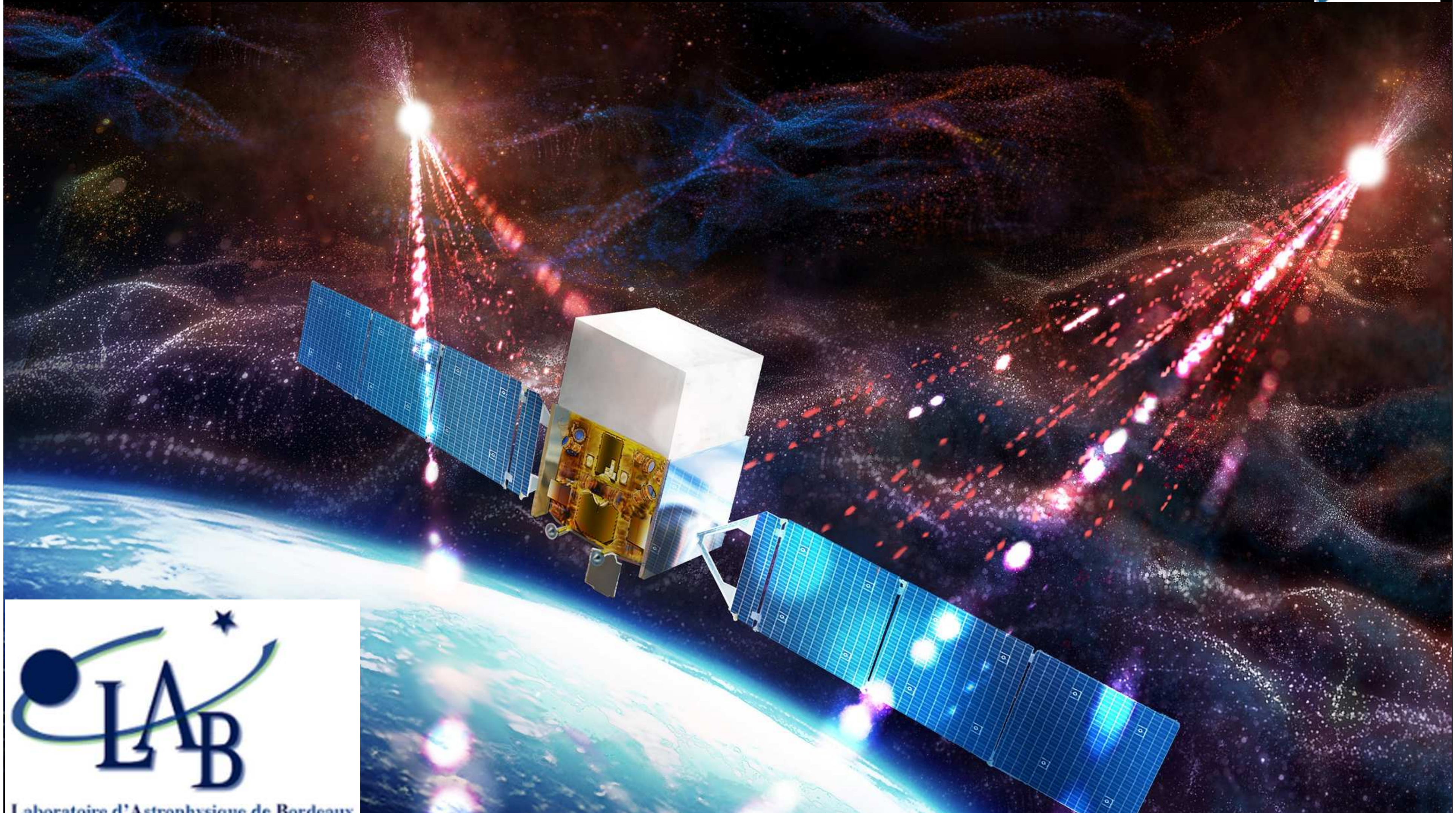
Using Many Gamma-ray Pulsars to Measure nHz Gravitational Waves

Fermi Gamma-ray Space Telescope



Laboratoire d'Astrophysique de Bordeaux

cnrs INSU Observer & comprendre

cnrs IN2P3 Les deux infinis

David.Smith@u-bordeaux.fr

19 November 2025

Remembering Berrie.

Basic Idea

1) ~All galaxies have a super-massive black hole (SMBH) in the middle.
Often, *two* SMBH's orbiting each other (due to galaxy mergers).
Massive binaries emit gravitational waves (GWs).

Individual binaries are too faint to "hear" – signal is the *sum* over the *population*.
Characterizing the signal informs about masses, mergers, history…

2) Millisecond pulsars (MSPs) are Nature's best clocks. See Pablo S-P's talk, tomorrow.
GW's delay and advance pulse times-of-arrival (ToA's).
The timing "noise" induced by the GW "noise" is the PTA *signal*.
PTA = Pulsar Timing Array.

3) Radio PTAs have tracked ("timed") arrays of MSPs for decades.
GW measurements (upper limits) since 2021 (2006).

4) Gamma-ray PTA: *who knew !?*
Less sensitive but i) useful upper limit (2023), a measurement in a few years, and
smaller and different systematic uncertainties.

5) **New** – improved gamma-ray analysis ; "Data Set II" in preparation.

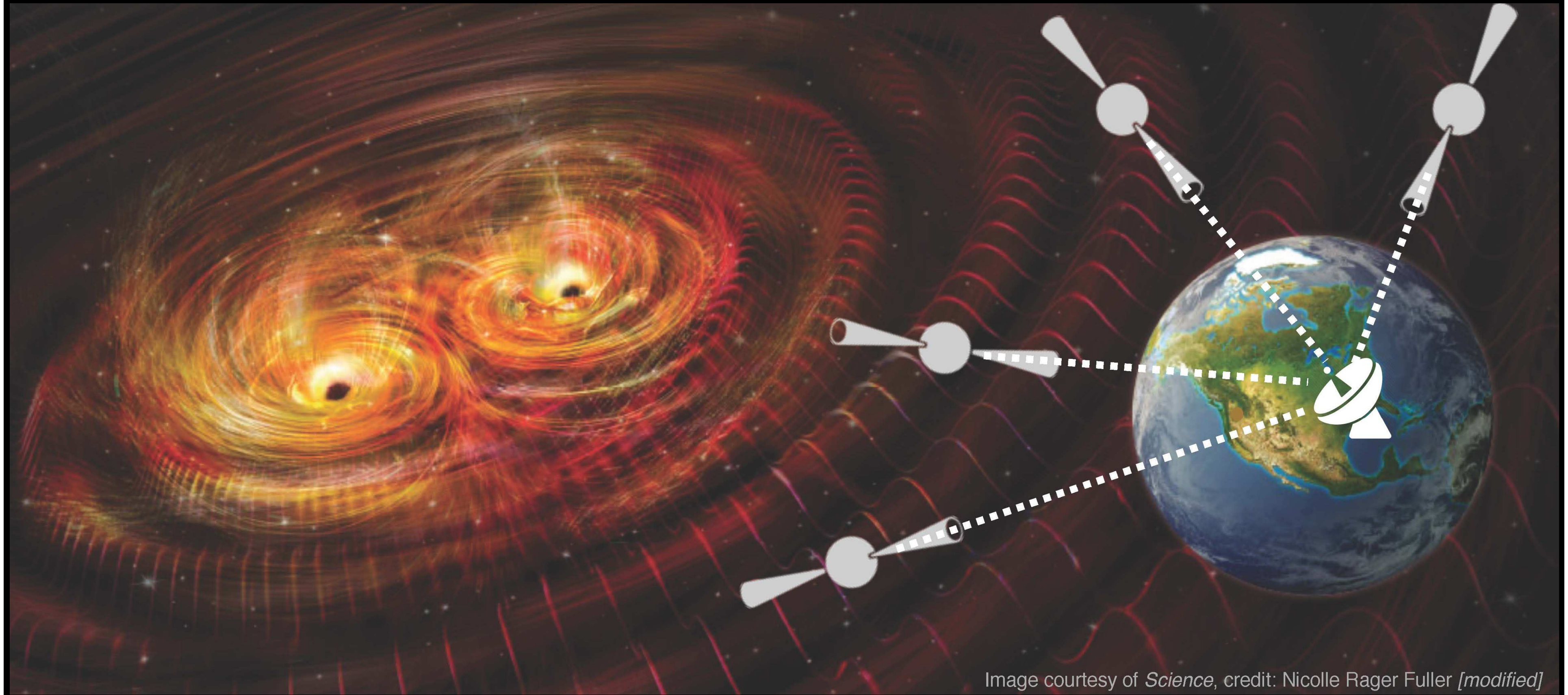
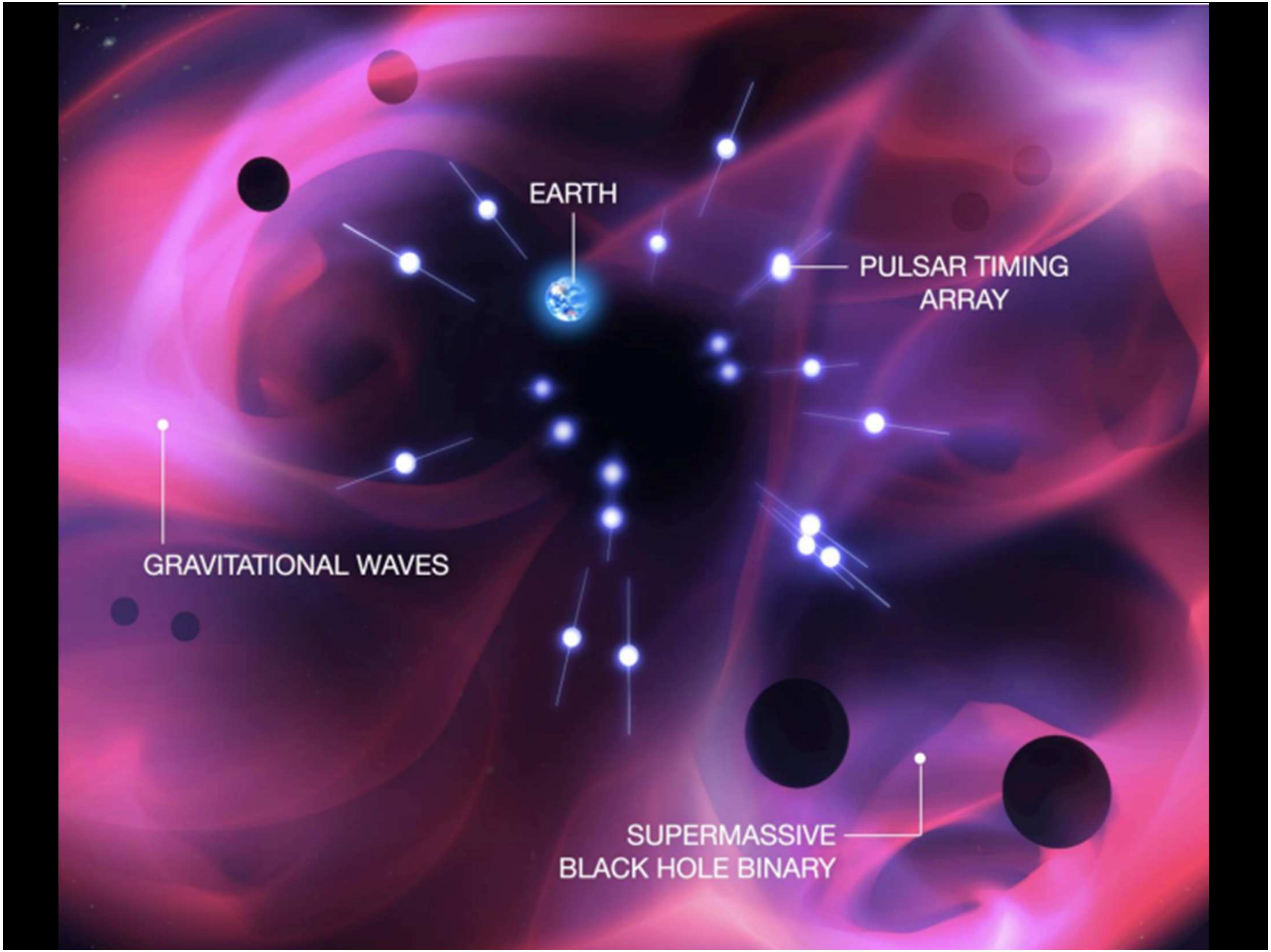


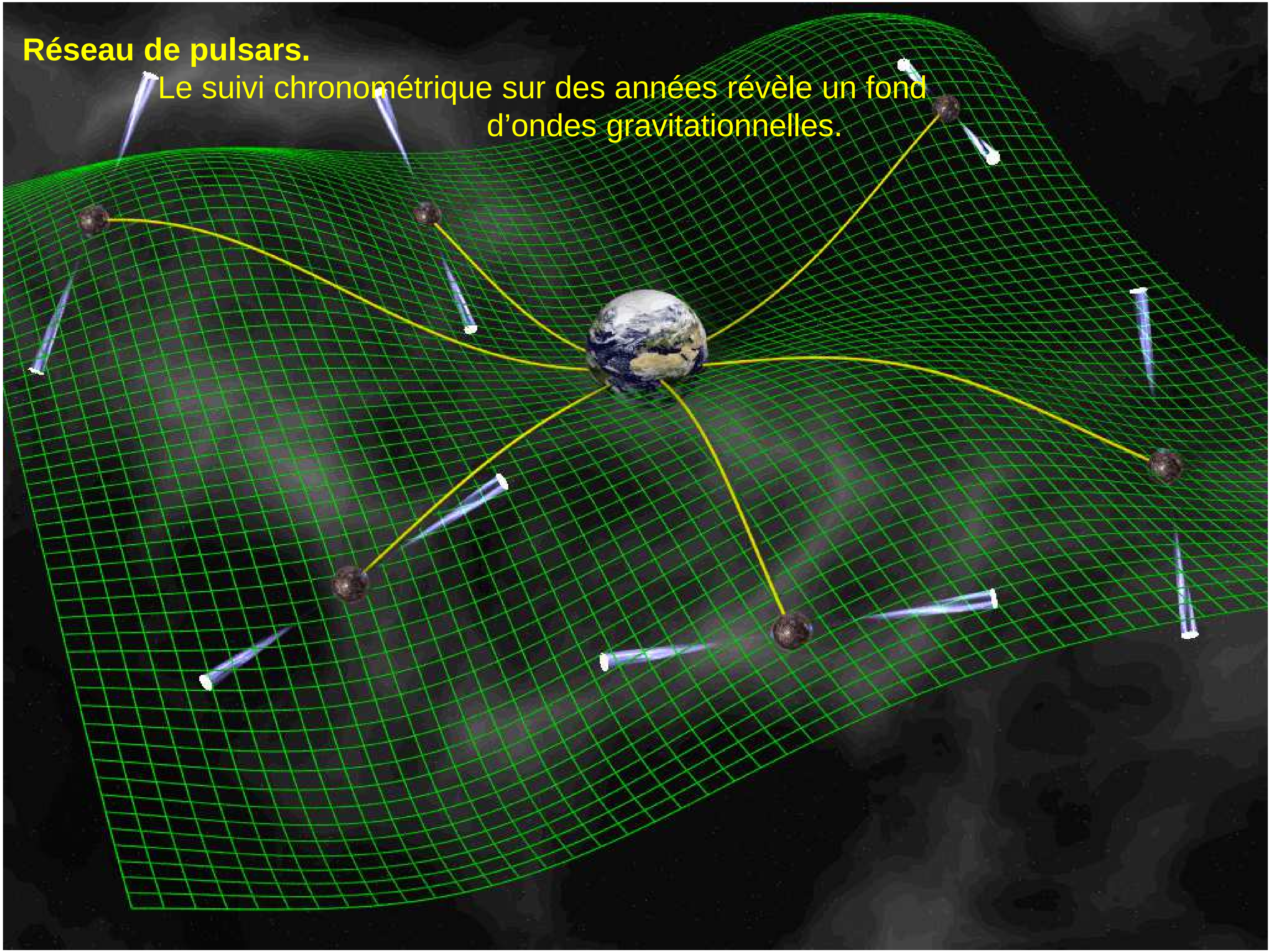
Image courtesy of *Science*, credit: Nicolle Rager Fuller *[modified]*


Les ondes gravitationnelles dilatent et comprimen l'espace-temps
Les plus grands radiotélescopes du monde mesurent le tic-tac des pulsars pendant des décennies, révélant le bruissement des trous noirs lointains
Des couples de trous noirs supermassifs dans l'Univers lointain émettent des ondes gravitationnelles
Les pulsars sont de véritables horloges cosmiques, enregistrant de subtiles variations de distance

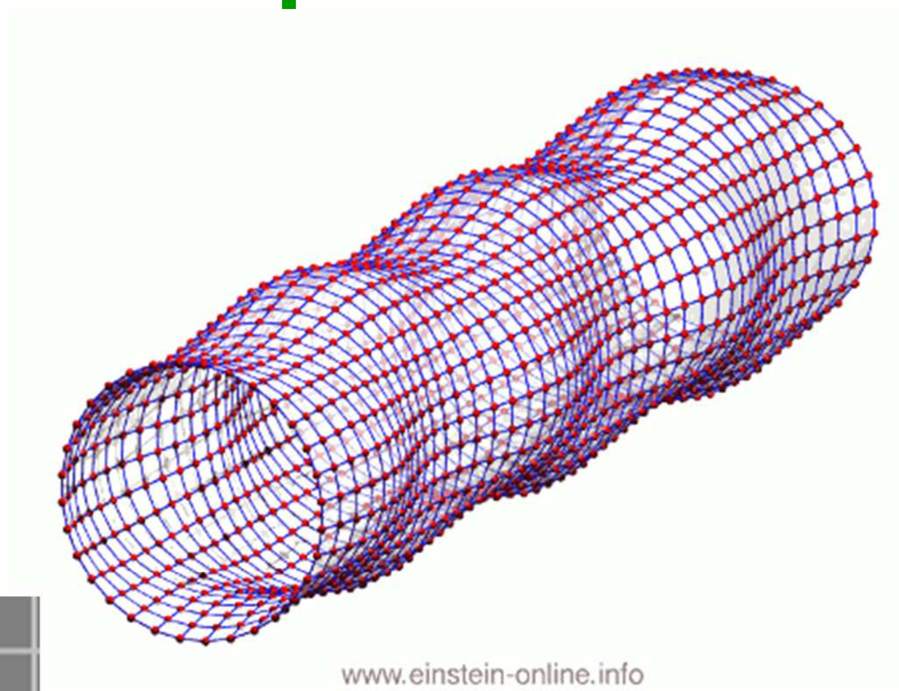

EARTH
PULSAR TIMING ARRAY
GRAVITATIONAL WAVES
SUPERMASSIVE BLACK HOLE BINARY

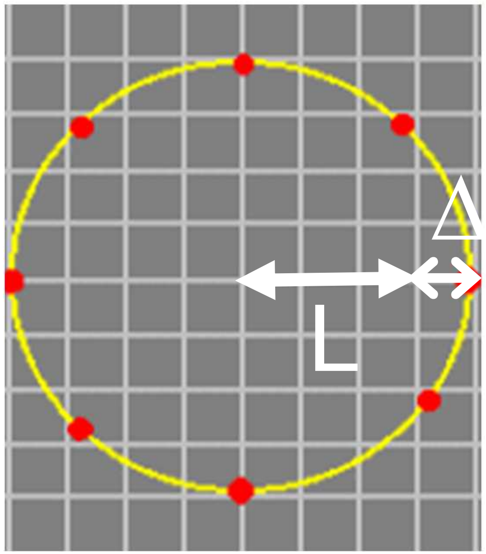
Réseau de pulsars.

Le suivi chronométrique sur des années révèle un fond d'ondes gravitationnelles.



Propagation d'une onde qui déforme l'espace.

