Phenomenology of Inflaton Fragmentation

Marcos A. G. García

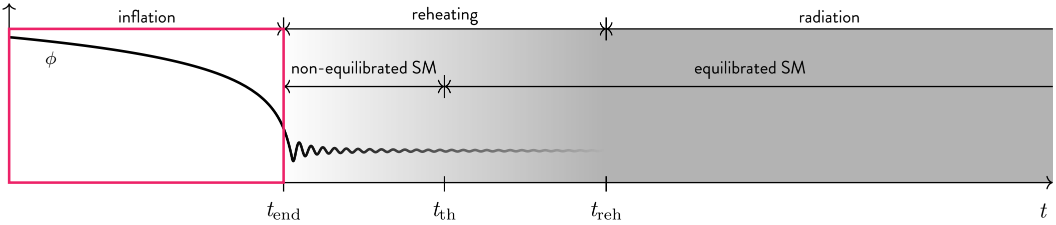
Universidad Nacional Autónoma de México

Ciencia y Tecnología
Secretaría de Ciencia, Humanidades, Tecnología e Innovación

INSTITUTO DE FÍSICA

física teórica IFUNAM

Inflation and reheating



Inflation: a slowly rolling scalar field

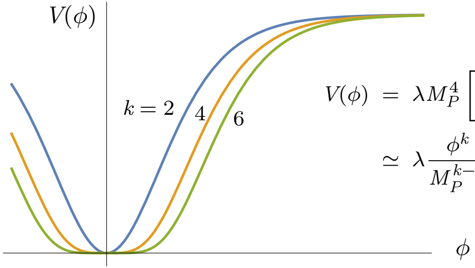
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H \equiv \frac{\dot{a}}{a} = \left(\frac{\rho_\phi}{3M_P^2}\right)^{1/2}$$

$$\frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho_\phi$$

$$\frac{1}{2}\dot{\phi}^2 - V(\phi) = p_\phi$$

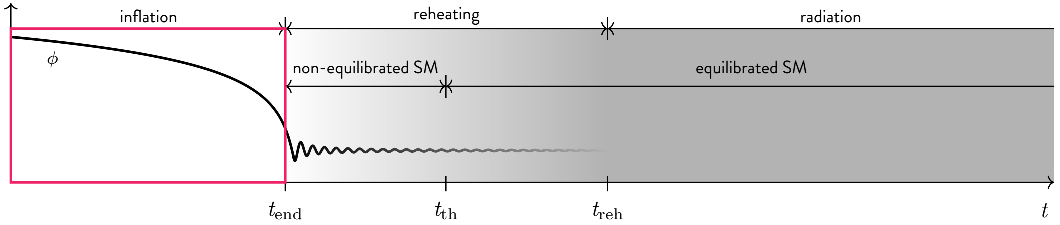
T-models (R. Kallosh and A. Linde, JCAP 10 (2013), 033)

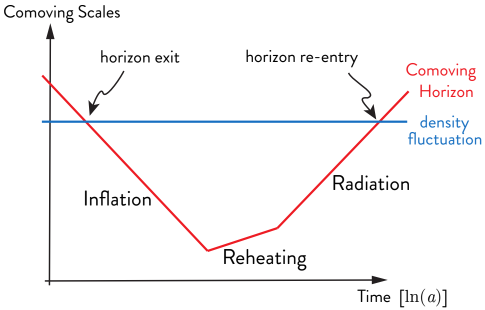


$$V(\phi) = \lambda M_P^4 \left[\sqrt{6\alpha}\tanh\left(\frac{\phi}{\sqrt{6\alpha}M_P}\right)\right]^k$$

$$\simeq \lambda \frac{\phi^k}{M_P^{k-4}} \quad (\phi \ll M_P)$$

Inflation and reheating





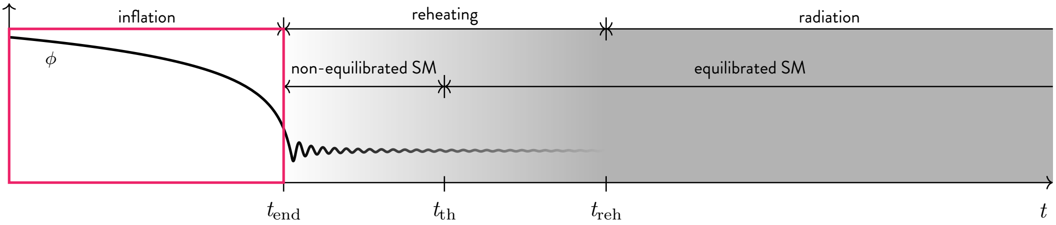
Curvature: $$\langle \mathcal{R}(k)\mathcal{R}^*(k')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_\mathcal{R}\delta(k-k')$$

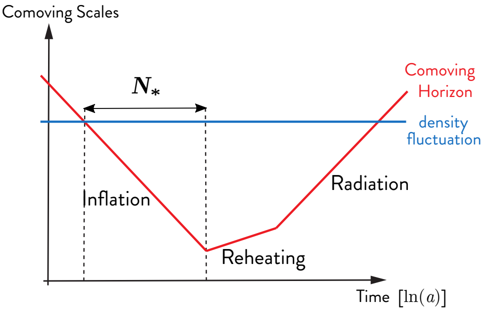
Tensors: $$\sum_{\gamma=+,\times}\langle h_{\mathbf{k},\gamma}h^*_{\mathbf{k}',\gamma}\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_\mathcal{T}\delta(k-k')$$

