Self-Gravity in Superradiance Clouds

Implications for Binary Dynamics and Observational Prospects

Hyungjin Kim (DESY)

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a minimally-coupled scalar field in Kerr background

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \mu^2 \phi^2 \right]$$

take the weak field and nonrelativistic limit

$$ds^2 \approx -(1 - r_s/r)dt^2 + (1 - r_s/r)^{-1}dr^2 + r^2 d\Omega^2$$

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$$\phi(t, \boldsymbol{x}) = \frac{1}{\sqrt{2\mu}} e^{-i\mu t} \psi(t, \boldsymbol{x}) + \text{h.c.}$$

the system effectively reduces to

$$i\dot{\psi} \approx \left(-\frac{\nabla^2}{2\mu} - \frac{\alpha}{r} \right) \psi$$

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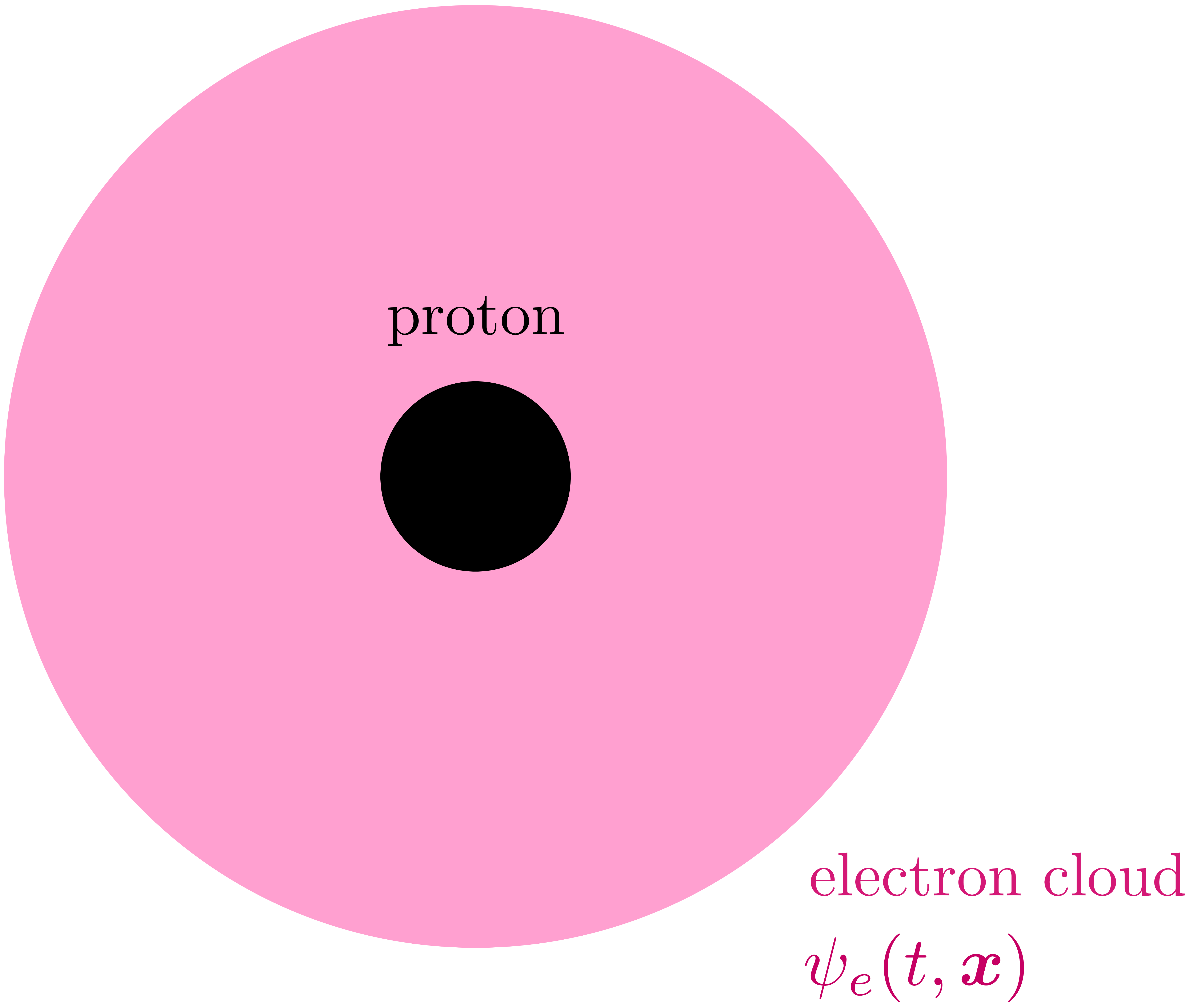
with a *fine structure constant*

$$\alpha = GM\mu$$

this is a hydrogen atom



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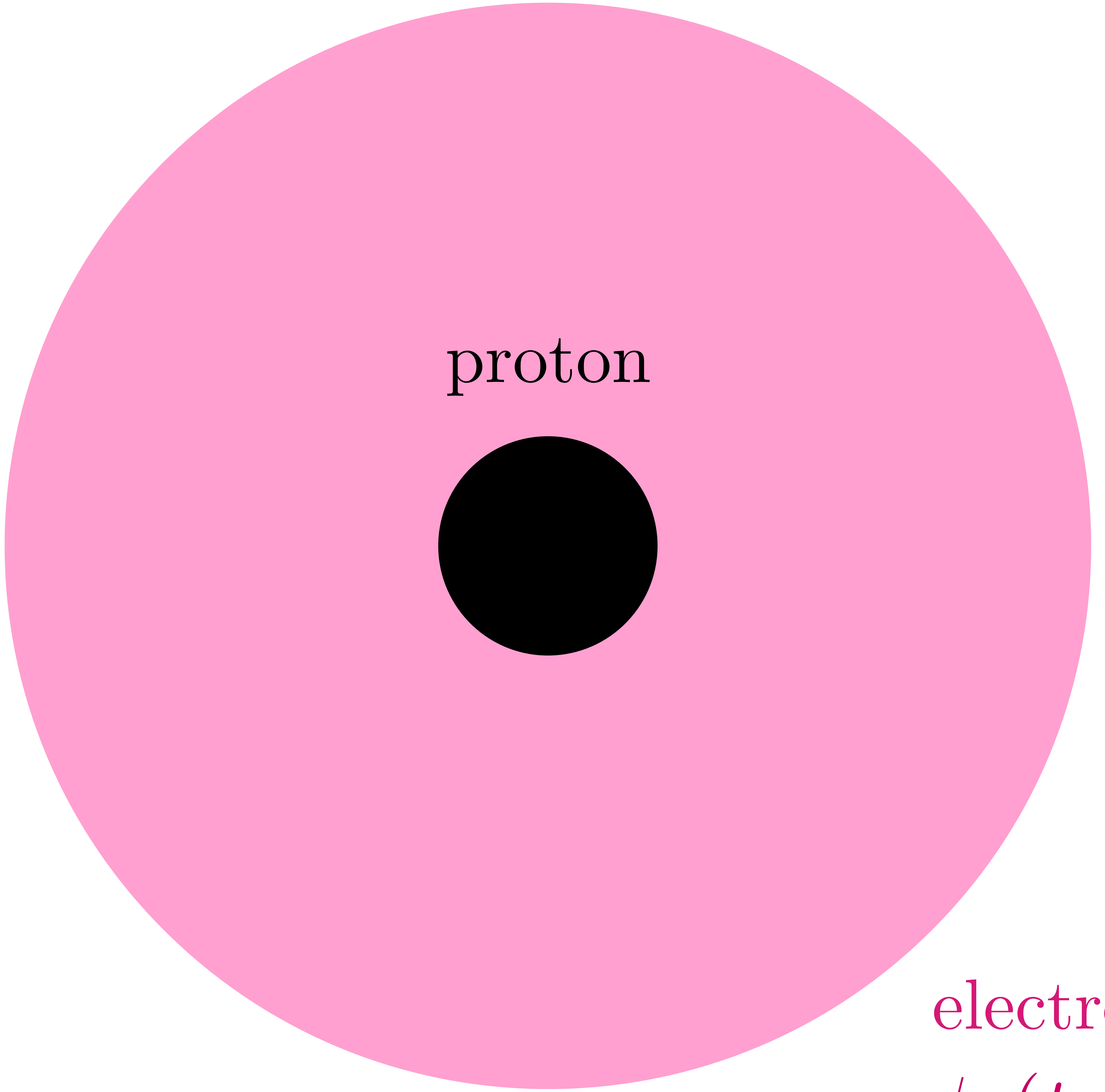
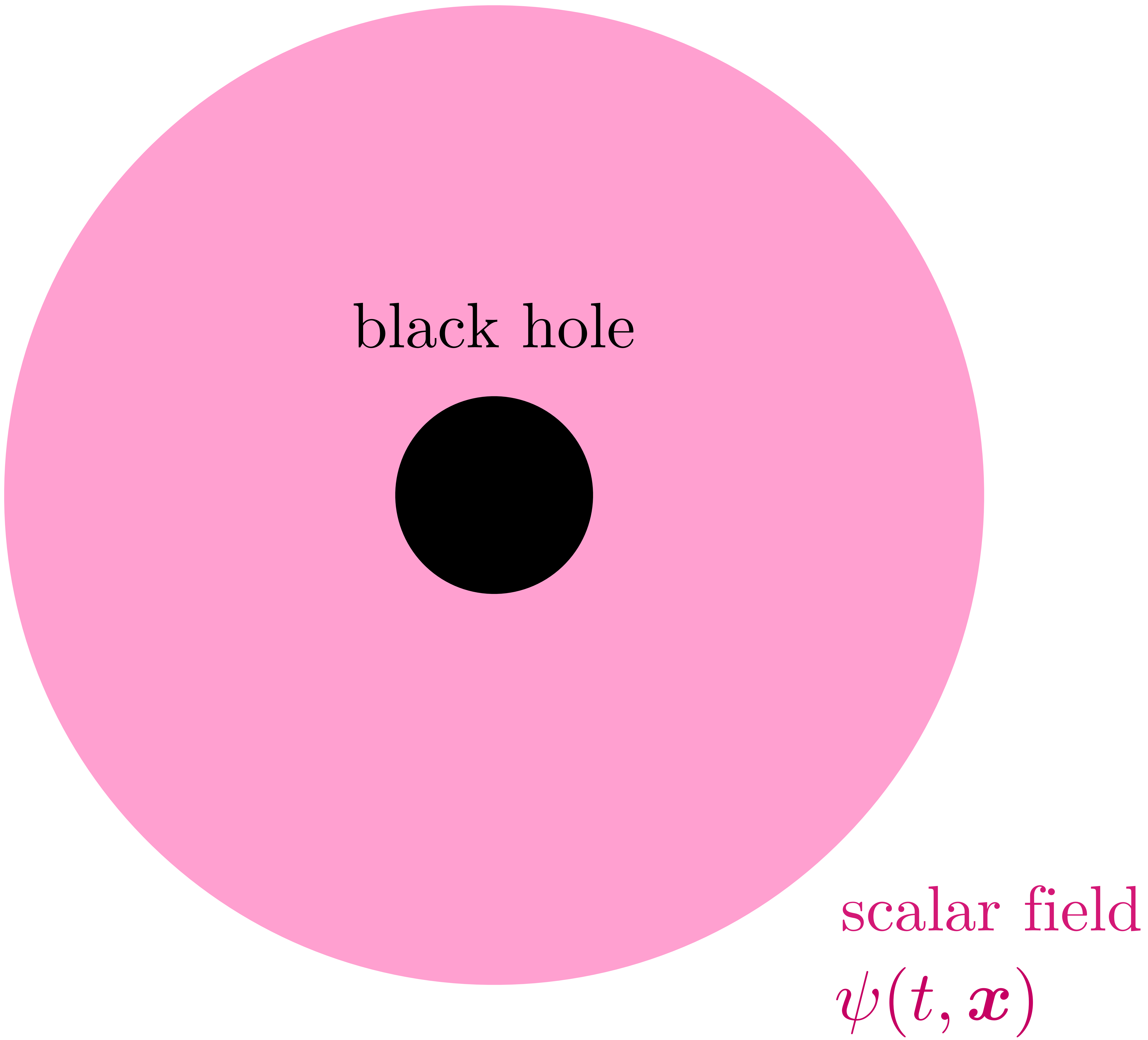
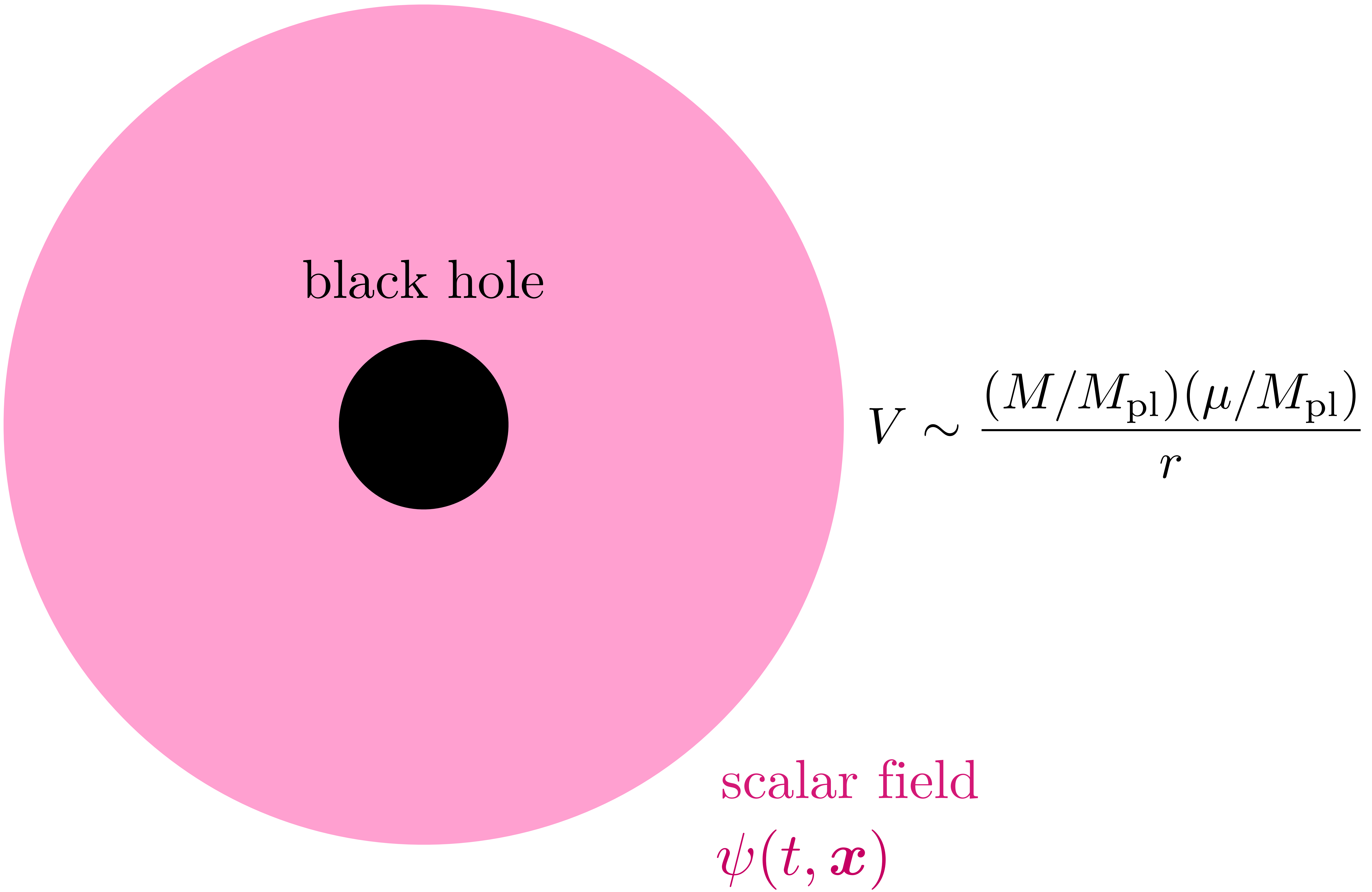


$$V \sim \frac{Q_p Q_e}{r}$$

electron cloud

$\psi_e(t, \boldsymbol{x})$


black hole
scalar field
$\psi(t, \boldsymbol{x})$


black hole
$V \sim \frac{(M/M_{\rm pl})(\mu/M_{\rm pl})}{r}$
scalar field
$\psi(t, \boldsymbol{x})$

from the Schrödinger equation

$$i\dot{\psi} = (H + V)\psi$$

energy levels can be found

$$E_{n\ell m} = -\frac{\alpha^2}{2n^2}$$

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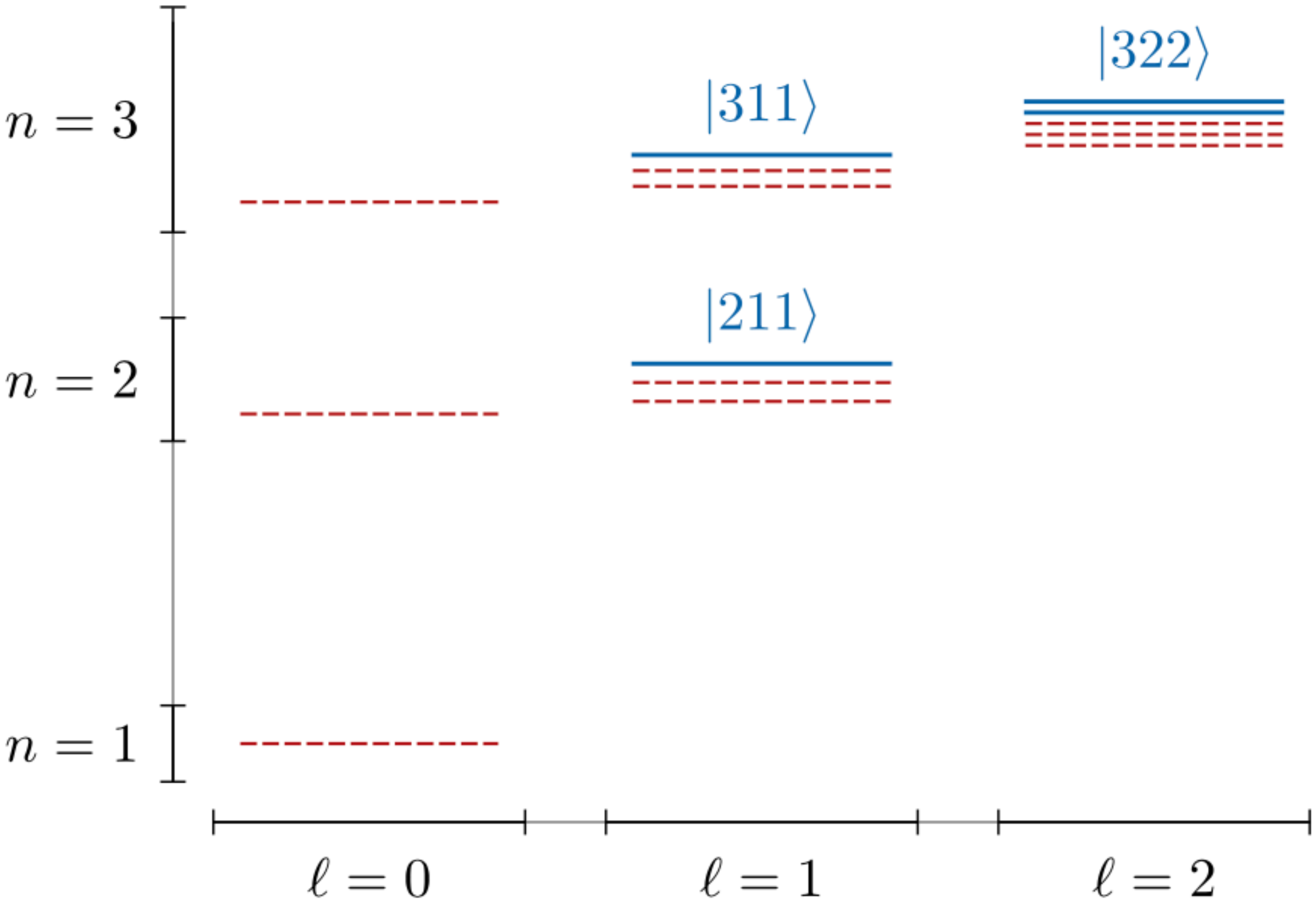
$$+\frac{2a_\star m\alpha^5}{n^3\ell(\ell + 1/2)(\ell + 1)} \text{(hyperfine splitting)}$$



