(P)reheating Fermions in a Quartic Inflaton Potential

Nabeen Bhusal

Based on hep-ph/2511.xxxxx in collaboration with

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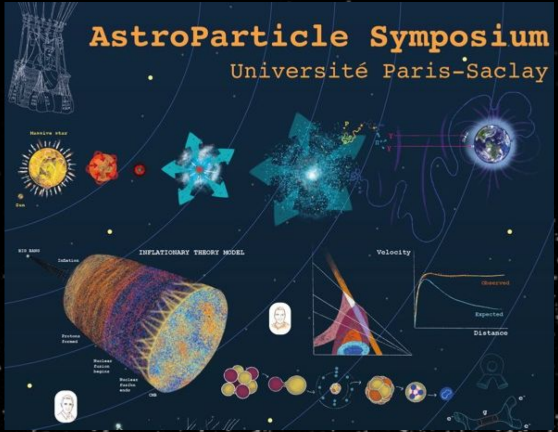

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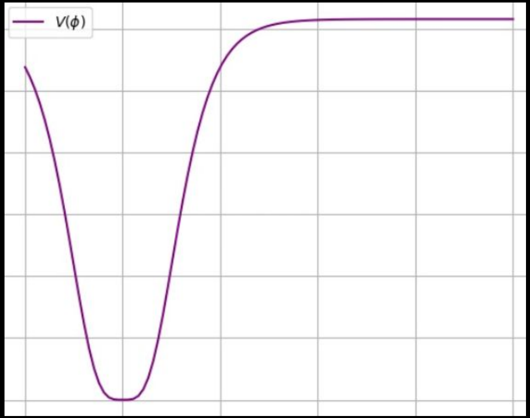
Content

Our setup : The inflaton sector

Consider $\mathcal{S} = \int d^4x \sqrt{-g}\left[-\frac{M_P^2}{2}R + \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \mathcal{L}_{\text{int}}\right]$

With $V(\phi) = \lambda M_P^4 \left(\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right)^k$ and $\mathcal{L}_{int} = y\phi\bar{\psi}\psi$, where we set k=4



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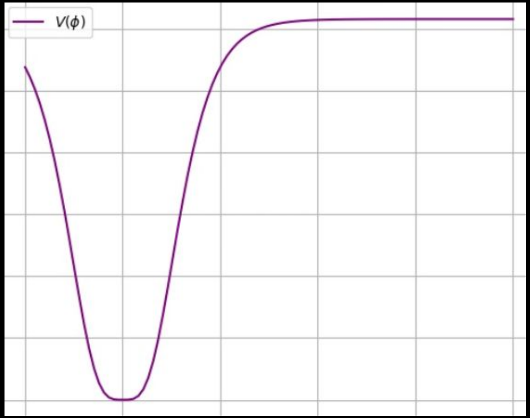
The inflaton mass is given then by: $m_\phi^2(t) = \lambda k(k-1) \phi_{\rm end}^{k-2} \left(\frac{a}{a_{\rm end}} \right)^{-6(k-2)/(k+2)}$

And the inflationary coupling $\lambda = \frac{18\pi^2 A_s}{6^{k/2} N_*^2}$



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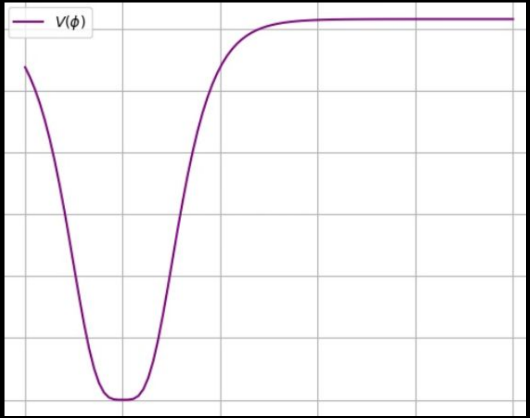
The inflation oscillates as $\phi(t) = \phi_0(t)\, \mathcal{P}(t) \simeq \phi_{\rm end} \left(\frac{a}{a_{\rm end}} \right)^{-6/(k+2)} \mathcal{P}(t)$

Where $\mathcal{P}(t) = {\rm sn}\,(t, -1)$ is the Jacobi sine function



Fragmentation

See the previous talk by Marcos!

- For our purpose: The instant when inflation fluctuations dominate the energy density over the zero mode
    - Occurs at ~ 180 a/a_{end} for quartic inflation [See JCAP 11 (2024) 004]
    - Quantum fluctuations are enhanced by quartic self coupling
    - Fermion production channel changes

Boltzmann vs. Bogoliubov

Boltzmann

Perturbative particle production from the oscillating inflation :

→ can only account for sub-horizon modes.
→ difficult to account for Pauli-blocking correctly.
→ subject to kinematics.