CMB observables

$$\mathcal{P}_\mathcal{R} \propto \left(\frac{k}{aH}\right)^{n_s-1}, \quad r \equiv \frac{\mathcal{P}_\mathcal{T}}{\mathcal{P}_\mathcal{R}}$$

Inflation and reheating

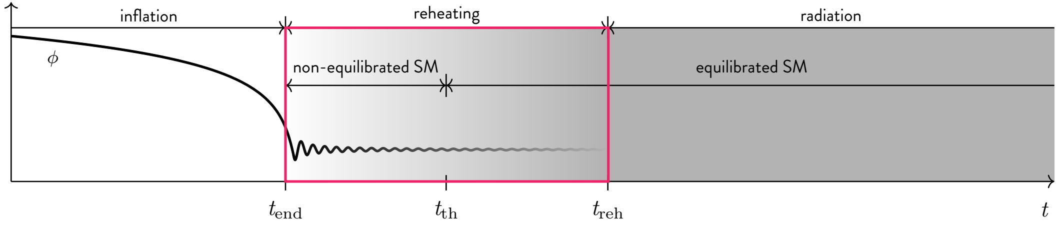




$$N_* = 66.89 - \ln\left(\frac{k_*}{a_0 H_0}\right) - \frac{1}{12}\ln g_{\rm reh} + \frac{1}{4}\ln\left(\frac{V_*^2}{\rho_{\rm end} M_P^4}\right) + \ln R_{\rm rad}$$

$$\ln R_{\rm rad} = \ln\left[\frac{a_{\rm end}}{a_{\rm rad}}\left(\frac{\rho_{\rm end}}{\rho_{\rm rad}}\right)^{1/4}\right]$$

Coherent oscillations



The end of inflation: $\ddot{a} = 0 \quad \Leftrightarrow \quad w = -1/3 \quad \Leftrightarrow \quad \dot{\phi}^2 = V(\phi) \; \propto \; \phi^k$

$$\phi(t) \;\simeq\; \phi_0(t)\mathcal{P}(t) \;=\; \phi_0(t)\sum_n \mathcal{P}_n e^{-in\omega_\phi t}$$

$$\langle\dot{\phi}^2\rangle \;\simeq\; \langle\phi V'(\phi)\rangle \;=\; k\langle V(\phi)\rangle$$

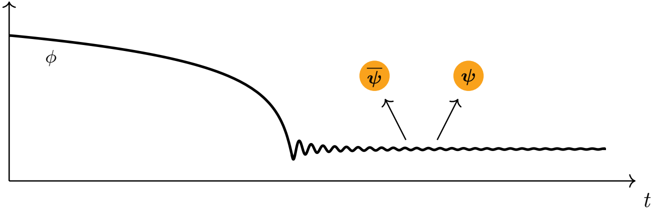
$$\rho_\phi \;\simeq\; \frac{k+2}{2}\langle V(\phi)\rangle \;=\; V(\phi_0)$$

$$p_\phi \;\simeq\; \left(\frac{k-2}{k+2}\right)\rho_\phi \;\equiv\; w_\phi \rho_\phi$$

Assuming the **coherence** of ϕ is maintained, **Fermi/Bose** effects are negligible, and no **resonant** growth, the perturbative (dissipative) approximation is valid,

$$\dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi \;=\; -\Gamma_\phi(1+w_\phi)\rho_\phi$$
$$\dot{\rho}_R + 4H\rho_R \;=\; \Gamma_\phi(1+w_\phi)\rho_\phi$$

Coherent oscillations



$$\phi(t)|0\rangle = \phi_0(t)\sum_n \mathcal{P}_n e^{-iK_n\cdot x}|0\rangle$$
$$K_n \equiv (n\omega_\phi, \mathbf{0})$$
$$\omega_\phi \propto m_\phi(t) \equiv \sqrt{V''(\phi_0(t))}$$
$$f_\phi(K) = (2\pi)^3 n_\phi(t)\delta^{(3)}(\boldsymbol{K})$$

$$\begin{aligned}\dot{\rho}_\psi + 4H\rho_\psi &= \sum_{n=1}^{\infty}\int \frac{d^3\boldsymbol{P}}{(2\pi)^3}\frac{d^3\boldsymbol{K}}{(2\pi)^3 n_\phi}\frac{d^3\boldsymbol{P'}}{(2\pi)^3 2P'^0}(2\pi)^4\delta^{(4)}(K_n - P - P')|\overline{\mathcal{M}_n}|^2 f_\phi(K)\\ &= \Gamma_\phi(1+w_\phi)\rho_\phi\end{aligned}$$

$$\boxed{\Gamma_\phi = \frac{1}{8\pi(1+w_\phi)\rho_\phi}\sum_{n=1}^{\infty}\left\langle|\overline{\mathcal{M}_n}|^2 n\omega_\phi\beta_n\right\rangle\,, \quad \beta_n = \sqrt{\left(1-\frac{(m_1+m_2)^2}{n^2\omega_\phi^2}\right)\left(1-\frac{(m_1-m_2)^2}{n^2\omega_\phi^2}\right)}}$$

MG, K. Kaneta, Y. Mambrini, K. Olive, JCAP 04 (2021), 012

Quadratic potential

Assume $V(\phi) \propto \phi^2$ and turn off all couplings to matter

Then, the inhomogeneous fluctuation $\delta\phi(t, \boldsymbol{x})$ satisfies, to first order,

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{\nabla^2 \delta\phi}{a^2} + \underbrace{V''(\phi(t))}_{m_\phi^2}\, \delta\phi \;=\; 0$$

Thus, no expectation of resonant growth ...?