Baumann et al (20)

some differences
from a quantum-mechanical hydrogen atom

1

the system is bosonic;

any level can be occupied by many particles;

the field will be treated as a classical field

2

due to the boundary condition

the energy eigenvalue develops an imarginary part

$$\omega_{n\ell m} = E_{n\ell m} + i\Gamma_{n\ell m}$$

$$\Gamma_{n\ell m} \propto \left(\frac{m a_\star}{2 r_+} - \omega_{n\ell m}\right) \alpha^{4\ell+5}$$

black hole spin

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outer horizon

black hole spin

$$\Gamma_{n\ell m} \propto \left(\frac{m a_\star}{2 r_+} - \omega_{n\ell m}\right) \alpha^{4\ell+5}$$

outer horizon

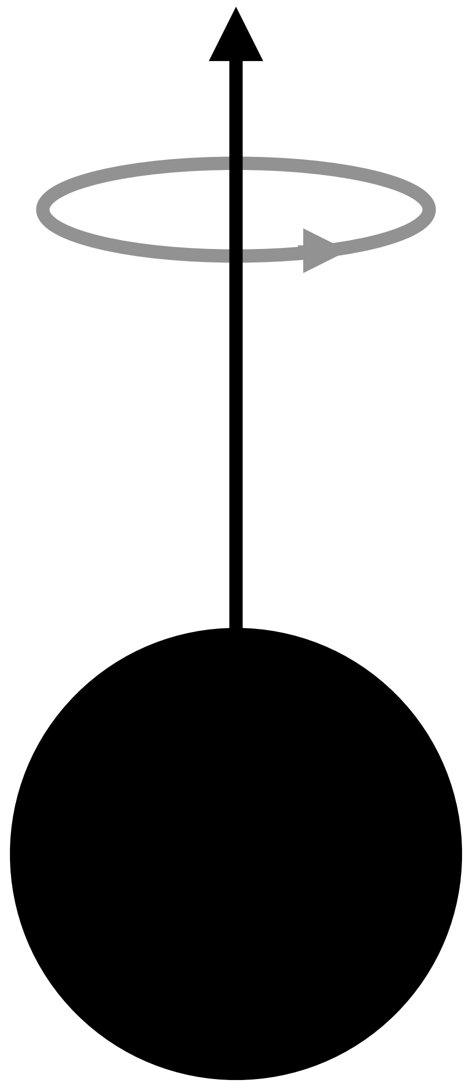
the imaginary part takes a positive sign when

$$0 < \omega_{n\ell m} \approx \mu < \frac{m a_\star}{2 r_+}$$

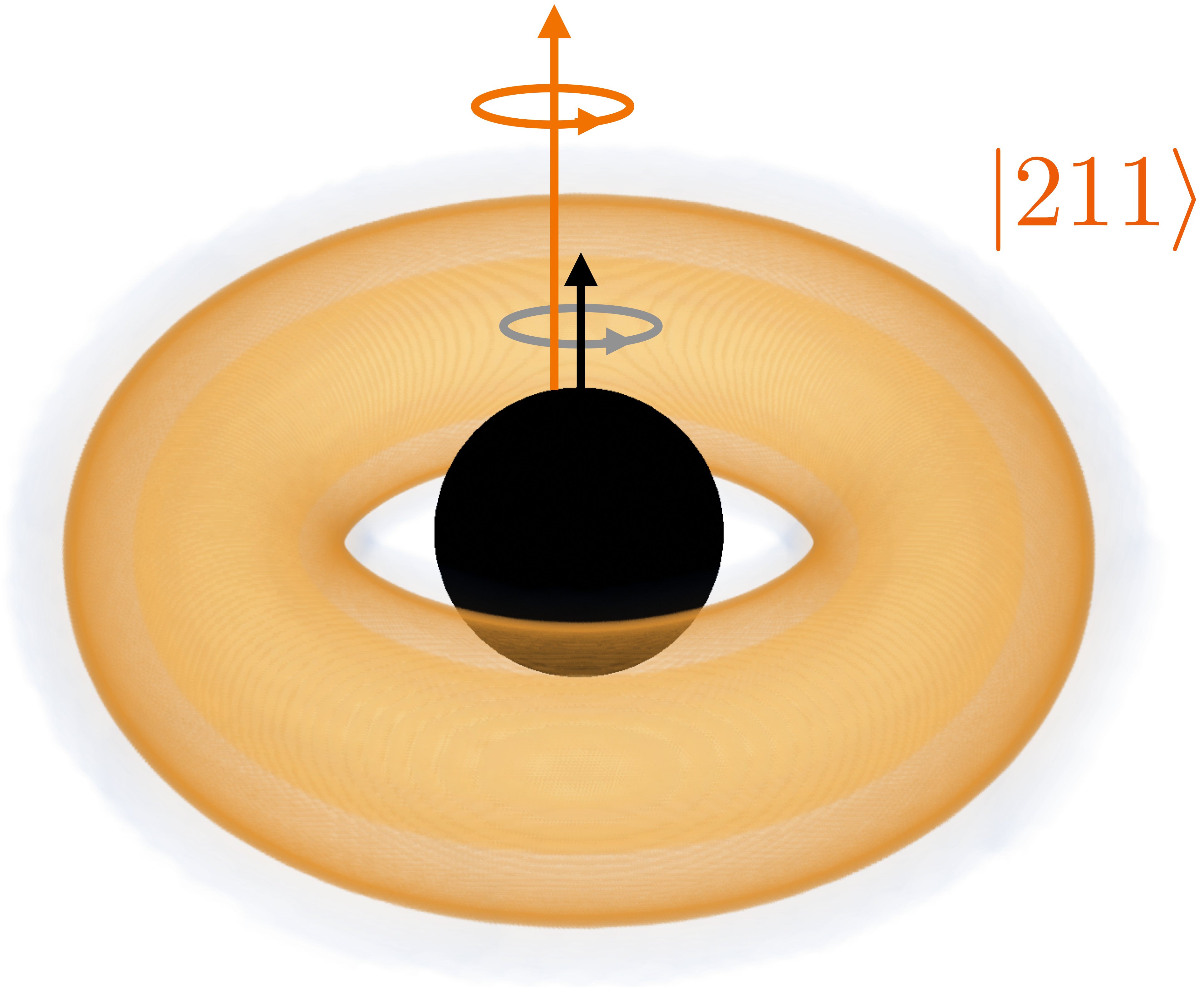
in which superradiance instability is triggered

when superradiance instability is triggered
the bosonic cloud is exponentially produced

$$\psi_{n\ell m}(t) \propto e^{-iE_{n\ell m}t} e^{\Gamma_{n\ell m}t}$$



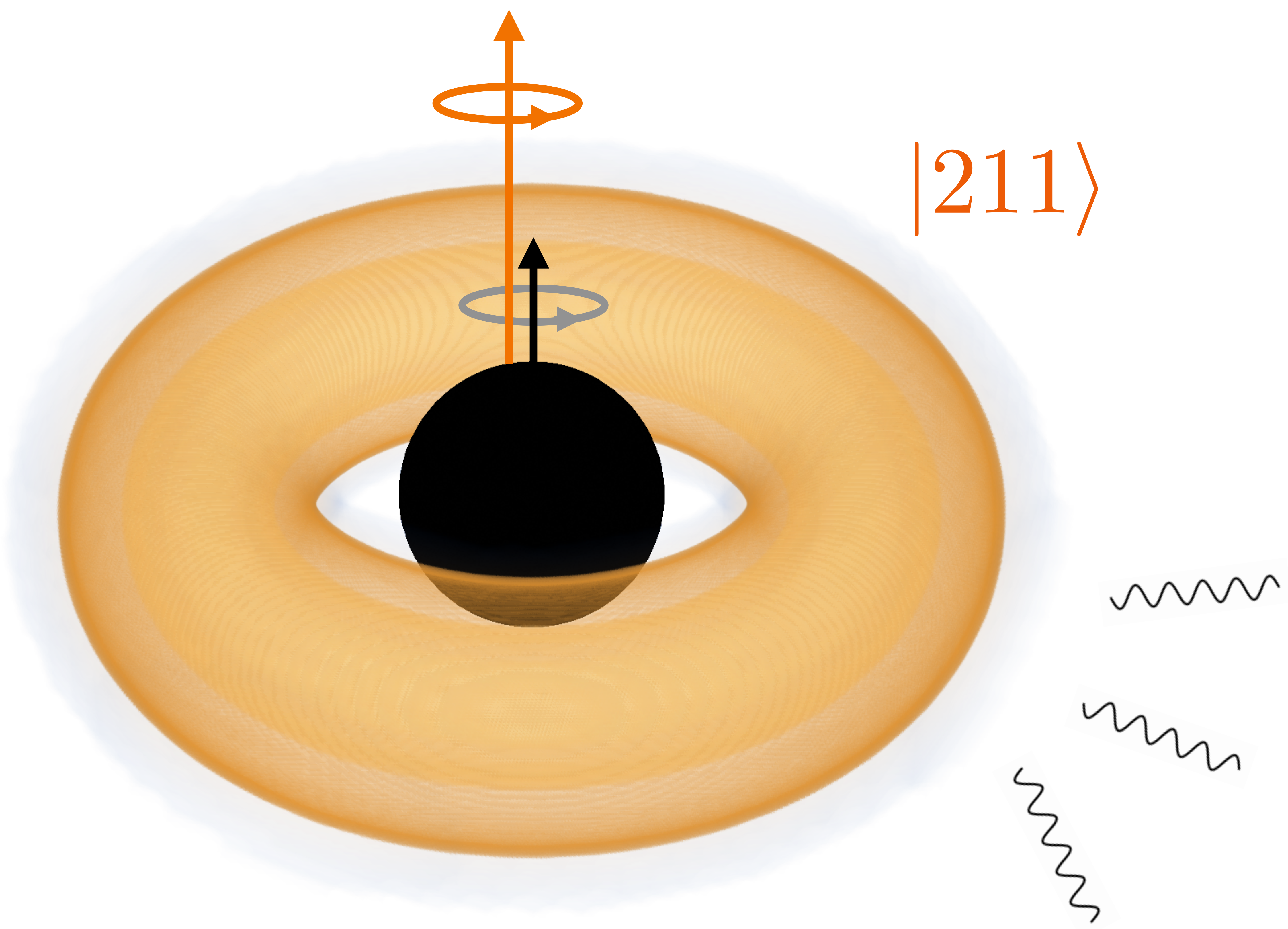
$$0 < \omega < \frac{m a_\star}{2 r_+}$$



Black hole spins down via superradiance

$$0 < \omega < \frac{m a_\star}{2 r_+}$$

Arvanitaki, Dubkovsky (11)
Arvanitaki et al (15)



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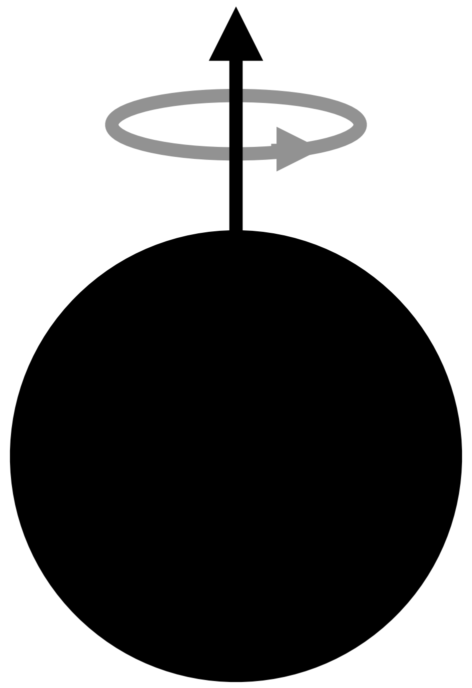