Bogoliubov

Non-perturbative fermion production from

1. The background
2. Non-adiabatic oscillations of the inflaton condensate

accounting for Pauli-blocking and all wavelengths :

→ can account for super-horizon modes.
→ Pauli-blocking is inherited from the fermion statistics.
→ can produce fermions out of equilibrium.

Motivation and Goals

Why fermions?

Reheating into bosons is well understood in all regimes and theoretical constraints on reheat temperature are solid. This is not the case for fermions.

Why Quartic?

It is conformal and inflaton fragments relatively early.
→ continuous conversion of condensate to inflation quanta which were expected to decay efficiently.

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- Perform a complete non-perturbative analysis to comment on whether reheating is realistically achievable before fragmentation
- Discuss post-fragmentation particle production in this setup

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In general, provide a "**more complete**" description of reheating with fermions.

Perturbative Fermion Production

Solving Boltzmann

The transition amplitude of the nth fourier mode of the coherently oscillating condensate is given by:

$$\left|\overline{\mathcal{M}_n}\right|^2 = \frac{2n^2\omega_\phi^2}{g_\psi}\bar{y}_n^2\beta_n^2\phi_0^2\left|\mathcal{P}_n\right|^2$$

where ,

$$\beta_n = \sqrt{1 - \frac{\mathcal{R}\mathcal{P}^2}{n^2}} \qquad \mathcal{R} \equiv \left.\frac{4m_\psi^2(t)}{\omega_\phi^2(t)}\right|_{\phi\to\phi_0} = \frac{4y^2\phi_{\rm end}^2}{\omega_{\rm end}^2}\left(\frac{a}{a_{\rm end}}\right)^{\frac{6(k-4)}{2+k}}$$

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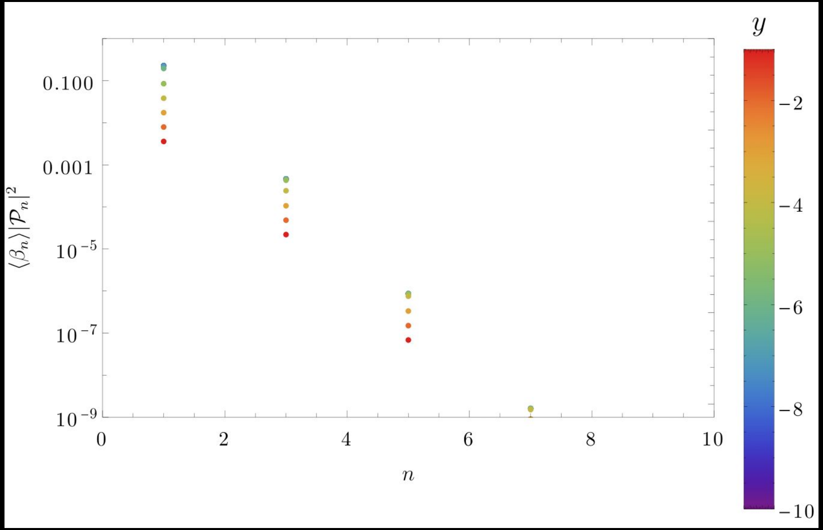

- For any Yukawa, the first fourier coefficient is the most dominant.

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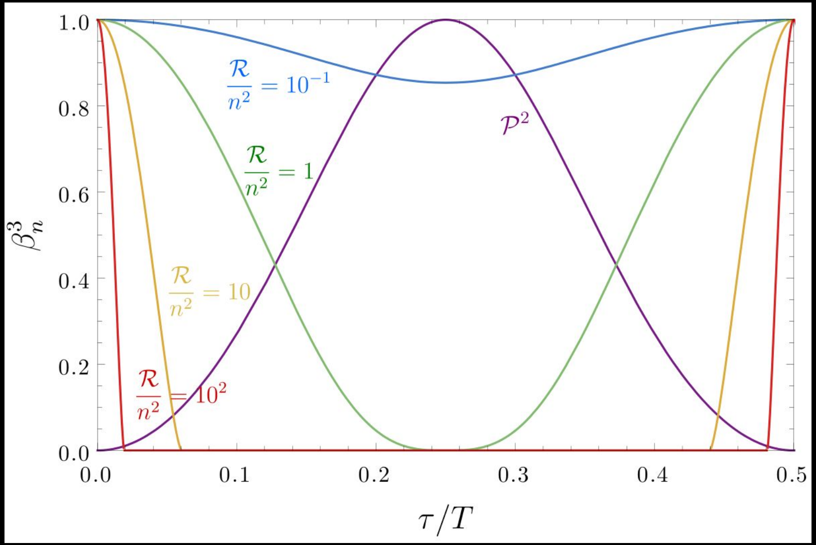