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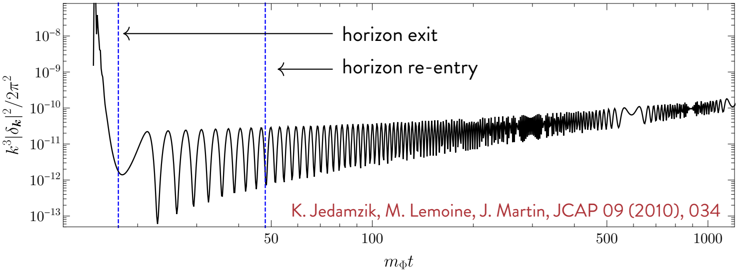
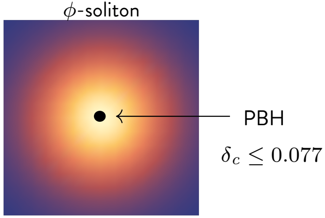
including **metric fluctuations.** Overdensities experience a narrow-resonance growth



K. Jedamzik, M. Lemoine, J. Martin, JCAP 09 (2010), 034

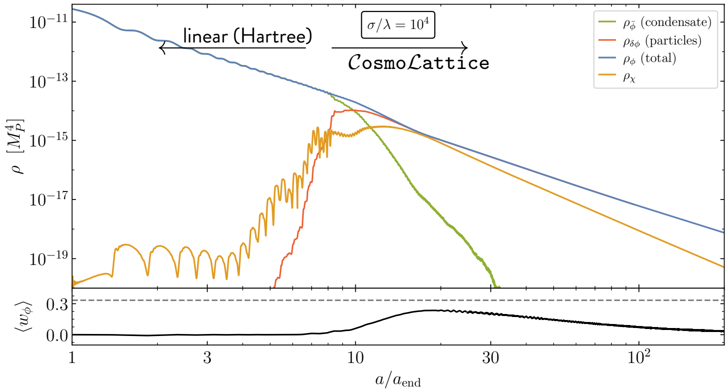


E. Milligan et al., JCAP 10 (2025), 025

Quadratic potential

Assume $V(\phi) \propto \phi^2$ and turn on an interaction with a (**dark sector**) scalar field $\mathcal{L}_{\rm int} = \frac{1}{2}\sigma\phi^2\chi^2$

$$\left(\frac{d^2}{dt^2} - \frac{\nabla^2}{a^2} + 3H\frac{d}{dt} + m_\chi^2 + \sigma\phi^2\right)\chi = 0 \quad \xrightarrow[\text{quantize}]{X = a\chi} \quad X_k'' + \Big[\underbrace{k^2 - \frac{a''}{a} + a^2 m_\chi^2 + \sigma a^2 \phi^2}_{\omega_k^2}\Big] X_k = 0$$



Resonant production of χ

$\Downarrow$

fragmentation of the ϕ condensate

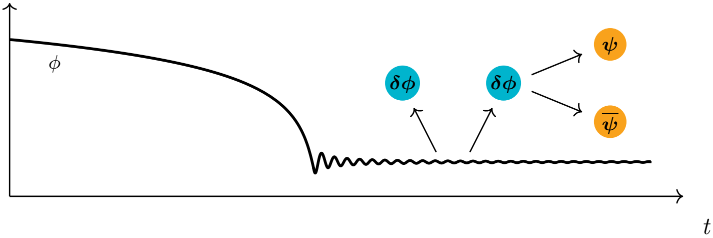
$$\rho_\phi = \overline{\tfrac{1}{2}\dot\phi^2 + \tfrac{1}{2a^2}(\nabla\phi)^2 + V(\phi)}$$

$$\rho_{\bar\phi} = \tfrac{1}{2}\dot{\bar\phi}^2 + V(\bar\phi)$$

$$\rho_{\delta\phi} = \rho_\phi - \rho_{\bar\phi}$$

MG, **A. Pereyra-Flores**, JCAP 08 (2024), 043

Decay of the fragmented inflaton



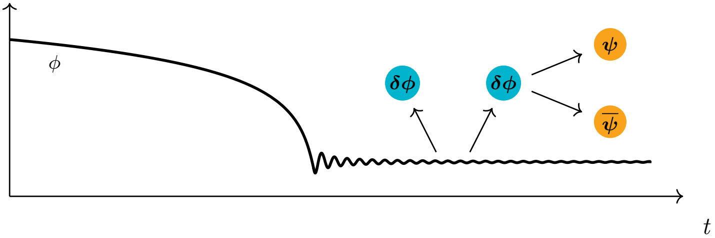
After fragmentation, the free inflaton quanta dominate the decays. Assuming **Fermi/Bose** effects are negligible,

$$\begin{aligned}\dot{\rho}_\psi + 4H\rho_\psi &= \int \frac{d^3\boldsymbol{P}}{(2\pi)^3}\frac{d^3\boldsymbol{K}}{(2\pi)^3 2K^0}\frac{d^3\boldsymbol{P}'}{(2\pi)^3 2P'^0}(2\pi)^4\delta^{(4)}(K-P-P')|\overline{\mathcal{M}}|^2 f_{\delta\phi}(K)\\ &= \Gamma_{\delta\phi} m_\phi n_{\delta\phi} \approx \Gamma_{\delta\phi}\left(\frac{m_\phi}{\bar{E}_\phi}\right)\rho_{\delta\phi}\end{aligned}$$

$$\Gamma_{\delta\phi} = \frac{|\mathcal{M}_{\delta\phi\to\bar{\psi}\psi}|^2}{16\pi m_\phi(t)}\sqrt{1-\frac{4m_\psi^2}{m_\phi(t)^2}}$$