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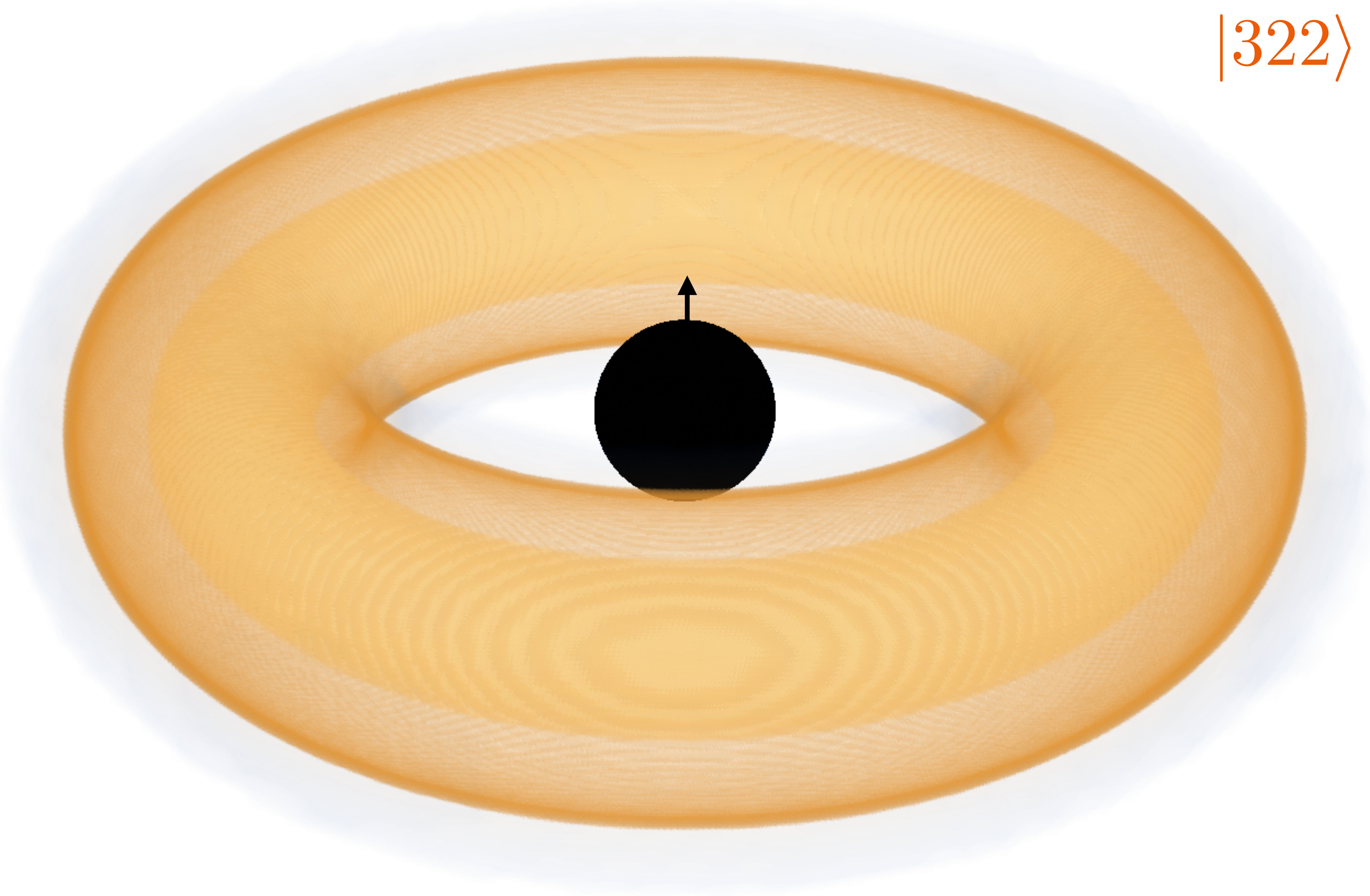
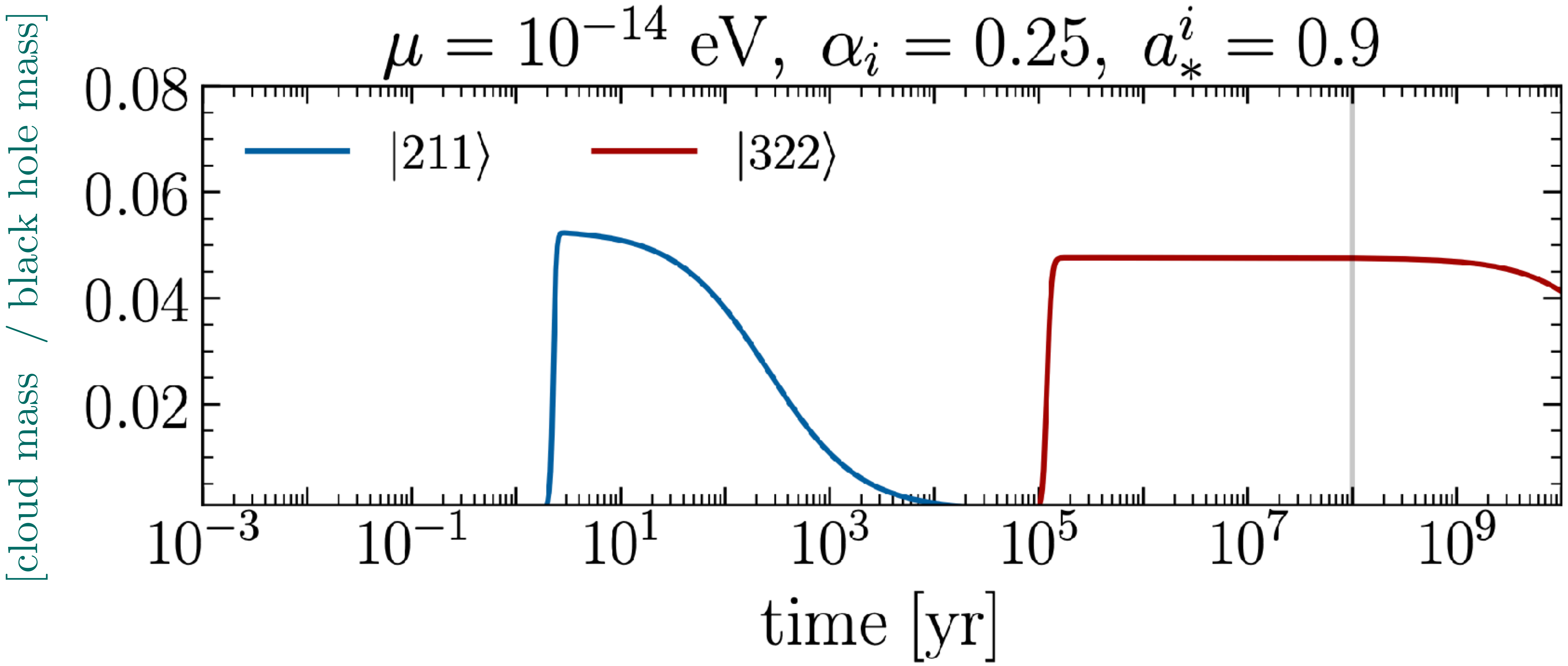
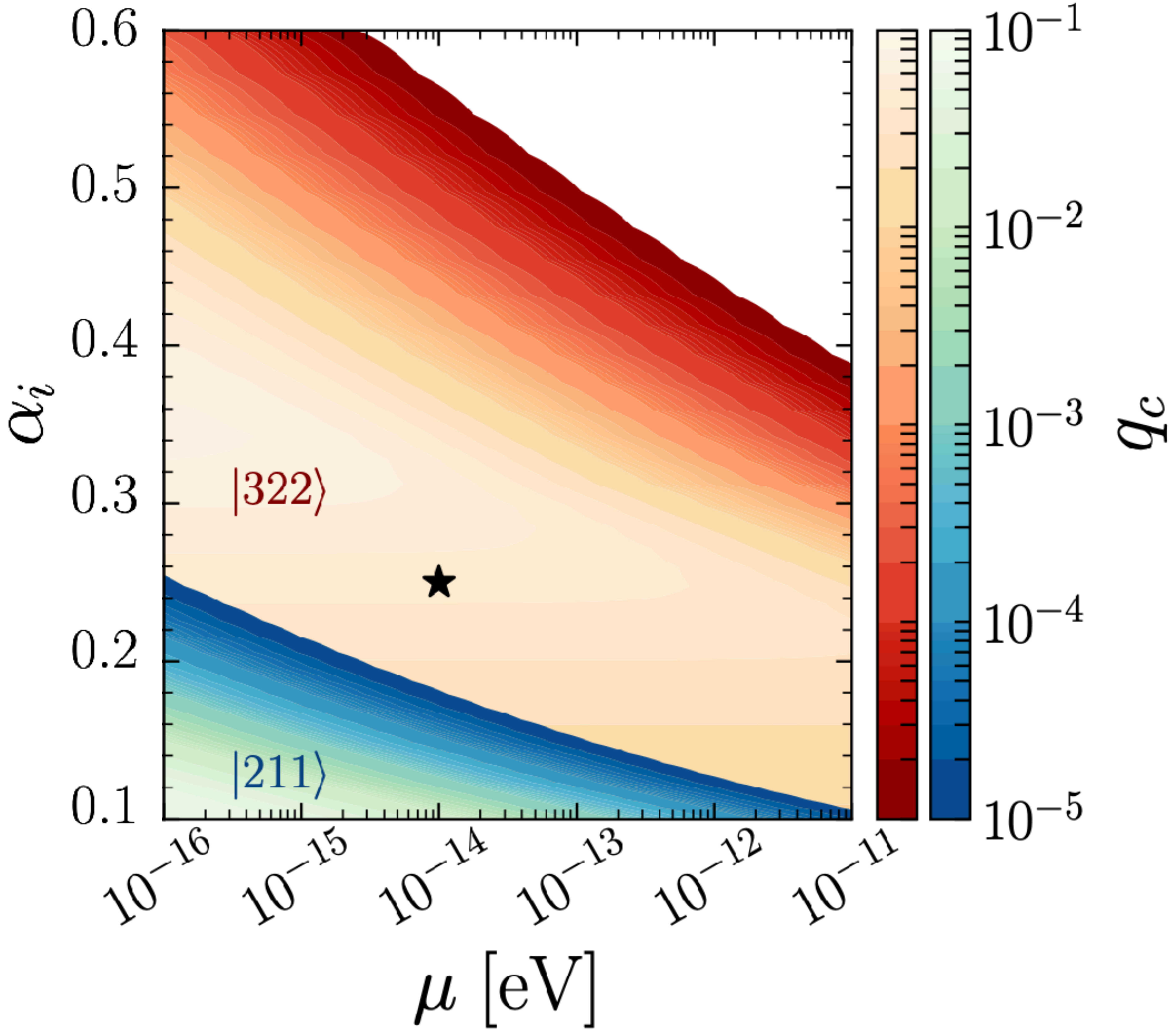

$|322\rangle$



$\mu = 10^{-14}$ eV, $\alpha_i = 0.25$, $a_*^i = 0.9$
|211⟩
|322⟩
0.08
0.06
0.04
0.02
10^{-3}
10^{-1}
10^{1}
10^{3}
10^{5}
10^{7}
10^{9}
time [yr]
[cloud mass / black hole mass]



0.6
0.5
0.4
0.3
0.2
0.1
α_i
$|322\rangle$
$|211\rangle$
10^{-16}
10^{-15}
10^{-14}
10^{-13}
10^{-12}
10^{-11}
μ [eV]
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
q_c

Another possibility:

when a black hole forms a binary with a secondary object


$|322\rangle$

when the orbital freq. matches to the level spacing of this atom

$$\Delta m \Omega = \Delta E$$


when the orbital freq. matches to the level spacing of this atom
$\Delta m \Omega = \Delta E$
$|322\rangle$
resonant transition may occur


$|320\rangle$

such transitions backreacts on the orbital dynamics
affecting the gravitational wave emission of the binary

$$i\dot{\psi} = \left(-\frac{\nabla^2}{2\mu} - \frac{\alpha}{r} + V_\star \right) \psi$$

$$V_\star \sim -\frac{G M_1 M_2}{|\boldsymbol{r} - \boldsymbol{r}_\star(t)|}$$

the secondary object is a time-dependent perturbation for a better (analytic) unerstanding of the system

- bosonic cloud dominantly in 322 state
- prograde, quasi-circular, equatorial orbit
- approximate the system to a 2-level system

$\phi_\star(t) \simeq \Omega t$

the potential can be expanded as

$$V_\star \sim -\frac{GM_1M_2}{|\boldsymbol{r} - \boldsymbol{r}_\star(t)|} = \sum_{\ell_\star m_\star} V_{\ell_\star m_\star} e^{-i m_\star \phi_\star(t)}$$

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$$V_\star \sim -\frac{GM_1M_2}{|\boldsymbol{r}-\boldsymbol{r}_\star(t)|} = \sum_{\ell_\star m_\star} V_{\ell_\star m_\star} e^{-im_\star \phi_\star(t)}$$

with a 2-level approximation
we may write a generic state as

$$|\psi(t)\rangle = c_1(t)|1\rangle + c_2(t)|2\rangle$$

from which we obtain the following Hamiltonian

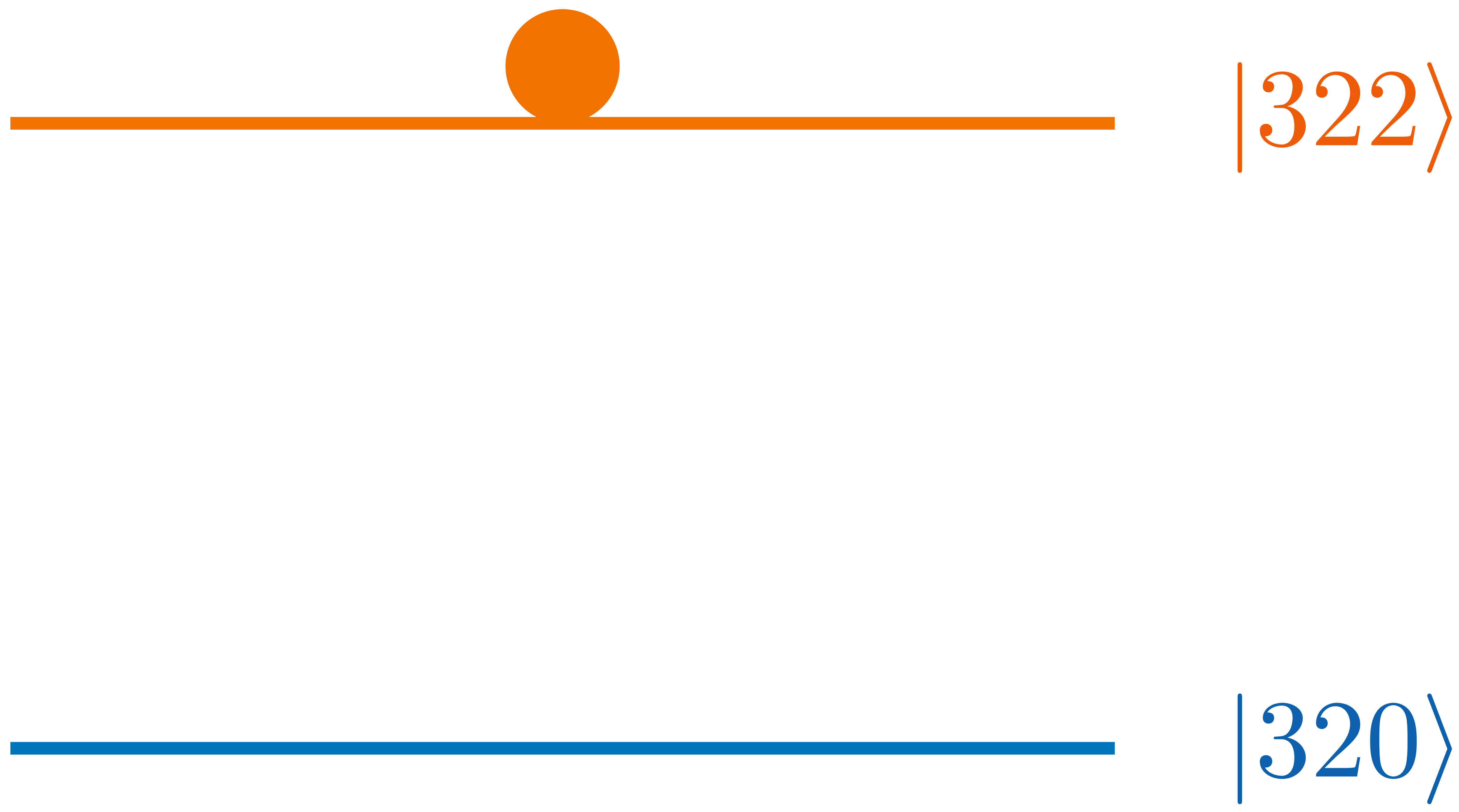
$$i\dot{\boldsymbol{c}} \simeq \begin{pmatrix} E_1 & \gamma e^{-i\Delta m\Omega t} \\ \gamma^* e^{i\Delta m\Omega t} & E_2 \end{pmatrix} \boldsymbol{c}$$

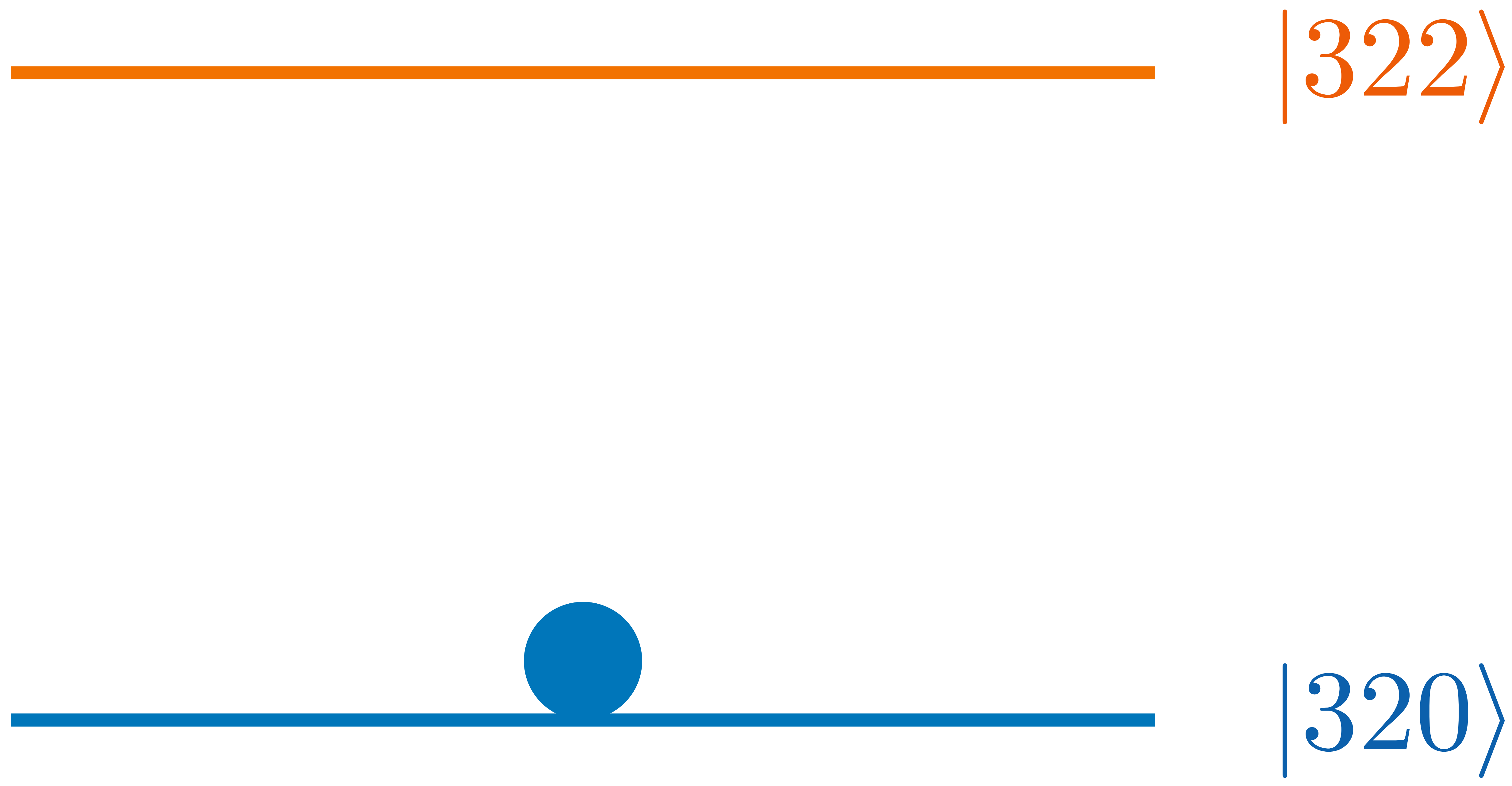
from which we obtain the following Hamiltonian

$$i\dot{\boldsymbol{c}} \simeq \begin{pmatrix} E_1 & \gamma e^{-i\Delta m\Omega t} \\ \gamma^* e^{i\Delta m\Omega t} & E_2 \end{pmatrix} \boldsymbol{c}$$