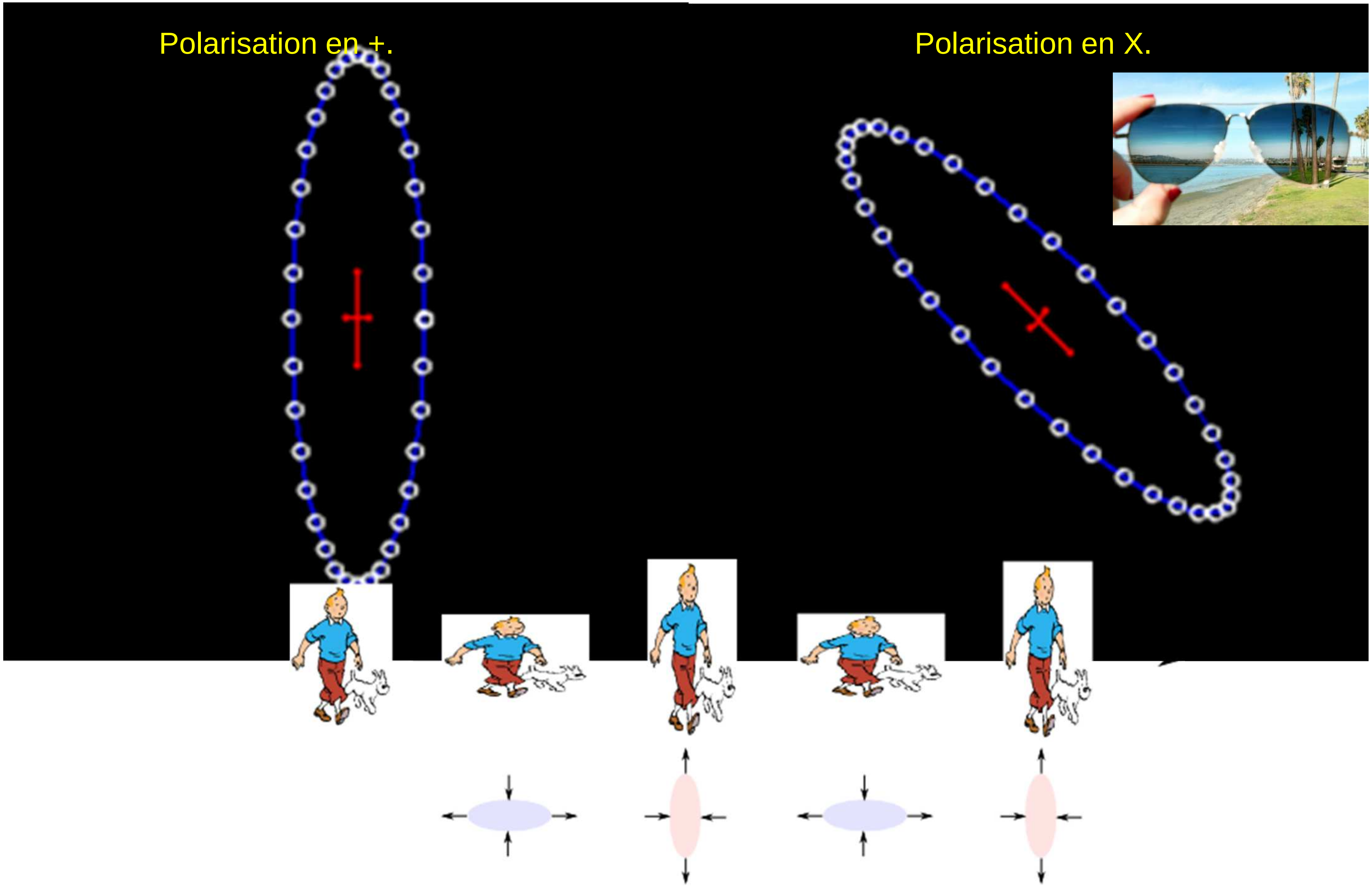




$$h = \frac{\Delta L}{L}$$

« Strain »

Anneaux de billes, déformés lors du passage d'une onde gravitationnelle:



Ann. Phys. (Berlin), 1–17 (2016) / DOI 10.1002/andp.201600209

annalen der physik

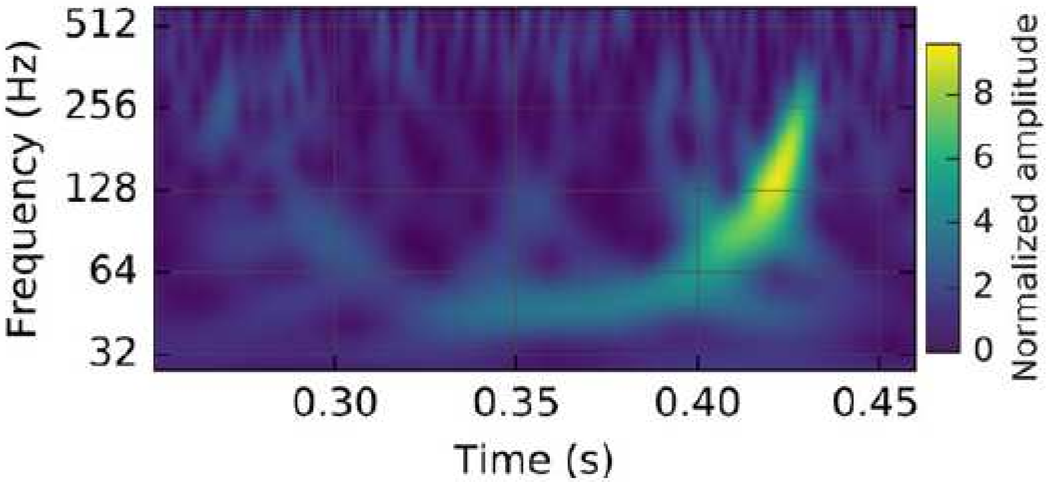
The basic physics of the binary black hole merger GW150914

LIGO Scientific and VIRGO Collaborations,***

https://doi.org/10.1002/andp.201600209

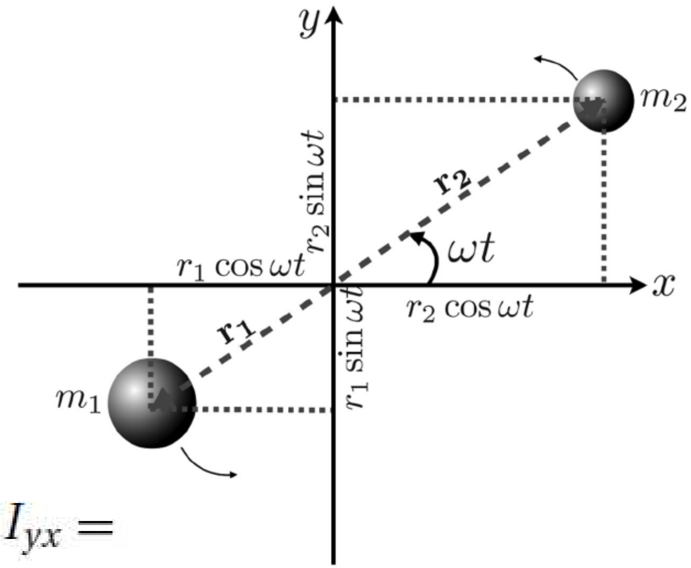
Ofek Birnholtz, contact author, with Bruce Allen, *pédagogue extraordinaire*.

Undergrad formulae to read masses, distance from plot.



Determining the mass scale: Einstein found [17] that the gravitational wave strain h at a (luminosity) distance d_L from a system whose traceless mass quadrupole moment is Q_{ij} (defined in App. A) is

$$h_{ij} = \frac{2\,G}{c^4\,d_L}\frac{\mathrm{d}^2 Q_{ij}}{\mathrm{d}t^2}, \qquad \text{(Eq. 4)}$$



$$Q_{ij}(t) = ½\,\mu r^2 I_{ij} \qquad \text{(Eq A3½) (moments of inertia)}$$

where $I_{xx} = \cos(2\omega t) + \frac{1}{3}$, $I_{yy} = \frac{1}{3} - \cos(2\omega t)$, $I_{xy} = I_{yx} = \sin(2\omega t)$ and $I_{zz} = -\frac{2}{3}$.

$$h \sim 4G\omega^2\mu r^2/c^4 d \qquad \text{(small!)}$$

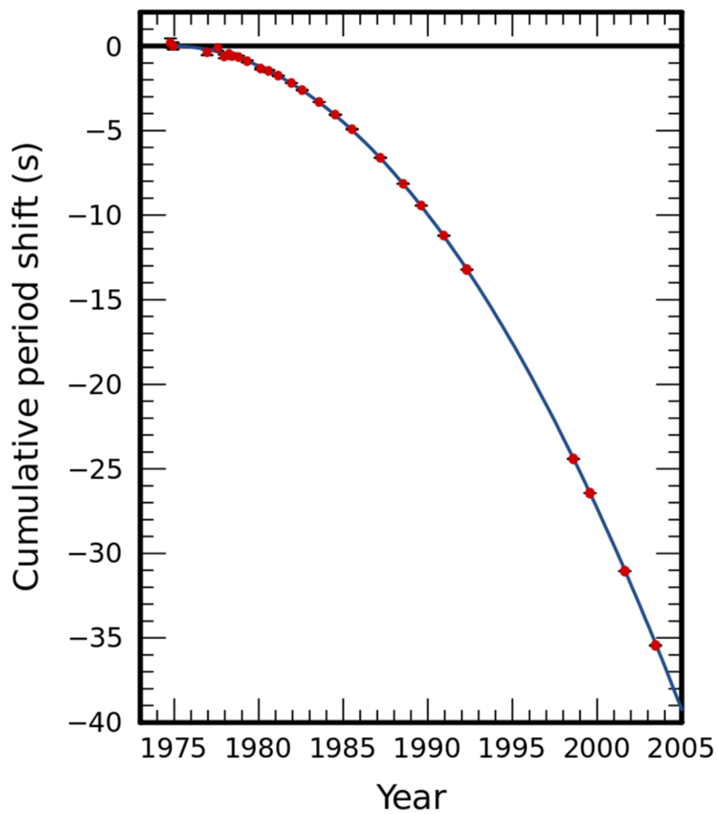
$\mu = m_1 m_2/(m_1+m_2) = ½$ pour $m_1=m_2$

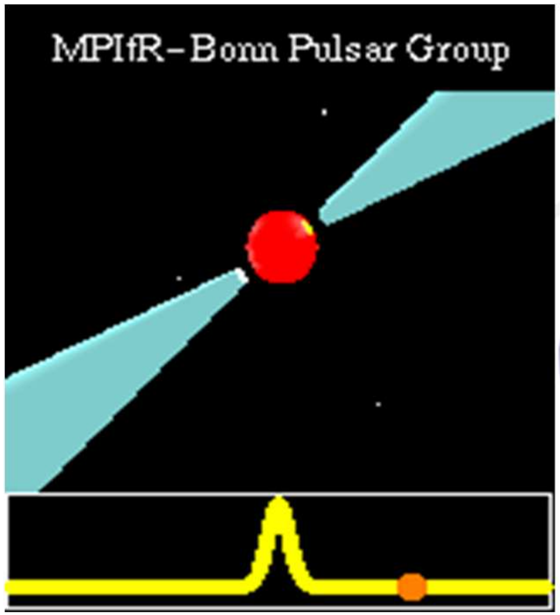
1974 – Hulse et Taylor discover PSR B1913+16, a pulsar -- neutron star binary.

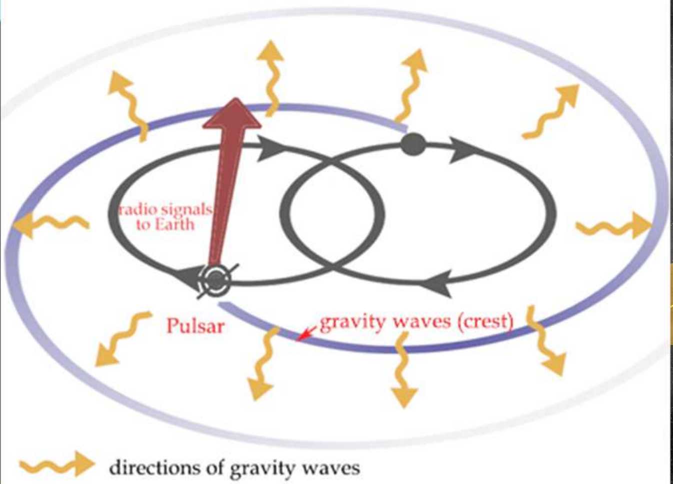
1993 Nobel prize – orbit shrinks as gravitational waves carry energy away.

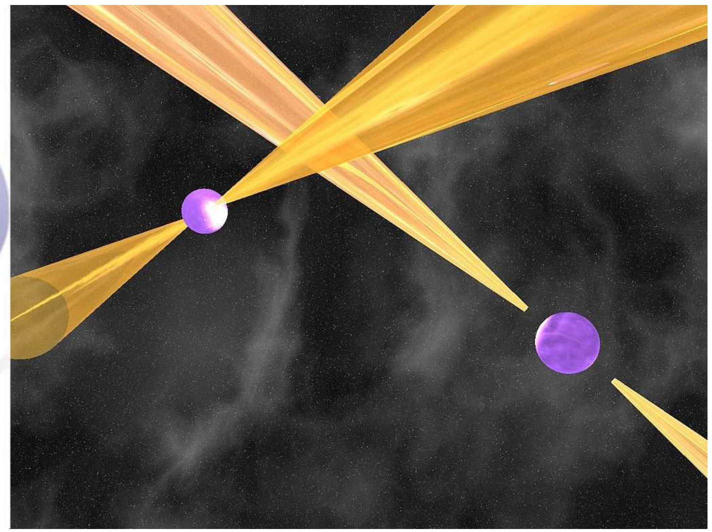
Curve: Gen’l Relativity. Dots: data. *Not a fit!*

2025 – Double pulsar J0737-3039A,B gives many of our best GR tests.







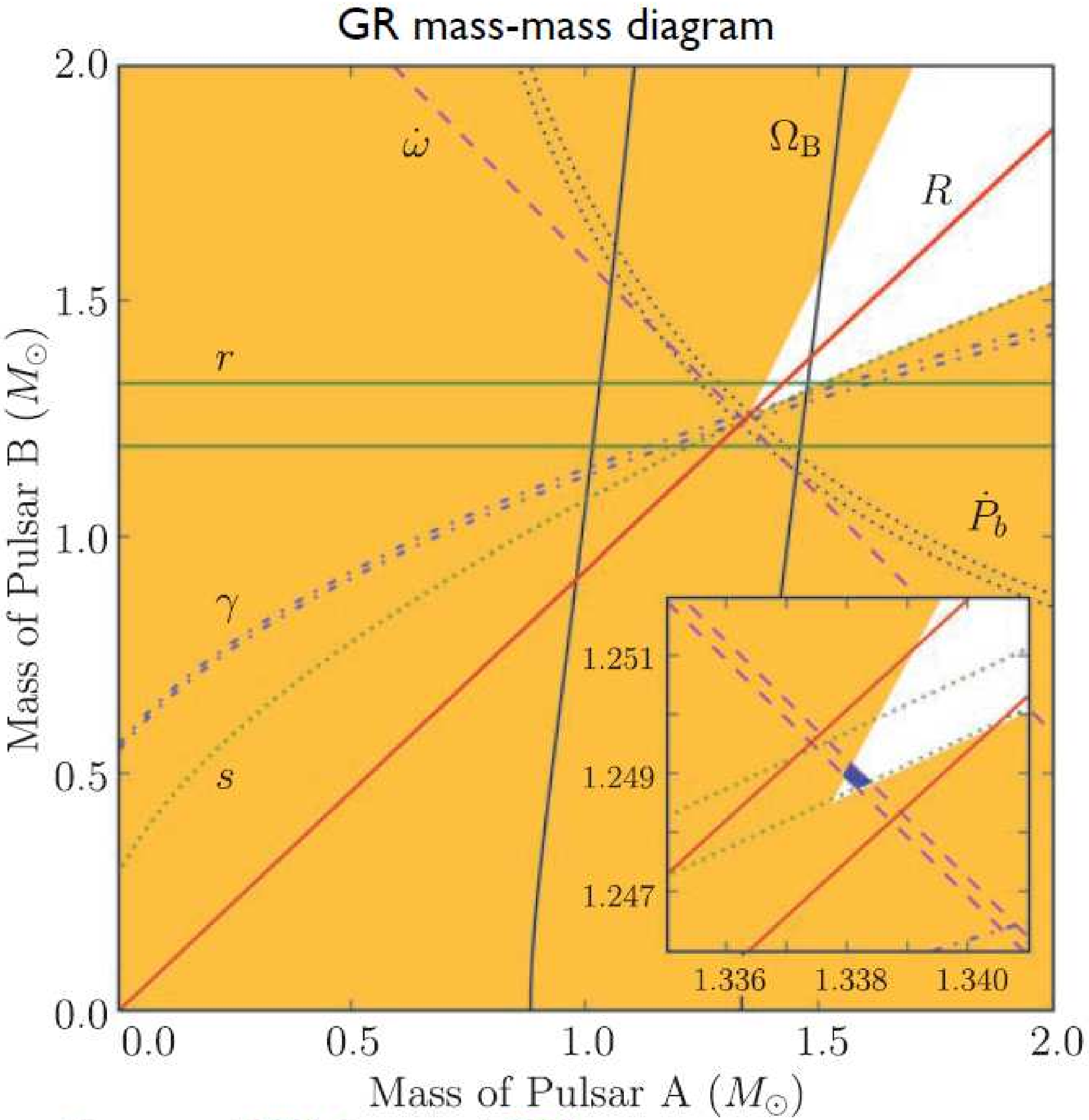


PSRs J0737-3039 A et B,
découvert en 2003.

Mesures affinées en permanence…

The Double Pulsar

La théorie d'Einstein reste la meilleure…



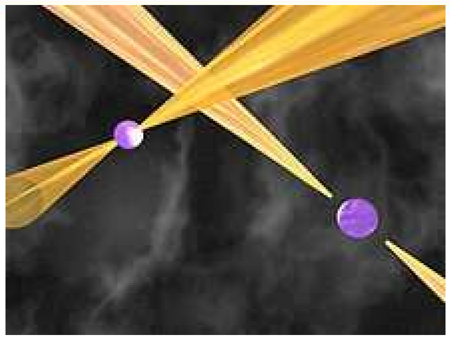
Kramer et al. 2006, Breton et al. 2008

Mass ratio m_A/m_B and
6 post-Keplerian parameters

- ➡ periastron precession
- ➡ time dilation
- ➡ range (r) and shape (s) of Shapiro delay
- ➡ geodetic precession
- ➡ gravitational wave damping

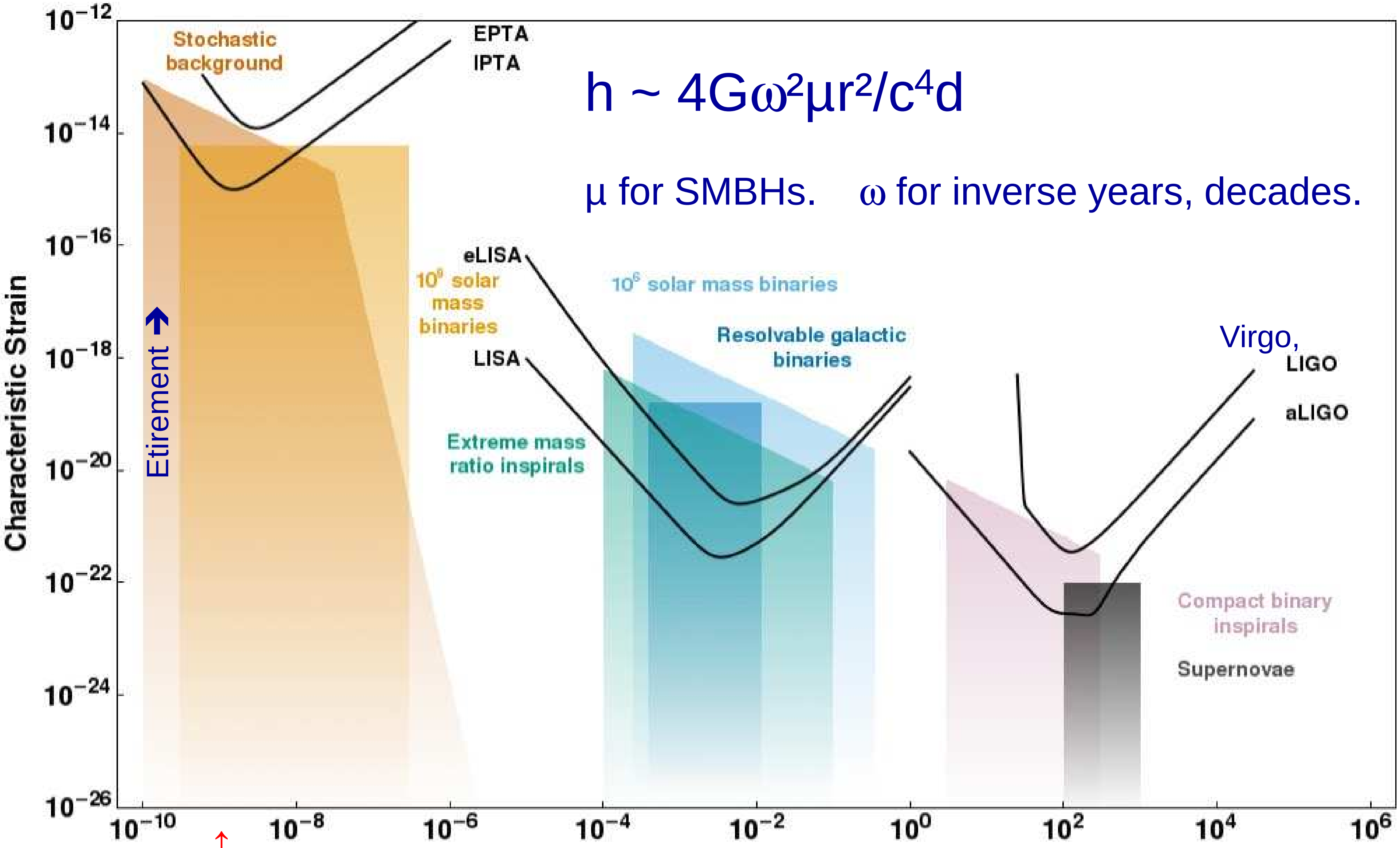
$$\mathcal{P}_{\rm pK} = f(\mathcal{P}_{\rm K}; m_A, m_B)$$

➜ *5 tests*



Pulsar timing is an amazingly sensitive technique
for measuring tiny things.