→ kinematic suppression for large effective fermion mass (large couplings)

The idea: Plug this into the collision term of a boltzmann equation

Non-Perturbative Fermion Production

Bogoliubov

We solve for the energy density of the fermions produced from the inflation as

$$\rho_\psi = \frac{2}{(2\pi)^3 a^4} \int d^3\boldsymbol{p}\, \omega_p n_p$$

Where the occupation number is given by $n_p = \frac{1}{2}\left|\left(1 + \frac{am_\psi}{\omega_p}\right)^{1/2} U_2 - \left(1 - \frac{am_\psi}{\omega_p}\right)^{1/2} U_1\right|^2$

In terms of the recast spinor mode equations $U_1'(\eta) = -ipU_2(\eta) + iam_\psi U_1(\eta).$ which are derived from dirac eqn.

$$U_2'(\eta) = -ipU_1(\eta) - iam_\psi U_2(\eta).$$

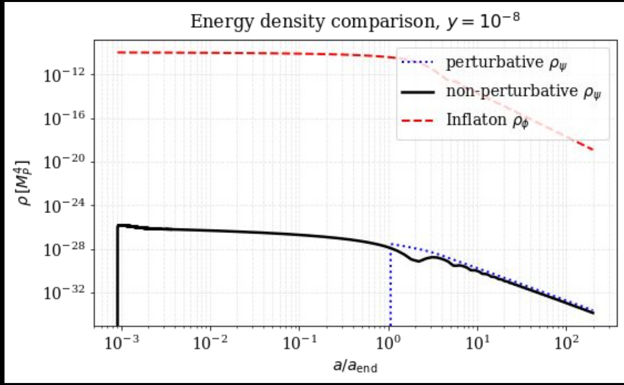
Modes initialized in the Bunch-Davies vacuum.

→ This is the 'Non-Perturbative' or 'Bogoliubov' approach
→ Valid until inflaton fragmentation

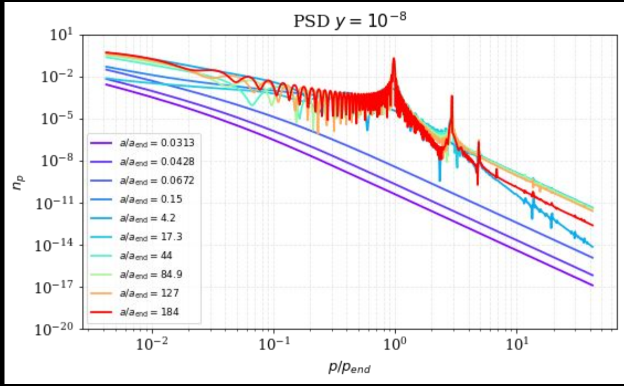
Reheating before fragmentation

In the **small coupling** regime fermion production is unsuppressed perturbatively [$y \lesssim 10^{-8}$]

- PSD is unaware of Pauli-statistics since occupation numbers are small → *Good approximation.*



The inflaton energy density scales as radiation i.e a^{-4}



Perturbative energy density of fermions scales as a^{-3}. Naively, reheating will occur…. But

Reheating before fragmentation

In the **small coupling** regime fermion production is unsuppressed perturbatively [$y \lesssim 10^{-8}$]