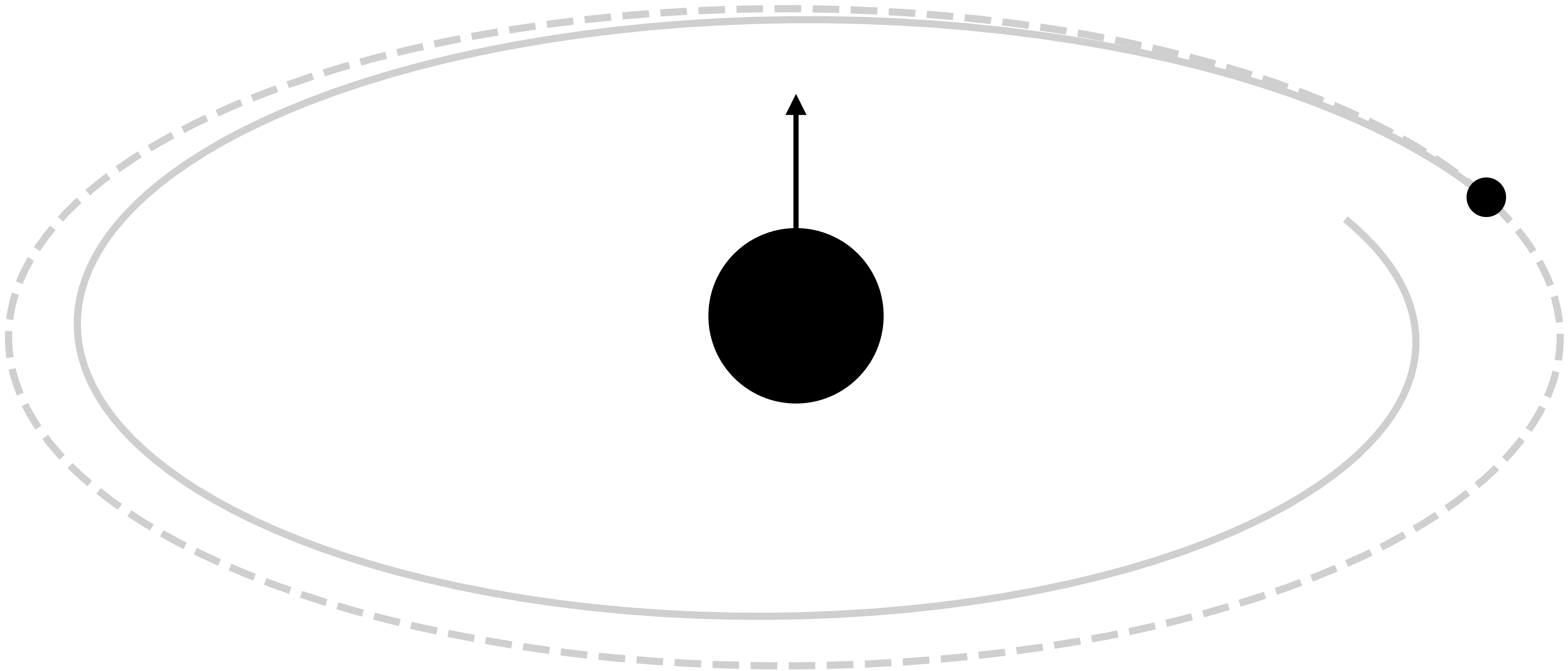
the secondary object acts as an external source
and triggers resonant transition when

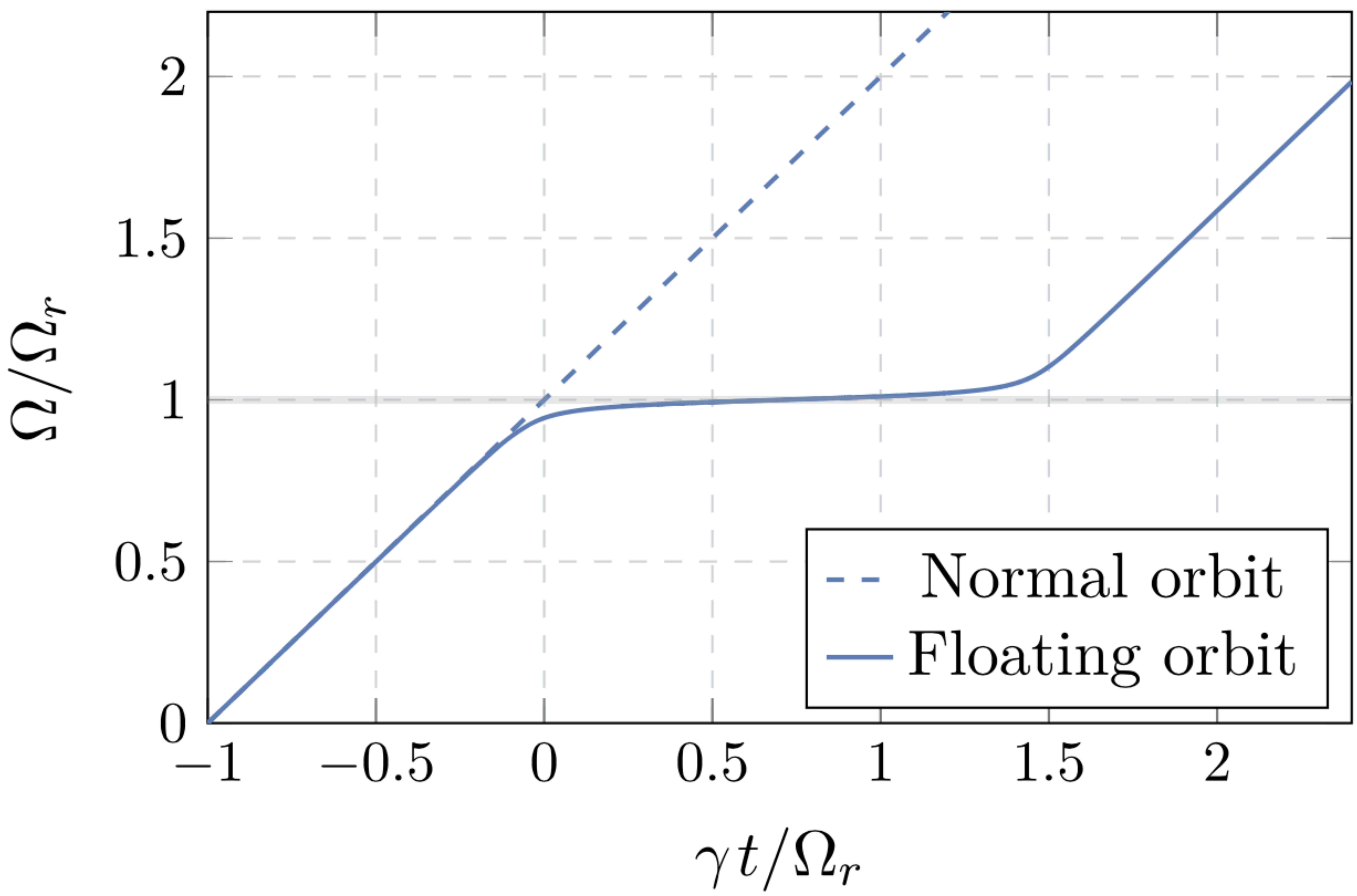
$$\Delta E = \Delta m\Omega$$


$|322\rangle$
$|320\rangle$



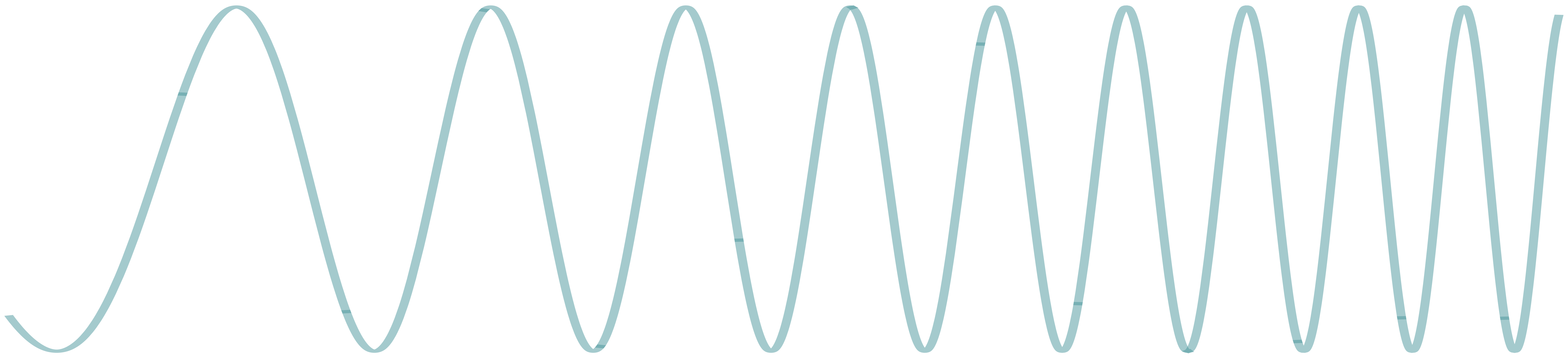
$|322\rangle$
$|320\rangle$





Baumann et al (19), (20)

schematically the waveform would look like



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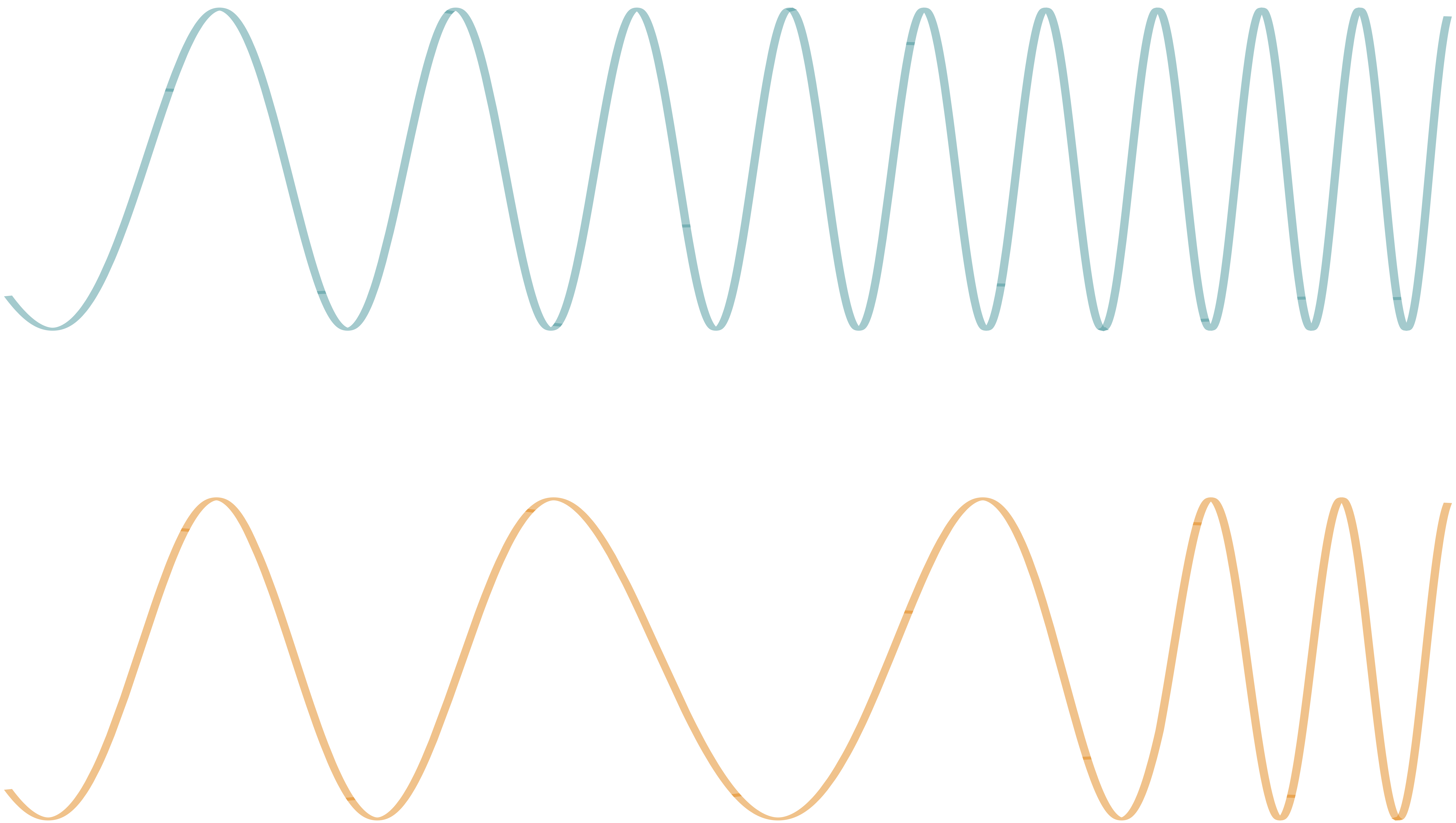

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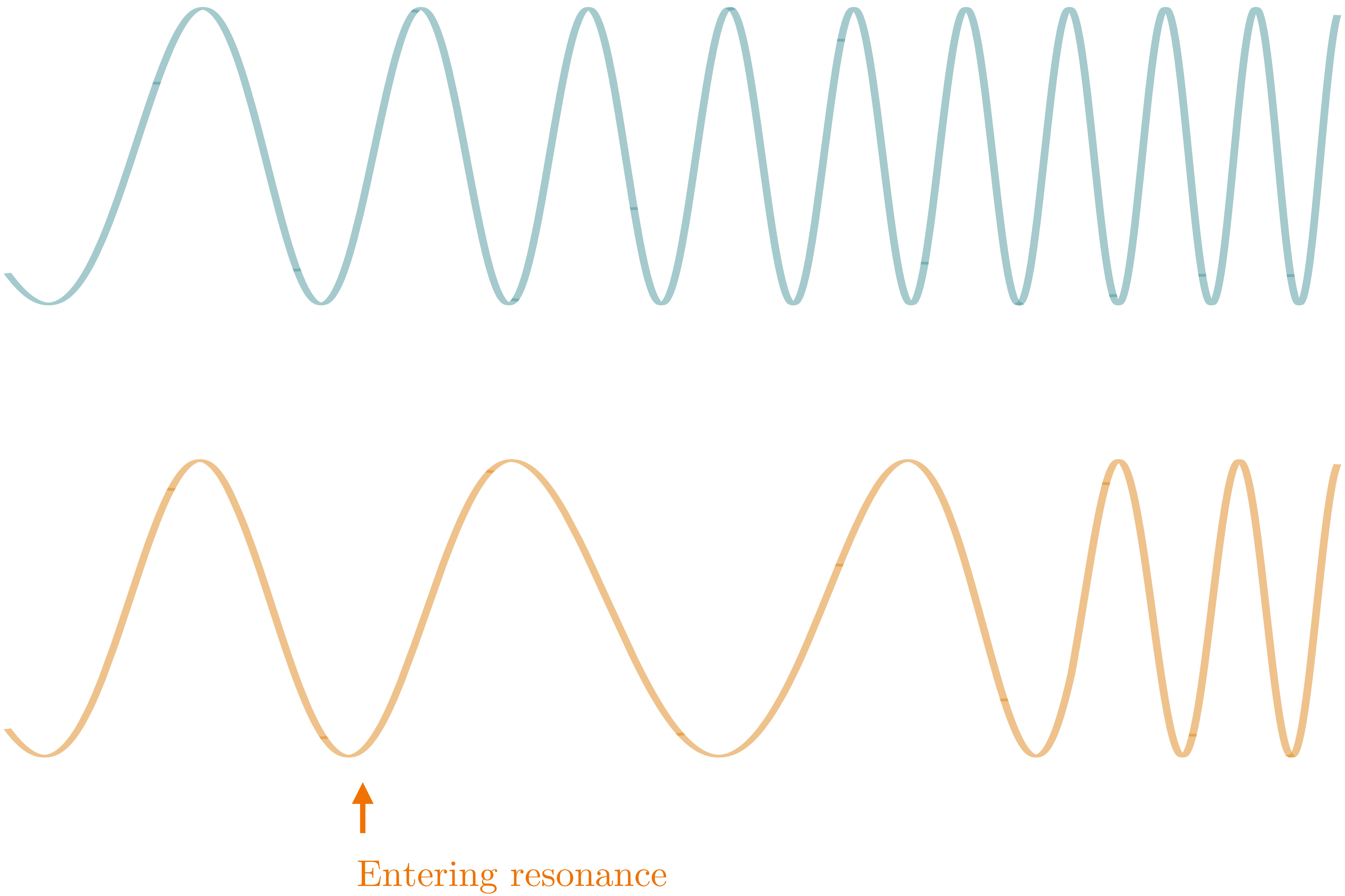

