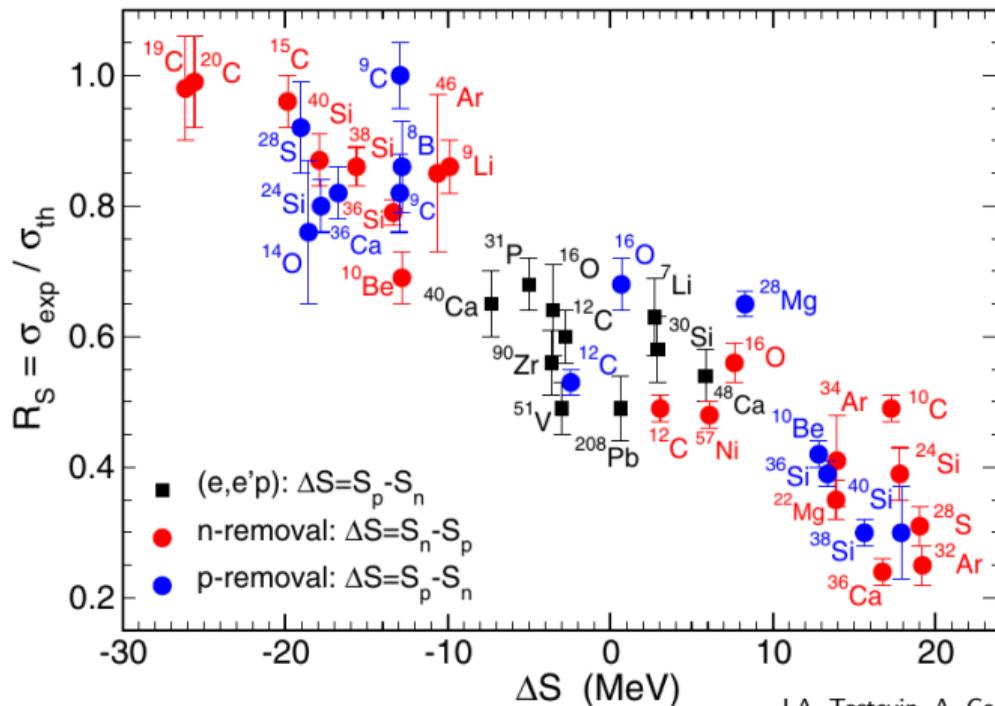


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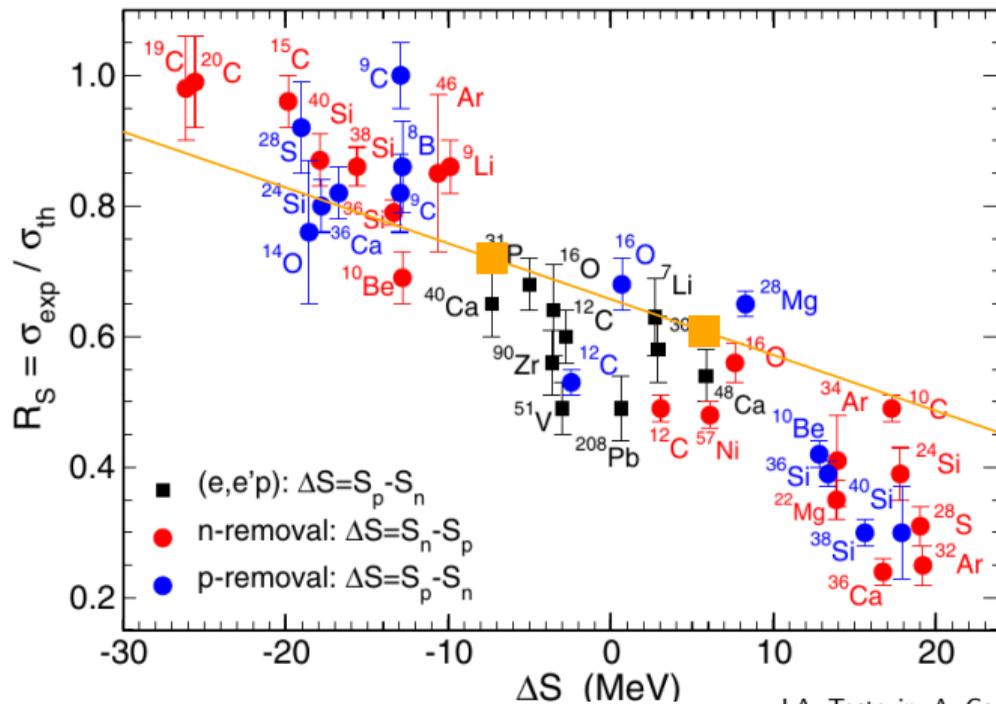