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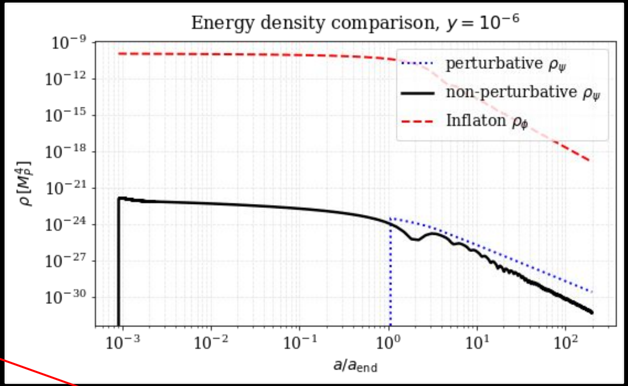
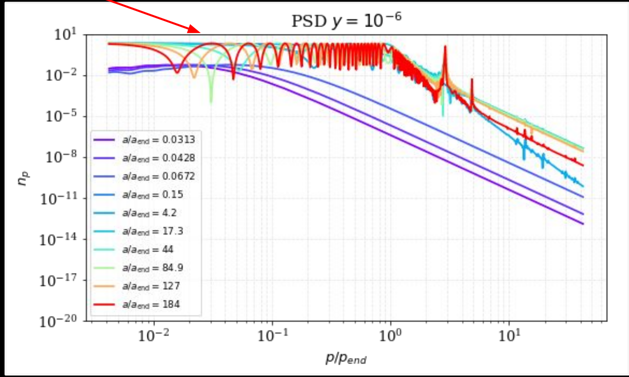
For **moderate couplings,** PSD saturates due to Pauli-blocking
→ *Regime where kinematic suppression is not large*
→ Perturbative calculation *overestimates fermion energy density*





Comparing the two

In the **small coupling** regime fermion production is unsuppressed

- PSD is unaware of Pauli-statistics but occupation numbers are small → *Good approximation*

For **moderate couplings** PSD saturates due to Pauli-blocking

→ *Regime where kinematic suppression is not large*

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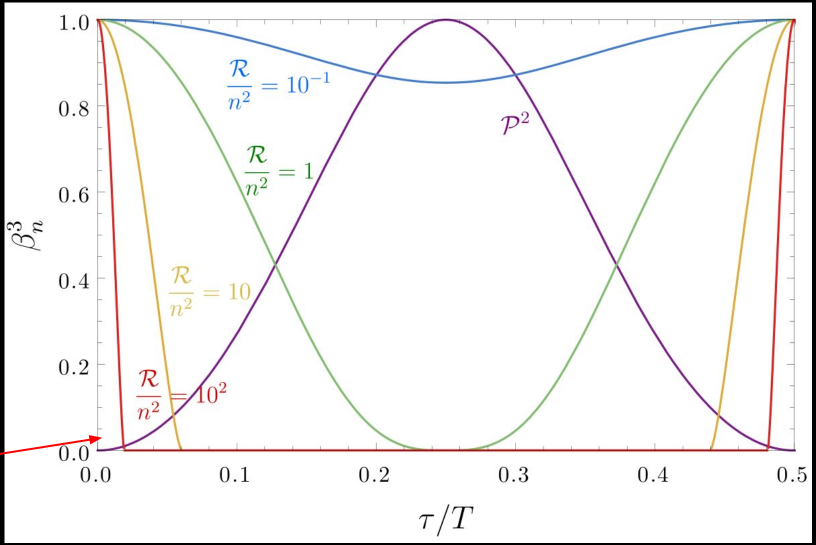
For **large couplings**, the perturbative picture breaks down

→ Dominated by kinematics (i.e. suppressed)

→ Production only at zero crossings of inflaton

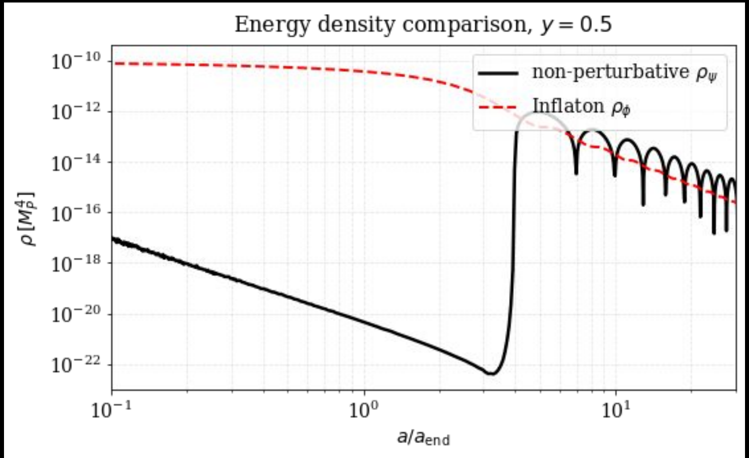


Reheating before fragmentation

- For couplings y$\gtrsim$ 0.4, seems to be possible.
- Occurs in first (few) oscillation(s).
- BUT
  Large yukawa can potentially spoil flatness.
  → for the model builders

Pre-fragmentation reheating → **CONSTRAINED coupling**

Need for post-fragmentation fermion production



Post-fragmentation production: The last hope?