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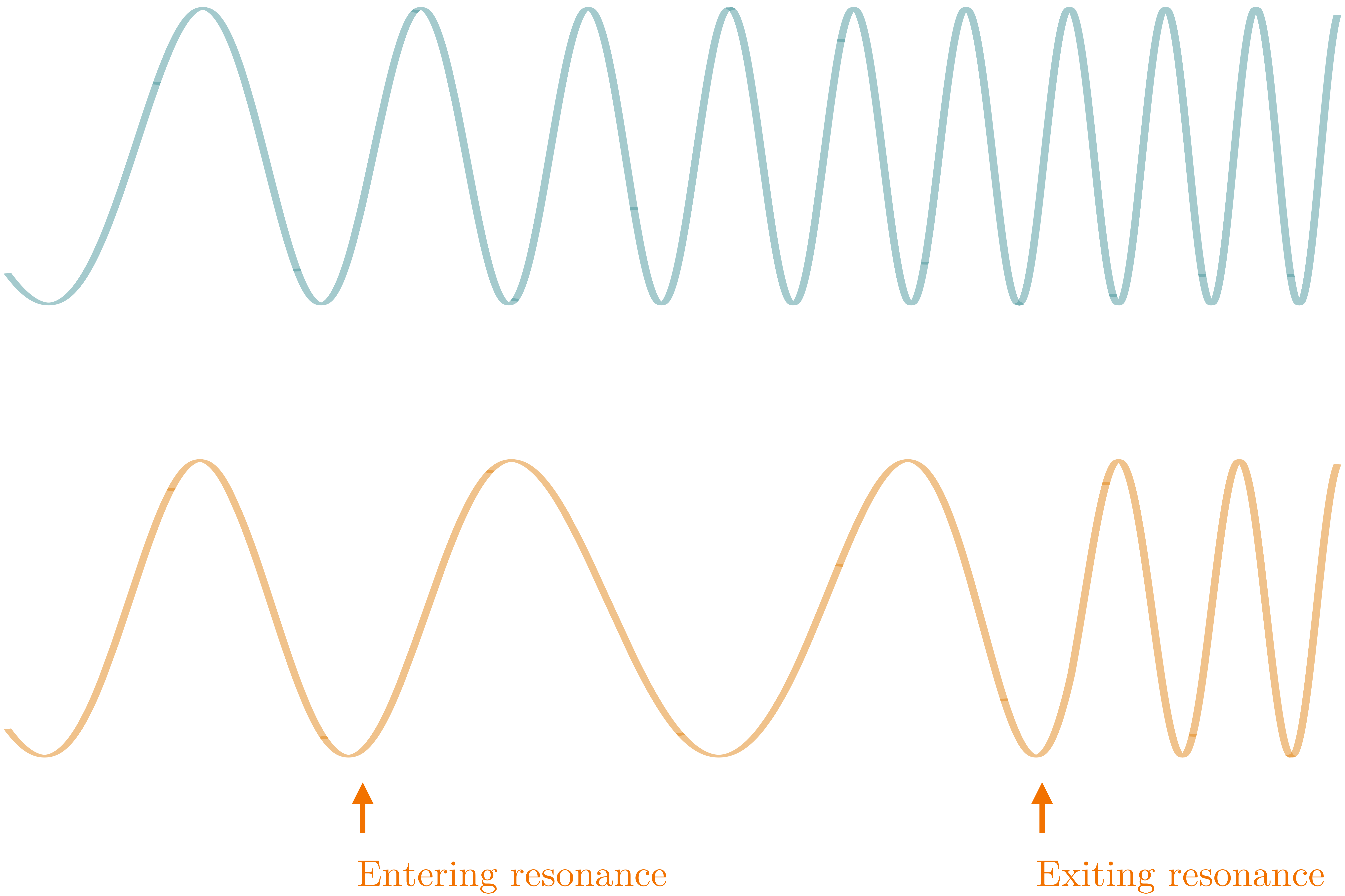
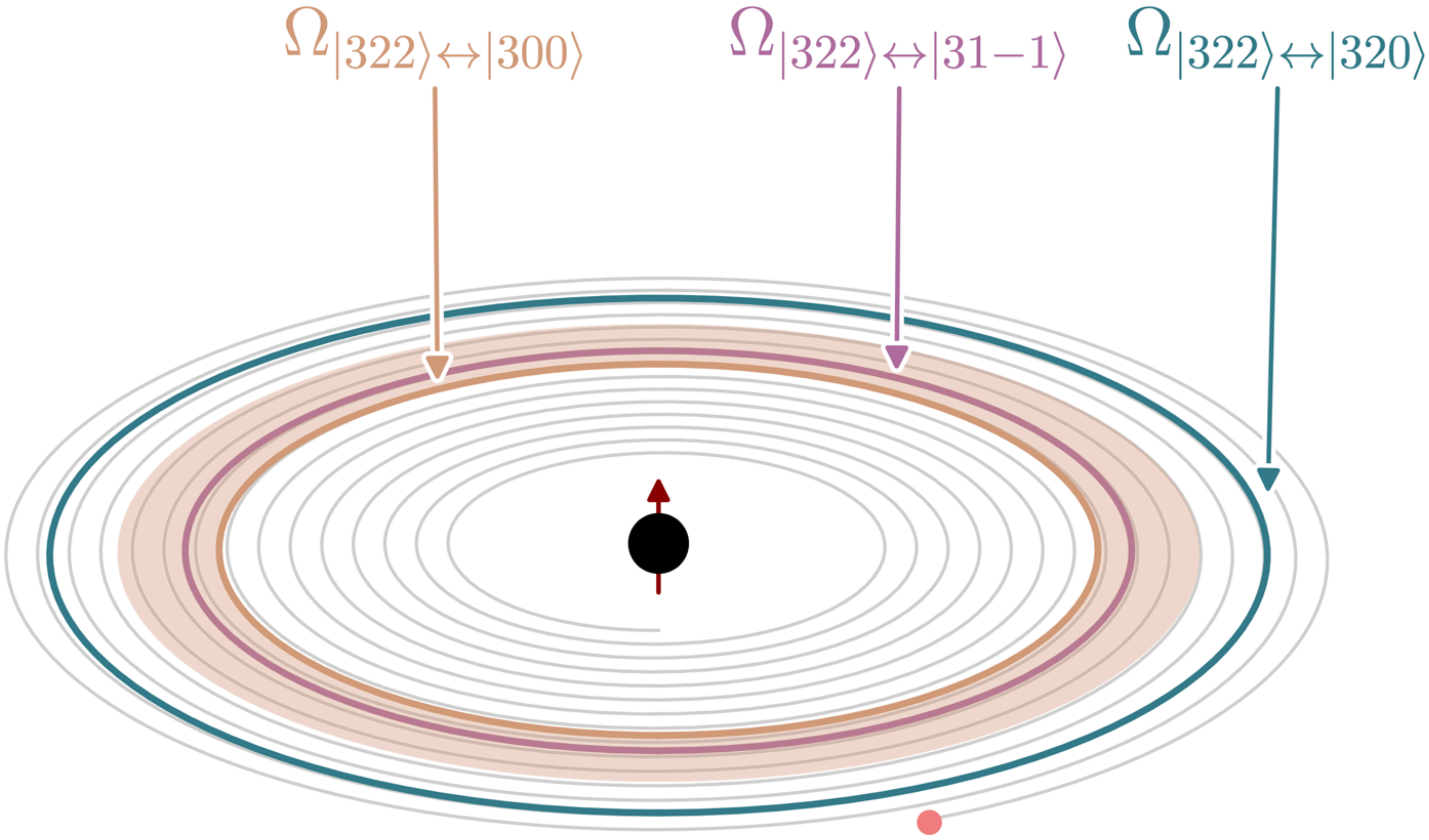
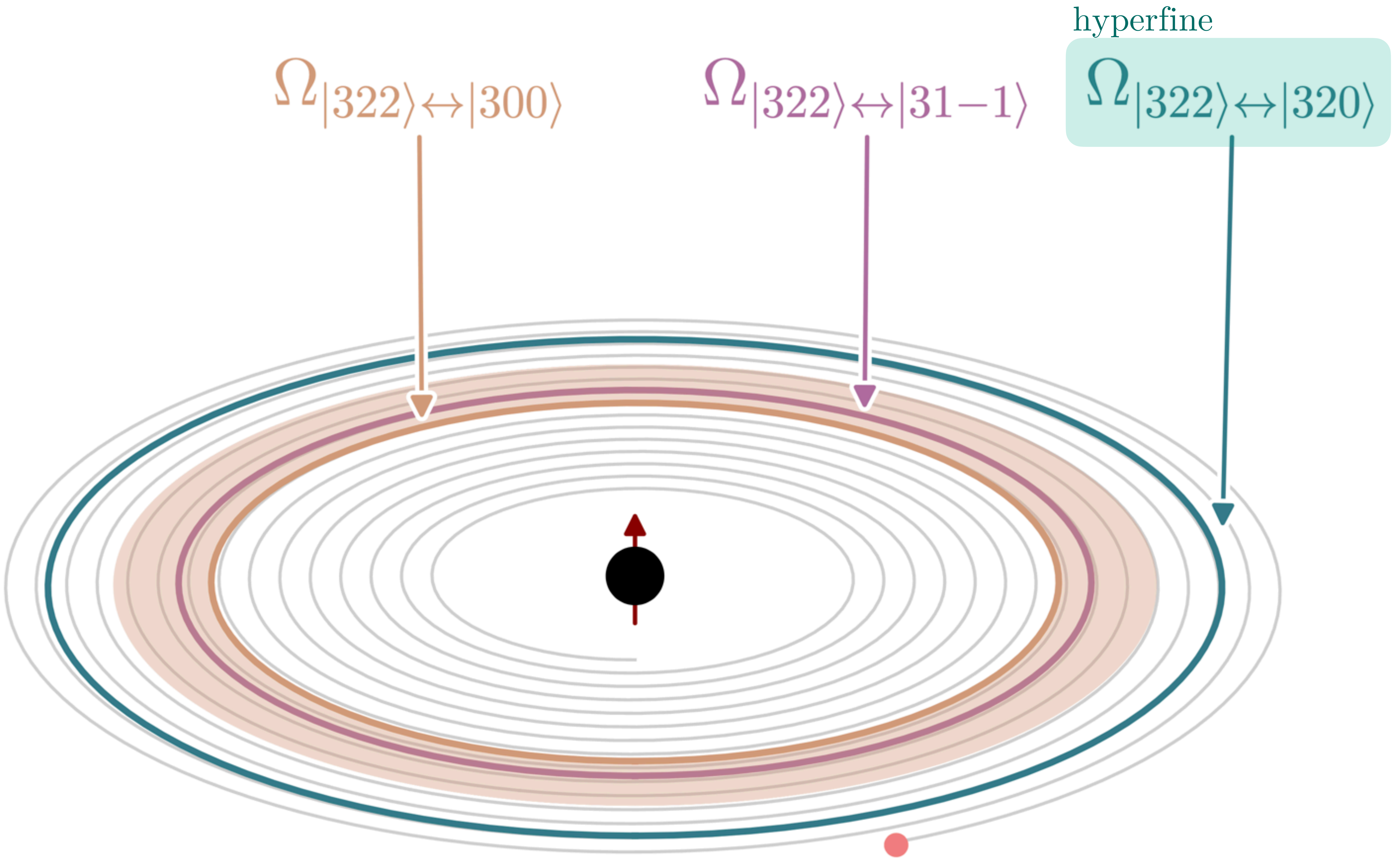
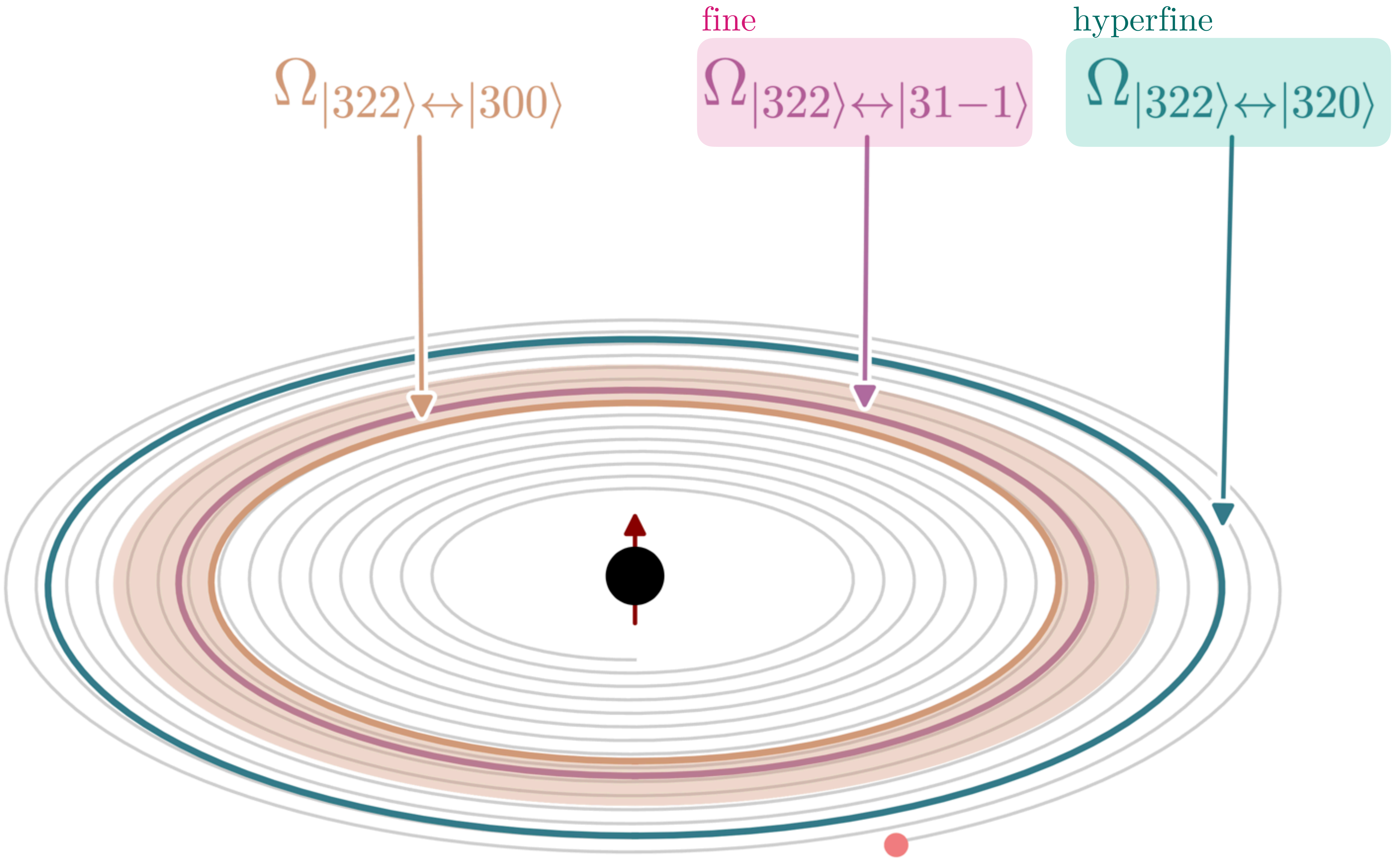




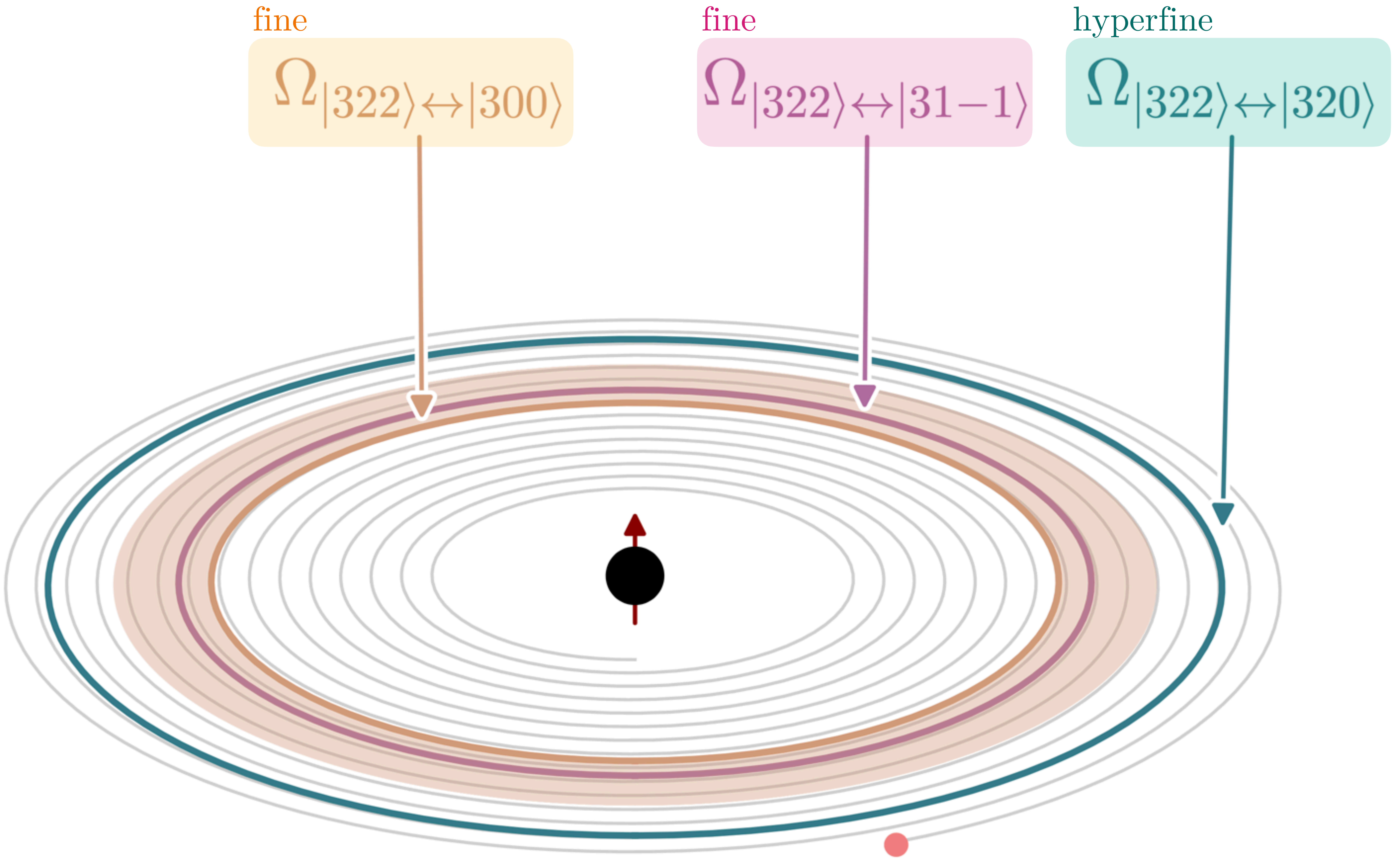
$\Omega_{|322\rangle\leftrightarrow|300\rangle}$
$\Omega_{|322\rangle\leftrightarrow|31-1\rangle}$
$\Omega_{|322\rangle\leftrightarrow|320\rangle}$



hyperfine
$\Omega_{|322\rangle\leftrightarrow|300\rangle}$
$\Omega_{|322\rangle\leftrightarrow|31-1\rangle}$
$\Omega_{|322\rangle\leftrightarrow|320\rangle}$