Quenching



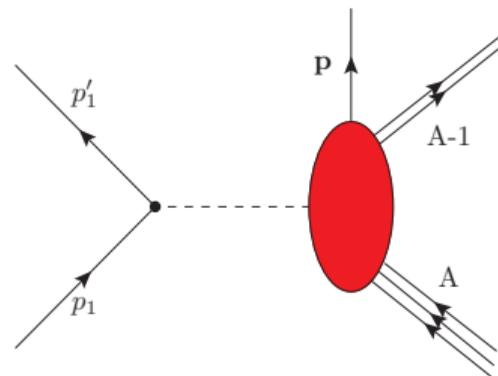
J.A. Tostevin, A. Gade *Phys. Rev. C* **90**, 057602 (2014)

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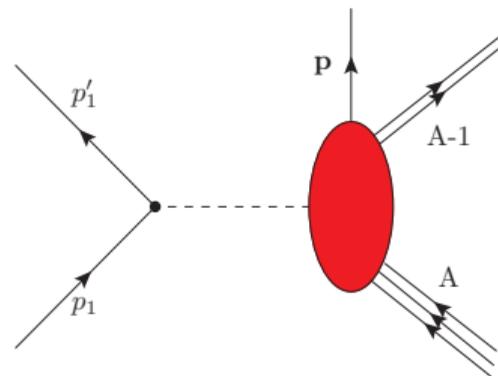
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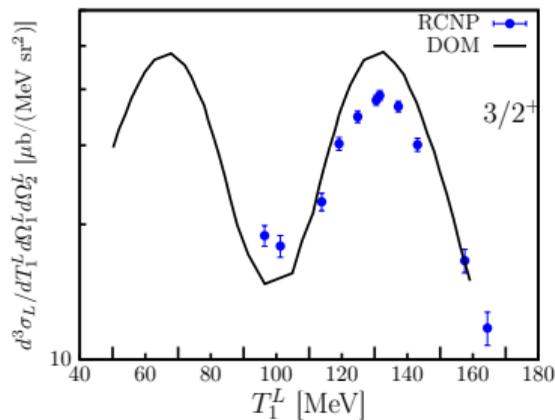
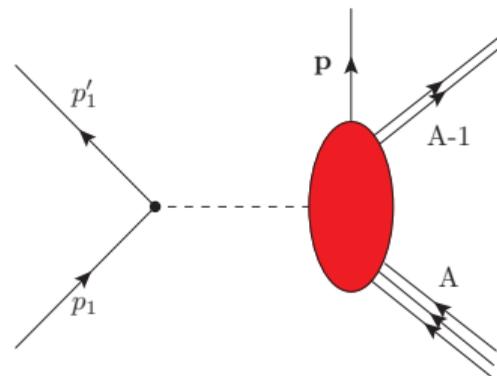
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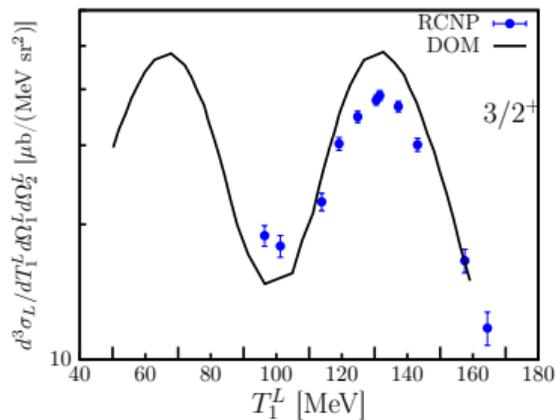
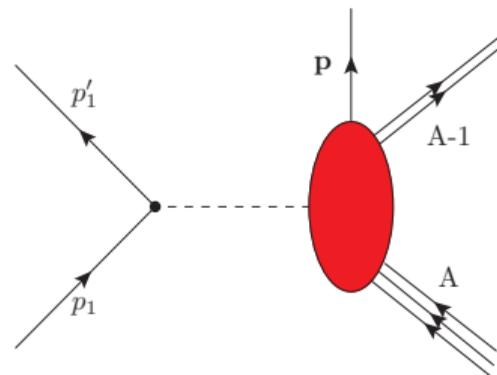