Prior to fragmentation:

Oscillating inflation condensate is dominant source
→ Non-perturbative description is correct

Post-fragmentation:

The condensate is sub-dominant to inflation fluctuations
→ we implement **perturbative** fluctuation decays to fermions

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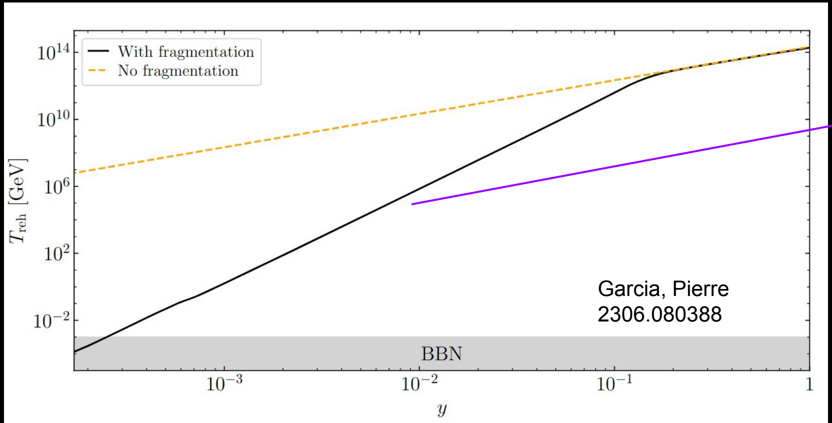
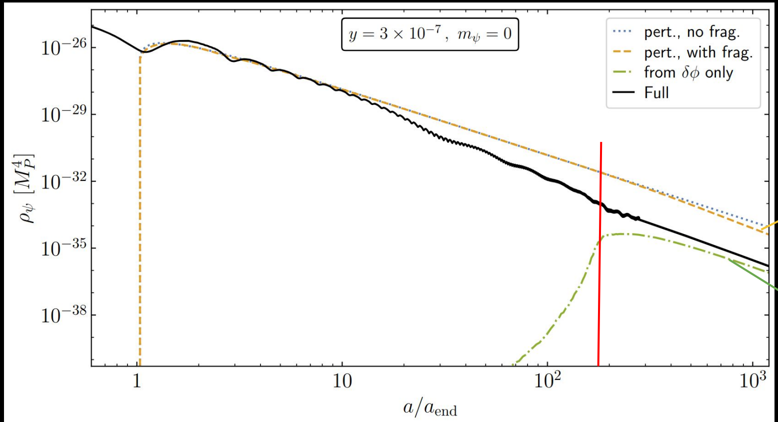
Post-fragmentation production: The last hope?

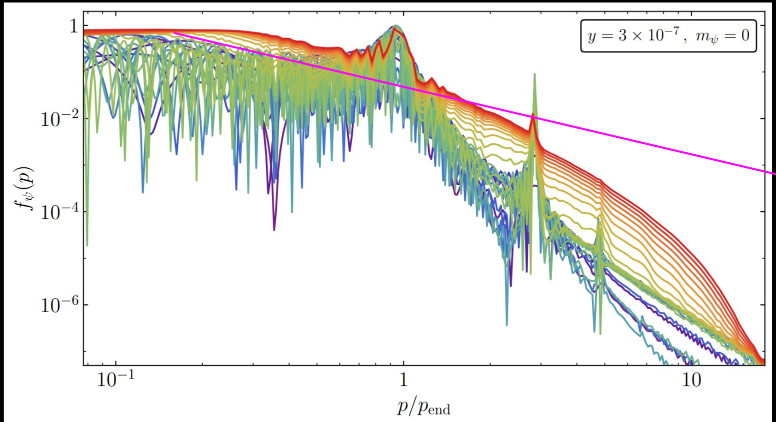
- For sufficiently large couplings [$y \gtrsim 10^{-8}$] kinematic blocking and Pauli suppression become relevant immediately → PSD is inherited from pre-fragmentation fermion distribution.
- Previous work did NOT account for blocking effects in the post-fragmentation production, thus overestimating post-fragmentation production.



Computed perturbatively

The "full" treatment





Plot from the last slide follows this treatment post-fragmentation

This work includes 1. Pauli-blocking
2. Kinematic suppression of fluctuation decays
→ Suppressed reheat temperature

PSD saturated for IR modes

Conclusion

In quartic-minimum inflaton potentials,

Prior to fragmentation:

- Perturbative particle production or the 'Boltzmann approach' is valid for very small couplings [$y \lesssim 10^{-8}$].
  → These couplings do not lead to reheating (BBN bounds).
- To really even speak of fermion reheating, it is necessary to be in the 'non-perturbative' or large yukawa limit.
  → Bogoliubov approach is necessary.
  → Possible reheating before fragmentation for O(0.5) couplings.