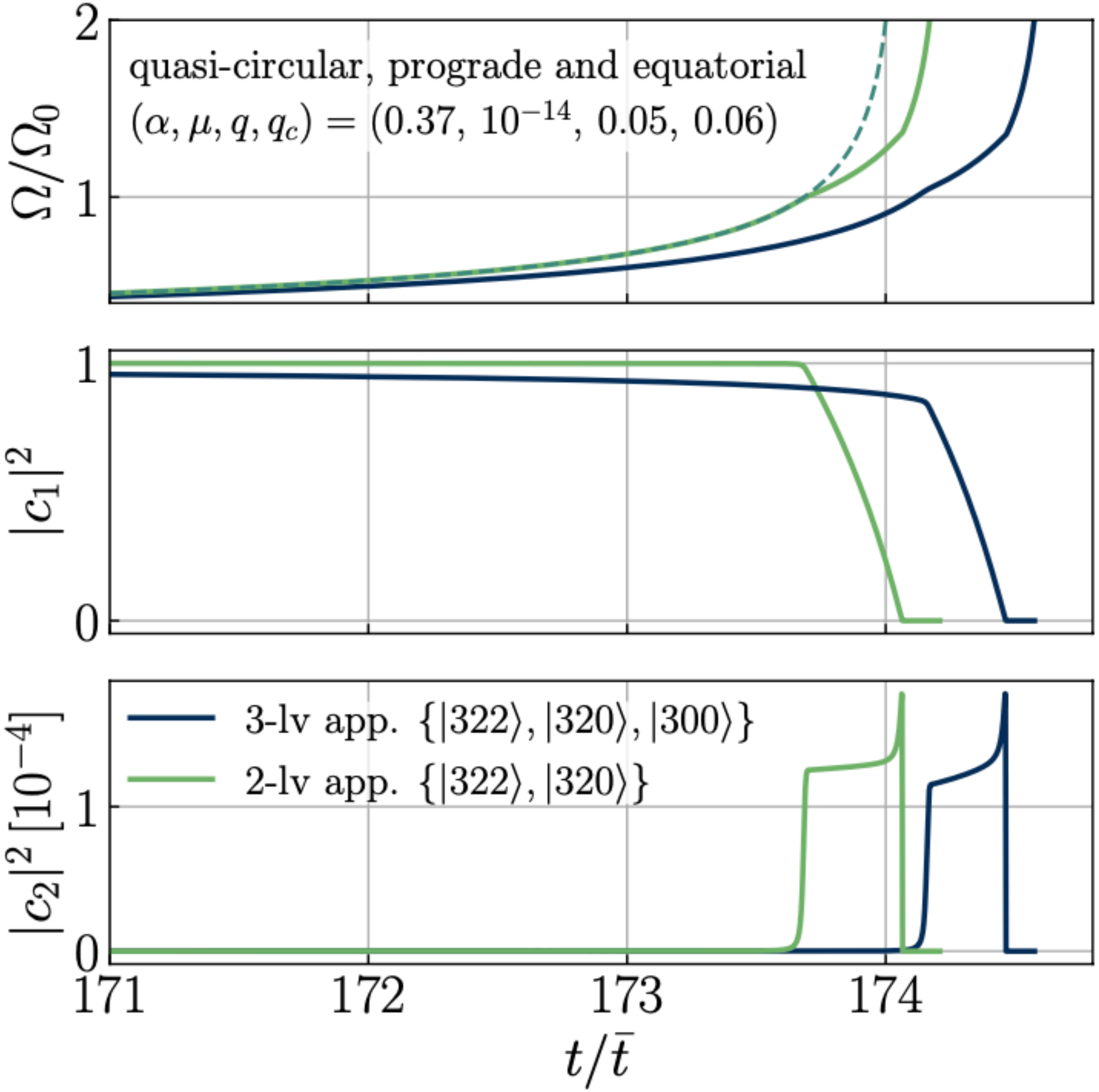
$\Omega_{|322\rangle \leftrightarrow |300\rangle}$
fine
$\Omega_{|322\rangle \leftrightarrow |31-1\rangle}$
hyperfine
$\Omega_{|322\rangle \leftrightarrow |320\rangle}$



fine
$\Omega_{|322\rangle \leftrightarrow |300\rangle}$
fine
$\Omega_{|322\rangle \leftrightarrow |31-1\rangle}$
hyperfine
$\Omega_{|322\rangle \leftrightarrow |320\rangle}$

$|322\rangle \leftrightarrow |320\rangle$

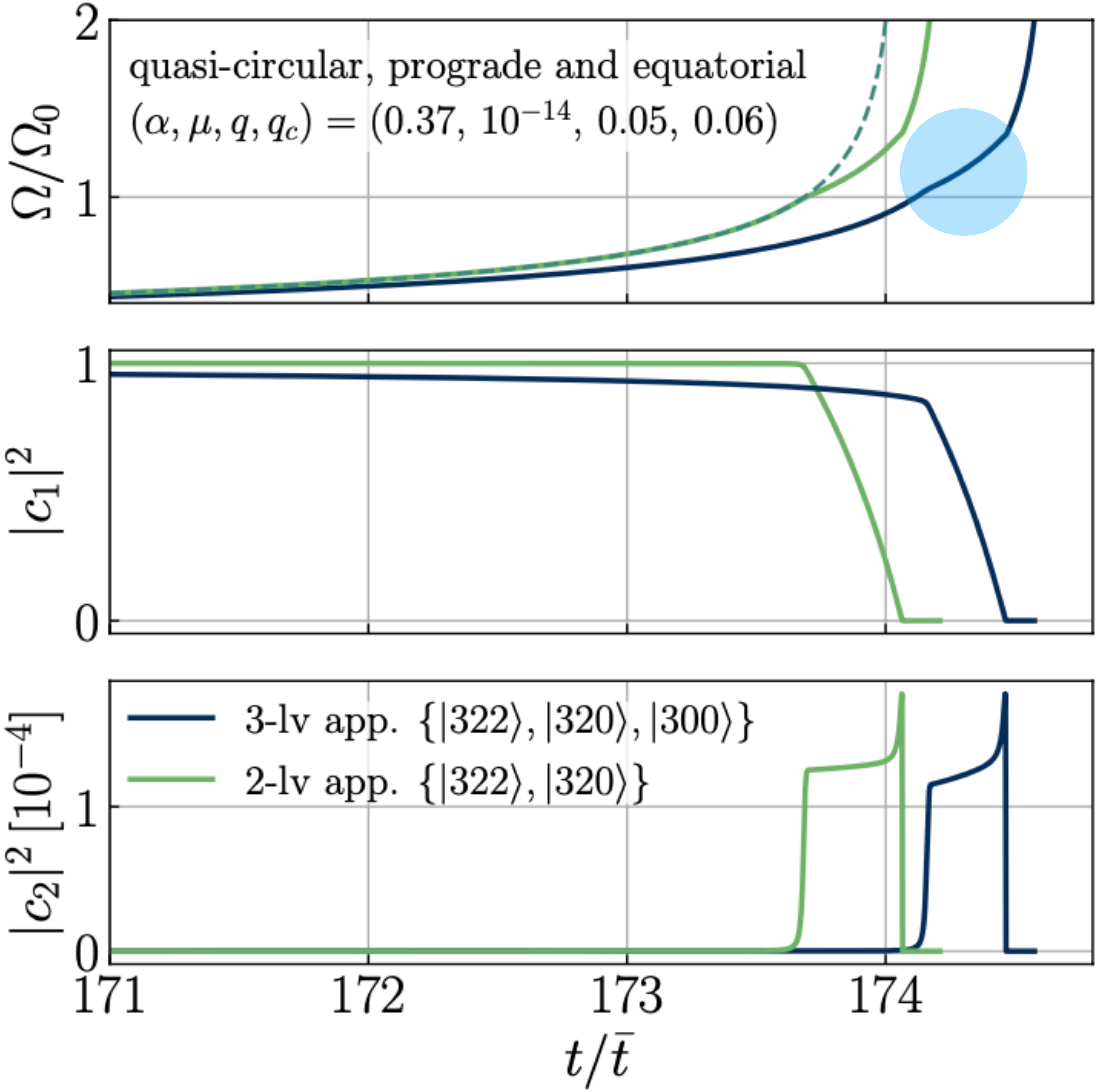
hyperfine



Kim, Lenoci (25)

$|322\rangle \leftrightarrow |320\rangle$

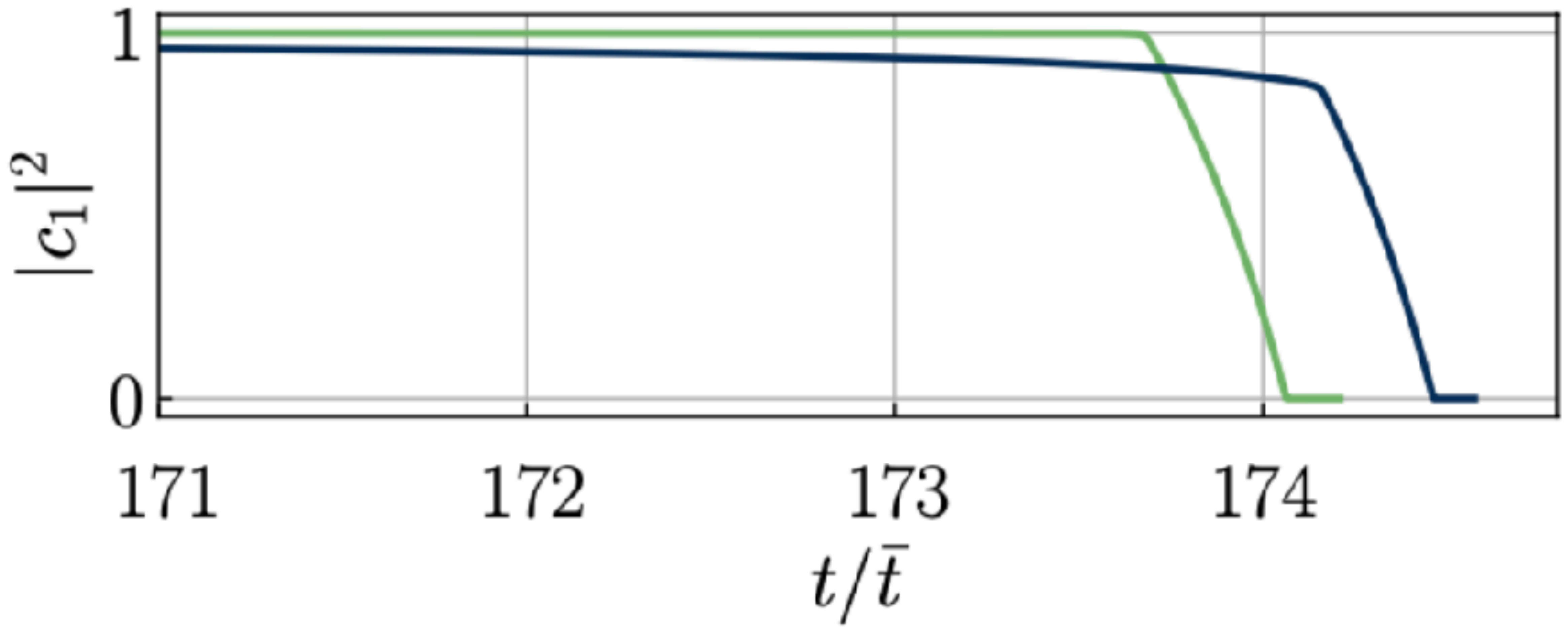
hyperfine



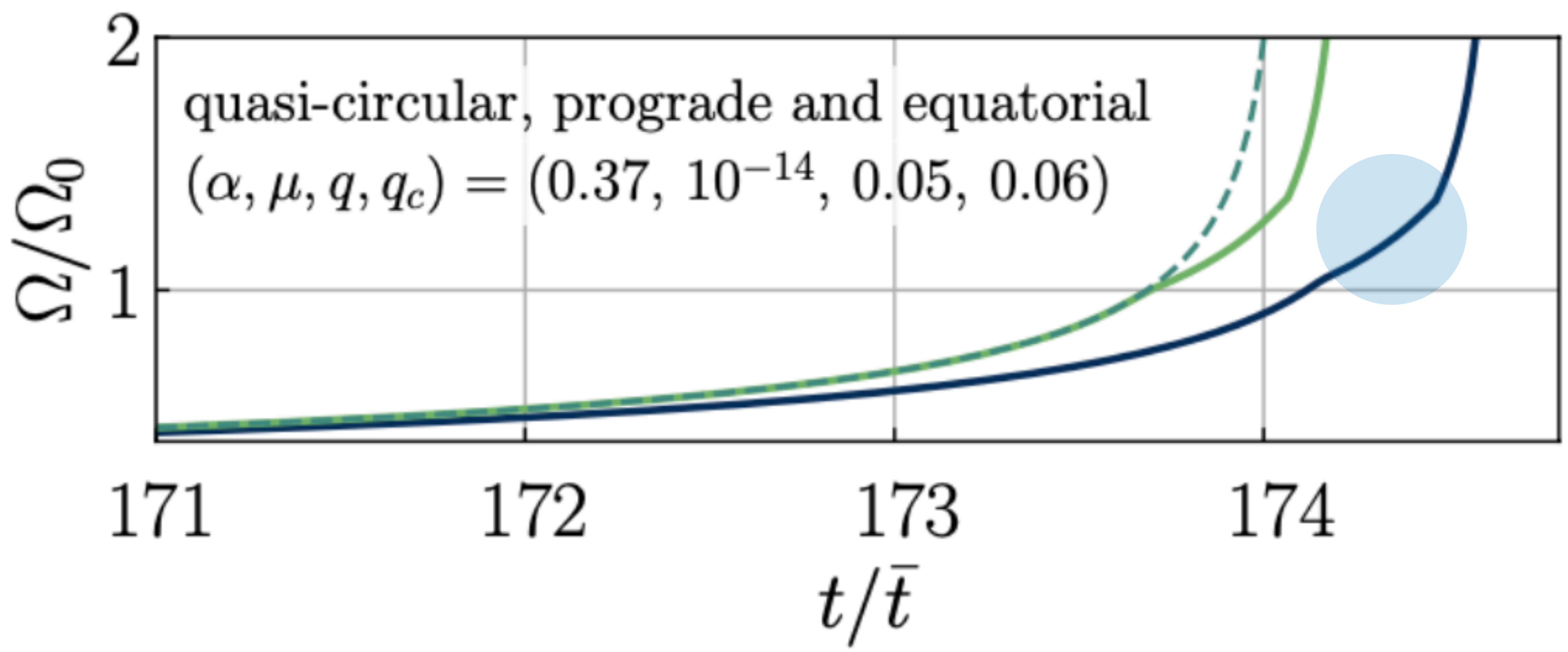
Kim, Lenoci (25)

this resonance is adiabatic

and the binary tends to exhaust the entire cloud



only one chance to measure
the existence of cloud from GWs



$$f_{\rm GW} \sim 10^{-3}\,{\rm Hz}$$

$$\dot{f}_{\rm GW} \sim 10^{-7}\,{\rm Hz/yr}$$

it falls into LISA band
but it is still challenging to measure
as it is almost like monochromatic signal

the sequence of resonance depends on
the energy levels obtained assuming

$$i\dot{\psi} \approx \left(-\frac{\nabla^2}{2\mu} - \frac{\alpha}{r}\right)\psi$$

as the boson cloud takes a significant fraction of mass from the black hole,

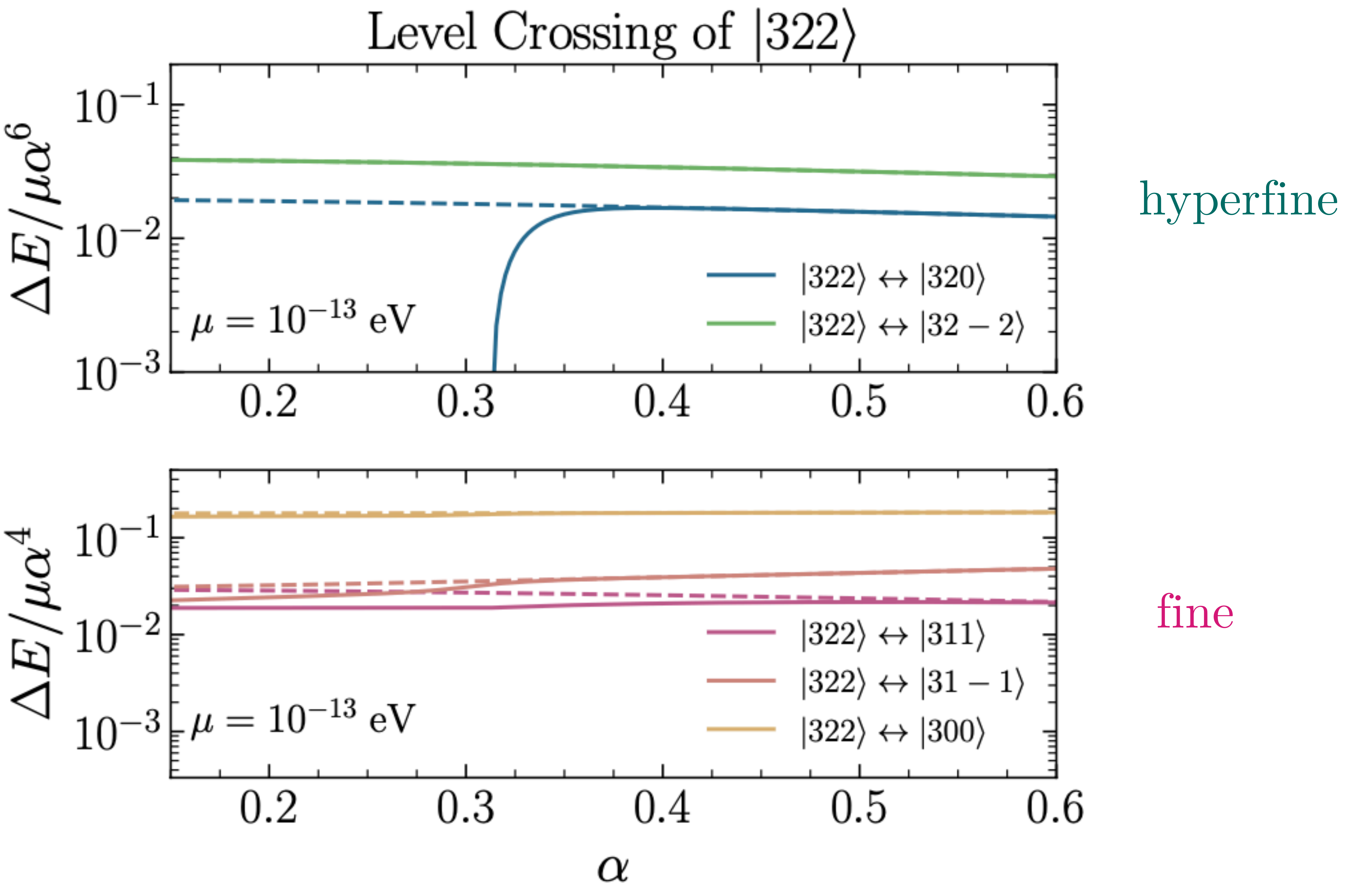
its own gravitational potential affects its own energy level