MG, M. Pierre, JCAP 11 (2023) 004; MG, M. Gross, Y. Mambrini, K. Olive, M. Pierre, J. Yoon, JCAP 12, 028 (2023)

Decay of the fragmented inflaton



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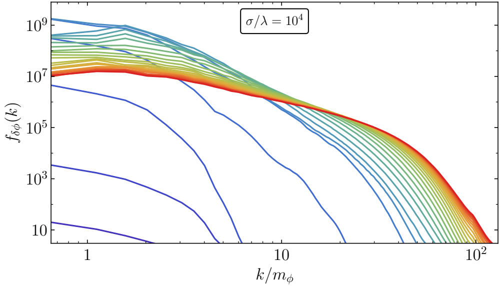
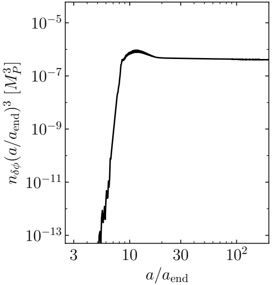
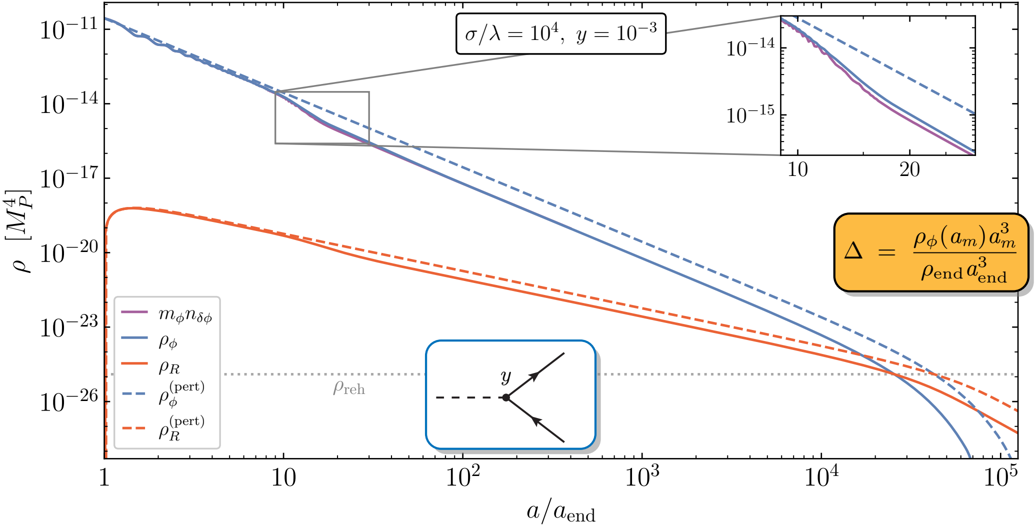




ϕ-χ interactions excite the inflaton UV modes (inflaton particles)

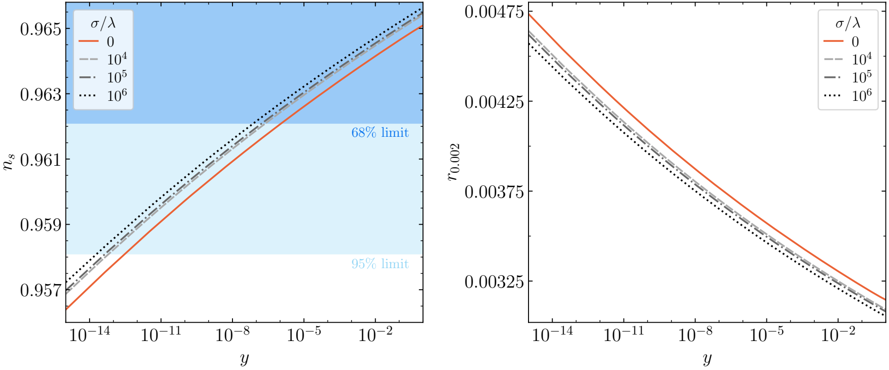
After backreaction the number of inflatons in a comoving volume is conserved, save for decays

Reheating after fragmentation



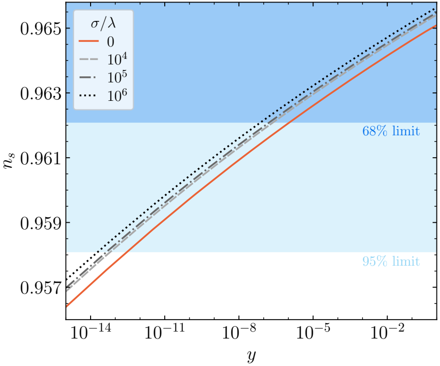
$$\Delta = \frac{\rho_\phi(a_m)\,a_m^3}{\rho_{\rm end}\,a_{\rm end}^3}$$

