

New Perspectives on the QCD axion

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Strong CP problem

$$\mathcal{L}_{QCD} \supset \theta G \tilde{G}$$

CP-odd term

Basis independent: $\bar{\theta} = \theta + \arg(\det \mathcal{M}_q)$

Observable effect

Neutron electric dipole moment: $d_N \simeq (5 \times 10^{-16} e \cdot \text{cm}) \bar{\theta}$ [Crewther, DiVecchia, Veneziano, Witten, 1979]

$$|d_N| \lesssim 3 \times 10^{-26} e \cdot \text{cm}$$



$$\bar{\theta} \lesssim 10^{-10}$$

Why is $\bar{\theta}$ so small?

$\bar{\theta}$ does not appear to be “anthropic”

Our Universe possible for $0 \lesssim \bar{\theta} \lesssim 0.1$

[Ubbaldi: 0811.1599][Dine, Haskins, Ubbaldi, Xu: 1801.03466]
[Lee, Meissner, Olive, Shifman, Vonk: 2006.12321]

Dynamical Solution: PQ mechanism and axion

[Peccei, Quinn 1977;
Weinberg 1978; Wilczek 1978]

Axion: $\Phi = \frac{1}{\sqrt{2}}(f_a + \rho)e^{i\frac{\rho}{f_a}}$ $U(1)_{\text{PQ}}$ spontaneously broken $\langle \Phi \rangle = \frac{f_a}{\sqrt{2}}$

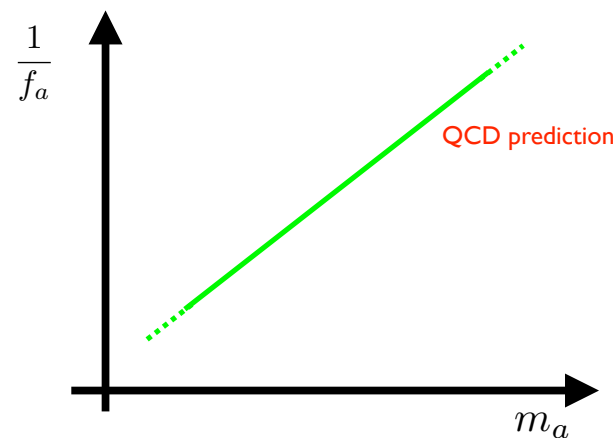
⇒ $\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a} \frac{\alpha_s}{8\pi} G\tilde{G} + \frac{1}{4}ag_{a\gamma\gamma}F\tilde{F} + \frac{1}{f_a}J^\mu\partial_\mu a$

Axion potential: $V_{\text{QCD}}(a) \simeq -m_a^2 f_a^2 \cos\left(\frac{a}{f_a} + \bar{\theta}\right) + \dots$ ⇒ minimum: $\theta_{\text{eff}} \equiv \frac{a}{f_a} + \bar{\theta} = 0!$

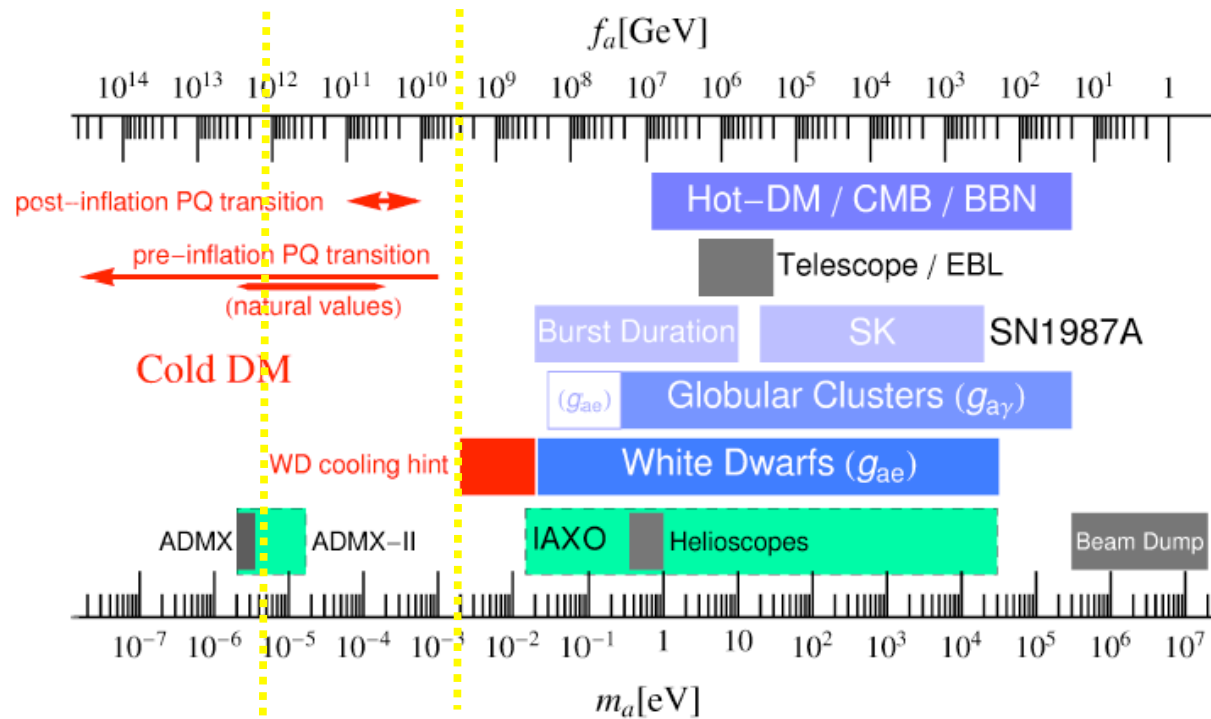
solves strong CP problem!