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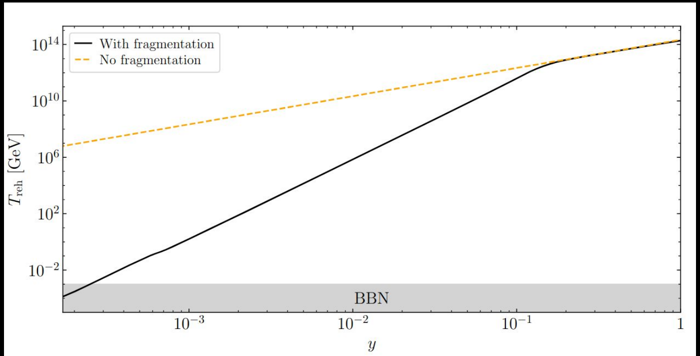
Post-fragmentation:

- Suppression effects (kinematic or Pauli-blocking) are important in the range of viable Yukawa couplings.
  → Reheat temperatures in previous work will be suppressed and couplings will be constrained.



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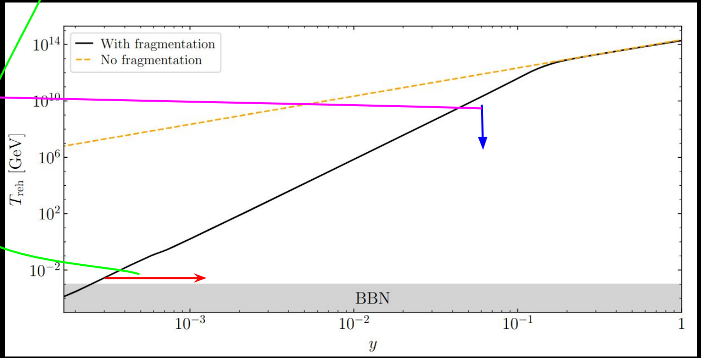

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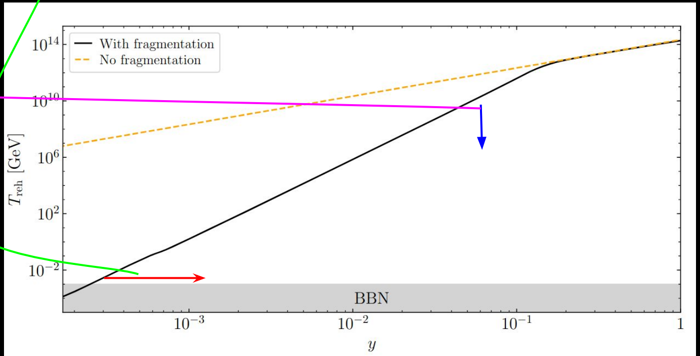
- Does axial coupling or a bare mass help?