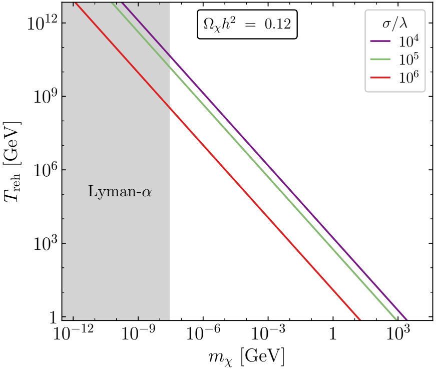
CMB observables



$$N_* - N_*^{(\mathrm{pert})} \simeq -\frac{1}{3}\ln\Delta \simeq \begin{cases} 0.51\,, & \sigma/\lambda = 10^4\,, \\ 0.63\,, & \sigma/\lambda = 10^5\,, \\ 0.90\,, & \sigma/\lambda = 10^6\,, \end{cases} \quad \Rightarrow \quad \begin{aligned} n_s &\simeq n_s^{(\mathrm{pert})} - \frac{2\ln\Delta}{3(N_*+2.5)^2} \\ r &\simeq r^{(\mathrm{pert})} + \frac{8\ln\Delta}{(N_*+2.5)^3} \end{aligned}$$

CMB observables



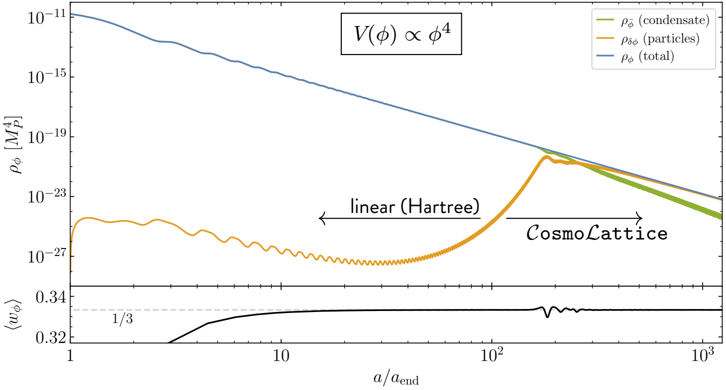


σ/λ	Lyman-α				CMB 68%	CMB 95%
0			$T_{\rm reh}$	$\gtrsim$	6.3×10^{8} GeV	(203 GeV)
10^4	4.5×10^{10} GeV	$\gtrsim$	$T_{\rm reh}$	$\gtrsim$	1.5×10^{8} GeV	(42 GeV)
10^5	1.5×10^{10} GeV	$\gtrsim$	$T_{\rm reh}$	$\gtrsim$	1.0×10^{8} GeV	(29 GeV)
10^6	3.4×10^{8} GeV	$\gtrsim$	$T_{\rm reh}$	$\gtrsim$	4.9×10^{7} GeV	(12 GeV)

Self-resonance

For **non-quadratic minima**, the oscillations of ϕ don't survive forever. The fluctuation $\delta\phi(t, \boldsymbol{x})$ satisfies

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{\nabla^2\delta\phi}{a^2} + V''(\phi(t))\,\delta\phi \;=\; 0 \quad \xrightarrow[\text{quantize}]{\Phi\,=\,a\phi} \quad \Phi_k'' + \Big[\underbrace{k^2 - \frac{a''}{a} + V''(\phi)a^2}_{\omega_k^2}\Big]\Phi_k \;=\; 0$$



Resonant production of $\delta\phi$

$\Downarrow$

fragmentation of the ϕ condensate

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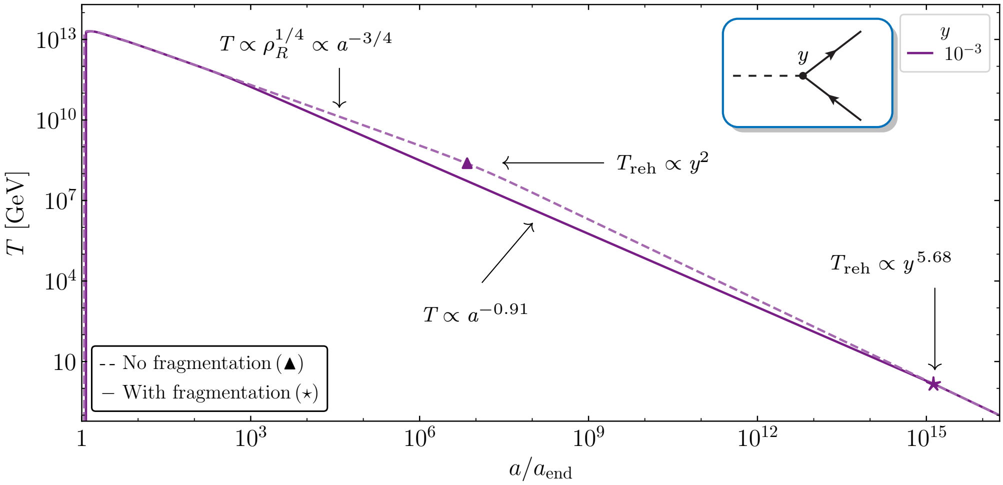
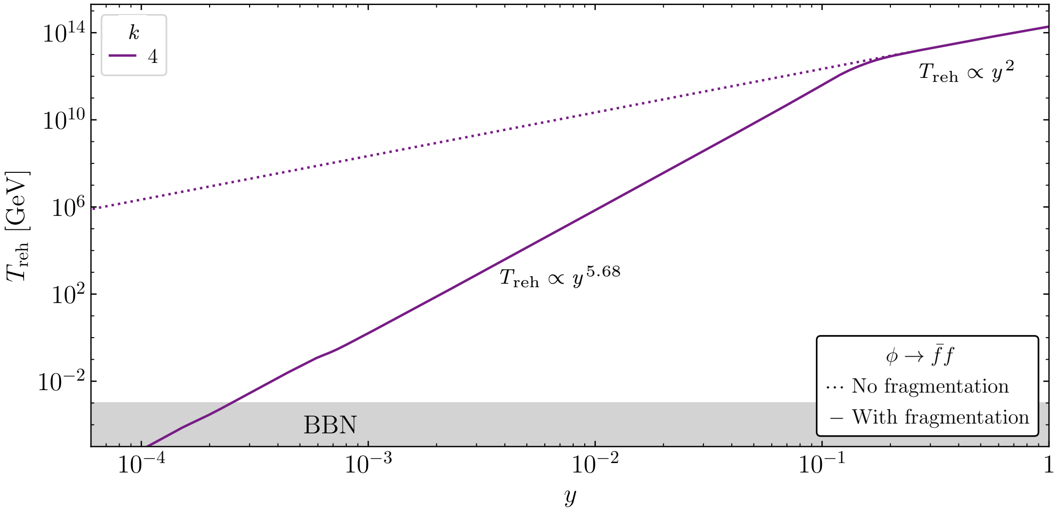
$$\boxed{\rho_{\delta\phi} = \rho_\phi - \rho_{\bar\phi}}$$