Back to nHz GW from binary SMBHs.



$$h \sim 4G\omega^2\mu r^2/c^4 d$$

↑
nHz (Today's talk)

- Many galaxies harbor binary SMBHs.
- Integrate strain ***h*** from an assumed population:
  Mass, period, and distance distributions.
- Predict ***h*** to follow a power-law frequency distribution, with $\alpha \approx -2/3$.

$$h_c(f) = A_{\mathrm{gwb}} \left(\frac{f}{\mathrm{yr}^{-1}} \right)^{\alpha}$$

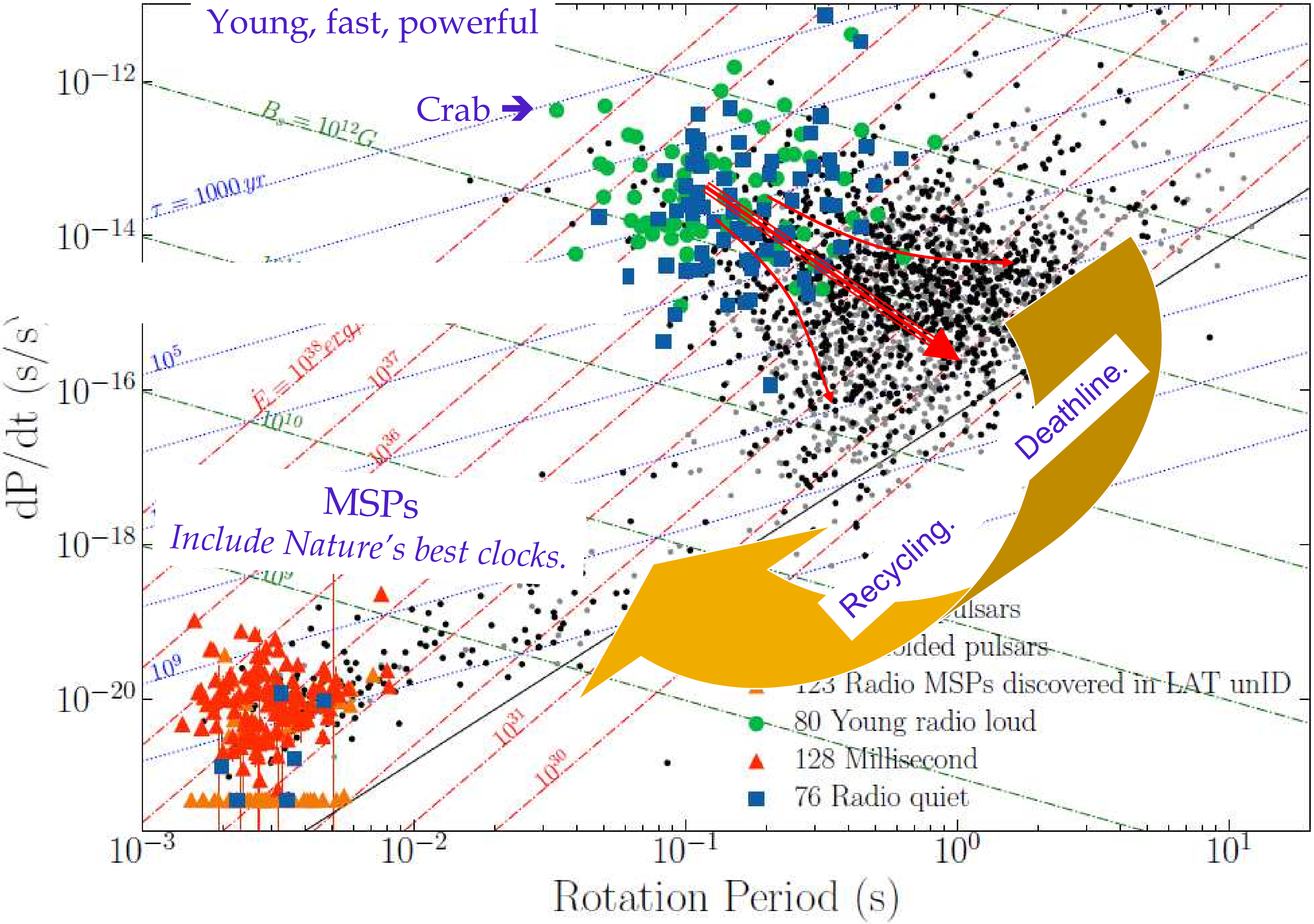
Using pulsars to detect $h_c(f)$

Basics of pulsar timing

Expect $\alpha \approx -2/3$.

$$h_c(f) = A_{\rm gwb} \left(\frac{f}{\rm yr^{-1}} \right)^{\alpha}$$

“P Pdot plot”



Young, fast, powerful
Crab
MSPs
Include Nature's best clocks.
Deathline.
Recycling.
123 Radio MSPs discovered in LAT unID
80 Young radio loud
128 Millisecond
76 Radio quiet
dP/dt (s/s)
Rotation Period (s)

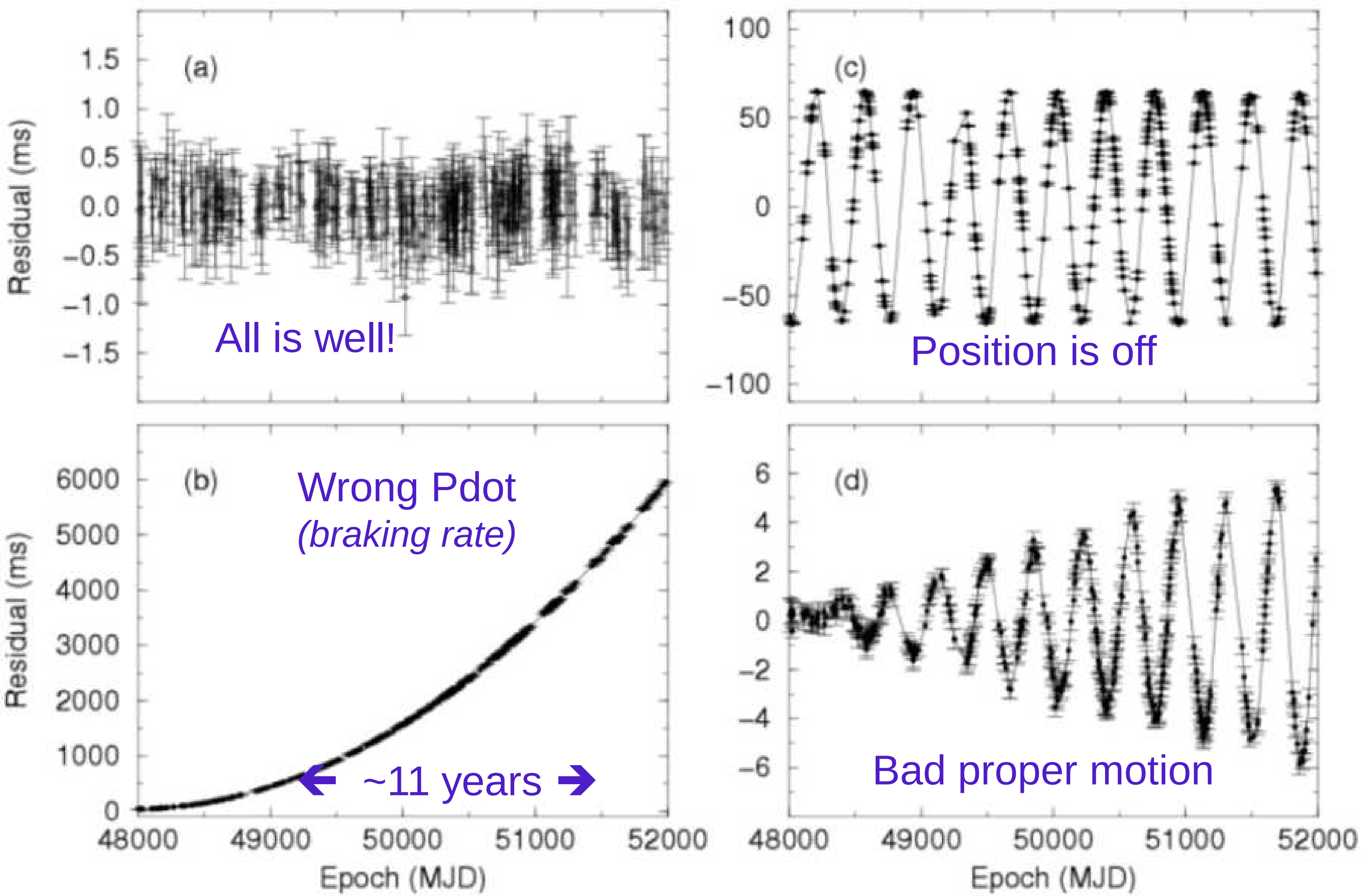
--- Pulsar Timing ---

Record pulse *times-of-arrival* (TOAs) weekly or monthly, for years & years.

Fit a rotation model (*ephemeris*) – position, spindown, binary orbit, DM.

Residuals = (predicted times – measured times).

Lorimer & Kramer (2005)



Radio rotation ephemeris includes the Dispersion Measure (DM)

Electron column density along the line of sight.

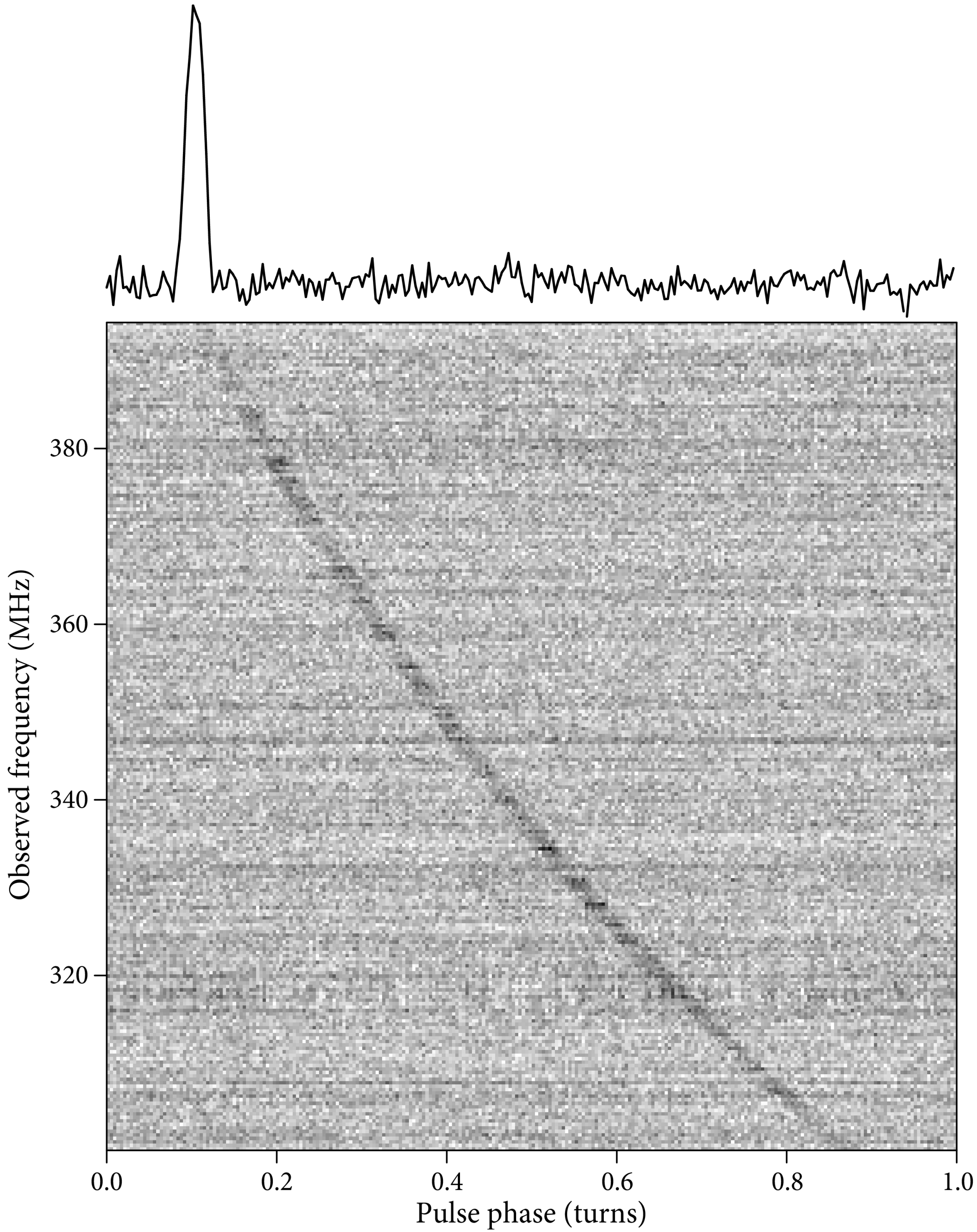
Space weather!

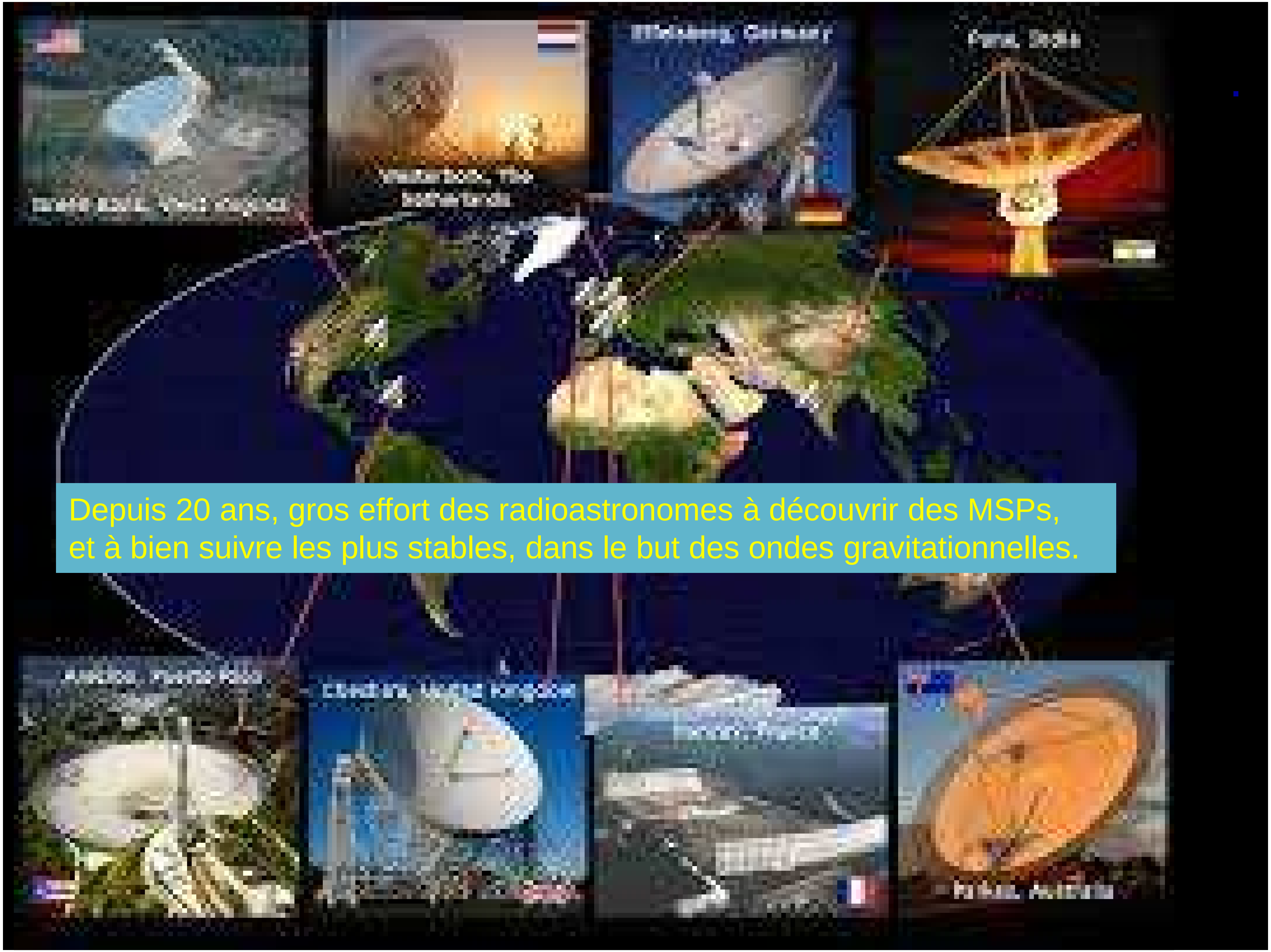
Interstellar plasma varies in time.

Possible nHz timescales.

$$t_d = \frac{e^2}{2\pi m_e c} \frac{\mathcal{DM}}{\nu^2} = 4140. \left(\frac{\mathcal{DM}}{\mathrm{cm}^{-3}\ \mathrm{pc}}\right) \left(\frac{\nu}{1\ \mathrm{MHz}}\right)^{-2} \mathrm{sec.}$$

$$\mathcal{DM} \equiv \int_0^d n_e \, dl$$





Depuis 20 ans, gros effort des radioastronomes à découvrir des MSPs,
et à bien suivre les plus stables, dans le but des ondes gravitationnelles.

« Good timers »

- Sharp peak x Fast spin rate
- Nearly Nothing perturbs its spinning & braking.

  Spiders need not apply
- Bright enough to allow repeated measurements.