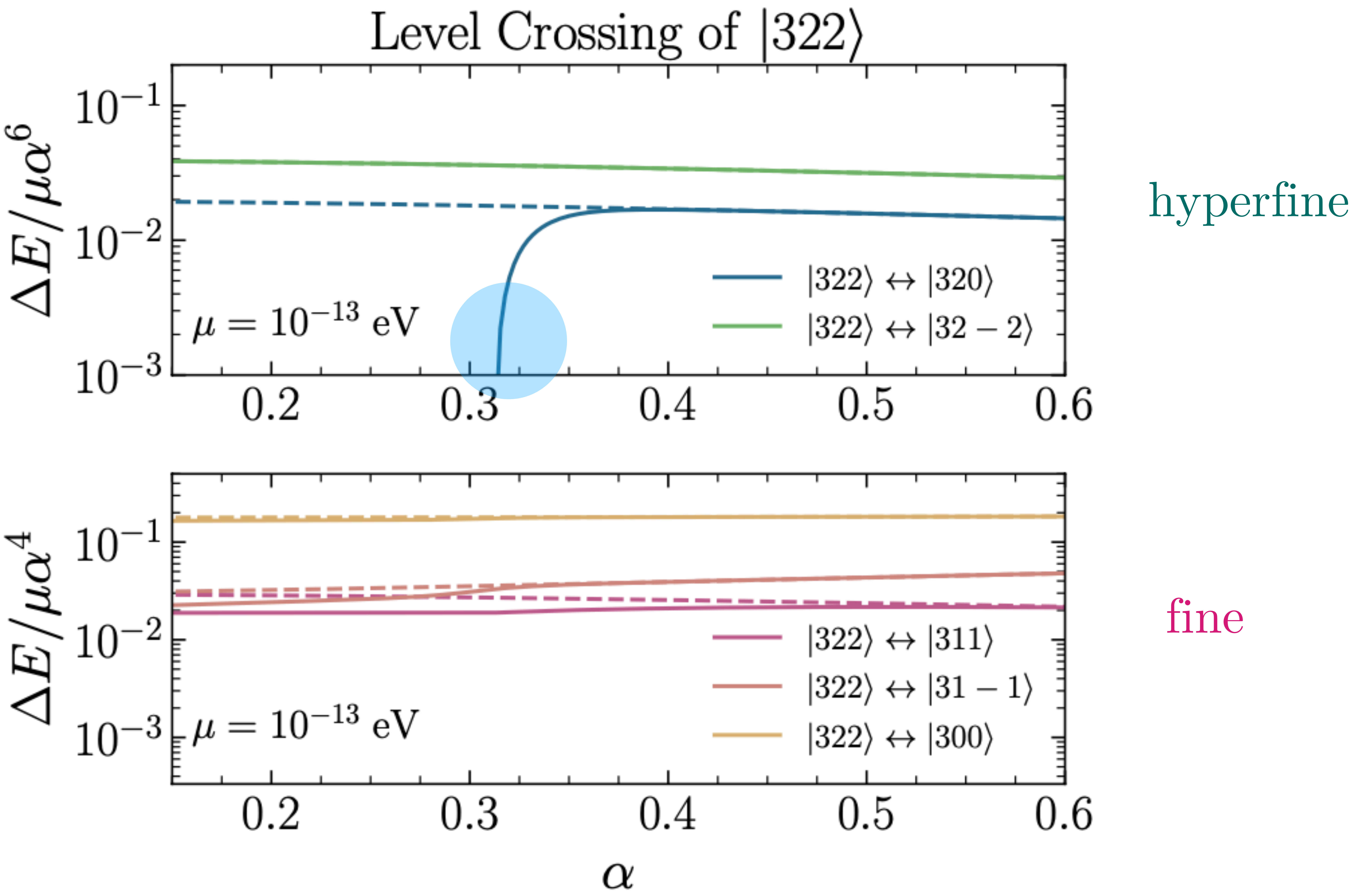
$$i\dot{\psi} \approx \left(-\frac{\nabla^2}{2\mu} - \frac{\alpha}{r} + V_s\right)\psi$$

$$V_s \sim -\int d^3r' \, |\psi(r')|^2 \frac{GM_c}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

this leads to

density-dependent corrections to energy levels


Level Crossing of $|322\rangle$
$\Delta E/\mu\alpha^6$
$\mu = 10^{-13}$ eV
$|322\rangle \leftrightarrow |320\rangle$
$|322\rangle \leftrightarrow |32-2\rangle$
hyperfine
$\Delta E/\mu\alpha^4$
$\mu = 10^{-13}$ eV
$|322\rangle \leftrightarrow |311\rangle$
$|322\rangle \leftrightarrow |31-1\rangle$
$|322\rangle \leftrightarrow |300\rangle$
fine
α



Level Crossing of $|322\rangle$
$\Delta E/\mu\alpha^6$
10^{-1}
10^{-2}
10^{-3}
$\mu = 10^{-13}$ eV
$|322\rangle \leftrightarrow |320\rangle$
$|322\rangle \leftrightarrow |32-2\rangle$
0.2
0.3
0.4
0.5
0.6
hyperfine
$\Delta E/\mu\alpha^4$
10^{-1}
10^{-2}
10^{-3}
$\mu = 10^{-13}$ eV
$|322\rangle \leftrightarrow |311\rangle$
$|322\rangle \leftrightarrow |31-1\rangle$
$|322\rangle \leftrightarrow |300\rangle$
0.2
0.3
0.4
0.5
0.6
α
fine

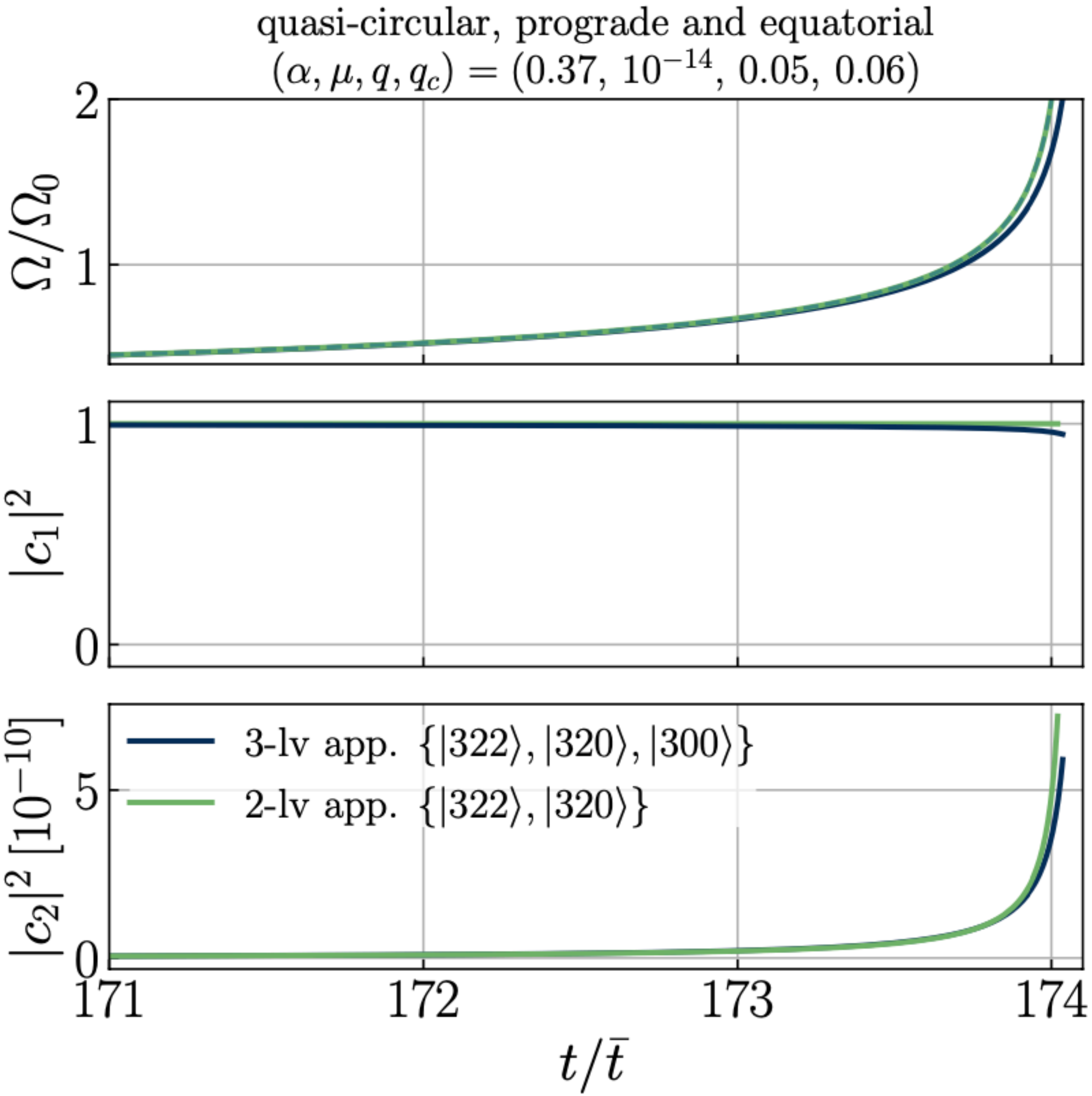
with the prograde orbit

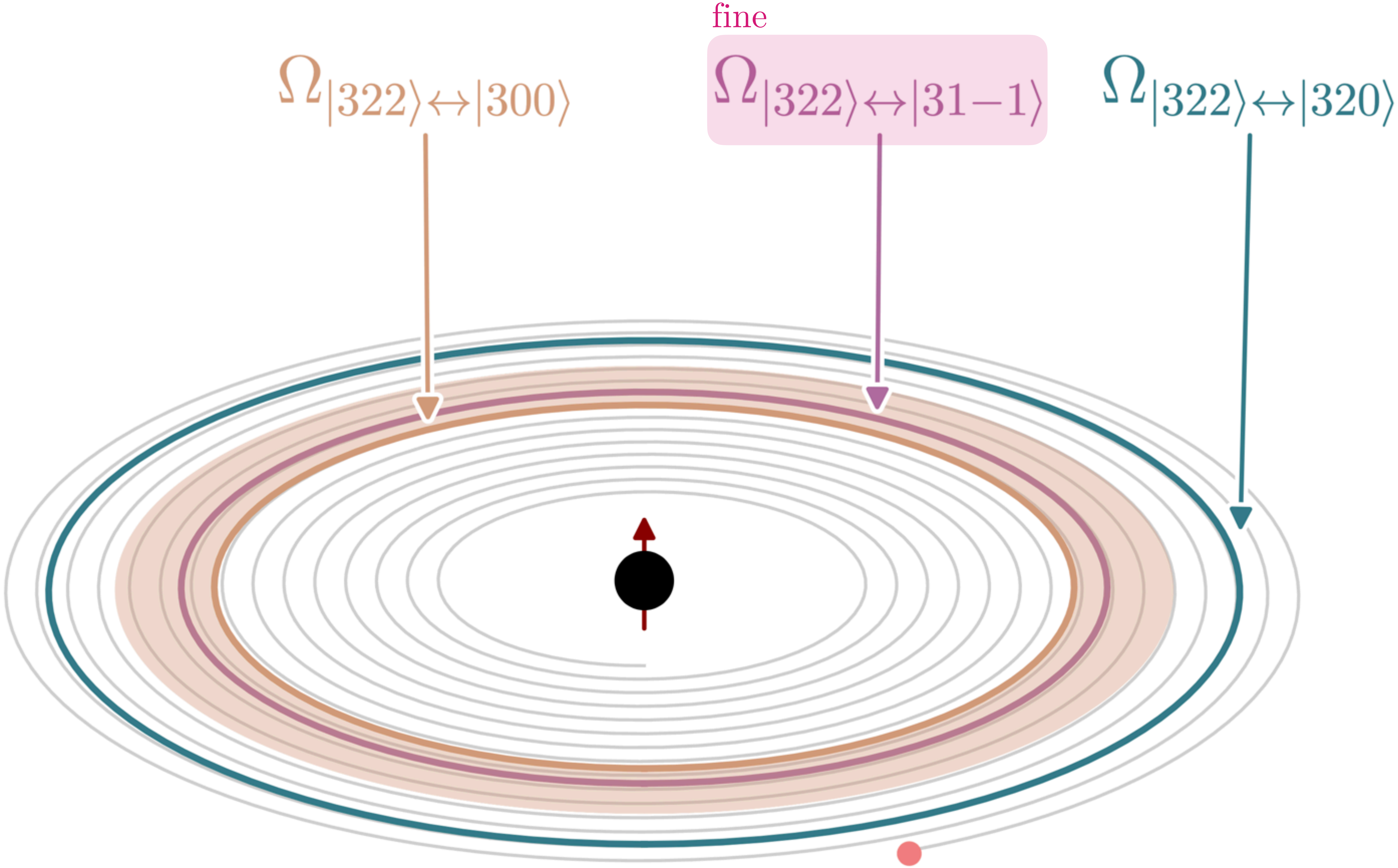
the resonance condition is no longer satisfied

$$\Delta E = \Delta m \Omega$$

$|322\rangle \leftrightarrow |320\rangle$

hyperfine

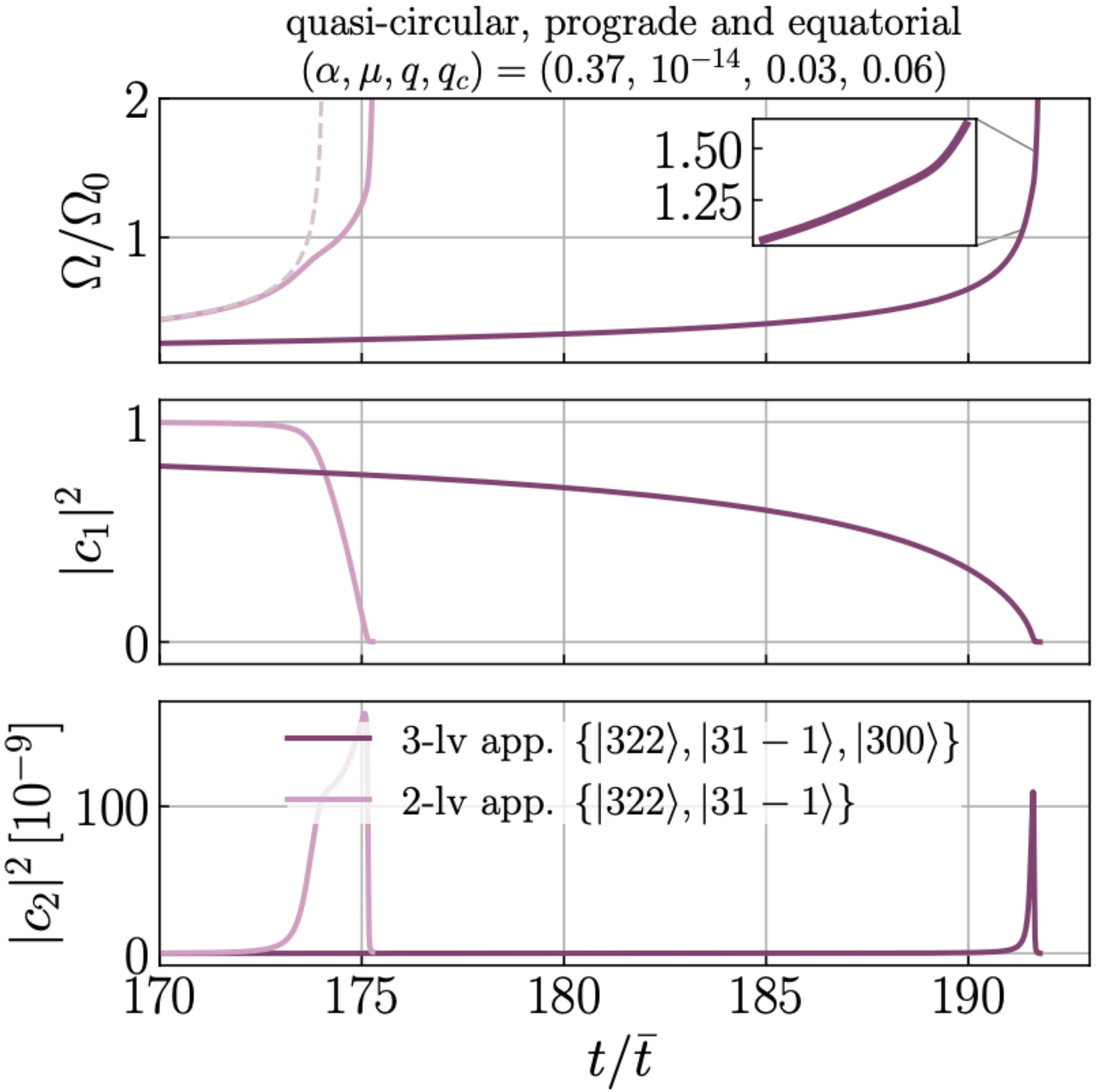




fine
$\Omega_{|322\rangle \leftrightarrow |300\rangle}$
$\Omega_{|322\rangle \leftrightarrow |31-1\rangle}$
$\Omega_{|322\rangle \leftrightarrow |320\rangle}$