MG, M. Pierre, JCAP 11 (2023) 004

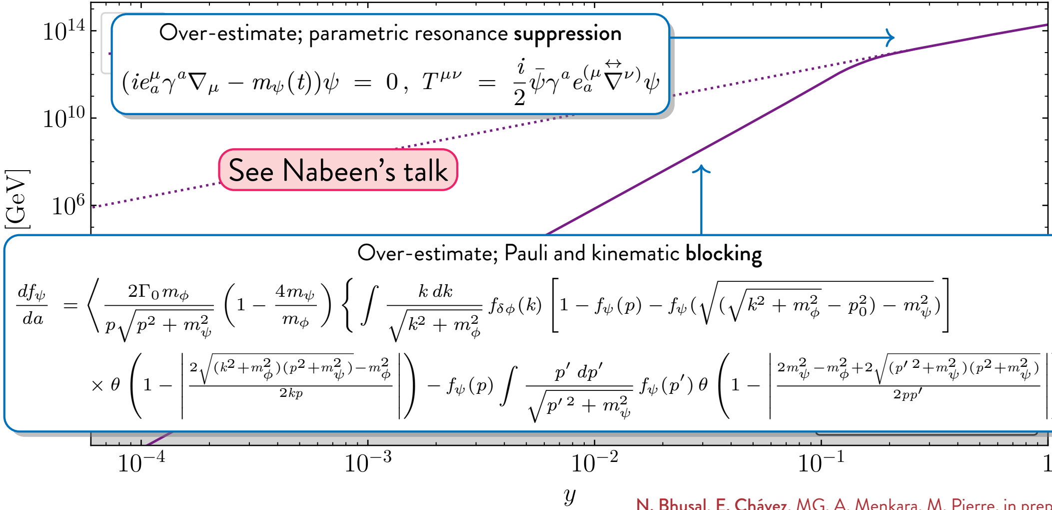
Reheating temperatures, self-resonance



Reheating temperatures, self-resonance



Reheating temperatures, self-resonance



Over-estimate; parametric resonance **suppression**

$$(i e^{\mu}_{a} \gamma^{a} \nabla_{\mu} - m_{\psi}(t))\psi = 0\,, \quad T^{\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^{a} e_{a}^{(\mu}\overleftrightarrow{\nabla}^{\nu)}\psi$$

See Nabeen's talk

Over-estimate; Pauli and kinematic **blocking**

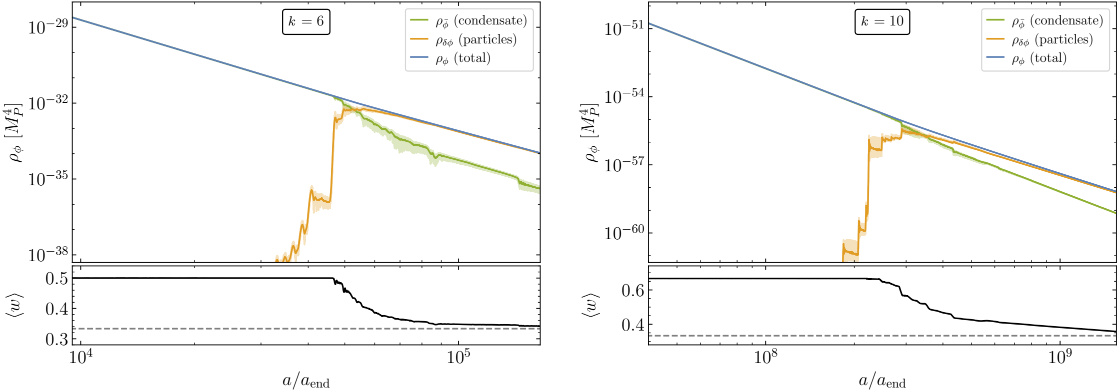
$$\frac{df_{\psi}}{da} = \Bigg\langle \frac{2\Gamma_0 m_{\phi}}{p\sqrt{p^2+m_{\psi}^2}}\left(1-\frac{4m_{\psi}}{m_{\phi}}\right)\Bigg\{\int \frac{k\,dk}{\sqrt{k^2+m_{\phi}^2}} f_{\delta\phi}(k)\left[1-f_{\psi}(p)-f_{\psi}\left(\sqrt{\left(\sqrt{k^2+m_{\phi}^2}-p_0^2\right)-m_{\psi}^2}\right)\right]$$

$$\times\,\theta\left(1-\left|\frac{2\sqrt{(k^2+m_{\phi}^2)(p^2+m_{\psi}^2)}-m_{\phi}^2}{2kp}\right|\right) - f_{\psi}(p)\int\frac{p'\,dp'}{\sqrt{p'^{\,2}+m_{\psi}^2}}\, f_{\psi}(p')\,\theta\left(1-\left|\frac{2m_{\psi}^2-m_{\phi}^2+2\sqrt{(p'^{\,2}+m_{\psi}^2)(p^2+m_{\psi}^2)}}{2pp'}\right|\right)\Bigg\}\Bigg\rangle$$