THANK YOU

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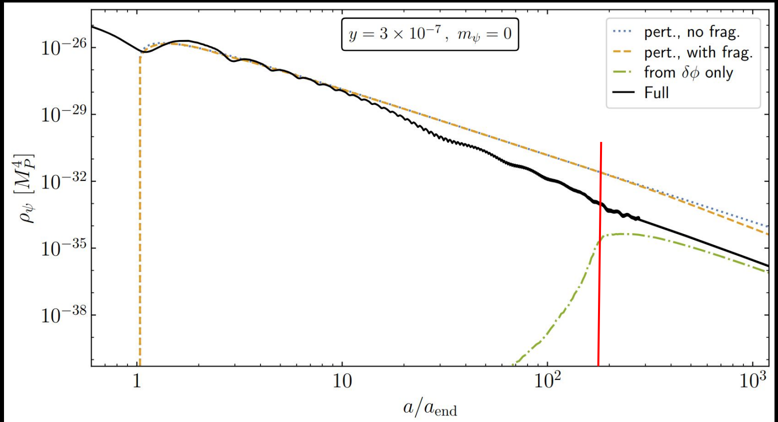
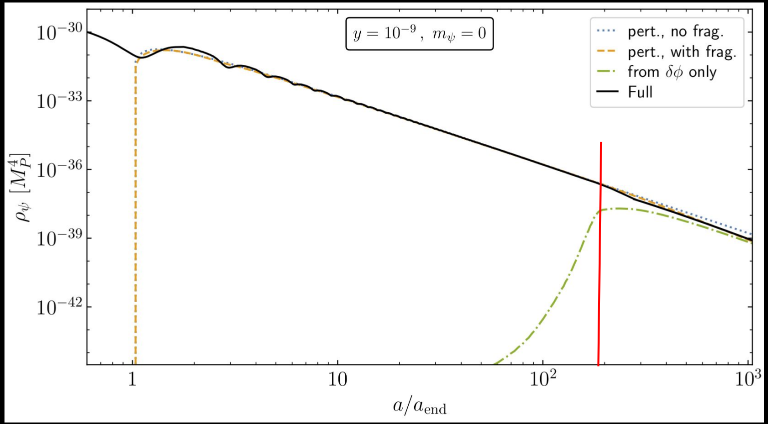
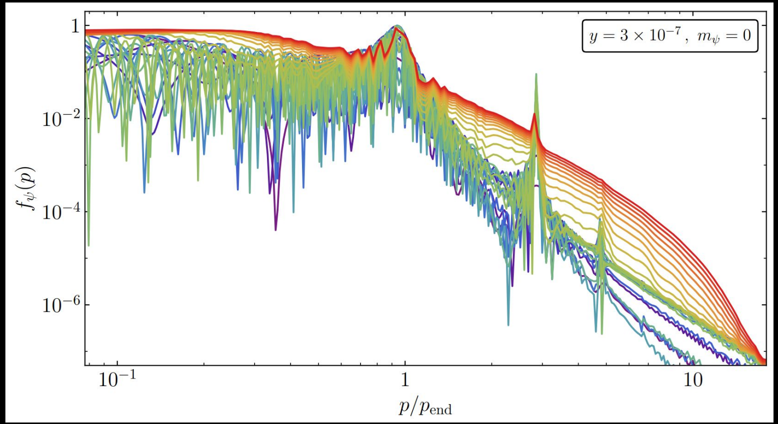
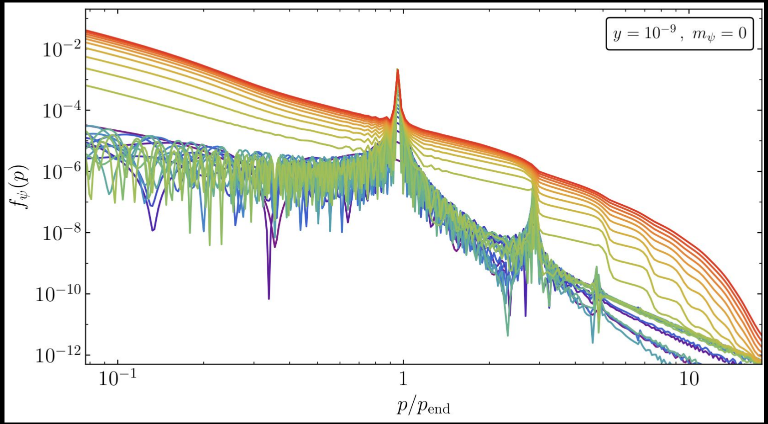
Backup slides