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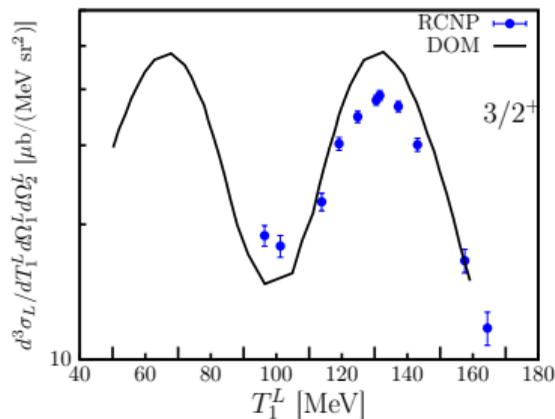
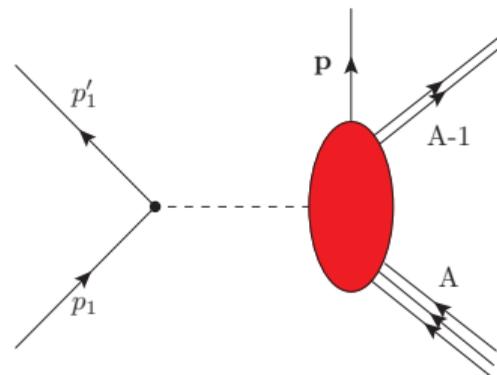
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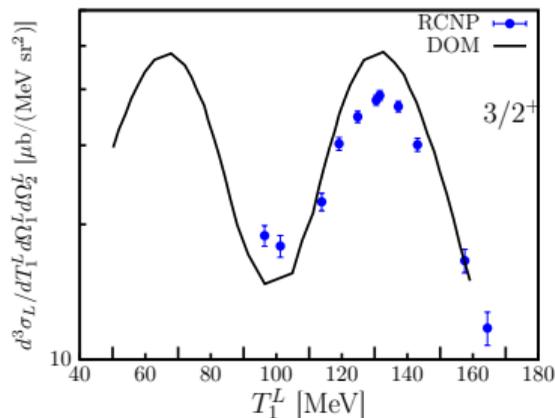
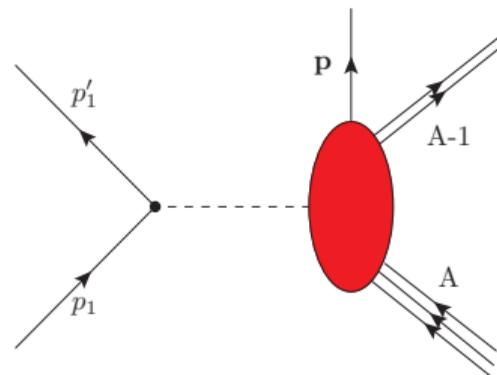
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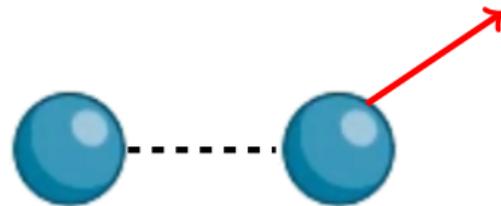
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- Remember that S_F comes directly from Σ_{DOM}^*
- Main difference is the probe \implies problem is likely V_{pp}

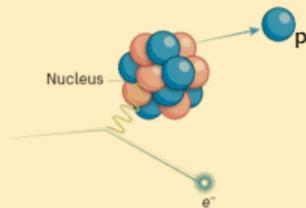
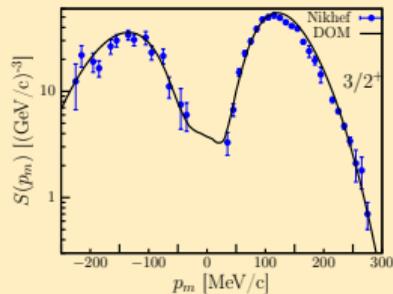


Free NN interaction is insufficient for knockout

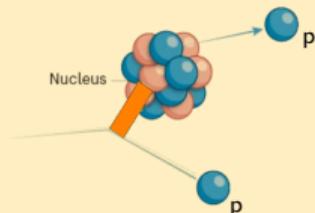
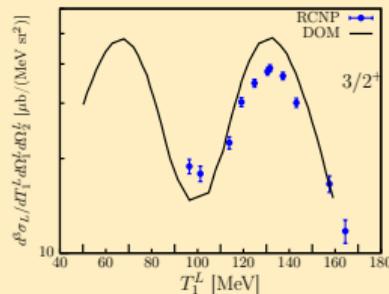
- The DOM provides consistent ingredients for knockout reactions
- Discrepancy between $^{40}\text{Ca}(e, e'p)^{39}\text{K}$ and $^{40}\text{Ca}(p, 2p)^{39}\text{K}$