⇒ $m_a^2 = \frac{\mathcal{T}}{f_a^2}$ $\mathcal{T} \equiv -i \int d^4x \langle 0 | T \left[\frac{1}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}(x), \frac{1}{32\pi^2} G_{\rho\sigma}^b \tilde{G}^{b\rho\sigma}(0) \right] | 0 \rangle$ topological susceptibility

⇒ $m_a^2 f_a^2 \sim m_q \Lambda_{\text{QCD}}^3 = \text{constant}$

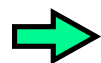


QCD axion limits



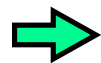
$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$$

$$\frac{1}{f_a} J^\mu \partial_\mu a$$



axion weakly-coupled - “invisible”

$$m_a^2 f_a^2 \sim m_q \Lambda_{\text{QCD}}^3$$



$$10^{-5} \text{ eV} \lesssim m_a \lesssim 10^{-3} \text{ eV}$$

Origin of dim-5 axion-gluon coupling?

Example: KSVZ [Kim '79; Shifman, Vainshtein, Zakharov '80]

(Also DFSZ [Dine, Fischler, Srednicki '81; Zhitnitsky '80])

$$U(1)_{PQ}$$

Dirac fermion: $\Psi \rightarrow e^{iq_\Psi \alpha \gamma_5} \Psi$

complex scalar: $\Phi \rightarrow e^{iq_\Phi \alpha} \Phi$

Scalar potential: $V(\Phi) = -m_{PQ}^2 |\Phi|^2 + \lambda_{PQ} |\Phi|^4$



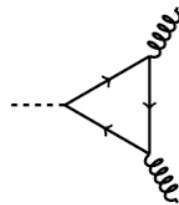
$$\langle \Phi \rangle = \frac{f_a}{\sqrt{2}} \quad f_a = \sqrt{\frac{m_{PQ}^2}{\lambda_{PQ}}}$$

Yukawa coupling:
($2q_\Psi + q_\Phi = 0$) $\Delta \mathcal{L} = h_{ij} \Phi \bar{\Psi}_{Ri} \Psi_{Lj} + \text{h.c.}$



$$m_\Psi \sim h \frac{f_a}{\sqrt{2}}$$

Integrate out
Dirac fermions:



$$\frac{g_s^2}{32\pi^2} \frac{a}{f_a} G \tilde{G}$$

BUT Origin of PQ potential? Why is $m_{PQ} \ll M_P$?

Axion quality:

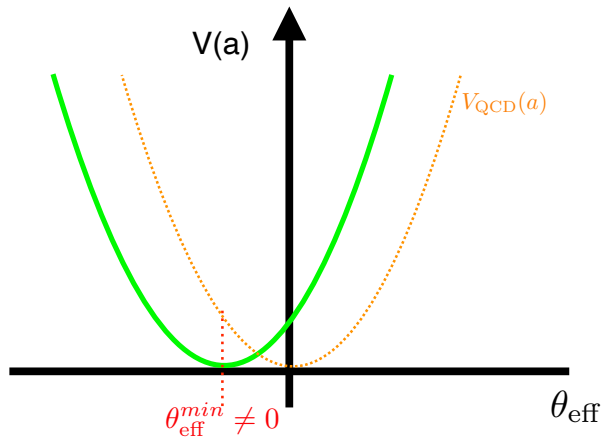
[Georgi, Hall, Wise 1981; Giddings, Strominger 1988; S.J.Rey 1989;
Holman et al 1992; March-Russell, Kamionkowski 1992]

