Inflationary particle production

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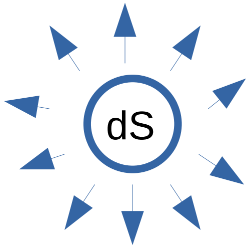
Parker, Grib, Zeldovich, Starobinsky,... 60-70's

– **inflation produces all particles:**



non-thermal dark matter

SM fields

etc.

– **background for any model bulding:**

Y(0) >> 0

initial conditions for freeze-in

SM radiation

– **yet many questions:**

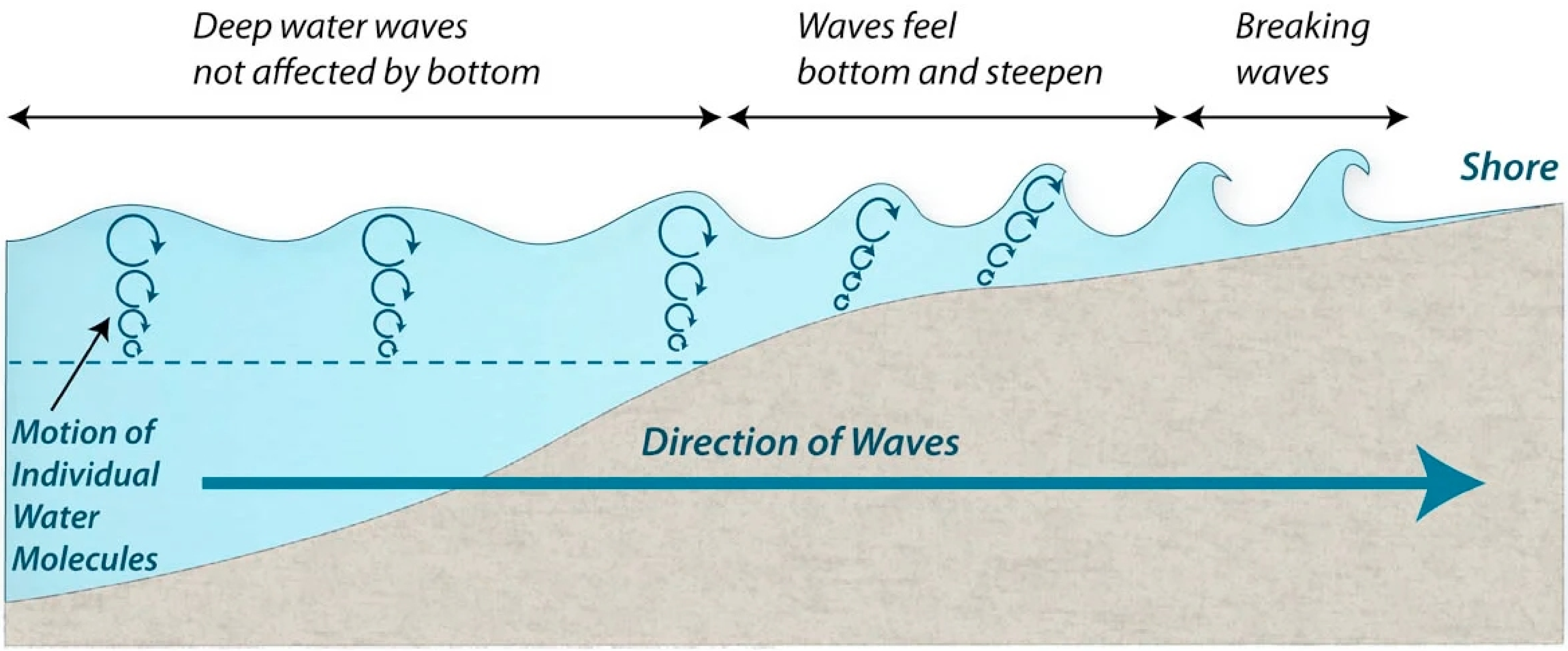
How efficient? How reliable?

Do different approaches agree?

How many quarks has inflation produced???