Electron probe: $^{40}\text{Ca}(e, e'p)^{39}\text{K}$

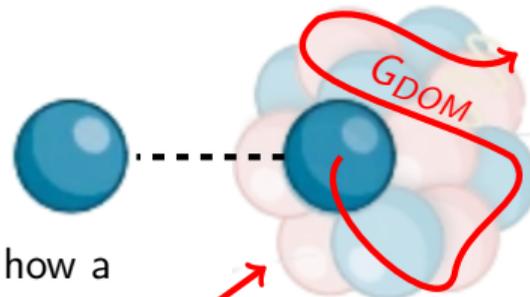


Proton probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$

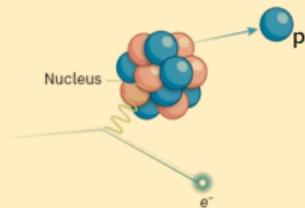
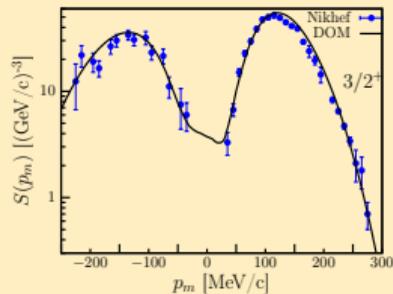


Free NN interaction is insufficient for knock-out

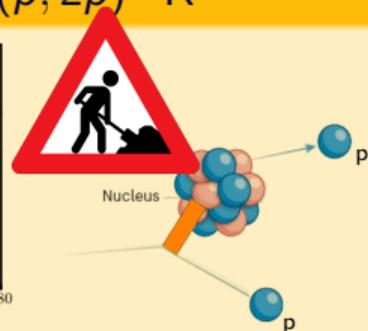
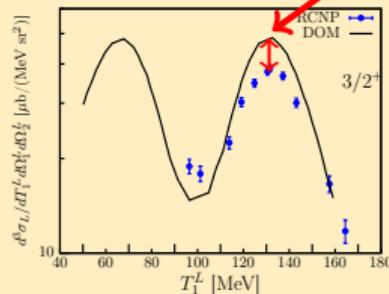
- The DOM provides consistent ingredients for knock-out reactions
- Discrepancy between $^{40}\text{Ca}(e, e'p)^{39}\text{K}$ and $^{40}\text{Ca}(p, 2p)^{39}\text{K}$
- Through Green's function formalism, the DOM can also describe how a proton propagates through the nucleus (propagator G_{DOM})



Electron probe: $^{40}\text{Ca}(e, e'p)^{39}\text{K}$



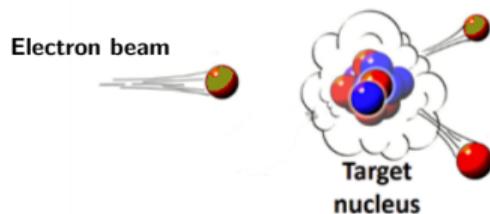
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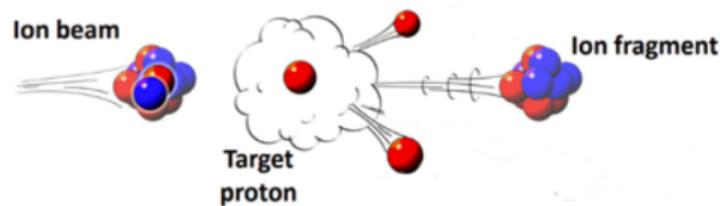
Why not just use $(e, e'p)$ for all single-knockout experiments?

Proton knockout reactions

Experimental sketch for **stable** nuclei



Experimental sketch for **exotic** nuclei (RIB)