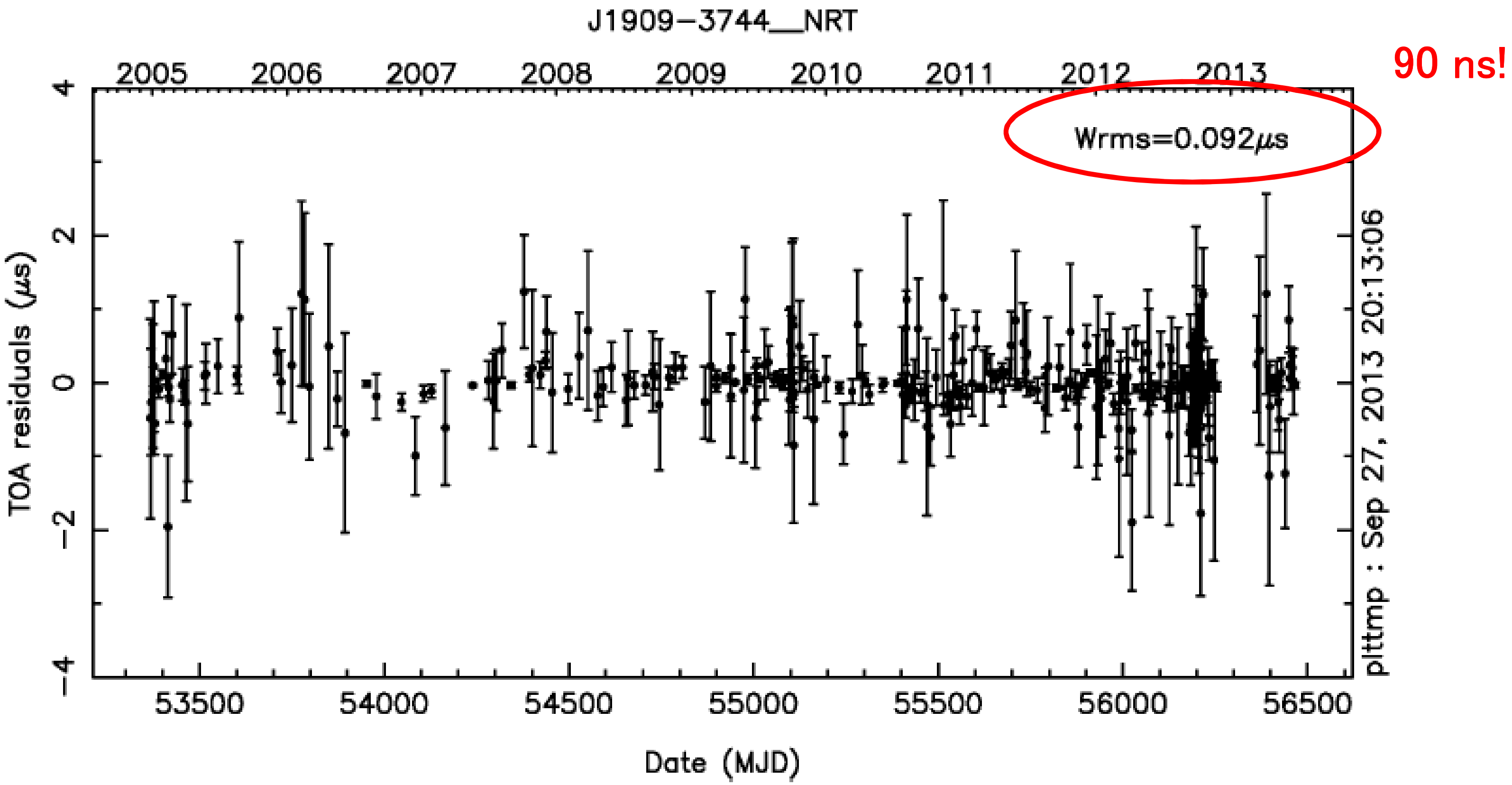
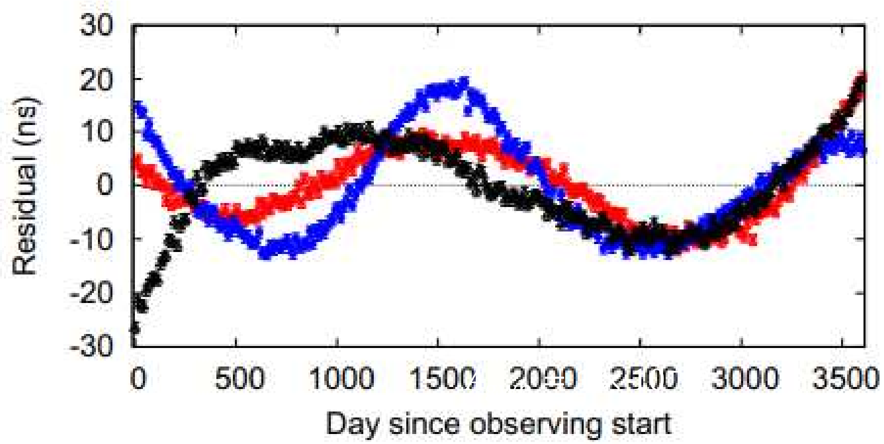


Fig. 2. Times of Arrival residuals obtained for the ultra-stable pulsar J1909-3744 with the Nançay radio telscope. The residuals obtained after substration of all the known propagation and rotational effects are characterized by a weighted rms of only ∼90ns over almost 10 years. Ismaël Cognard, LPC2E Orléans/Nançay.

Once the cosmic clock is « calibrated », search for *deviations*.

Here: prediction for 3 MSPs in different sky directions.



Delays/advances are perturbations from gravitational wave.

Too small to see for individual pulsars.

For an MSP *array*,

GW-induced residuals should have a power spectrum with Γ = 13/3,

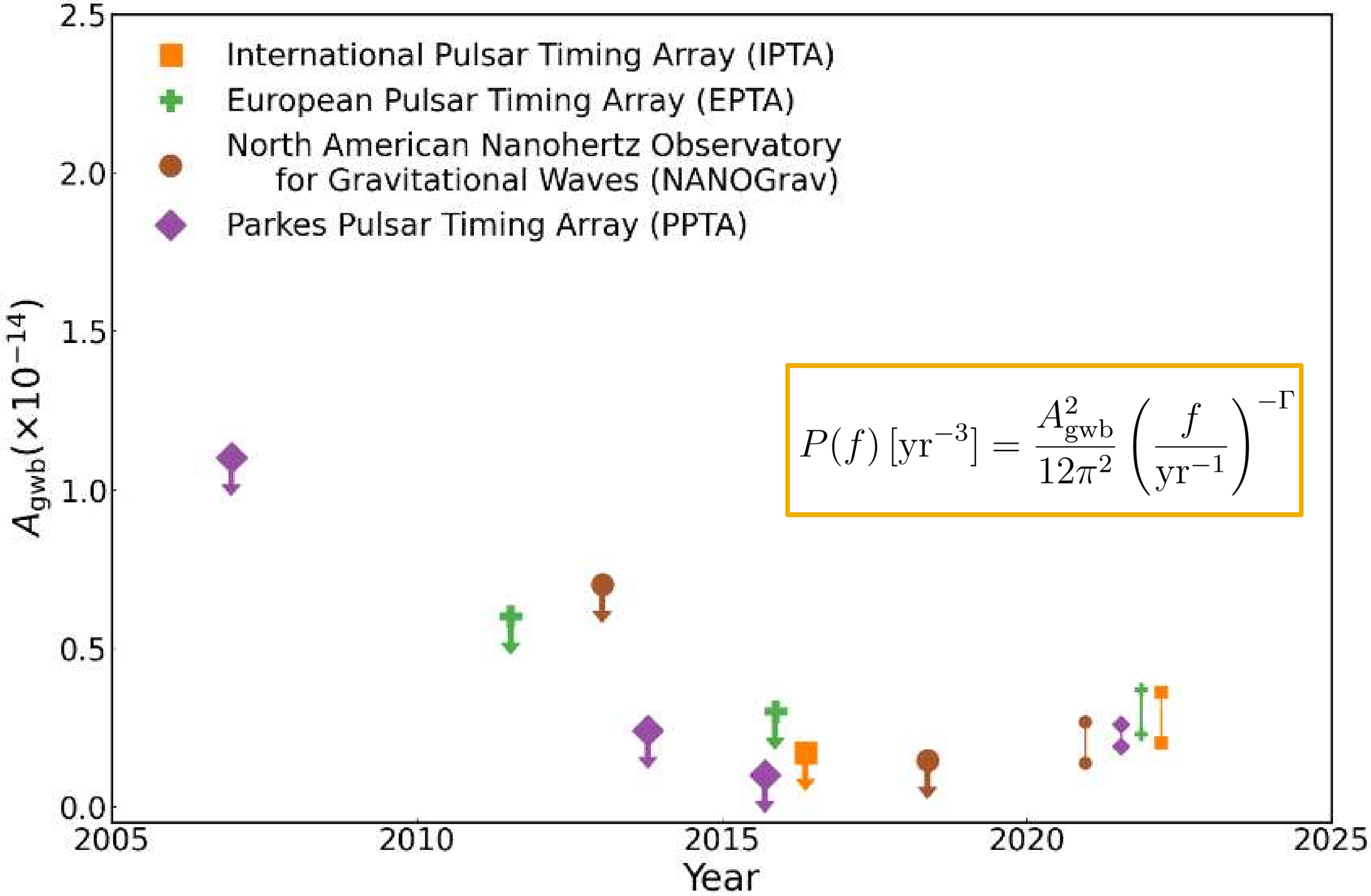
$$P(f)\,[\mathrm{yr}^{-3}] = \frac{A_{\mathrm{gwb}}^2}{12\pi^2}\left(\frac{f}{\mathrm{yr}^{-1}}\right)^{-\Gamma}$$

This is the first signature that the PTA's search for.

Upper limits from the radio consortia since 2007.

2021: First published measurement.

Freeze Γ = 13/3 to find A_{gwb}.



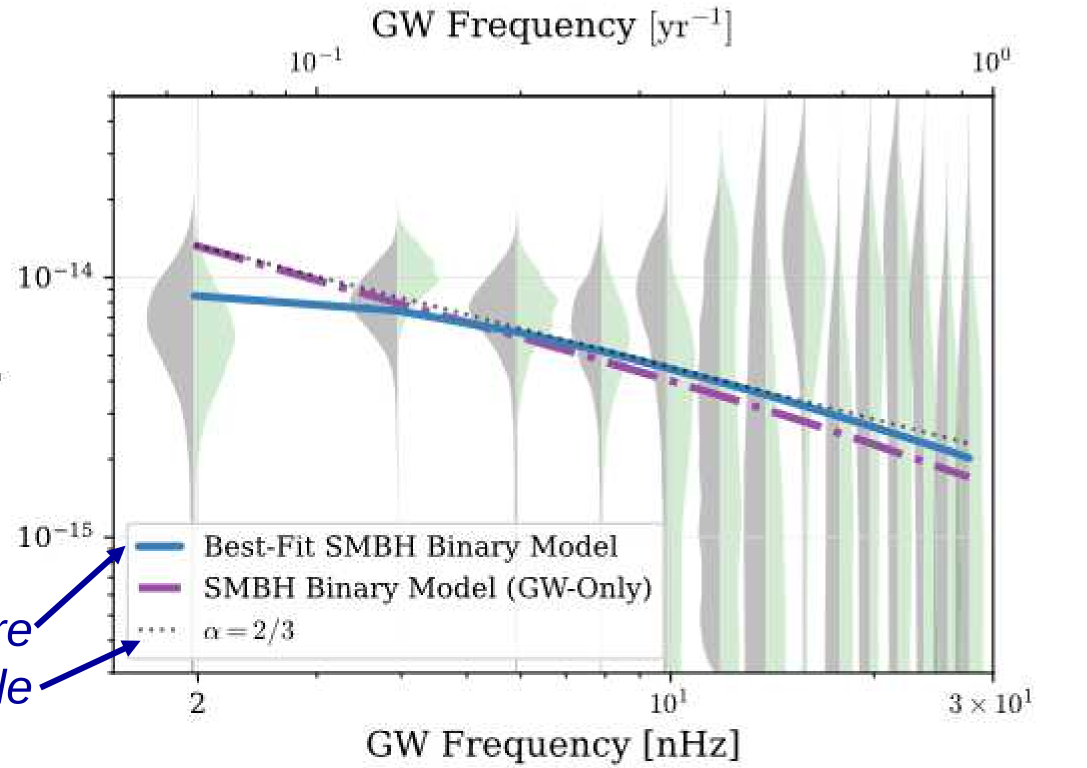
THE ASTROPHYSICAL JOURNAL LETTERS, 952:L37 (30pp), 2023 August 1 https://doi.org/10.3847/2041-8213/ace18b

The NANOGrav 15 yr Data Set: Constraints on Supermassive Black Hole Binaries from the Gravitational-wave Background

Gabriella Agazie[1], Akash Anumarlapudi[1], A

Previous results:
Assume an idealized population.

Here: Free up some parameters,
start probing the true population.



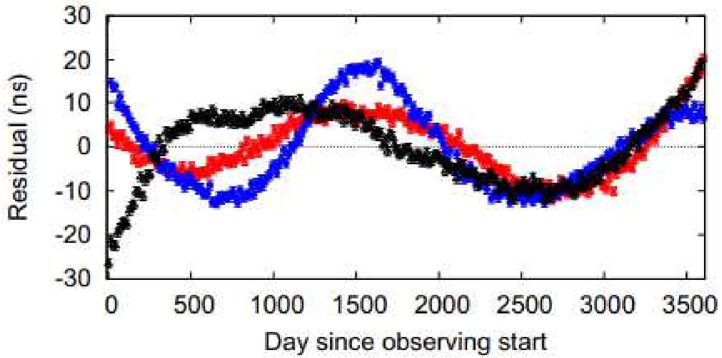
Mesure
Idéale

Low frequency sensitivity increases as the years go by.

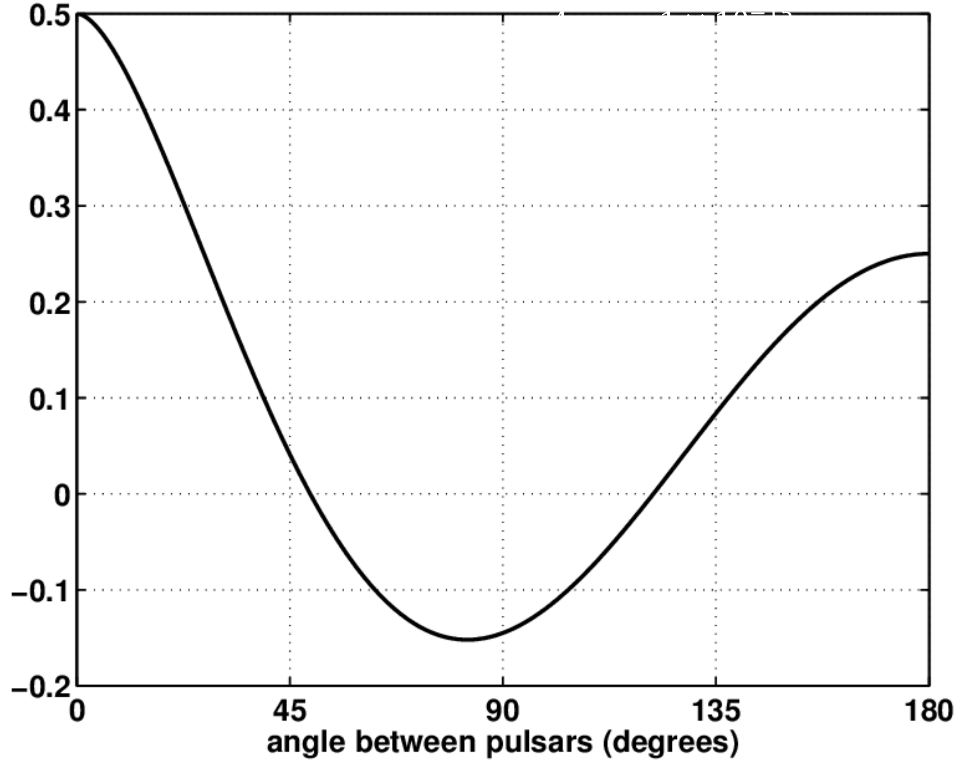
But… is it gravity or is it noise?

Towards a smoking gun :

For two pulsars, the correlation in their residuals depends on their angular separation across the sky.



For N pairs,
build the *Hellings-Downs* curve ➔

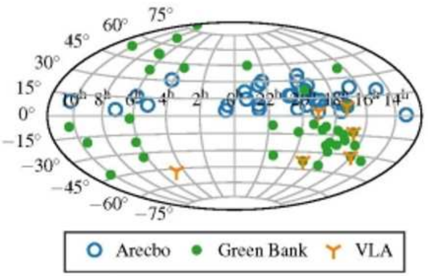


The field's Holy Grail.

The Astrophysical Journal Letters, 951:L8 (24pp), 2023 July 1 https://doi.org/10.3847/2041-8213/acdac6

The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background

Gabriella Agazie[1], Akash Anumarlapudi[1], Anne M. Archibald[2], Zaven Arzoumanian[3], Paul T. Baker[4], Bence Bécsy[5], Laura Blecha[6], Adam Brazier[7,8], Paul R. Brook[9], Sarah Burke-Spolaor[10,11], Rand Burnette[5], Robin Case[5]



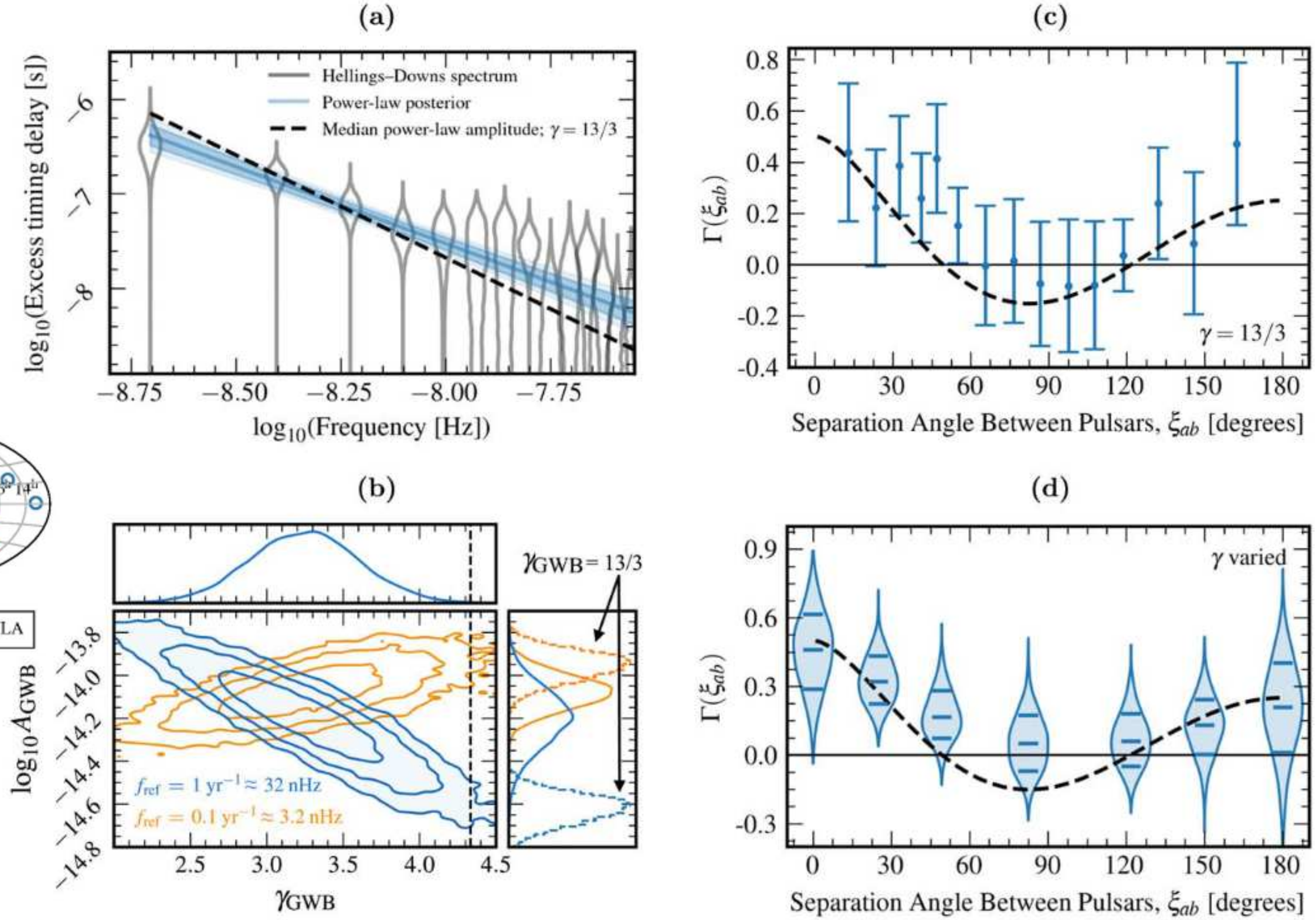
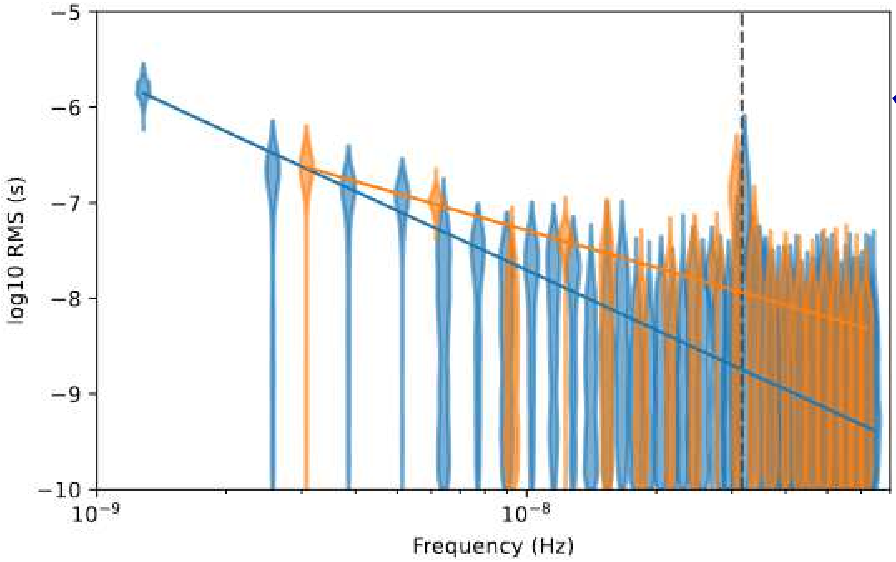