The "full" treatment









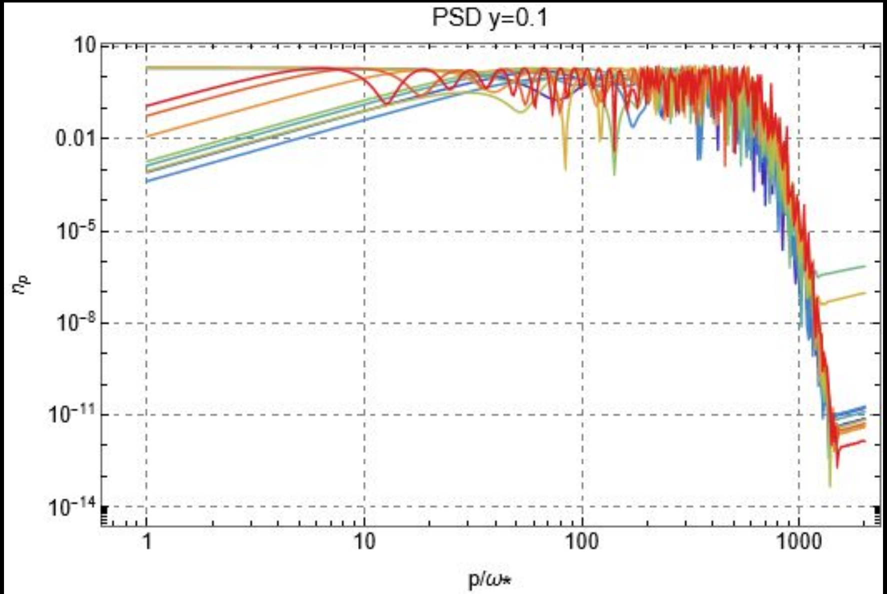
Need for Non-Perturbativity

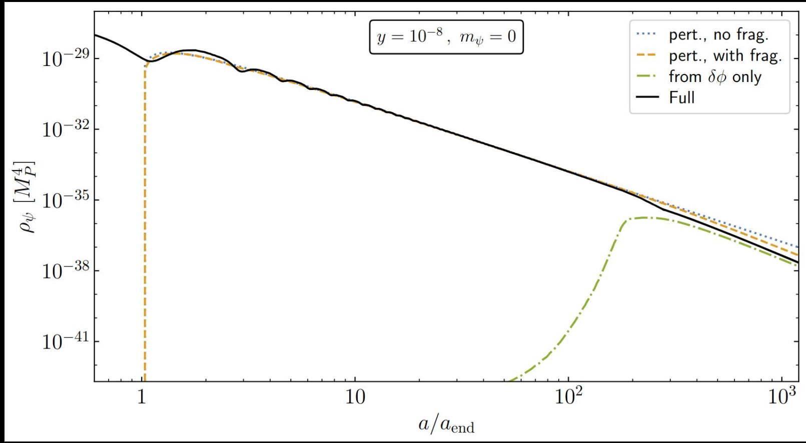
PSD for **large** Yukawas show exponential tails in the UV where the 'perturbative' calculations show a power law behavior:

→ Smaller coupling showed power law UV tails.

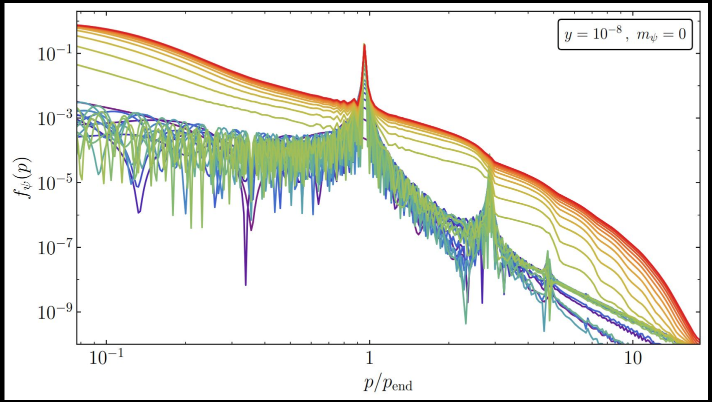
→ Indicative of the breakdown of the perturbative approach since even UV modes are 'non-perturbative'.

- Notice saturation of occupation number upto large momenta.

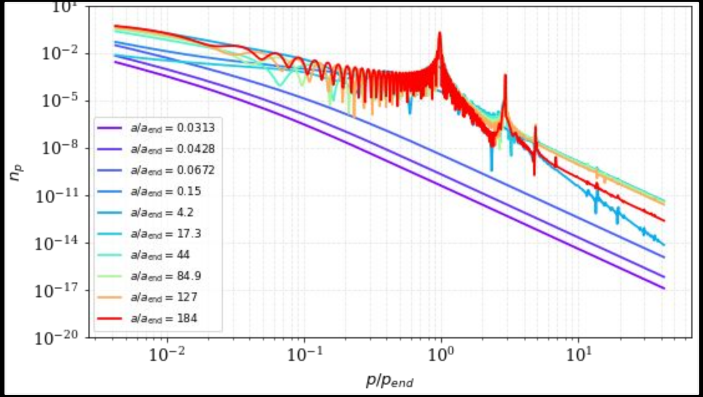




$y = 10^{-8},\ m_\psi = 0$
pert., no frag.
pert., with frag.
from $\delta\phi$ only
Full
$\rho_\psi\ [M_P^4]$
10^{-29}
10^{-32}
10^{-35}
10^{-38}
10^{-41}
1
10
10^2
10^3
$a/a_{\rm end}$



$y = 10^{-8},\ m_\psi = 0$
$f_\psi(p)$
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-1}
1
10
$p/p_{\rm end}$



n_p
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
10^{-20}
$a/a_{end} = 0.0313$
$a/a_{end} = 0.0428$
$a/a_{end} = 0.0672$
$a/a_{end} = 0.15$
$a/a_{end} = 4.2$
$a/a_{end} = 17.3$
$a/a_{end} = 44$
$a/a_{end} = 84.9$
$a/a_{end} = 127$
$a/a_{end} = 184$
10^{-2}
10^{-1}
10^{0}
10^{1}
p/p_{end}