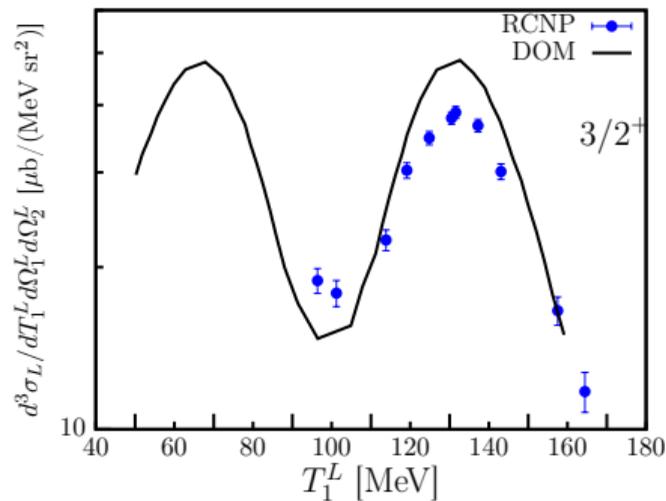


Reaction mechanism well-understood



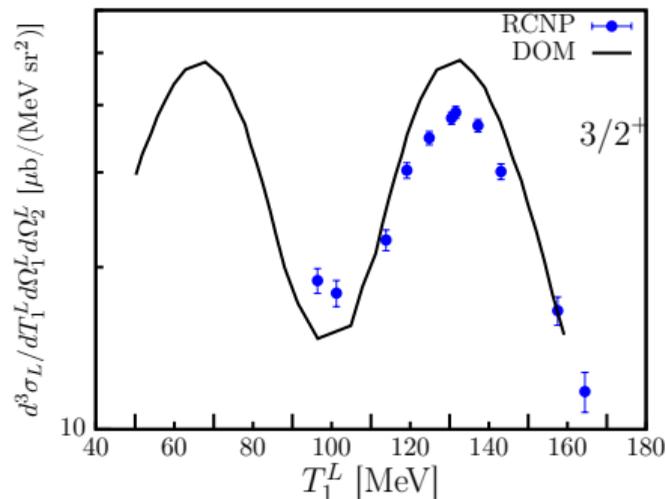
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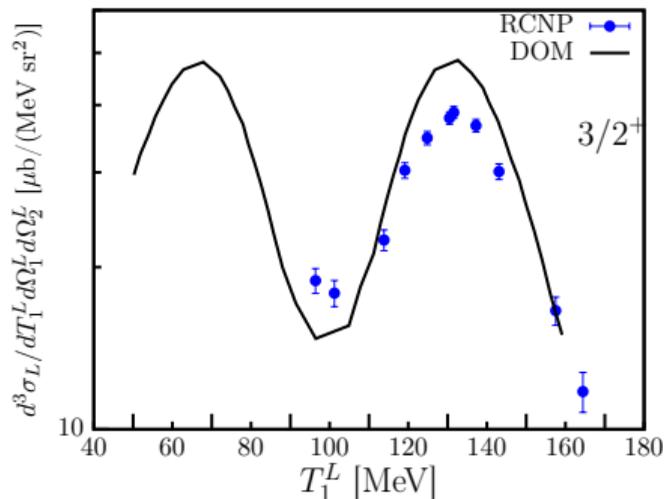
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DOM	FL	0.560 ± 0.05
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DOM	Mel (free)	0.515 ± 0.05