Figure 1. Summary of the main Bayesian and optimal-statistic analyses presented in this paper, which establish multiple lines of evidence for the presence of

Version européenne:

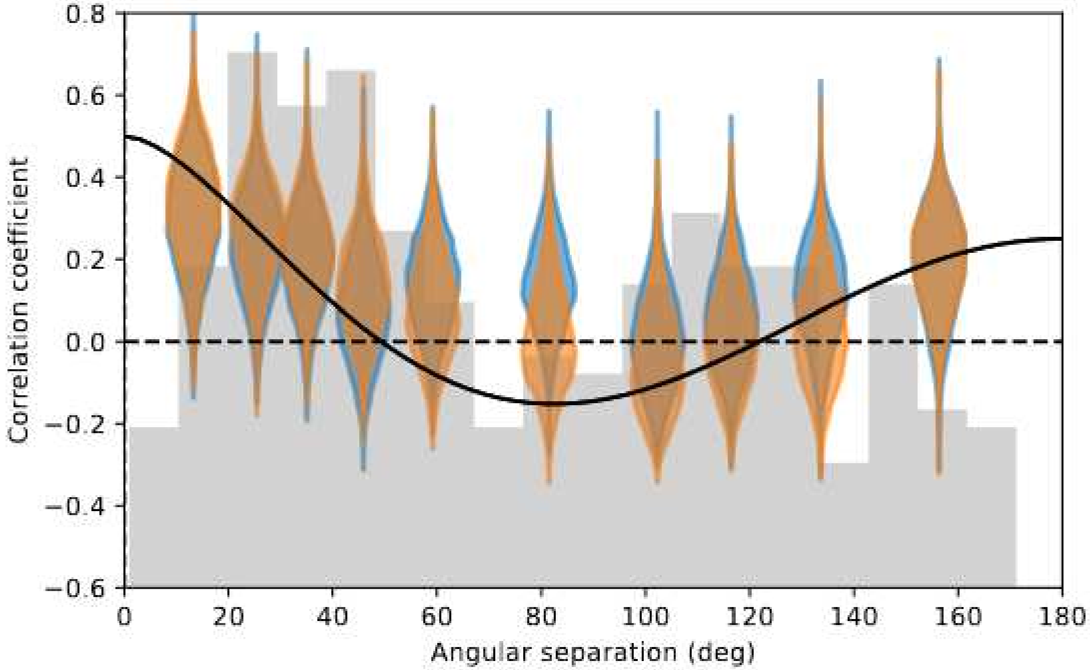
EPTA Collaboration and InPTA Collaboration: A&A 678, A50 (2023)



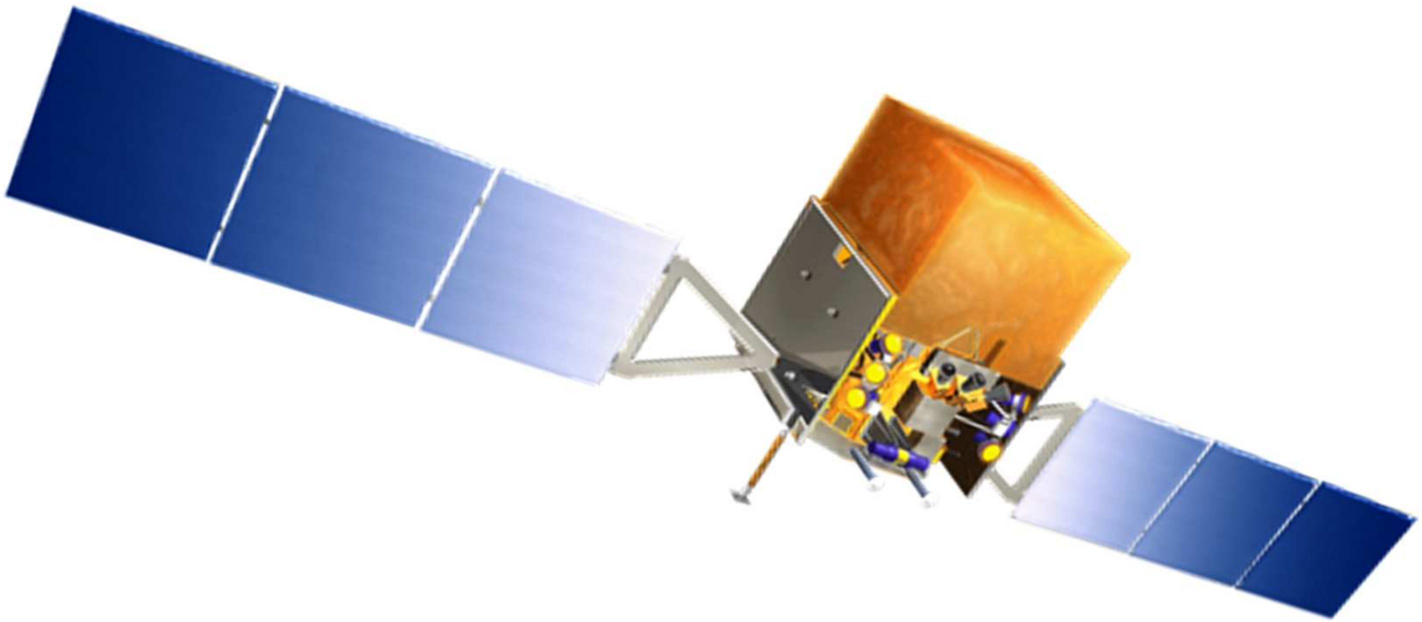
🡐*Spectre des amplitudes* du bruit induit par les ondes gravitationnelles

Blue: pente de 13/3 'naïve' attendue.
Orange: affinage.

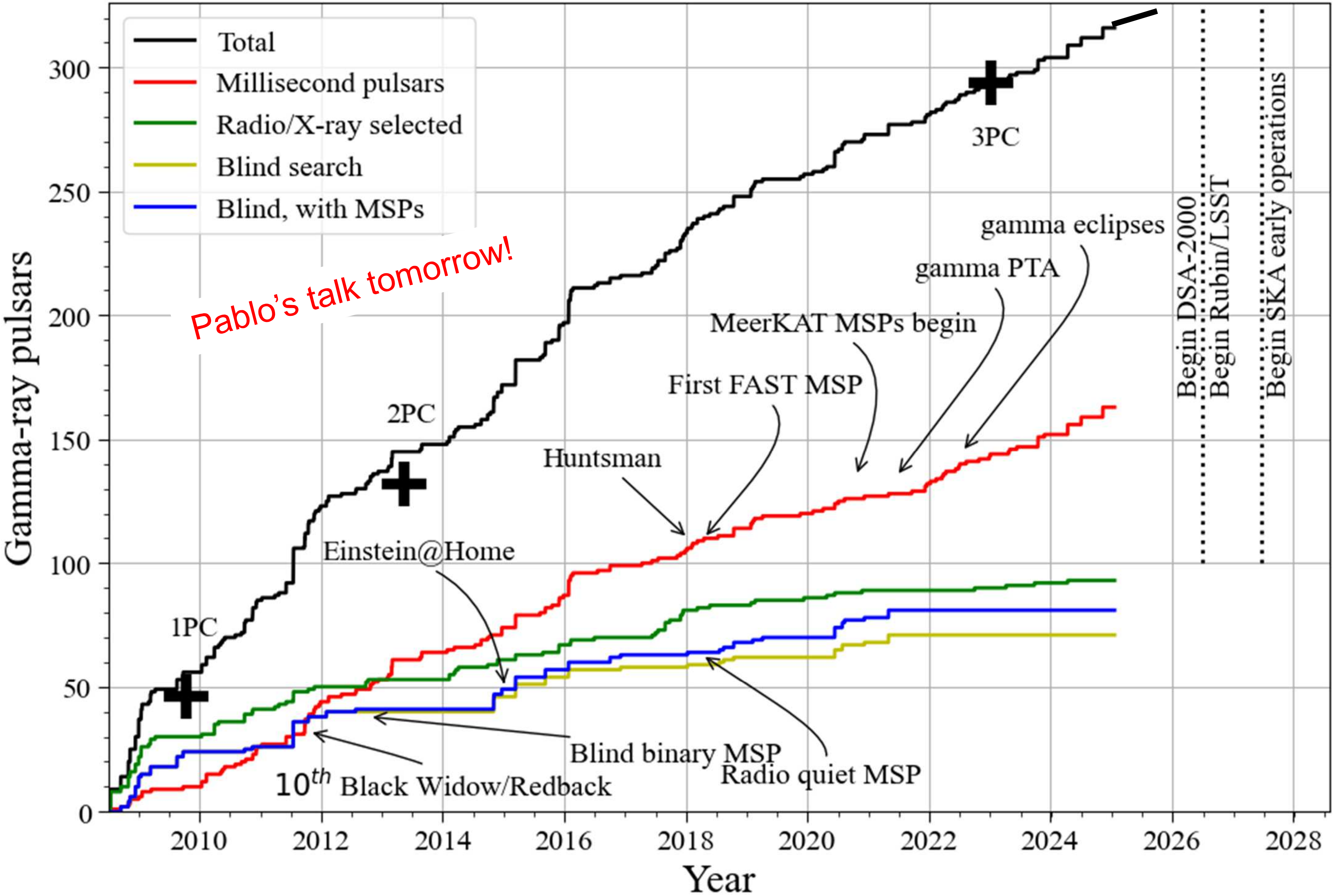
Corrélation mesurés ➔
pour toutes les combinaisons de 30 paires de pulsars.



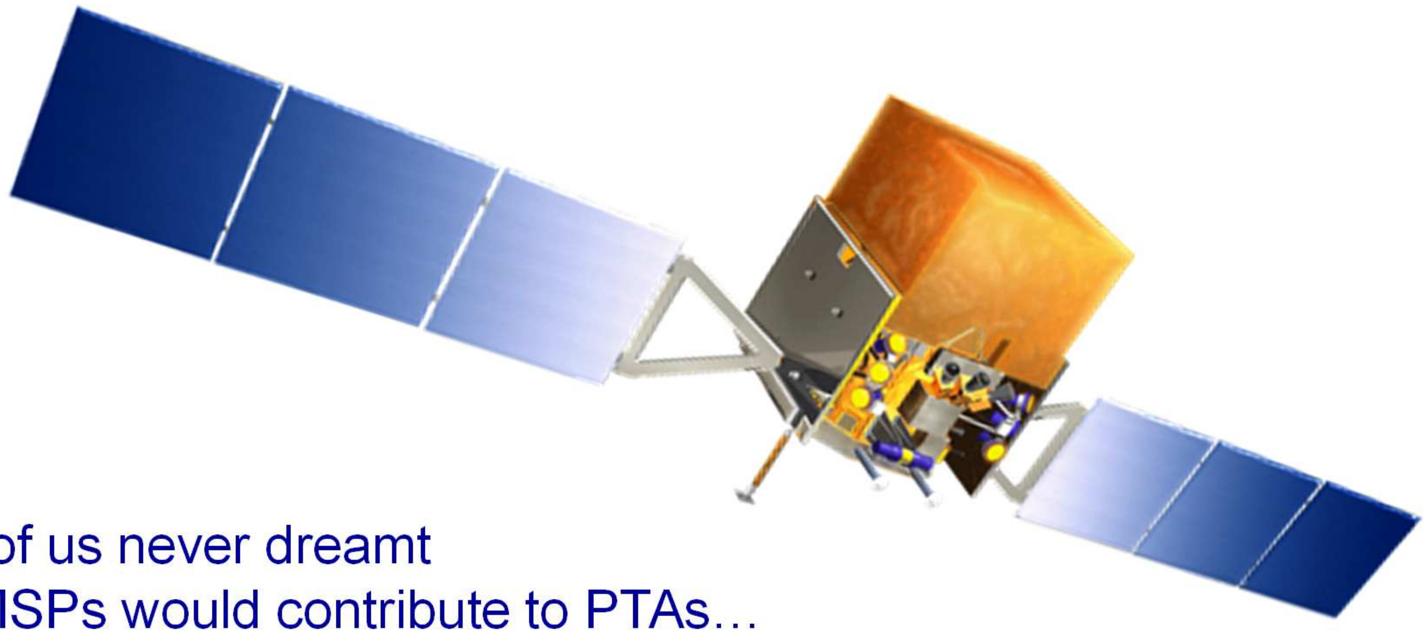
Et les rayons gamma dans tout ça?



Now >328 with γ pulses.



Surprise: ½ of γ pulsars are MSPs.
~30 are good timers.



Most of us never dreamt
LAT MSPs would contribute to PTAs…

Matthew Kerr, Aditya Parthasarathy et al did it!!

LAT time stamp accuracy:

- Ground tests showed ~300 ns accuracy for the GPS-based dates.

First ever space mission with accurate timing.

Here: 10 years on-orbit, MSPs and Crab confirm the ground tests.

THE ASTROPHYSICAL JOURNAL SUPPLEMENT SERIES, 256:12 (25pp), 2021 September Ajello et al.

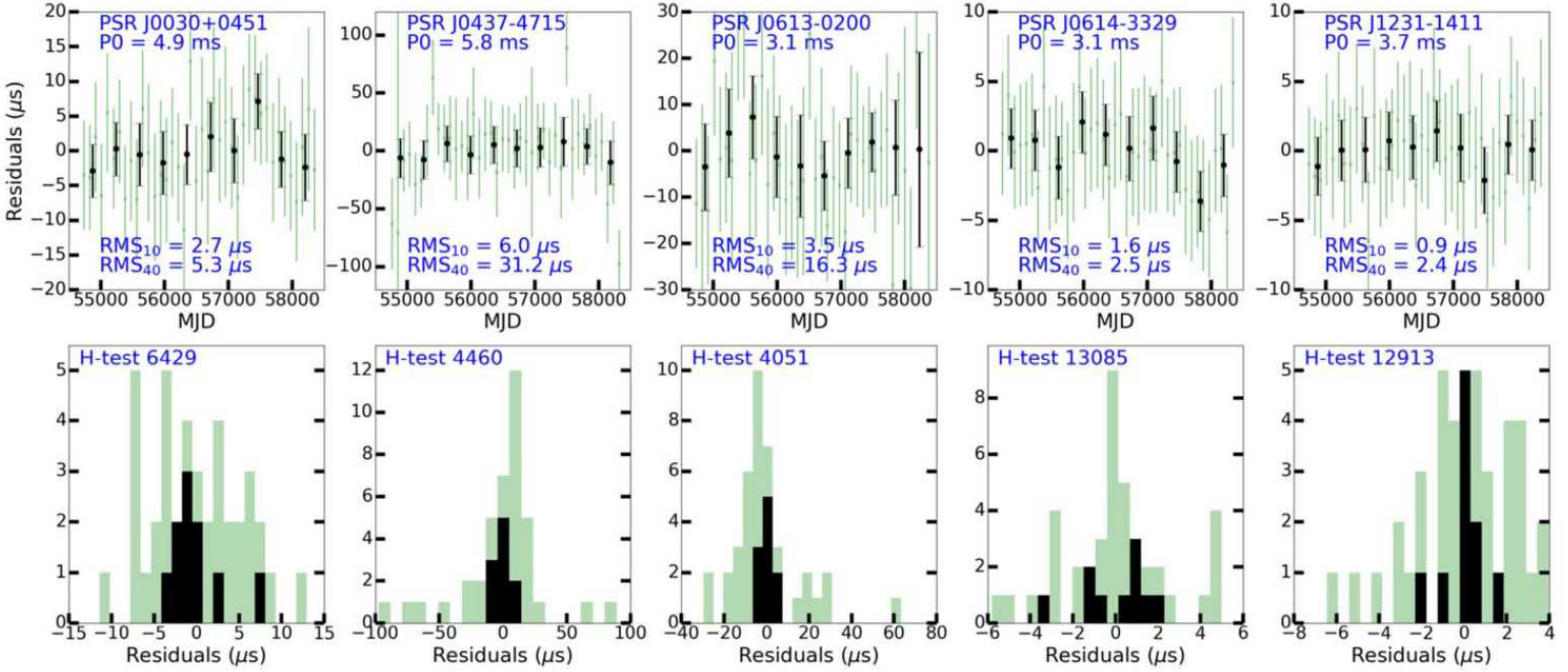


Figure 25. Stability of gamma-ray millisecond pulse arrival times compared to radio references. Top: black dots show ten one-year average values, and green crosses show 40 three-month average values. The rms variations are dominated by the statistical uncertainties in both the gamma-ray and radio data. Bottom: the same points, displayed as histograms. The H-test indicates the statistical significance of the pulsed signal (see text).

Science

A gamma-ray pulsar timing array constrains the nanohertz gravitational wave background

Contrainte du fond d'ondes gravitationnelles nHz
par la chronométrie en rayons gamma
d'un réseau de pulsars.