Inflationary particle production:

fluctuations → *particles*



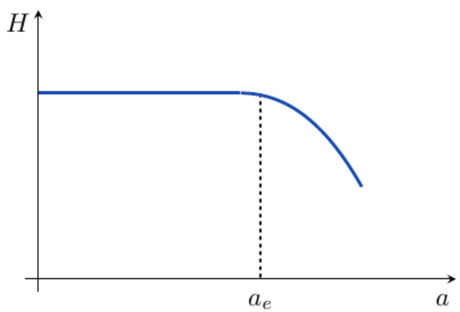
Bogolyubov

Wavefunction evolution with in- and out-boundary conditions

$$\chi_k'' + \omega_k^2 \chi_k = 0$$

Past:

| **in** >



Future:

| **out** >

Particle number:

$$\langle 0^{\rm in} | N^{\rm out} | 0^{\rm in} \rangle = V \int \frac{d^3\mathbf{k}}{(2\pi)^3} |\beta_k|^2$$

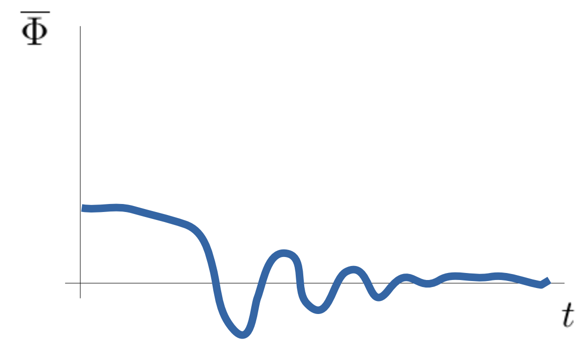
$$\beta_k = i(\chi_k^{\rm out\,\prime} \chi_k^{\rm in} - \chi_k^{\rm out} \chi_k^{\rm in\,\prime})$$

Starobinsky

Evolution of the “condensate” with quantum noise

$$\frac{d}{dt}\langle \overline{\Phi}^2 \rangle = -\frac{2}{3}\frac{m^2}{H}\langle \overline{\Phi}^2 \rangle + \frac{H^3}{4\pi^2}$$

After inflation:
condensate → *particles*



Particle number:

$$n = \frac{\rho}{m} = \frac{1}{2} m \overline{\Phi}^2$$

Bogolyubov approach :

$$\phi(\eta,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[a_{\mathbf{k}}\,\chi_k(\eta)\,e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}}\,\chi^*_k(\eta)\,e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

free light scalar

$$\chi_k'' + \omega_k^2\,\chi_k = 0\ ,$$

$$\omega_k^2(\eta) = k^2 + a^2(\eta)\,m^2 + \left(\frac{1}{6}-\xi\right)\,a^2(\eta)\,R(\eta)$$

Asymptotic solutions : ("Minkowski-like")

$$\chi_k^{\mathrm{in}}(\eta\to-\infty) \to \frac{e^{-i\int^\eta d\eta'\omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}}\ ,$$

$$\chi_k^{\mathrm{out}}(\eta\to+\infty) \to \frac{e^{-i\int^\eta d\eta'\omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}}\ .$$

Bogolyubov transform :

$$\begin{cases} \chi_k^{\mathrm{in}}(\eta) = \alpha_k\,\chi_k^{\mathrm{out}}(\eta) + \beta_k\,\chi_k^{\mathrm{out}\,*}(\eta) \\ a^{\mathrm{in}}_{\mathbf{k}} = \alpha_k^*\,a^{\mathrm{out}}_{\mathbf{k}} - \beta_k^*\,a^{\mathrm{out}\,\dagger}_{-\mathbf{k}} \end{cases}$$

Bunch-Davies vacuum :
(no particles initially)

$$a_{\mathbf{k}}^{\text{in}}\,|0^{\text{in}}\rangle = 0$$

Particle number at late times:

$$\langle 0^{\text{in}}|N^{\text{out}}|0^{\text{in}}\rangle = V \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,|\beta_k|^2$$

$$\beta_k = i(\chi_k^{\text{out}\,\prime}\chi_k^{\text{in}} - \chi_k^{\text{out}}\chi_k^{\text{in}\,\prime})$$