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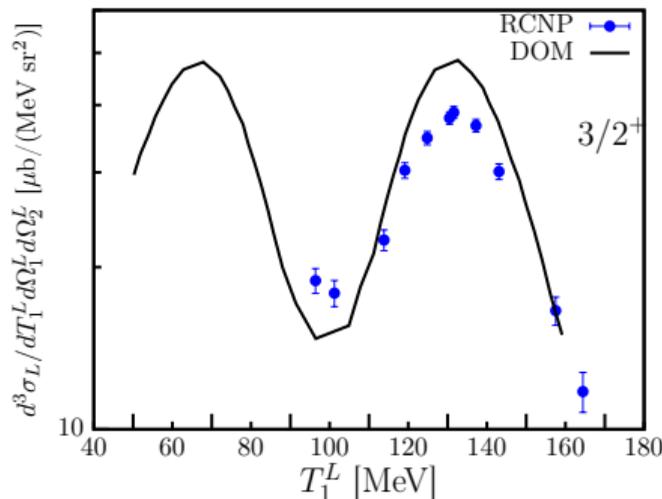
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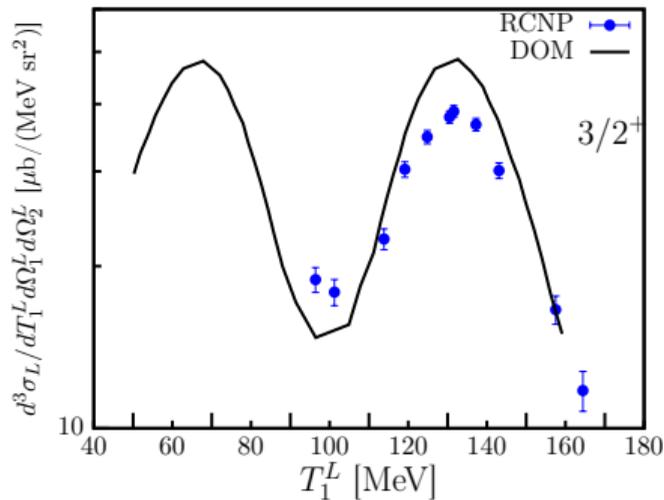
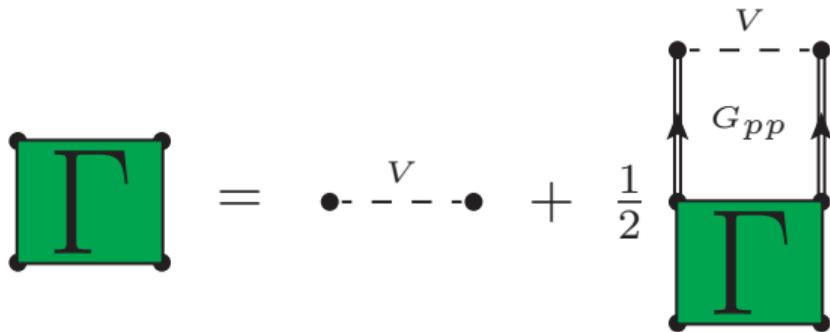
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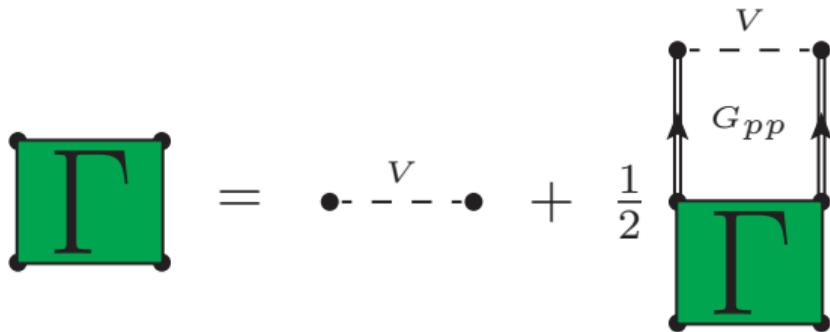
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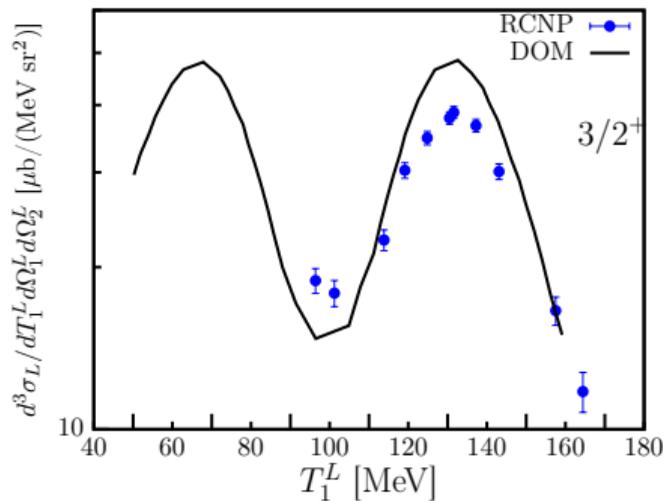
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- $G_{pp} \approx \int G_{\text{DOM}} \times G_{\text{DOM}}$



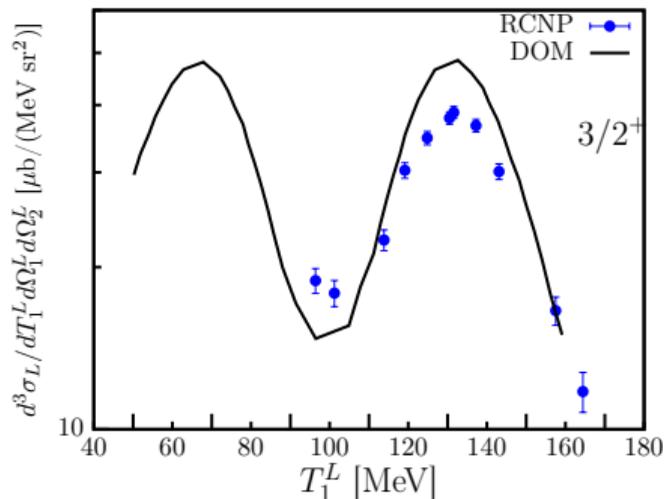
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$$\Gamma = \text{---} \overset{V}{\text{---}} \text{---} + \frac{1}{2} \left[\text{---} \overset{V}{\text{---}} \text{---} \right] \Gamma$$