Gravitational
violation of $U(1)_{\text{PQ}}$

$$\frac{|c_n| e^{i\delta_n}}{M_P^{n-4}} \Phi^n + h.c.$$



$$V(a) = V_{\text{QCD}}(a) - \frac{|c_n| f_a^n}{M_P^{n-4}} \cos\left(\frac{na}{f_a} + \delta_n\right)$$



minimum: $\theta_{\text{eff}} \neq 0 !$

axion quality
problem

Terms must be suppressed to very high order!

$$n \geq 10$$

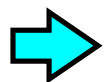
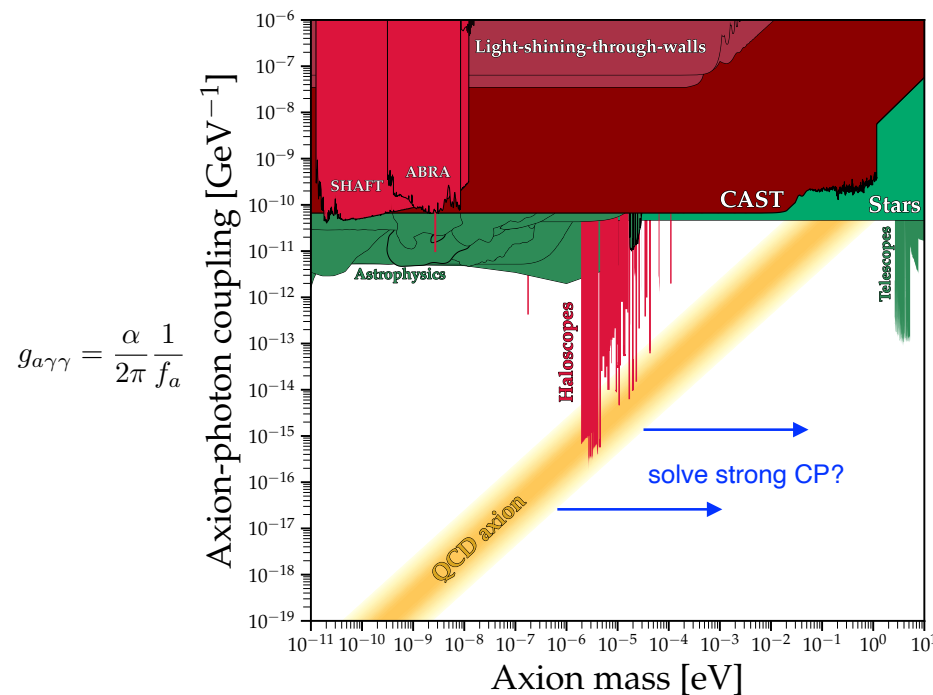
($c_n \sim 1$)

[Note: gravitational instantons $c_n \sim e^{-S} \rightarrow S \geq 200$ but $S \sim \log \frac{M_P}{f_a \sqrt{g_s}} \sim \frac{3}{2} \log \frac{1}{g_s}$ for dynamical radial mode]

[Alvey, Escudero: 2009.03917]

Questions

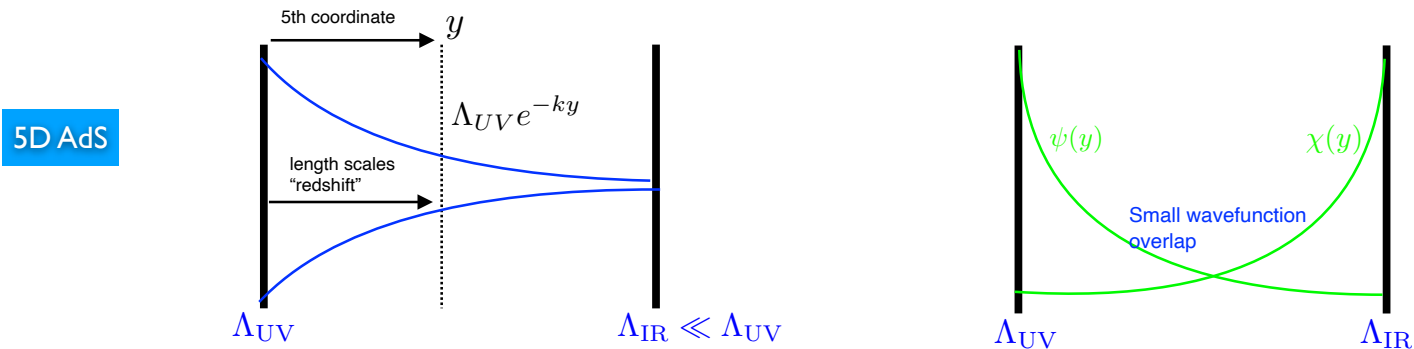
- Origin of the PQ breaking potential and spontaneous breaking scale f_a ?
- How to solve the axion quality problem? Connection to GUT or supersymmetry?
- Can the QCD axion mass be different?