Example: inflation followed by radiation-domination

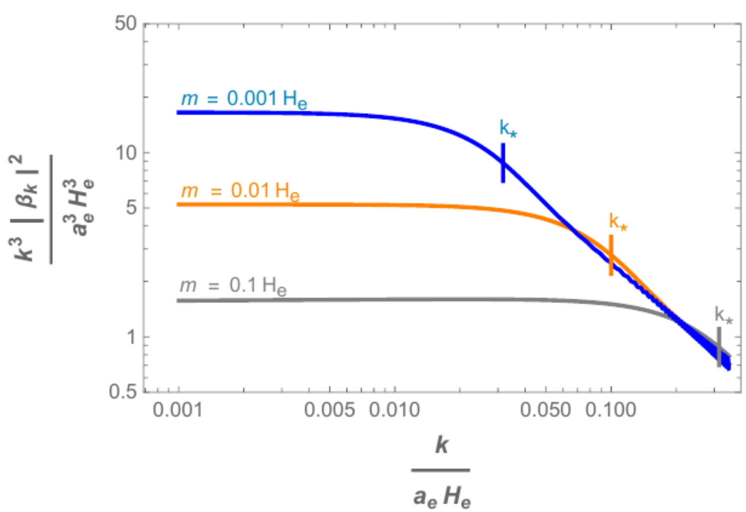
Feiteira, OL' 25

Integral converges, dominated by momenta k < k_*

$$a^3 n = \frac{3\kappa^2\, a_e^3}{4\pi^2}\frac{H_e^{11/2}}{m^{5/2}}$$

finite, but huge for light m

$$k_* = \sqrt{m a_e^2 H_e}$$



$$\langle\Phi^2\rangle_{\text{IR}} = \frac{1}{a^2(\eta)}\int^{\epsilon aH}\frac{d^3\mathbf{k}}{(2\pi)^3}\left|\chi_k^{\text{in}}(\eta)\right|^2 = \frac{3}{8\pi^2}\frac{H^4}{m^2}$$

anomalously large average field at the end of inflation (correlator / condensate)

Starobinsky

$$\Phi = \overline{\Phi}(t,\mathbf{r}) + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\theta(k-\epsilon a(t)H)\,\left(a_{\mathbf{k}}\chi_k(t)\,e^{i\mathbf{k}\cdot\mathbf{r}} + a^\dagger_{\mathbf{k}}\chi^*_k(t)\,e^{-i\mathbf{k}\cdot\mathbf{r}}\right)$$

long wavelength *short wavelength*

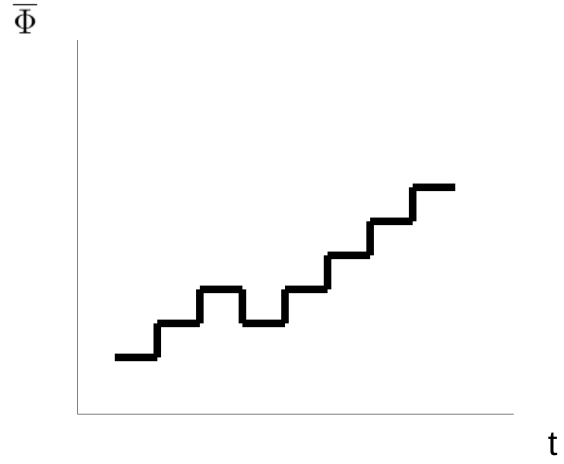
Random walk:

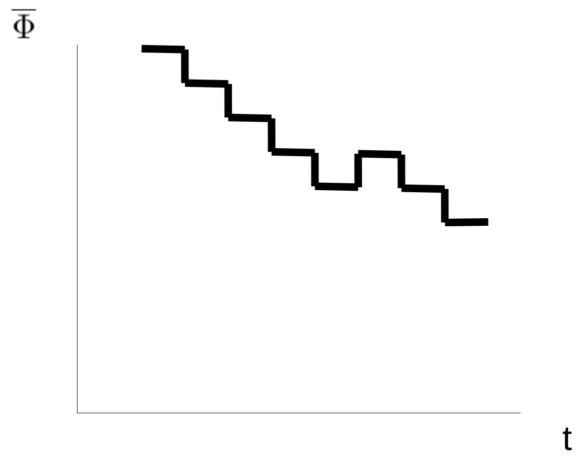
$$\frac{d}{dt}\langle\overline{\Phi}^2\rangle = -\frac{2}{3}\,\frac{m^2}{H}\,\langle\overline{\Phi}^2\rangle + \frac{H^3}{4\pi^2}$$

$$\langle\overline{\Phi}^2\rangle = \frac{3H^4}{8\pi^2 m^2} + \left(\langle\overline{\Phi}^2\rangle_0 - \frac{3H^4}{8\pi^2 m^2}\right)\,e^{-\frac{2m^2}{3H}(t-t_0)} \rightarrow \frac{3}{8\pi^2}\,\frac{H^4}{m^2}$$

Timescale:

$$\#\text{ of e-folds} > \mathcal{O}\left(\frac{H^2}{m^2}\right)$$ >>> 1 (e.g. 10^{26})





Bogolyubov

infinitely long inflation

$$a_{\mathbf{k}}^{\text{in}} |0^{\text{in}}\rangle = 0$$

$$\eta \to -\infty$$

Starobinsky

finite inflation

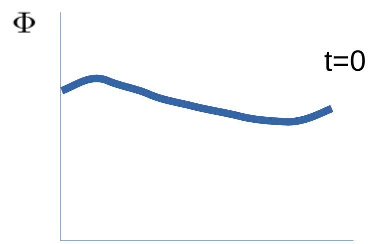
non-trivial initial conditions

$$\langle \overline{\Phi}^2 \rangle_0$$

Agree if inflation is super-long !