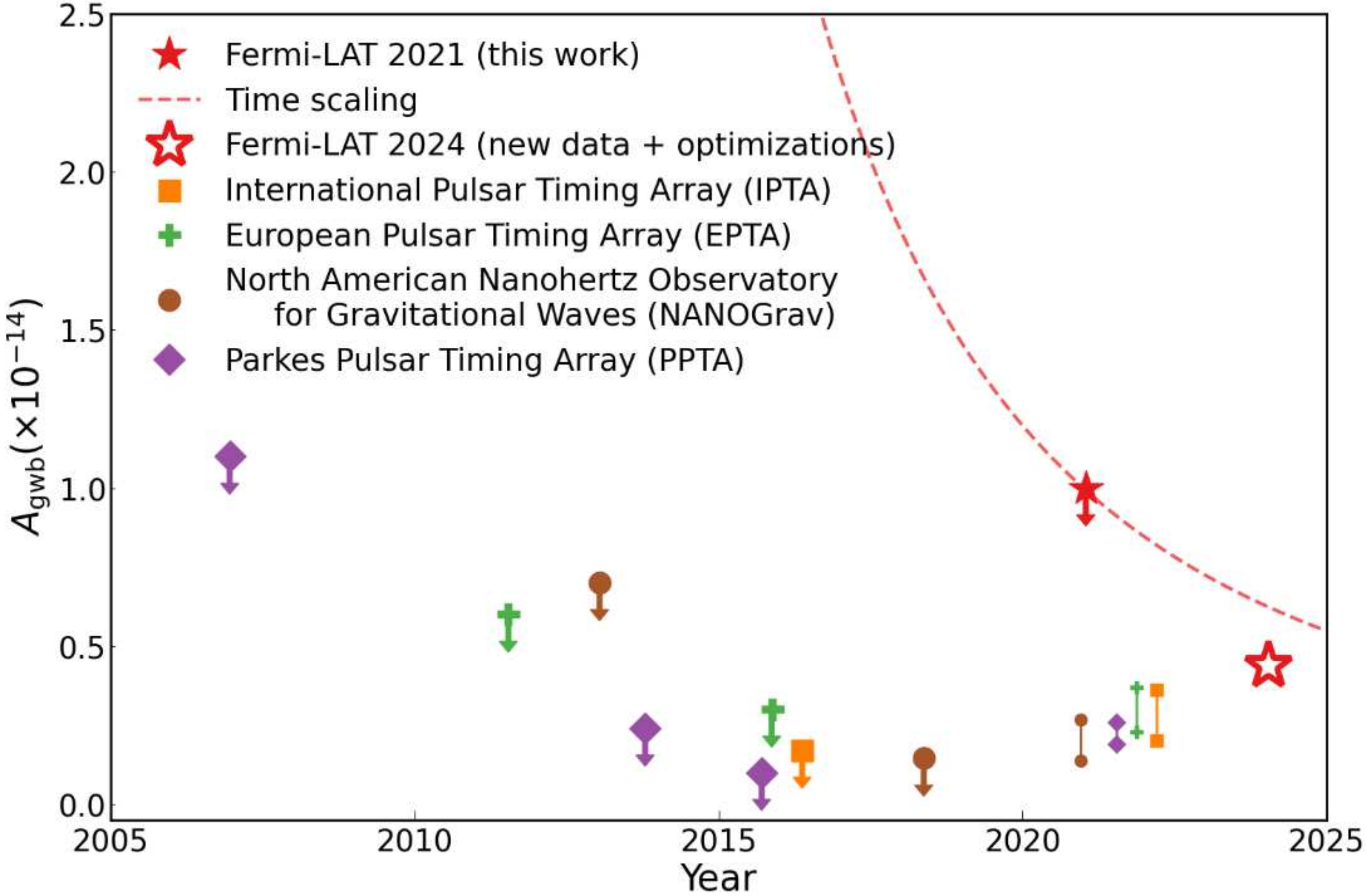
Ajello, *Science* **376**, 521–523 (2022)   29 April 2022

*Fermi-LAT Collaboration authors and affiliations are listed in the supplementary materials.
†Corresponding authors: Matthew Kerr (matthew.kerr@gmail.com); Aditya Parthasarathy (adityapartha3112@gmail.com)

nHz = nano Hertz = un milliardième de cycle par second = un cycle en 32 ans.
« plusieurs années »

Gamma Result: $A_{gwb} < 1.0 \times 10^{-14}$

Gamma-ray measurement (not just UL) should come in
5 (10) years to reach the top (bottom) of the radio uncertainty range.



Why the gamma result is important

- PTAs are probes of galaxies in the early Universe.

- Radio PTAs suffer from difficult systematic effects:
  - DM variations (space weather)
  - Human-generated radio background noise (RFI)
  - Frequent radio-telescope improvements
  - Data sets from diverse instruments, with varying sampling

Long author lists because there is a lot of hard work.

- *Fermi* LAT data:
  - No propagation effects.
  - Wonderfully stable instrument & sky-survey strategy.
  - Mostly different set of MSPs.
  - Discover a new γ MSP today? 17 years of timing tomorrow!

Gammas to provide useful confirmation of difficult, important measurements.

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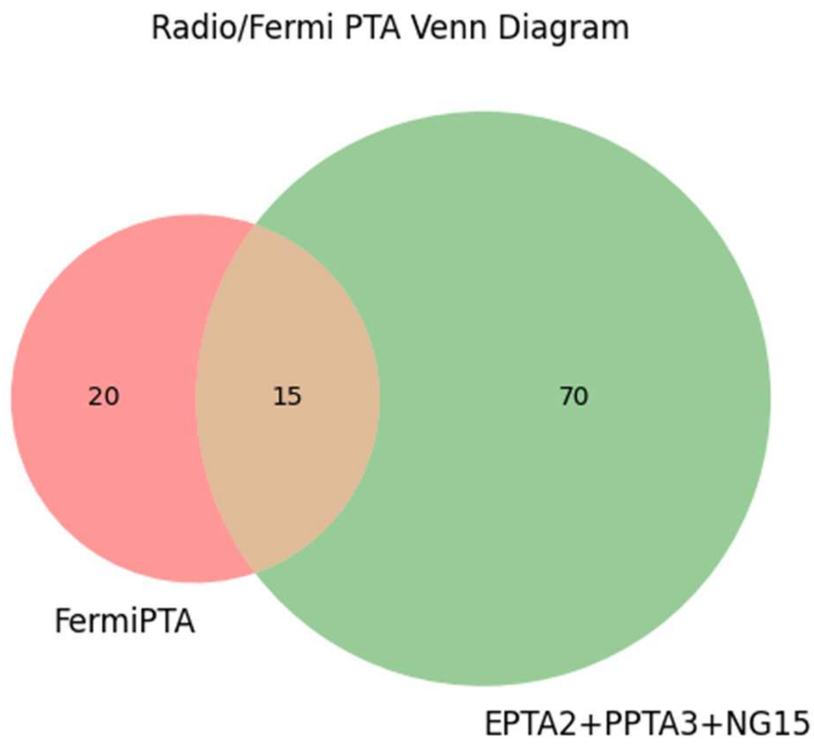

Gammas to provide useful confirmation of difficult, important measurements.

Basic Idea

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Often, *two* SMBH's orbiting each other (due to galaxy mergers).
Massive binaries emit gravitational waves (GWs).

Individual binaries are too faint to "hear" – signal is the *sum* over the *population*.
Characterizing the signal informs about masses, mergers, history…

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The timing "noise" induced by the GW "noise" is the PTA *signal*.
PTA = Pulsar Timing Array.

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GW measurements (upper limits) since 2021 (2006).

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Less sensitive but i) useful upper limit (2023), a measurement in a few years, and
smaller and different systematic uncertainties.

5) **New** – improved gamma-ray analysis ; "Data Set II" in preparation.

MAX-PLANCK-INSTITUT FÜR GRAVITATIONSPHYSIK (Albert-Einstein-Institut)
Leibniz Universität Hannover

An updated constraint for the Gravitational Wave background from the Gamma-ray Pulsar Timing Array

Serena Valtolina
Colin Clark
Rutger van Haasteren
Aurélien Chalumeau
Thankful Cromartie
Matthew Kerr
Lars Nieder
Aditya Parthasarathy

27th October 2025

Pulsar 2025 conference in Sardinia

ApJ (submitted)

They re-thought & re-coded the analysis scheme.

Likelihood in Fourier domain
(methods paper)

Published

**Regularizing the pulsar timing array likelihood:
A path toward Fourier space**

Serena Valtolina* and Rutger van Haasteren†

In preparation

Timing Gamma-ray Pulsars with Gibbs Sampling

C. J. Clark, S. Valtolina, L. Nieder, and R. van Haasteren,

New sampling scheme for photon
timing (shoogle)

A new PTA likelihood in Fourier domain

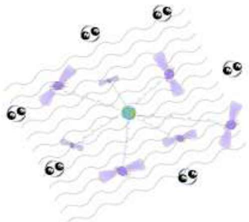
Regularizing the pulsar timing array likelihood: A path toward Fourier space

Serena Valtolina* and Rutger van Haasteren†

- It allows us to divide the PTA analysis in **2 steps**:

1. analyze each pulsar *individually*
   focus on signals NOT covariant with the GWB (WN, det. signals…)

Posterior of the Fourier coefficients describing the signals covariant with GWB



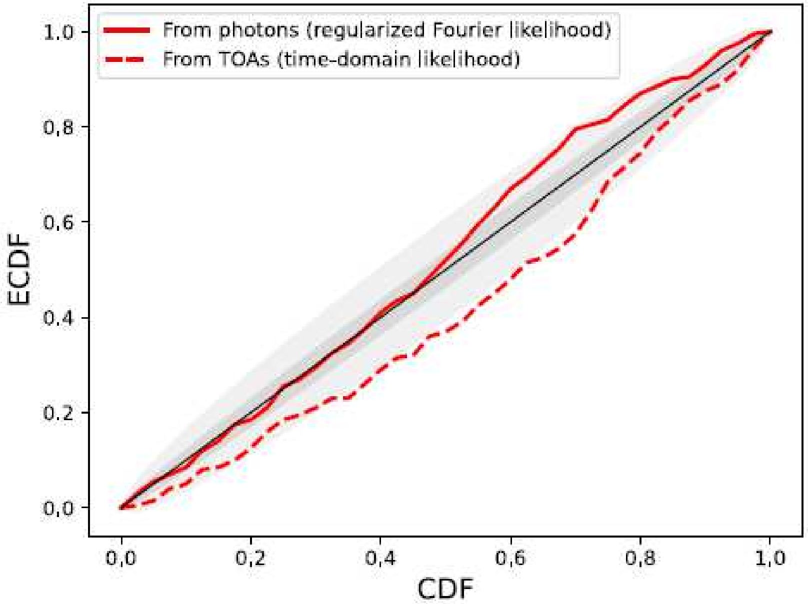
2. analyze the *full array* of pulsars
   focus on the GWB and the signals covariant with it (RN, DMvar …)

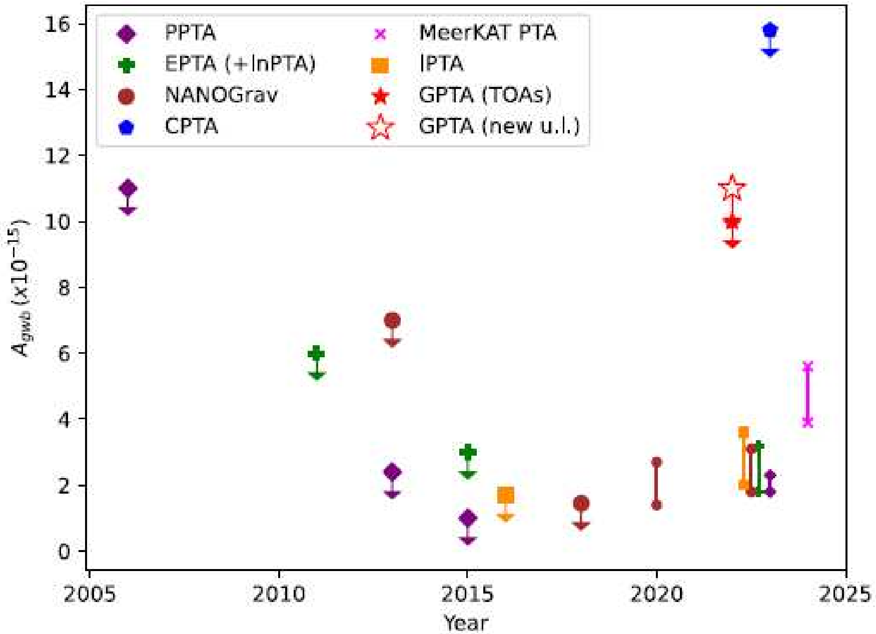
- It can be applied to both **Gamma-ray PTA** and radio PTA data!

Same (epsilon higher) UL, using same data set.

New method based on **single photons + regularized likelihood** proved to be *statistically more robust* than the TOAs method for gamma-ray PTA.

Updated upper limit: $A_{gwb} < 1.2 \times 10^{-14}$





Robustness of the analysis, reliability of the uncertainties are clearly demonstrated.

Work-in-progress

Apply method to « Data Set II » (more gamma MSPs).

I'm quite curious to see what will come out of it…

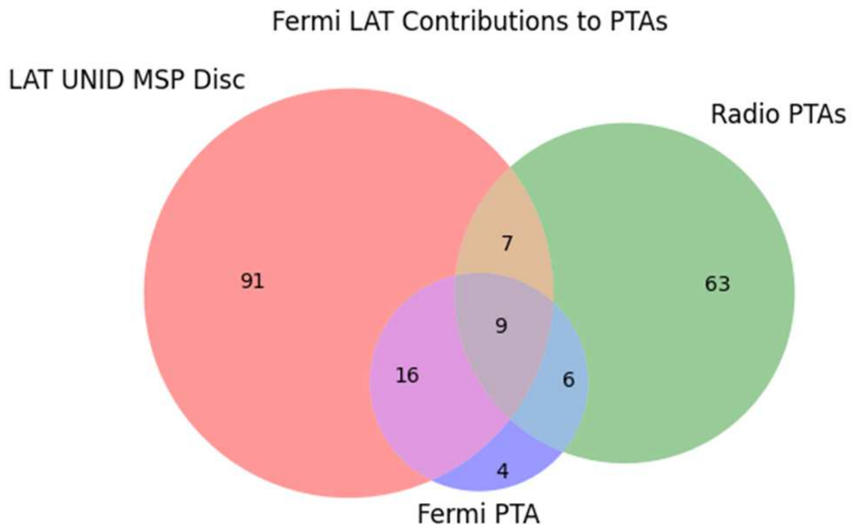
Conclusions

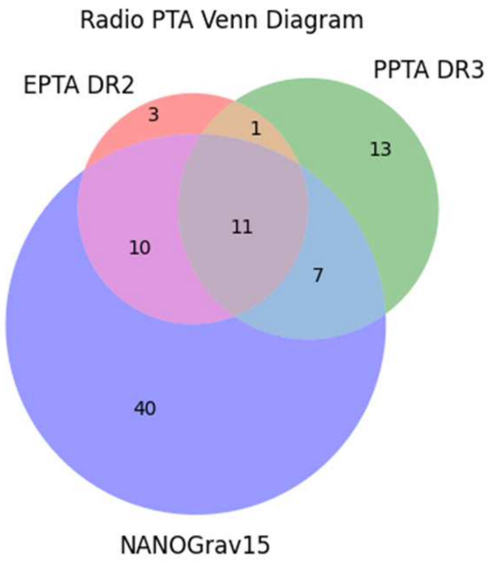
- Decades-long radio timing of stable MSPs opening a new window for gravitational wave astronomy.
- Characterize galaxies in the early Universe
- Search for exotic signals (not mentioned today)

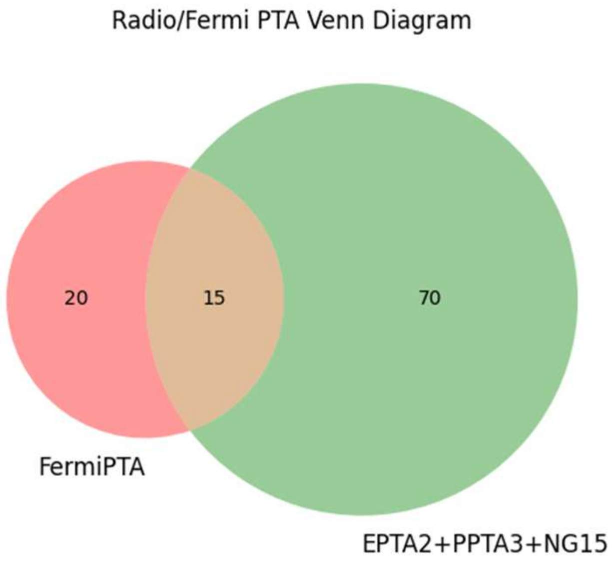
- Fermi LAT gamma-ray MSP timing provides complementary measurements.

- Methodology improving,
- gamma-ray MSP sample growing,
- glorious upcoming results hoped for.

Venn diagrams



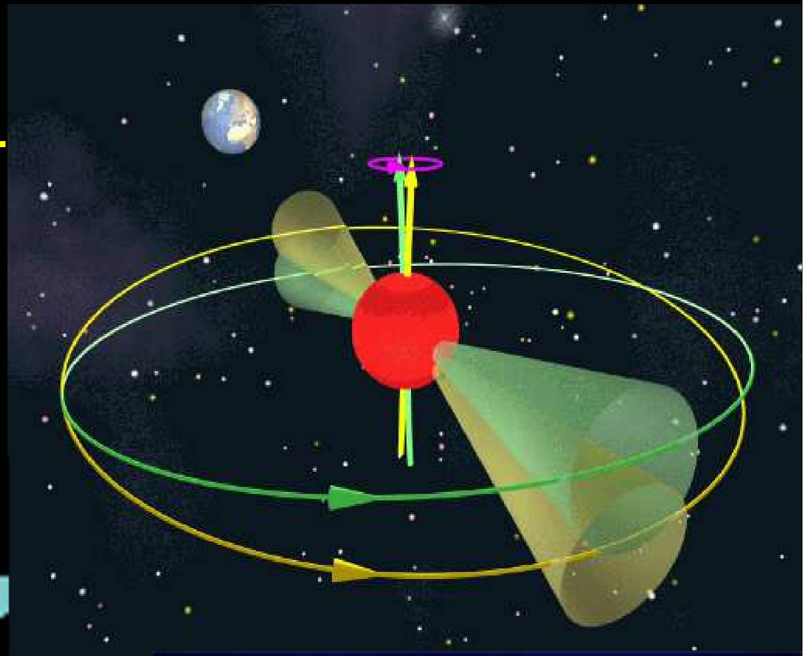




Pulsar: une étoile à neutrons en rotation

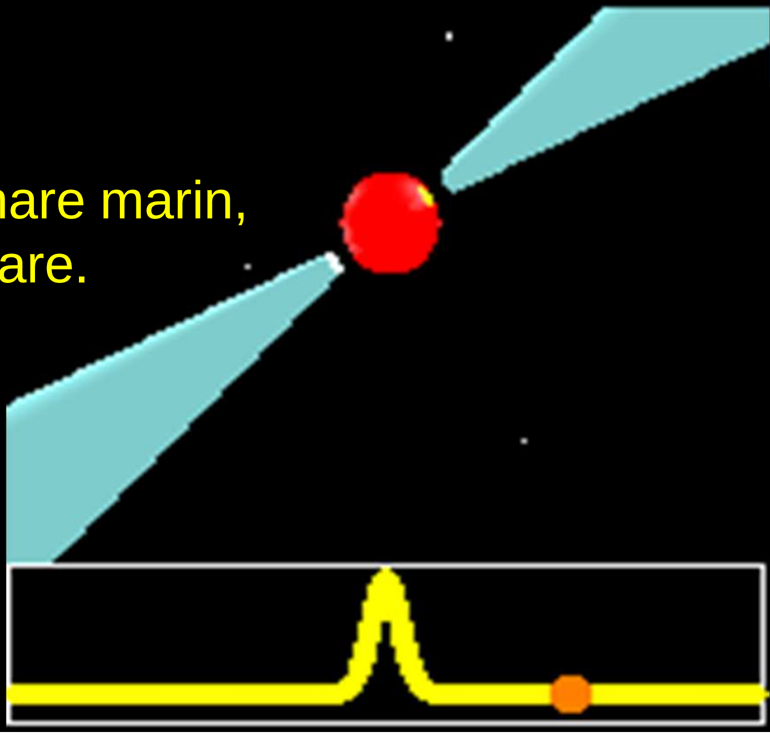
Une supernova démarre quand le cœur d'une étoile massive se transforme en neutrons.