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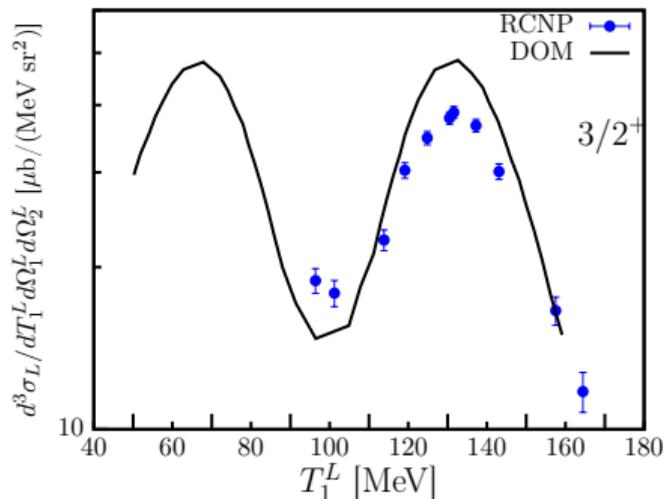
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- $G_{pp} \approx \int G_{\text{DOM}} \times G_{\text{DOM}}$
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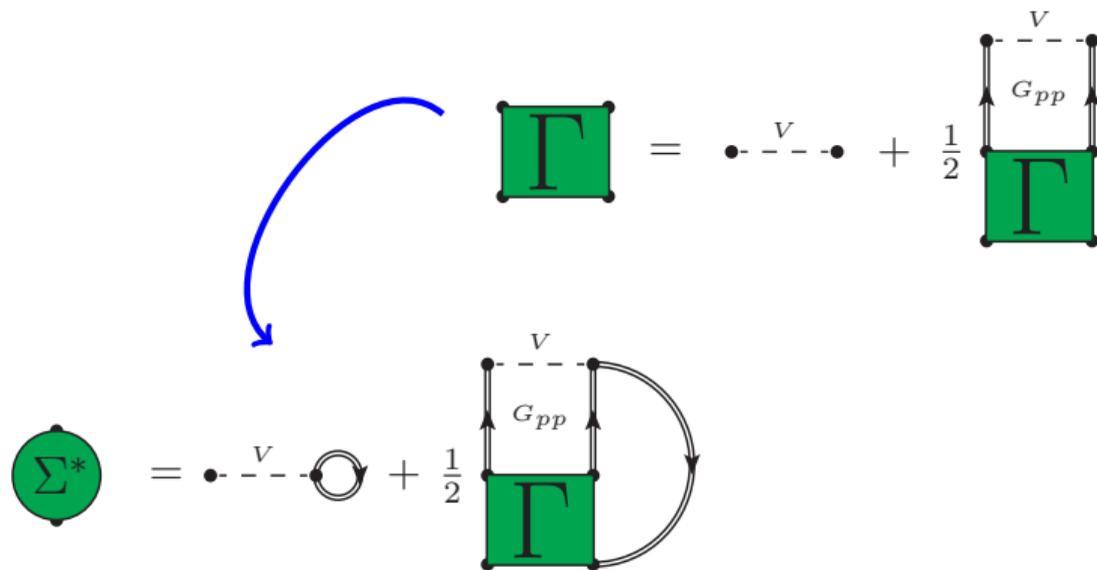
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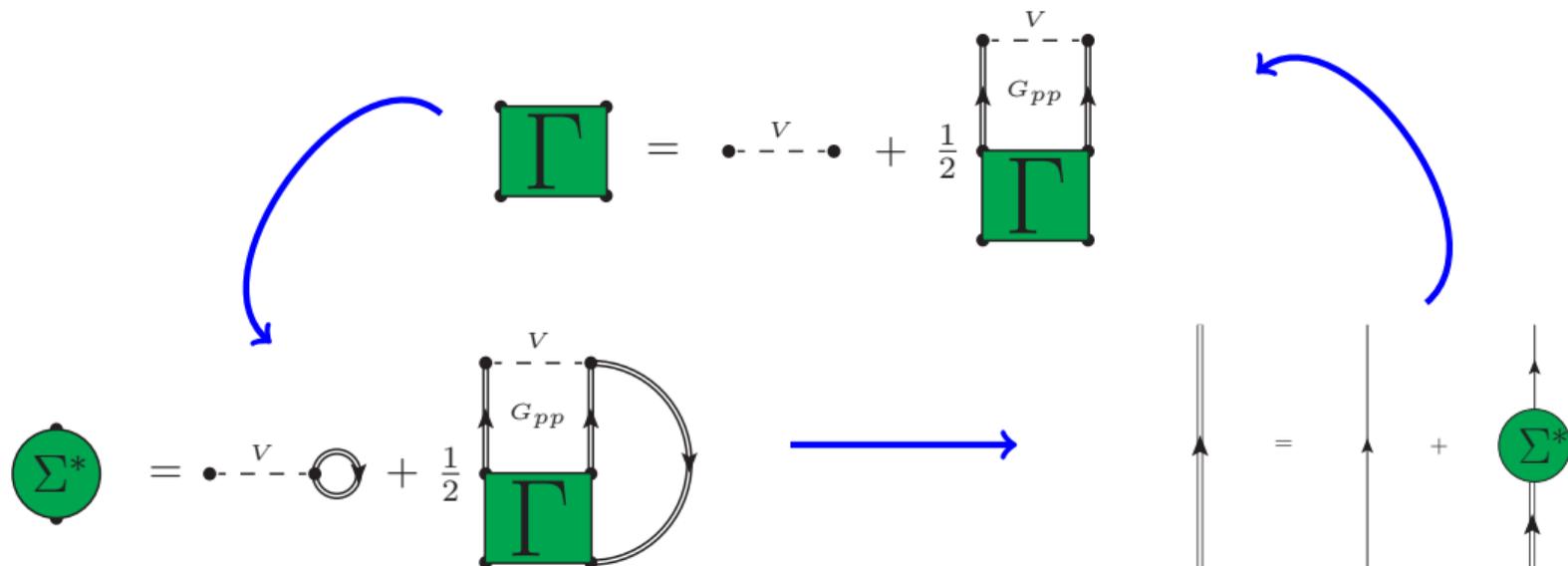
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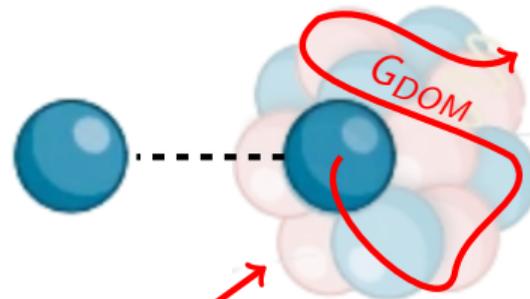
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- Γ can also be used to calculate the nucleon self-energy (OMP)