N. Bhusal, E. Chávez, MG, A. Menkara, M. Pierre, in preparation

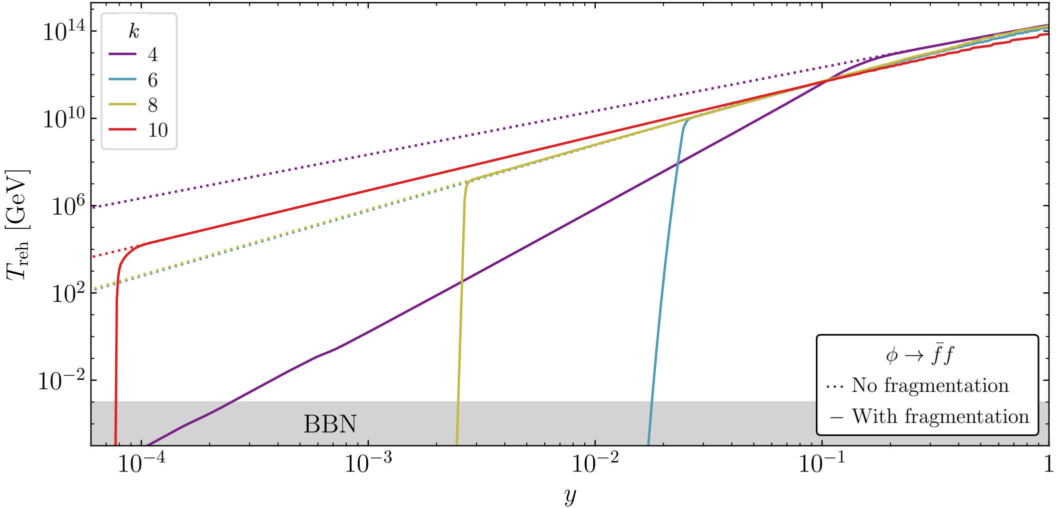
Self-resonance: $V(\phi) \propto \phi^k, k > 4$

Fragmentation leads to radiation domination, $w = \dfrac{k-2}{k+2} \longrightarrow \dfrac{1}{3}$ (not the same as reheating)

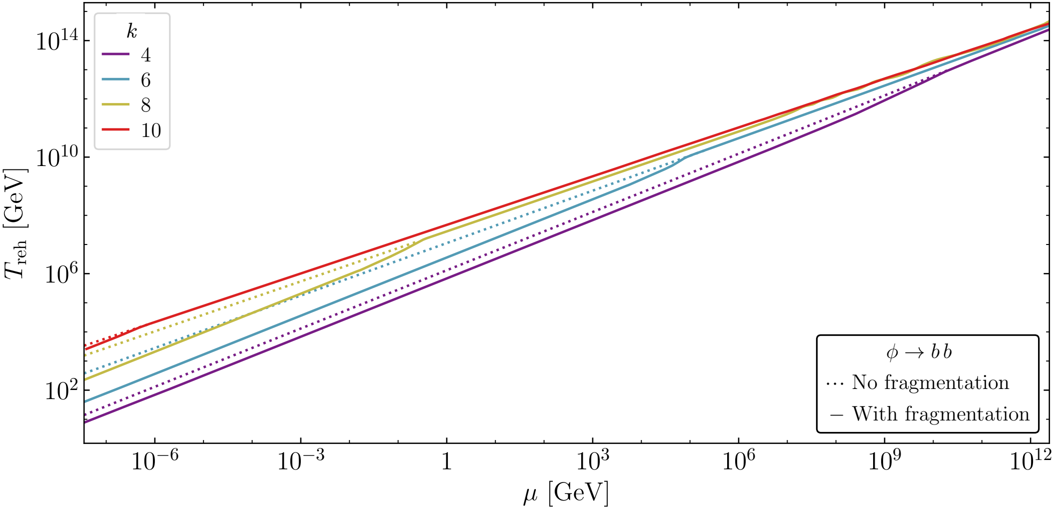


K. Lozanov, M. Amin, PRL 119, 061301 (2017); MG, M. Gross, Y. Mambrini, K. Olive, M. Pierre, J. Yoon, JCAP 12, 028 (2023)