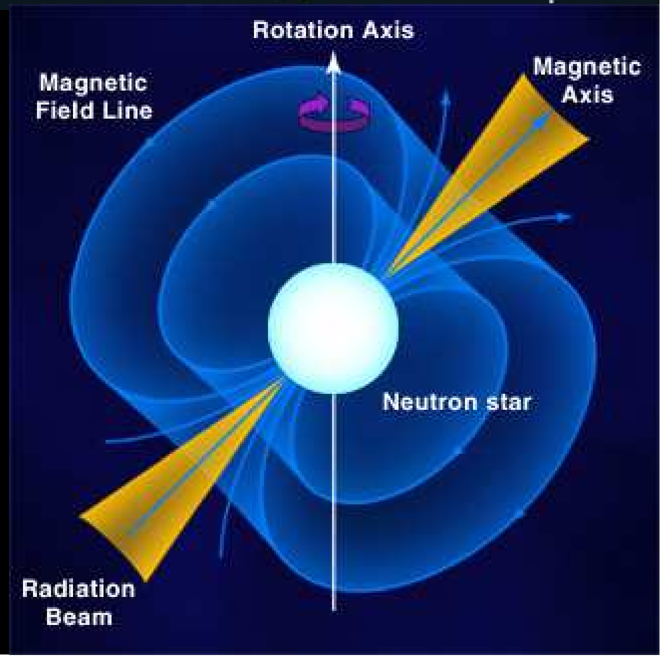
- ~ 26 km de diamètre
- ~1.4 masses solaires
- 1.6 ms à 10 s par rotation
- Champ magnétique intense

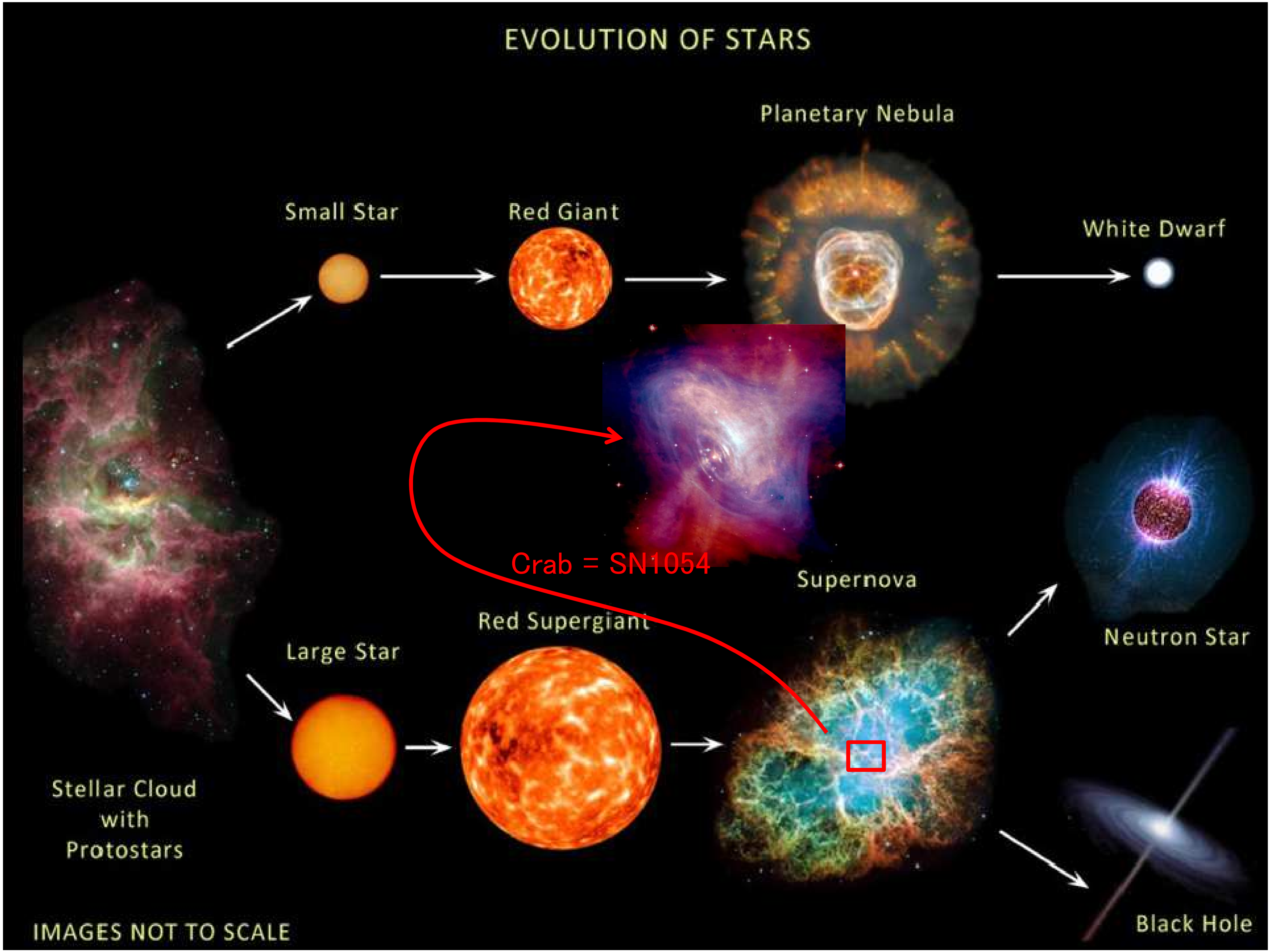


Ces '*dynamos cosmiques*' rayonnent des faisceaux.

Pulsations:
comme un phare marin,
ou un gyrophare.






EVOLUTION OF STARS
Planetary Nebula
Small Star
Red Giant
White Dwarf
Crab = SN1054
Supernova
Red Supergiant
Large Star
Neutron Star
Stellar Cloud
with
Protostars
Black Hole
IMAGES NOT TO SCALE

Equation d'Einstein (1915)

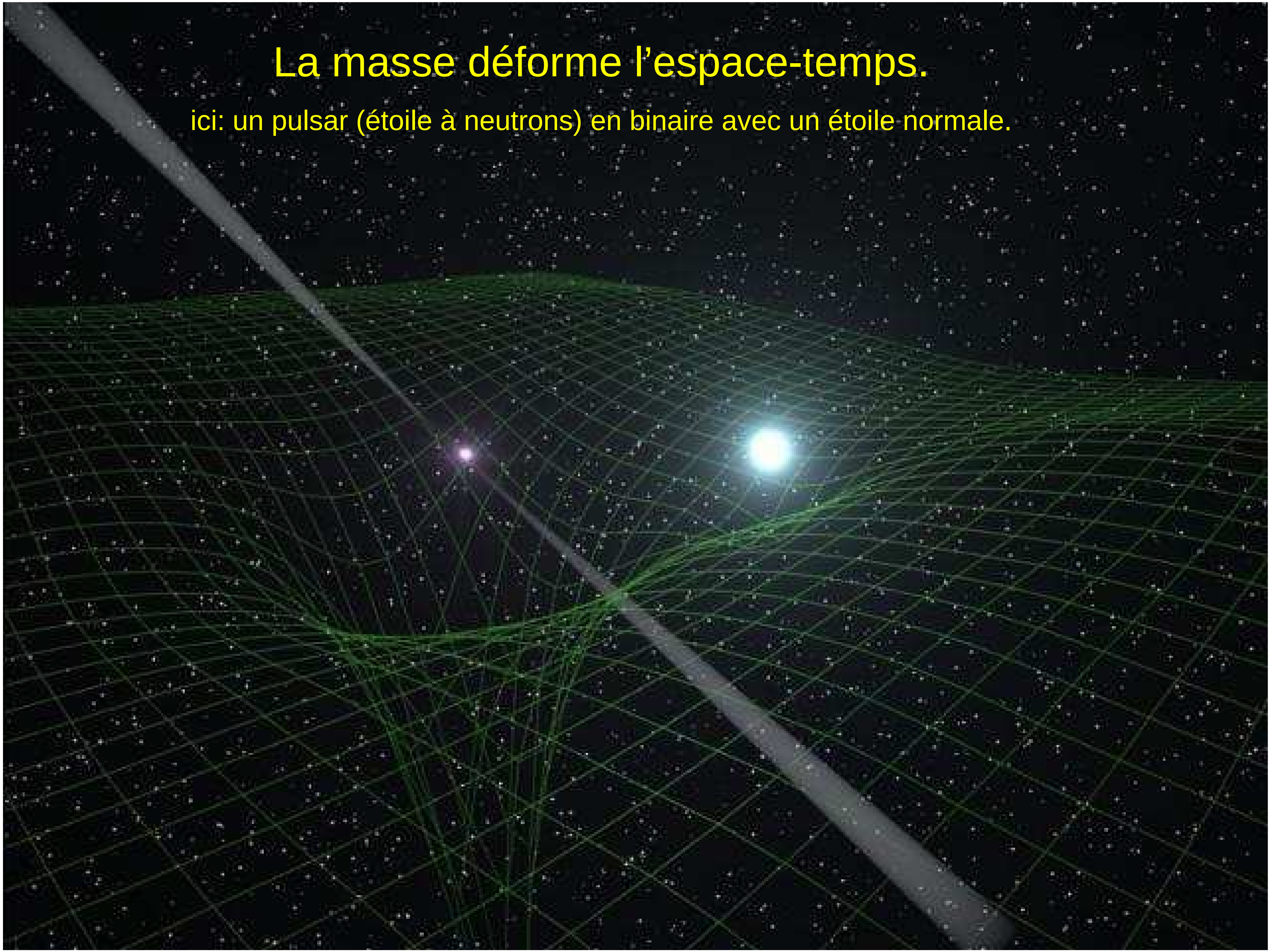
Se simplifie à la gravitation de Newton pour les cas « *normaux* » …

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} + \Lambda \, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

où :

- $\boldsymbol{R_{\mu\nu}}$ est le tenseur de Ricci ; (R : la courbure d'espace-temps)
- $\boldsymbol{R}$ est la courbure scalaire ;
- $\boldsymbol{g_{\mu\nu}}$ est le tenseur métrique de signature (+,-,-,-) ;
- $\boldsymbol{\Lambda}$ est la constante cosmologique ;
- $\boldsymbol{T_{\mu\nu}}$ est le tenseur énergie-impulsion ;

https://fr.wikipedia.org/wiki/équation_d'Einstein


La masse déforme l’espace-temps.
ici: un pulsar (étoile à neutrons) en binaire avec un étoile normale.

Approximative Champ

Näherungsweise Integration der Feldgleichungen der Gravitation.

Equations

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformatio-
gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen
handelt werden können, deren Quadrate und Produkte gegen die ers
Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} =$
je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

planes ondes

§ 2. Ebene Gravitationswellen.

Aus den Gleichungen (6) und (9) folgt, daß sich Gravitationsfelder stets mit der Geschwindigkeit 1, d. h. mit Lichtgeschwindigkeit, fortpflanzen. Ebene, nach der positiven x-Achse fortschreitende Gravitationswellen sind daher durch den Ansatz zu finden

$$\gamma'_{\mu\nu} = \alpha_{\mu\nu} f(x_1 + i x_4) = \alpha_{\mu\nu} f(x - t) . \qquad (15)$$

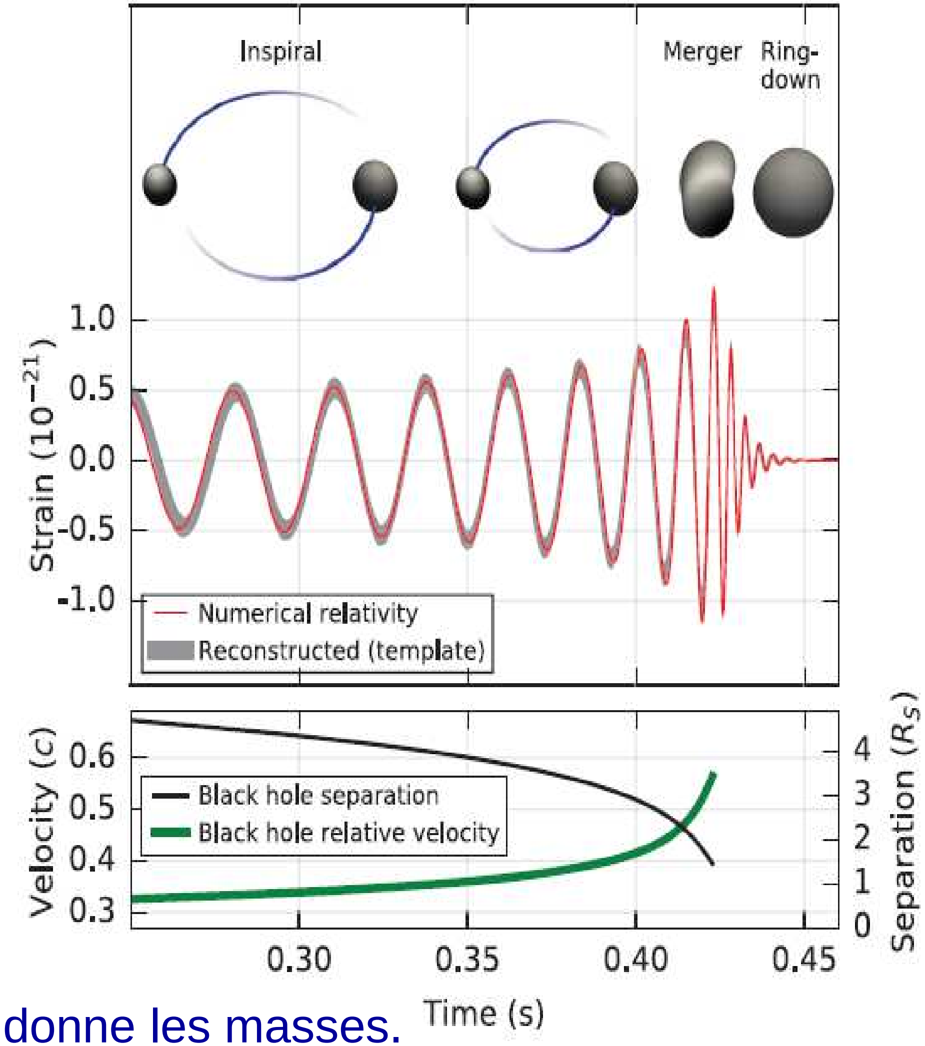
Dabei sind die $\alpha_{\mu\nu}$ Konstante: f ist eine Funktion des Arguments $x - t$. Ist der betrachtete Raum frei von Materie, d. h. verschwinden die $T_{\mu\nu}$, so sind die Gleichungen (6) durch diesen Ansatz erfüllt. Die

$E = 3M_{\odot}c^2$ en ondes gravs!

Plus puissant que le reste de l'Univers pendant un dixième de seconde.

TABLE I. Source parameters for GW150914. We report median values with 90% credible intervals that include statistical errors, and systematic errors from averaging the results of different waveform models. Masses are given in the source frame; to convert to the detector frame multiply by $(1+z)$ [90]. The source redshift assumes standard cosmology [91].

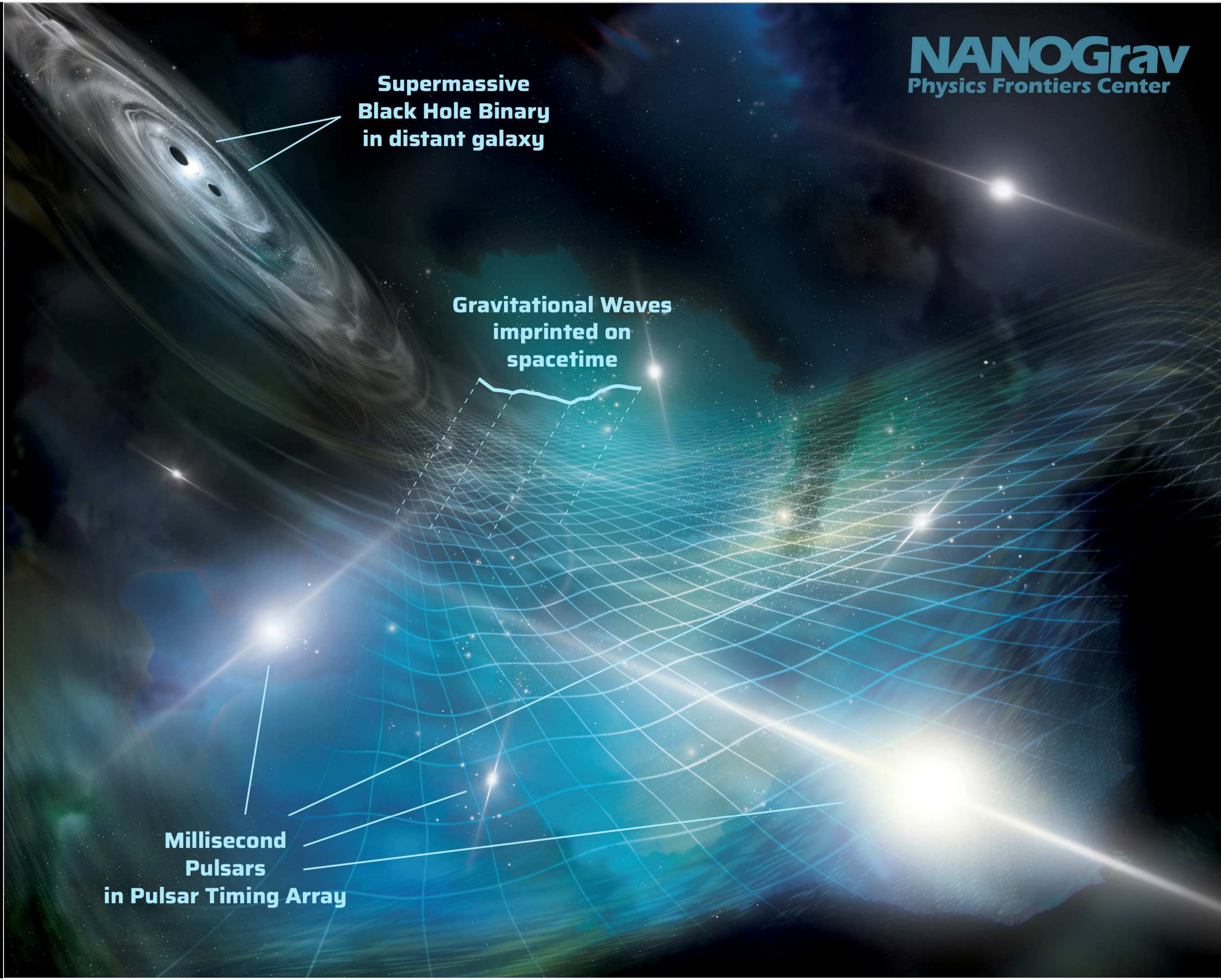
Primary black hole mass	$36^{+5}_{-4} M_{\odot}$
Secondary black hole mass	$29^{+4}_{-4} M_{\odot}$
Final black hole mass	$62^{+4}_{-4} M_{\odot}$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$



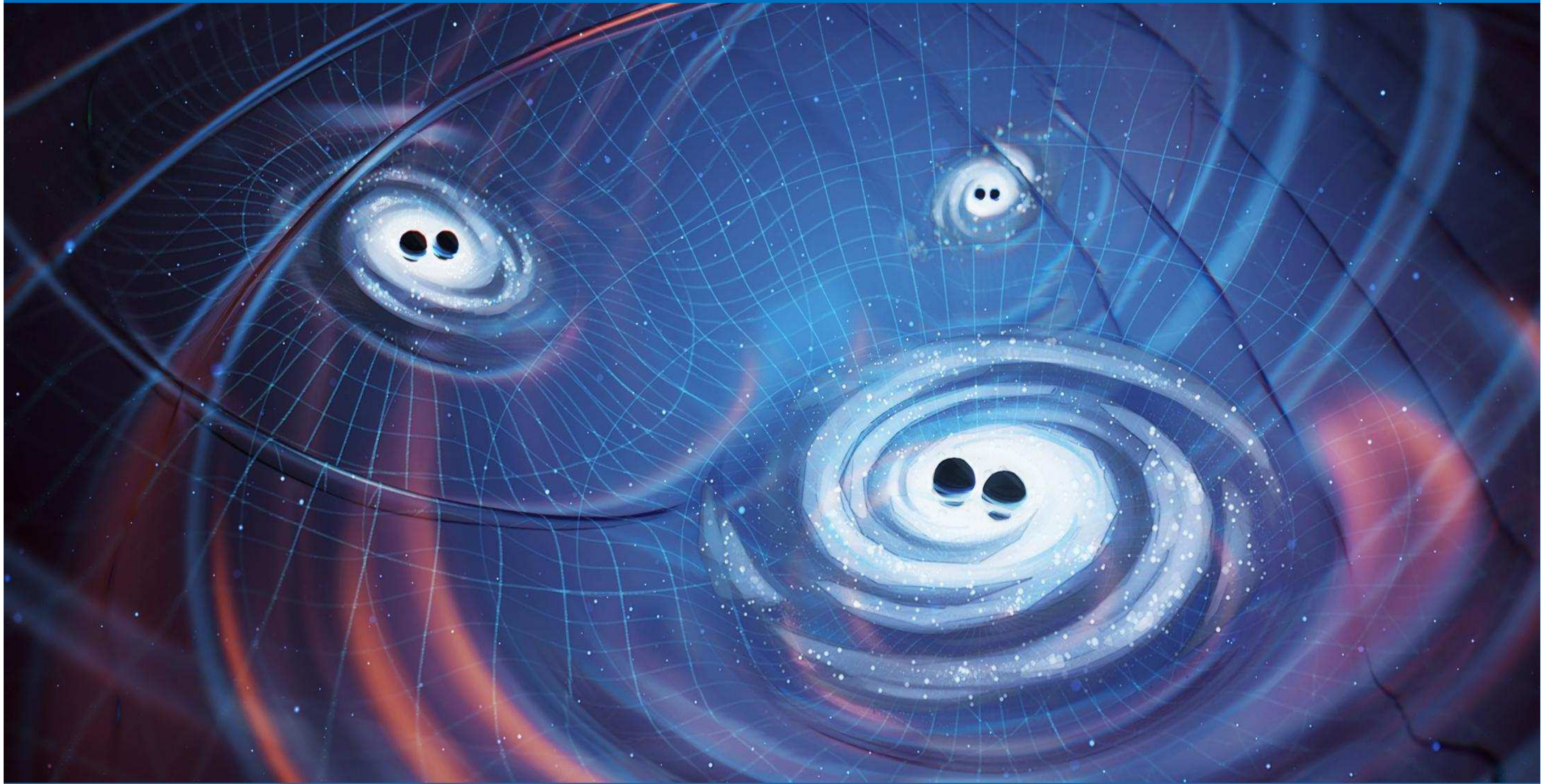
"Luminosity distance": la forme du signal donne les masses.