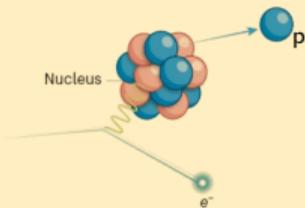
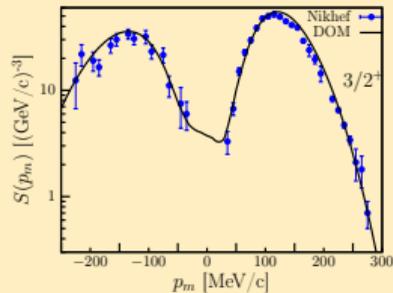


Summary

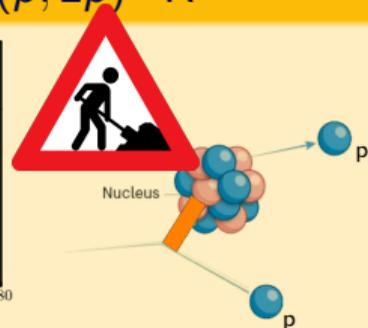
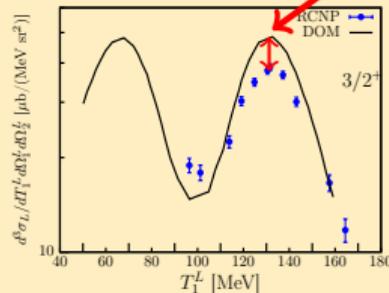
- The DOM provides consistent ingredients for knockout reactions
- The DOM provides a path toward improvement through the nucleus-informed NN interaction Γ_{NN}
- Combine this capability with a global DOM to extend the reach



Electron probe: $^{40}\text{Ca}(e, e'p)^{39}\text{K}$



Proton probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$



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