$$\langle \overline{\Phi}^2 \rangle \to \frac{3}{8\pi^2} \frac{H^4}{m^2}$$

Inflaton is not in the Bunch-Davies vacuum, why should any other scalar be ?



parametrize particle abundance in terms of unknown "condensate" value at the end of inflation $\overline{\Phi}$

$$Y \simeq 0.07 \times \frac{\overline{\Phi}^2}{m^{1/2} M_{\text{Pl}}^{3/2}}$$

$$\overline{\Phi} > H_e$$

$$Y \le 4.4 \times 10^{-10} \frac{\text{GeV}}{m}$$

$$m \ll \text{ eV}$$

General result for *inflationary* scalar production:

Feiteira, OL' 25

$$Y \propto \frac{\overline{\Phi}^2}{m_{\rm eff}^{1/2}} \times \left(\frac{H_R}{m_{\rm eff}}\right)^{\gamma}$$

γ = 0 (radiation dom.) or ½ (matter dom.)

Includes

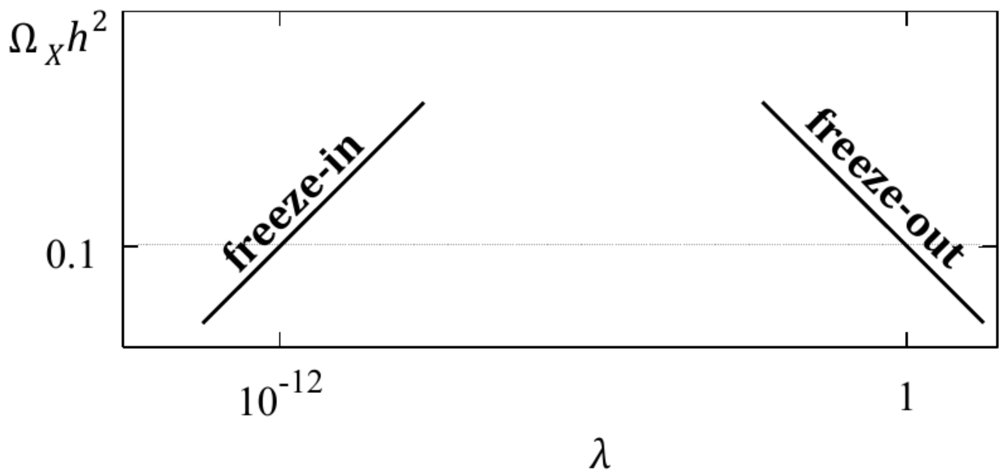
- weak self-coupling λ
- small non-minimal coupling to gravity ξ
- general initial conditions

Intermediate conclusion:

- ***scalar abundance cannot be predicted even within classical gravity !***
- ***very strong lower bounds exist (m << eV or T_R < GeV)***
- ***even worse in quantum gravity*** $\frac{\varphi^4\Phi^2}{M_{\rm Pl}^2}, \frac{\varphi^6\Phi^2}{M_{\rm Pl}^4}, \frac{\varphi^8\Phi^2}{M_{\rm Pl}^6}$

Dark Matter freeze-in problematic unless T_R is low

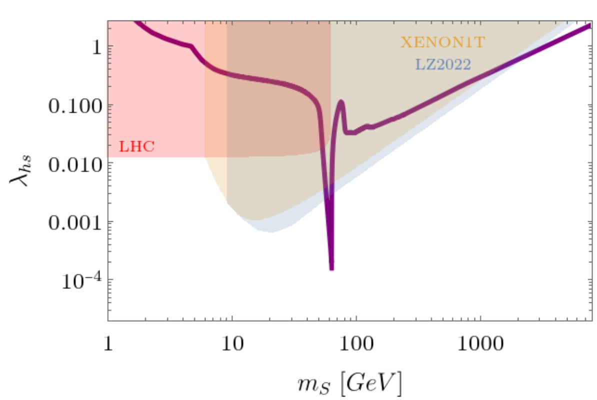
Cosme,Costa,OL ‘23

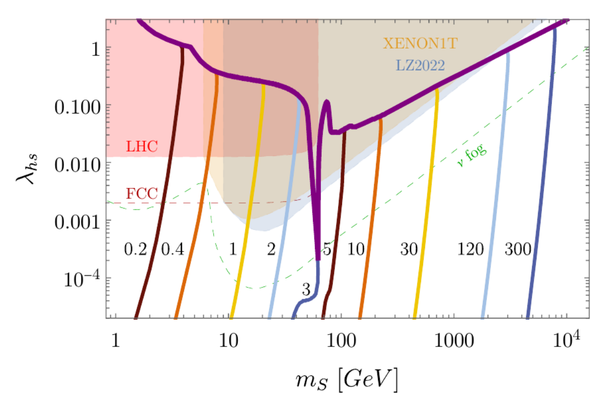


freeze-in at stronger coupling

Arcadi,Costa,Goudelis,OL ‘24

“excluded”