$|322\rangle \leftrightarrow |31{-}1\rangle$

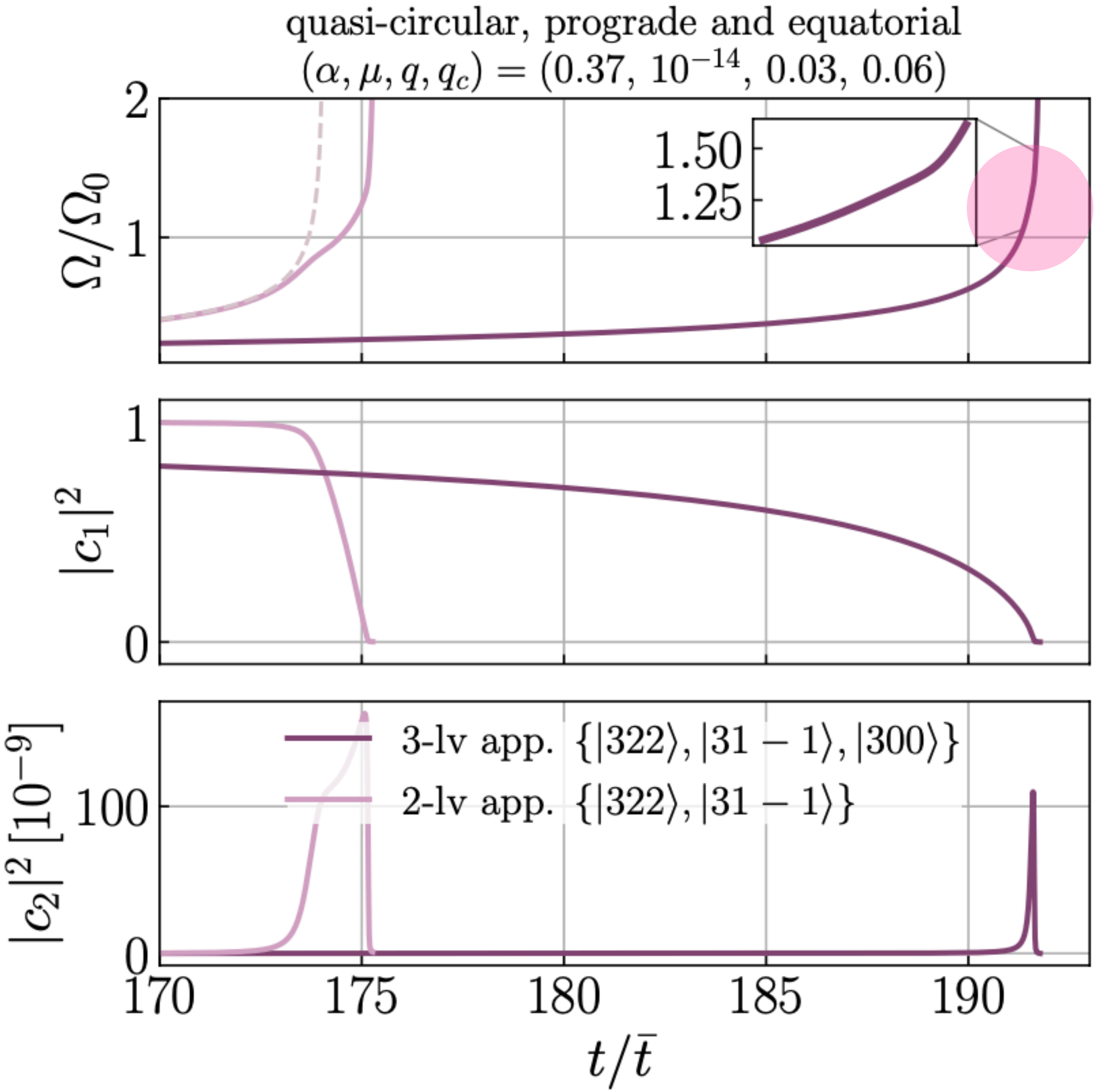
fine



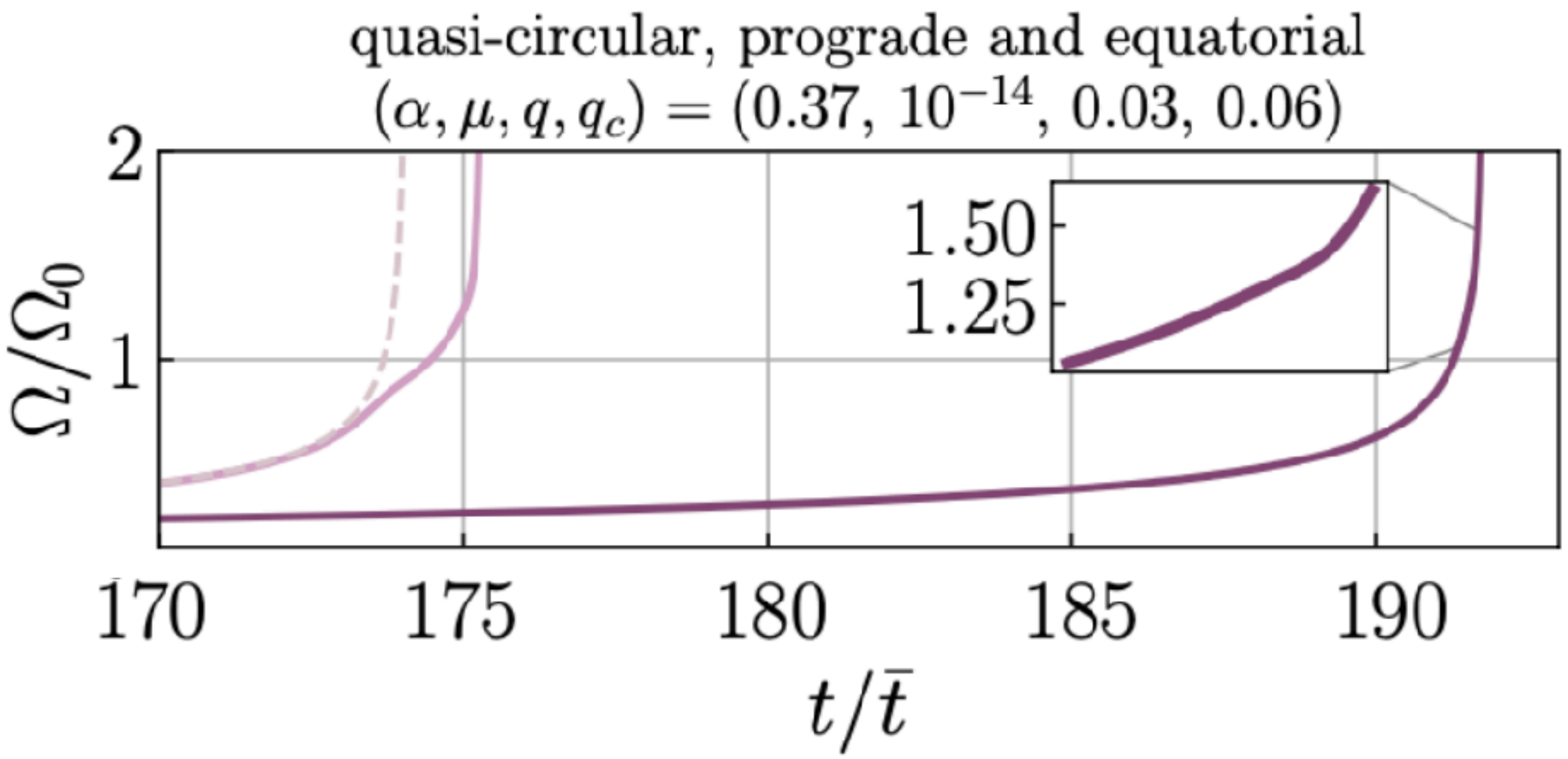
Kim, Lenoci (25)

$|322\rangle \leftrightarrow |31{-}1\rangle$

fine



Kim, Lenoci (25)



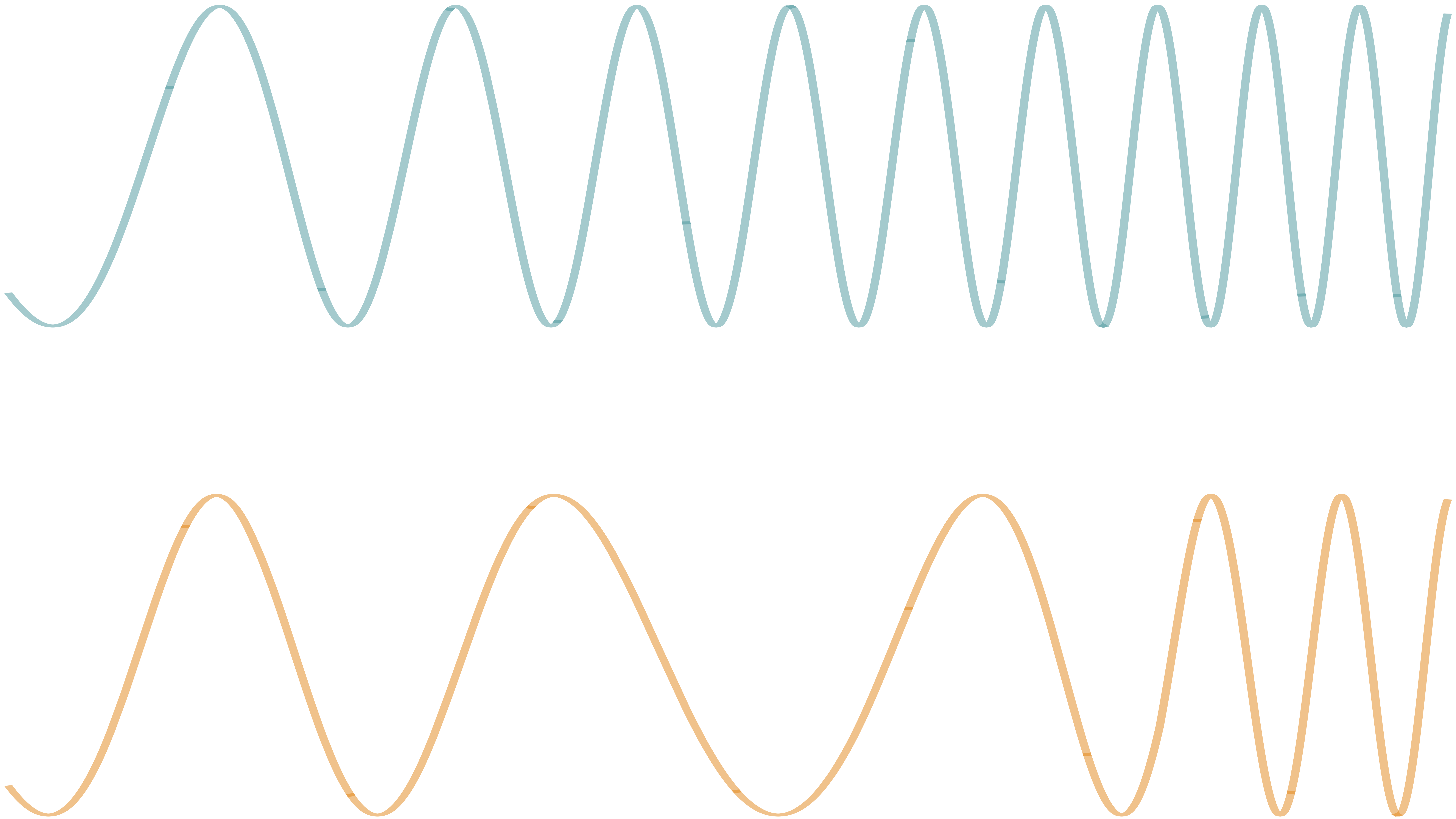
$$f_{\rm GW} \sim 10^{-2}\,{\rm Hz}$$

$$\dot{f}_{\rm GW} \sim 10^{-3}\,{\rm Hz/yr}$$

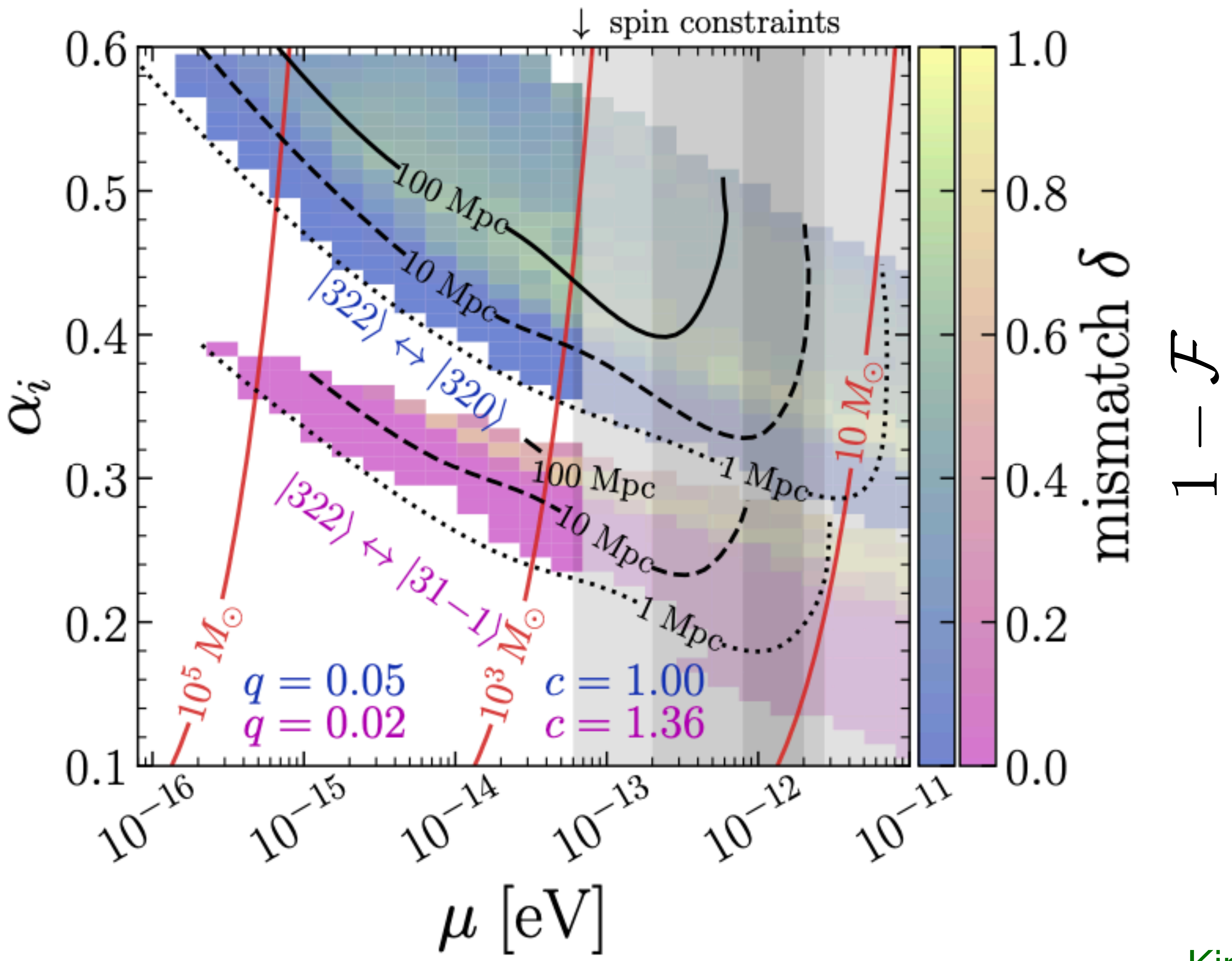
still falls into LISA band

and exhibits a faster freq. drift !

do we have a chance to measure it with LISA?



$$\mathcal{F} = \max_{\boldsymbol{\theta}_v} \frac{\left(h(\boldsymbol{\theta}_{\rm true}) \,\middle|\, h(\boldsymbol{\theta}_v) \right)}{\sqrt{\left(h(\boldsymbol{\theta}_{\rm true}) \,\middle|\, h(\boldsymbol{\theta}_{\rm true}) \right)\left(h(\boldsymbol{\theta}_v) \,\middle|\, h(\boldsymbol{\theta}_v) \right)}}$$



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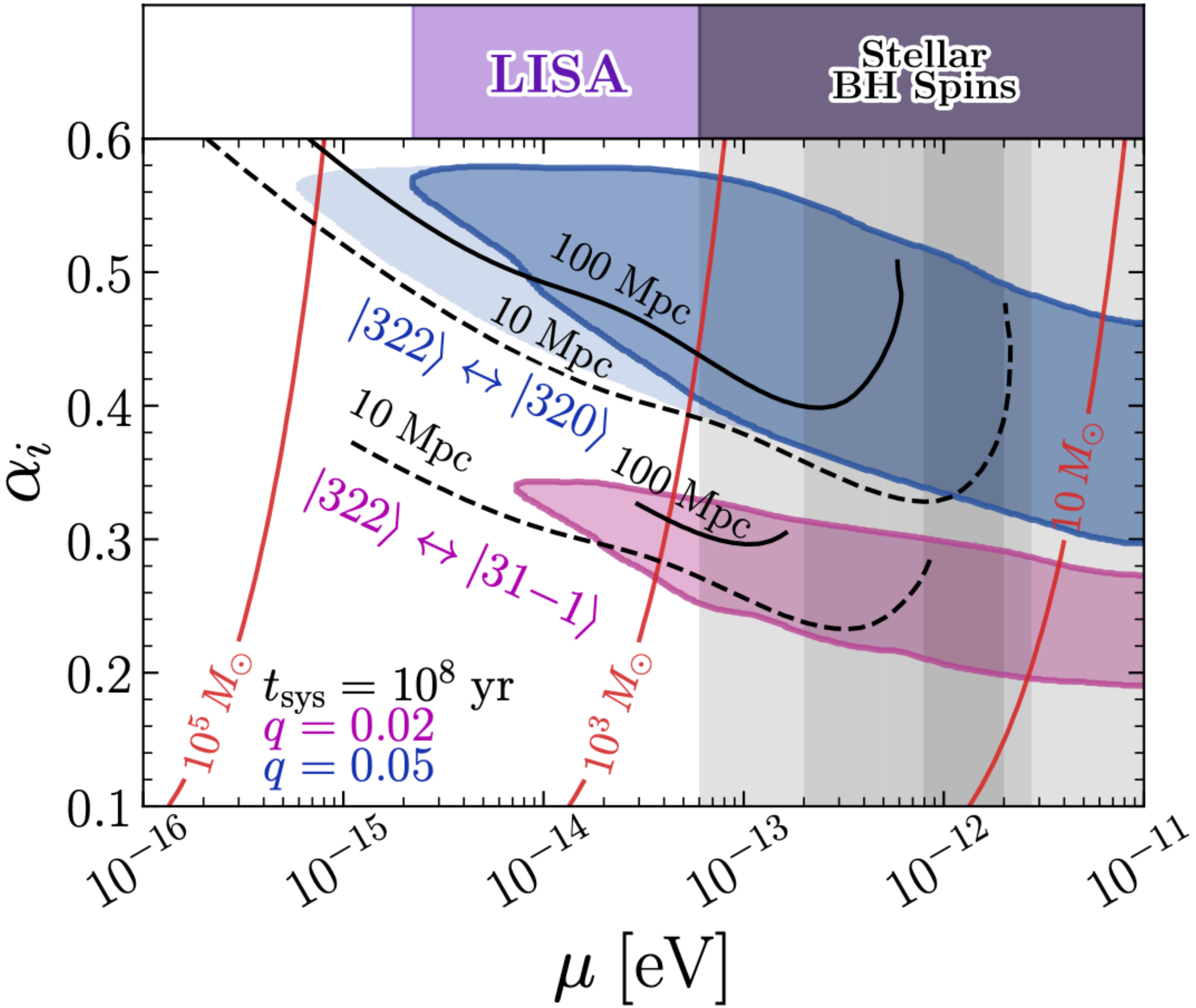

LISA
Stellar BH Spins
0.6
0.5
0.4
0.3
0.2
0.1
α_i
10^{-16}
10^{-15}
10^{-14}
10^{-13}
10^{-12}
10^{-11}
μ [eV]
100 Mpc
10 Mpc
$|322\rangle \leftrightarrow |320\rangle$
10 Mpc
100 Mpc
$|322\rangle \leftrightarrow |31{-}1\rangle$
$10^5 M_\odot$
$10^3 M_\odot$
$10\, M_\odot$
$t_{\rm sys} = 10^8$ yr
$q = 0.02$
$q = 0.05$