La relativé générale d'Einstein donne l'intensité à la source.

Intensité observée ➔ distance.


NANOGrav
Physics Frontiers Center
Supermassive
Black Hole Binary
in distant galaxy
Gravitational Waves
imprinted on
spacetime
Millisecond
Pulsars
in Pulsar Timing Array

On capte la *somme* des ondes de *beaucoup* de paires de trous noirs.



Une étoile à neutron est un volant d'inertie terrible!

Toutes tournent très régulièrement.

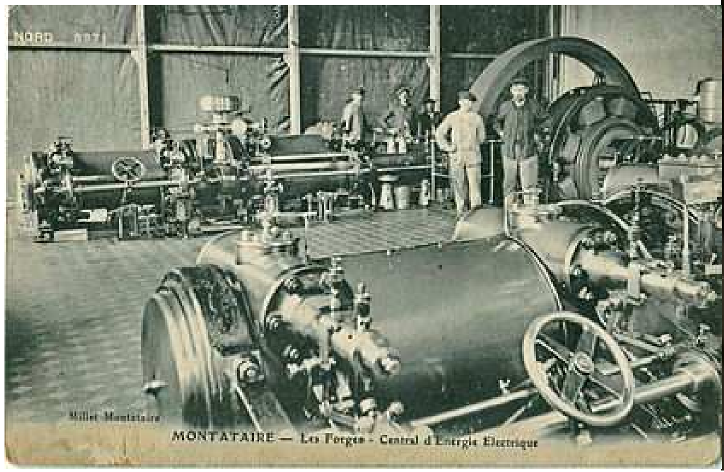
Mais… les pulsars jeunes ont un ralentissement saccadé:

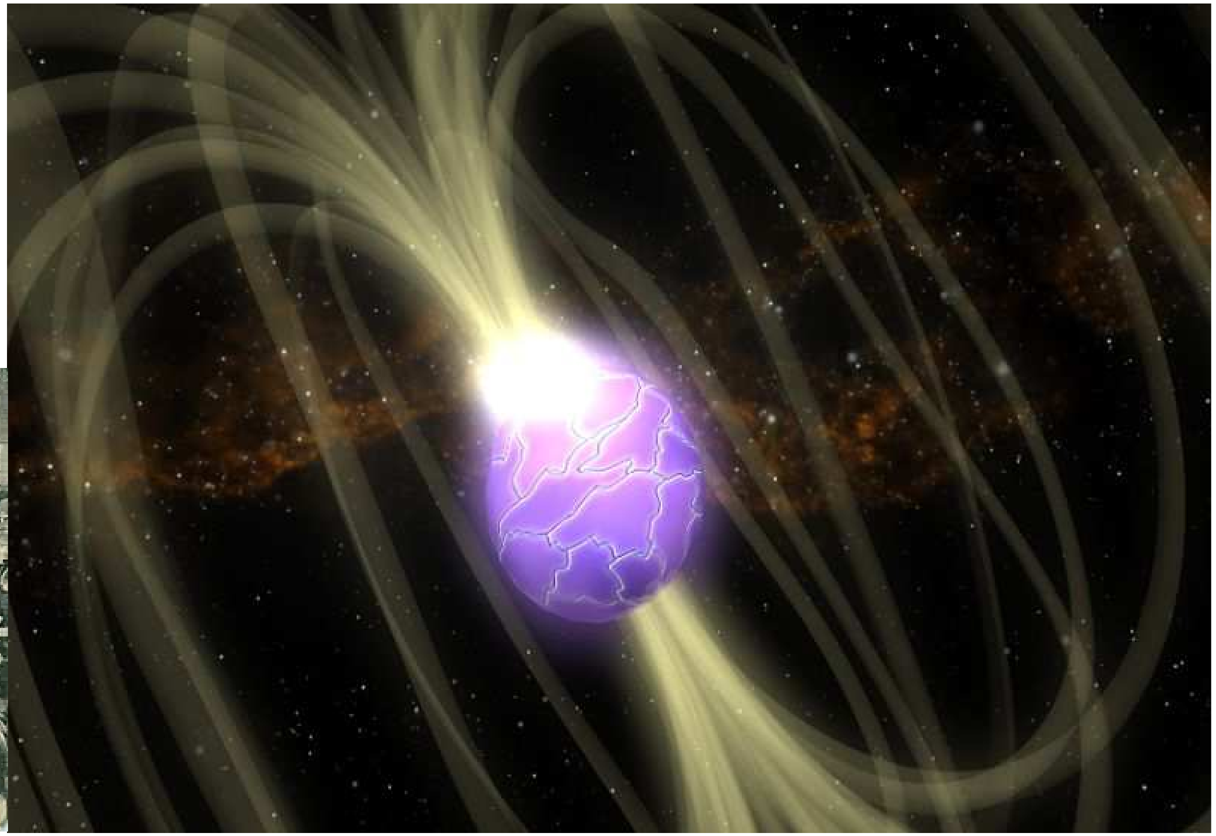
Tremblements d'étoile? La croûte en fer se fracture? Turbulences dans le vent du pulsar?

Fermi LAT a découvert des dizaines de '*veuves noires'* : le vent du pulsar « mange » l'étoile compagne ➔ orbite perturbé.

Pour une trentaine de MSPs, rien ne « frotte » leur rotation

➔ merveilleusement stables!





Toutes

Mais…

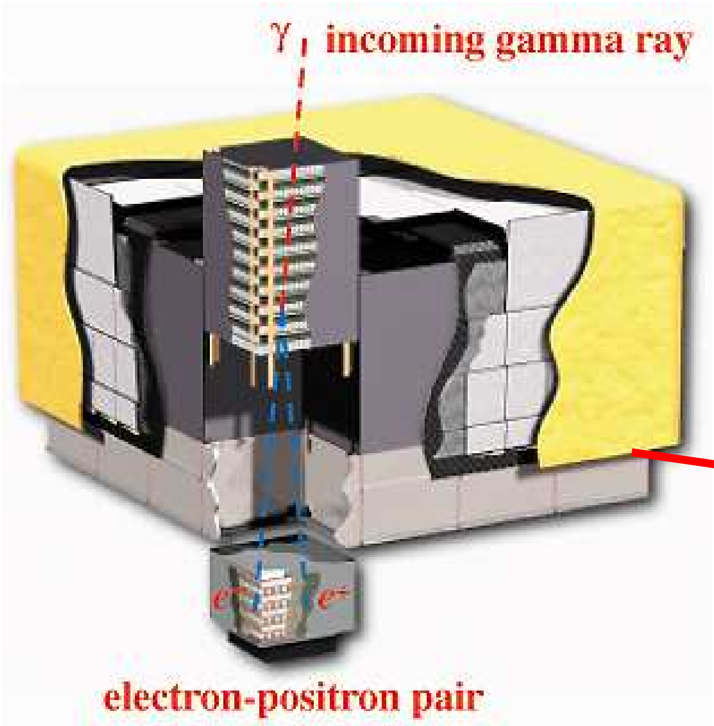
Tre ar?

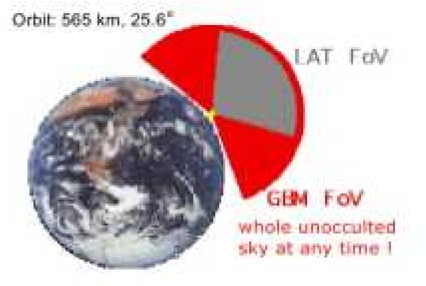
Fermi I
« mang

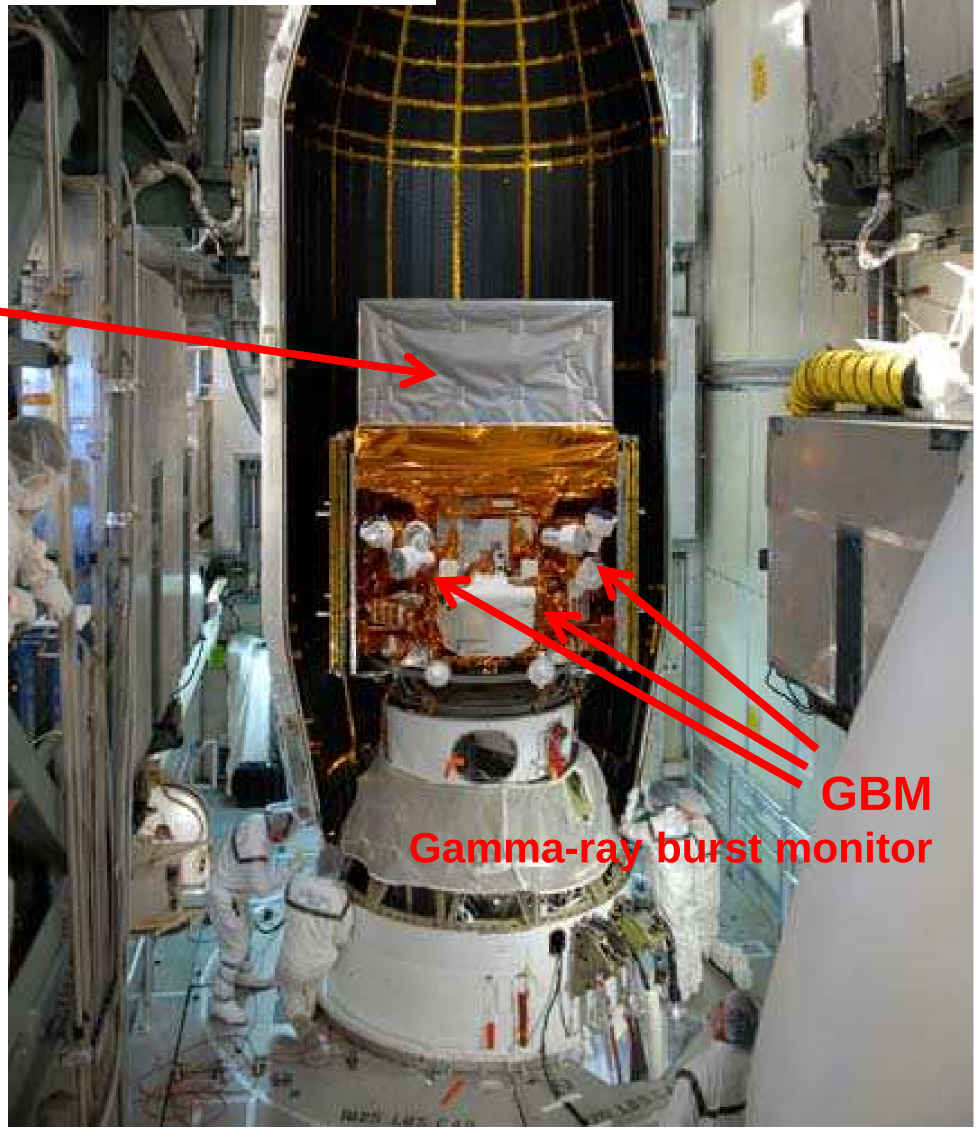
Pour u
rien ne

➔ men

Fermi LAT et GBM



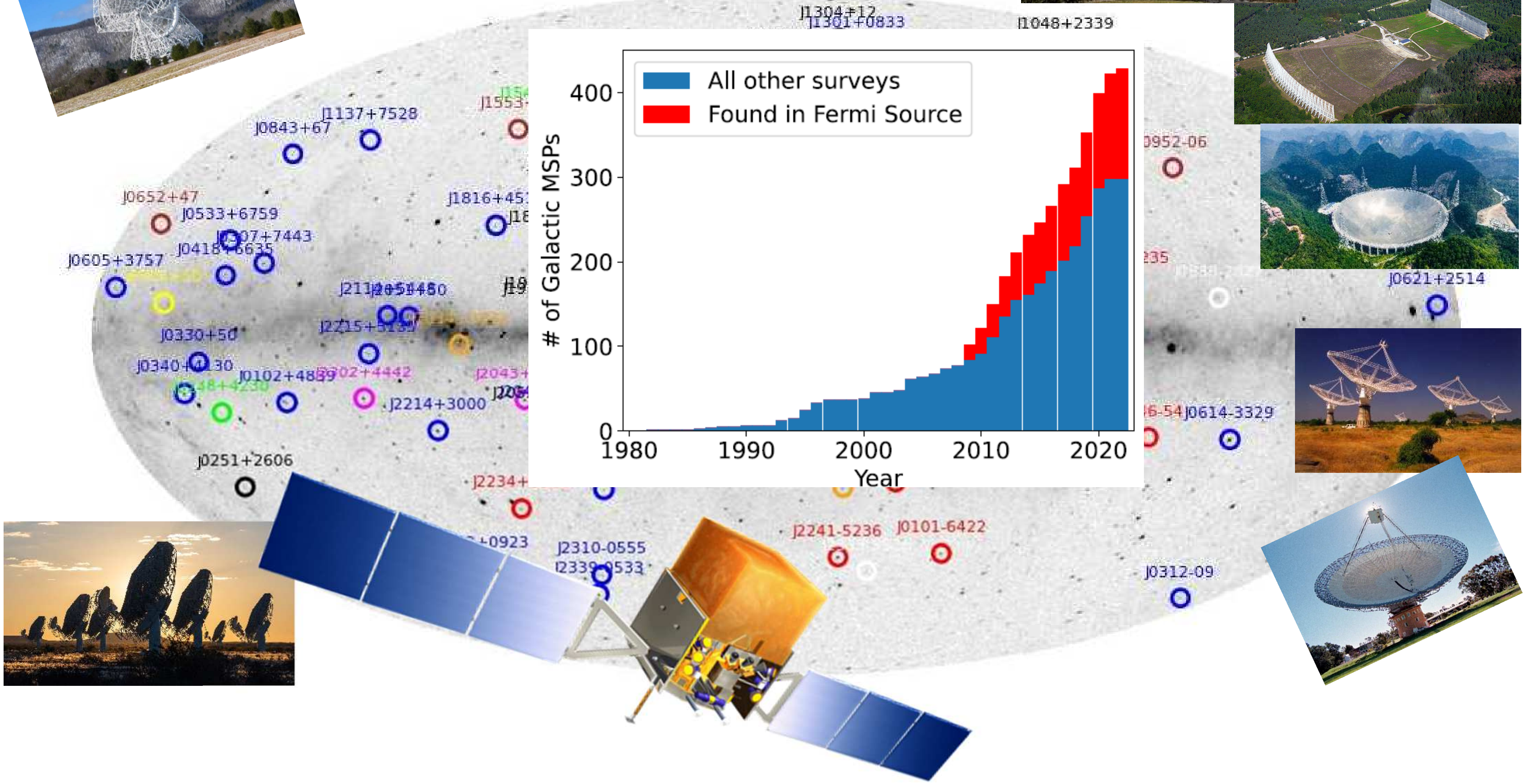




Nous observons *tous* les pulsars du ciel,
connus ou pas,
détectables ou pas,
en rayons gamma du GeV, depuis 2008.

Depuis 20 ans, gros effort des radioastronomes à découvrir des MSPs,
et à bien suivre les plus stables, dans le but des ondes gravitationnelles.

***Fermi* LAT fournit une carte à trésors.**



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https://doi.org/10.3847/2041-8213/acdac6

CrossMark

The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background

Gabriella Agazie[1], Akash Anumarlapudi[1], Anne M. Archibald[2], Zaven Arzoumanian[3], Paul T. Baker[4], Bence Bécsy[5], Laura Blecha[6], Adam Brazier[7,8], Paul R. Brook[9], Sarah Burke-Spolaor[10,11], Rand Burnette[5], Robin Case[5],

Summary of the main Bayesian and optimal-statistic analyses presented in this paper, which establish multiple lines of evidence for the presence of owns correlations in the 15 yr NANOGrav data set. Throughout we refer to the 68.3%, 95.4%, and 99.7% regions of distributions as $1\sigma/2\sigma/3\sigma$ regions, dimensions. (a) Bayesian "free-spectrum" analysis, showing posteriors (gray violins) of independent variance parameters for a Hellings–Downs-correlated rocess at frequencies i/T, with T the total data set time span. The blue represents the posterior median and $1\sigma/2\sigma$ posterior bands for a power-law model; black line corresponds to a $\gamma = 13/3$ (SMBHB-like) power law, plotted with the median posterior amplitude. See Section 3 for more details. (b) Posterior distribution of GWB amplitude and spectral exponent in an HD power-law model, showing $1\sigma/2\sigma/3\sigma$ credible regions. The value $\gamma_{\rm GWB} = 13/3$ (dashed is included in the 99% credible region. The amplitude is referenced to $f_{\rm ref} = 1$ yr^{-1} (blue) and 0.1 yr^{-1} (orange). The dashed blue and orange curves in the subpanel show its marginal posterior density for a $\gamma = 13/3$ model, with $f_{\rm ref} = 1$ yr^{-1} and $f_{\rm ref} = 0.1$ yr^{-1}, respectively. See Section 3 for more details. (c) paration-binned interpulsar correlations, measured from 2211 distinct pairings in our 67-pulsar array using the frequentist optimal statistic, assuming a-posteriori pulsar noise parameters and $\gamma = 13/3$ common-process amplitude from a Bayesian inference analysis. The bin widths are chosen so that each proximately the same number of pulsar pairs, and central bin locations avoid zeros of the Hellings–Downs curve. This binned reconstruction accounts for between pulsar pairs (Romano et al. 2021; Allen & Romano 2022). The dashed black line shows the Hellings–Downs correlation pattern, and the binned ormalized by the amplitude of the $\gamma = 13/3$ common process to be on the same scale. Note that we do not employ binning of interpulsar correlations in our tatistics; this panel serves as a visual consistency check only. See Section 4 for more frequentist results. (d) Bayesian reconstruction of normalized correlations, modeled as a cubic spline within a variable-exponent power-law model. The violins plot the marginal posterior densities (plus median and le values) of the correlations at the knots. The knot positions are fixed and are chosen on the basis of features of the Hellings–Downs curve (also shown as a k line for reference): they include the maximum and minimum angular separations, the two zero-crossings of the Hellings–Downs curve, and the position n correlation. See Section 3 for more details.