Feiteira, Koutroulis, OL, Pokorski ‘25

Quark / Lepton inflationary production

By conformal transformation:

$$(i\gamma^\mu \partial_\mu - a(\eta)M)\,\Psi = 0$$

↓

Parker ‘ 71
Chung, Everett, Yoo, Zhou ‘11

$$Y \propto \left(\frac{M}{M_{\rm Pl}}\right)^{3/2}$$

tiny for standard fermion masses!

Starobinsky, Yokoyama ‘94

But for the minimally coupled Higgs:

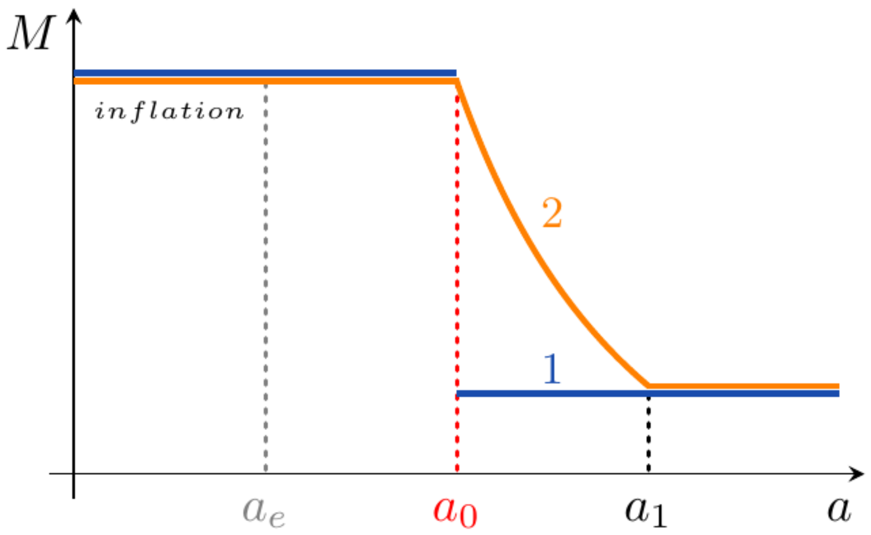
$$\langle h^2 \rangle \to 0.1 \frac{H^2}{\sqrt{\lambda_h}}$$

$$M_f = \frac{1}{\sqrt{2}} Y_f \langle h \rangle$$

→ SM fermions super-heavy

solve the Dirac equation in a time-dependent background with a time-dependent mass

Mass Profiles**:**



Result:

$$Y^{\rm SM} \sim 10^{-3} \times \frac{Y_f^{7/2}}{\lambda_h} \left(\frac{H_e}{M_{\rm Pl}}\right)^{3/2}$$

up to 10^{20} above naive estimate

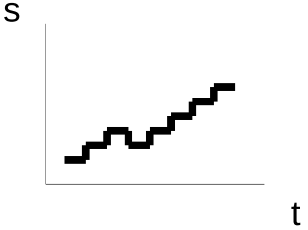
Non-reheating SM production

R-neutrino inflationary production

Majorana mass via a singlet:

$$\Delta\mathcal{L} = \frac{1}{2}\,\mathcal{Y}^s\, s\, \nu_R \nu_R + \text{h.c.}$$

$$\langle s^2 \rangle \to 0.1\,\frac{H^2}{\sqrt{\lambda_s}}$$



Long-lived condensate produces effectively “heavy” neutrinos:

$$\mathcal{Y}^s s_e$$

$$\mathcal{Y}^s \gg \sqrt{\lambda_s} \quad \Rightarrow \quad Y_0 \simeq 5\times 10^{-3}\left(\frac{M}{M_{\rm Pl}}\right)^{3/2}$$

Matches the DM abundance for

$$m_\nu \sim 10\ \text{GeV} \times \left(\frac{10^{13}\ \text{GeV}}{\mathcal{Y}^s s_e}\right)^{3/2}$$

Classical gravity!!

CONCLUSION

- *inflation is efficient in particle production (uncertainty!)*
- *dark relics are (over)produced during/after inflation*
- *traditional freeze-in models problematic*
- *SM fermions produced by inflation*
- *Bogolyubov*  *Starobinsky*