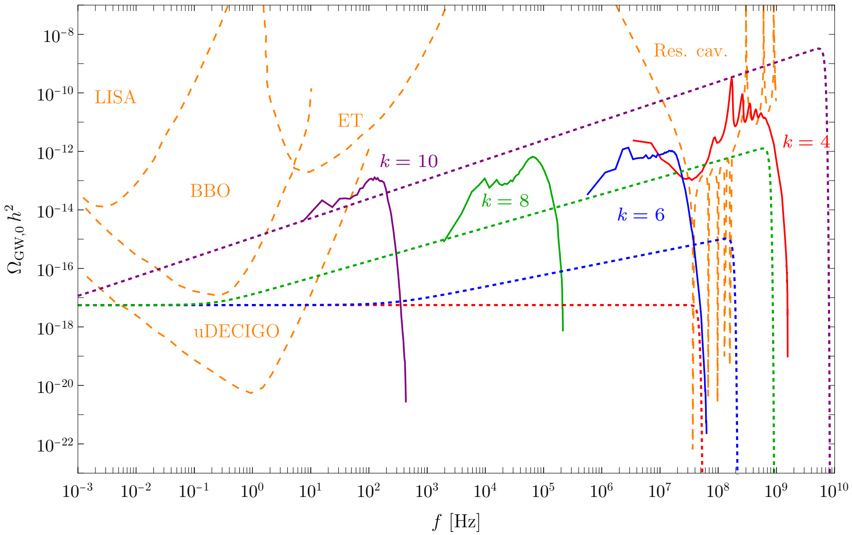
Reheating temperatures, self-resonance



Reheating temperatures, self-resonance



Induced gravitational waves

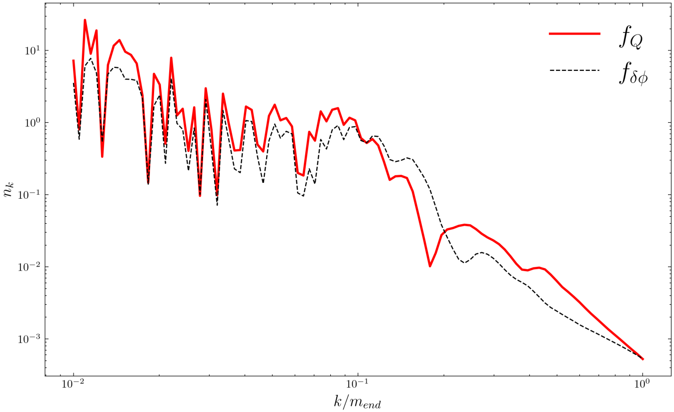


MG, M. Pierre, JCAP 12 (2023) 028
MG, M. Pierre, JCAP 09 (2024), 054

Gravitational fragmentation

Can gravitational effects hasten inflaton fragmentation?

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{\nabla^2\delta\phi}{a^2} + V''(\phi(t))\,\delta\phi \;=\; 4\dot{\phi}\dot{\Psi} - 2\,V'(\phi(t))\Psi$$

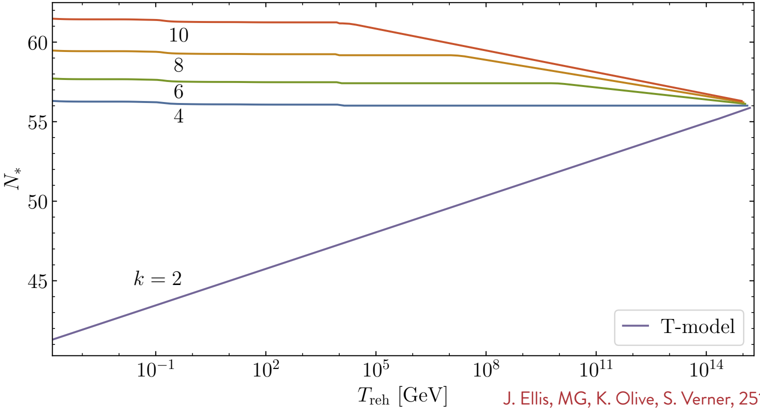


M. Gómez Quintanar, MG, preliminary

CMB observables

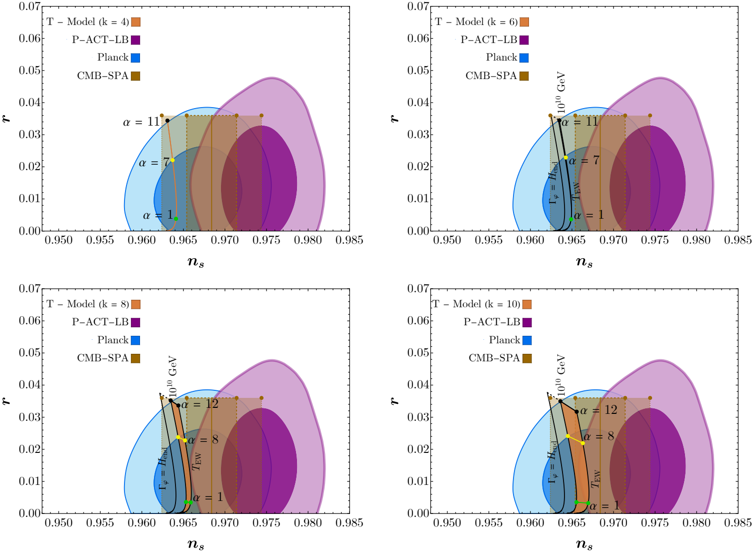
The stiff equation of state boosts N_* only in a limited range of reheating temperatures,

$$N_* \simeq N_*^{(\mathrm{inf})} - \frac{k+2}{6k}\ln\left(\frac{V_{\mathrm{end}}}{\lambda M_P^4}\right) - \frac{k-4}{3k}\ln\left(\frac{\max[T_{\mathrm{reh}}, T_{\mathrm{frag}}]}{M_P}\right) - \frac{k-2}{6k}\ln g_{\mathrm{reh}}\,,$$



J. Ellis, MG, K. Olive, S. Verner, 2510.18656

CMB observables



J. Ellis, MG, K. Olive, S. Verner, 2510.18656

Final thoughts

Summary

- Combined lattice + Boltzmann approach allows a description of post-fragmentation reheating
- Quantifiable impact on observables
- More to do to include fluctuation + dissipation
- (Bosonic) preheating calls for (fermionic) preheating
- Gravitational fragmentation?

Conclusion

There's still a lot to do!