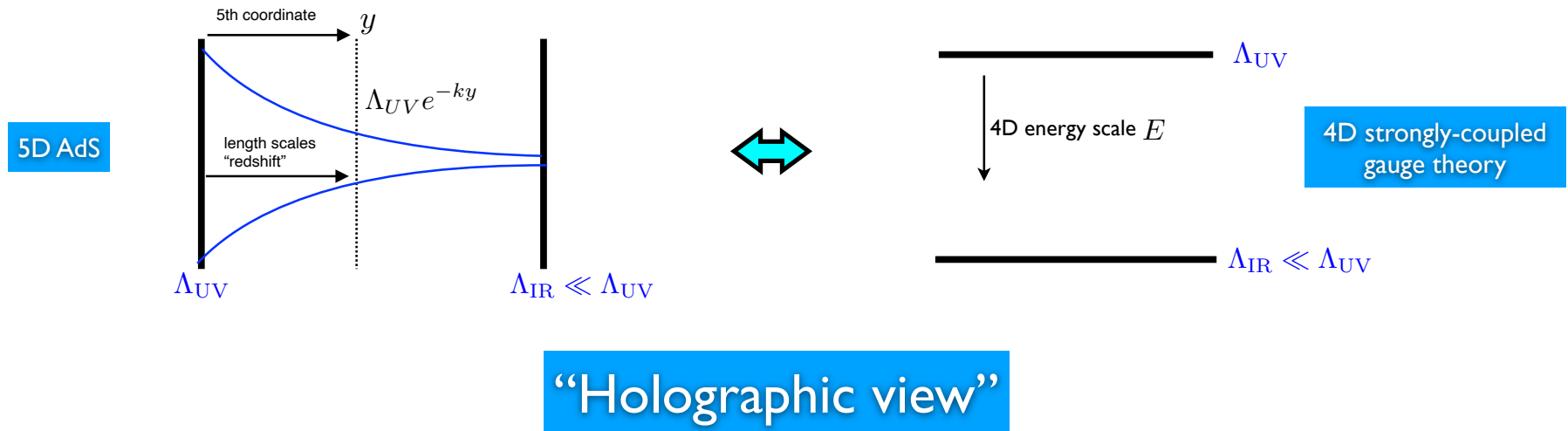
Use 5th dimension to address these questions!

Why use the 5th dimension?

- ◆ Can generate a hierarchy of scales and small couplings! 😊

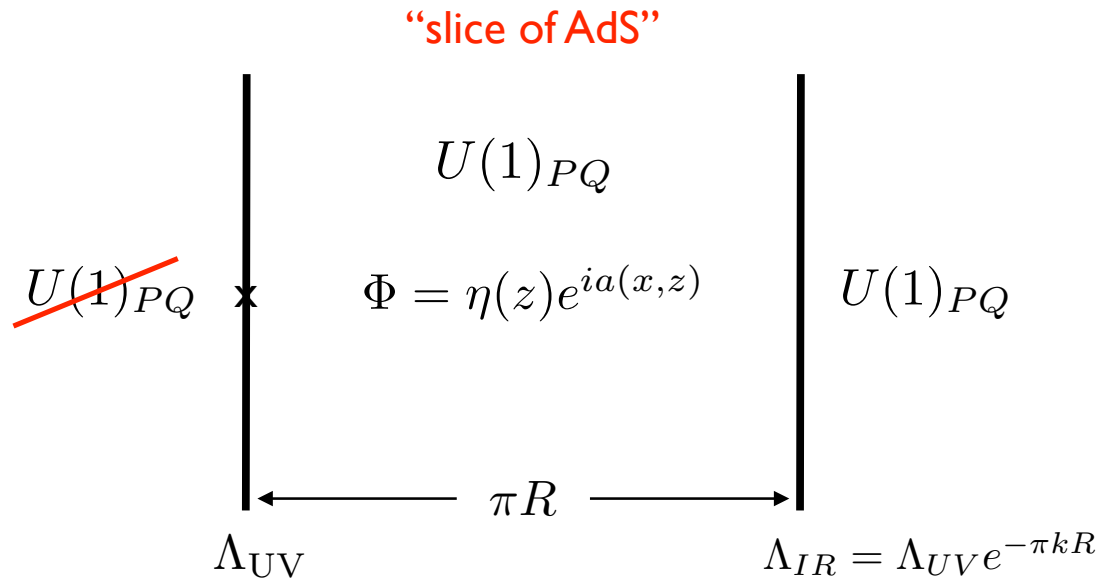


- ◆ “Warped” dimension is dual to 4D strong dynamics! 😊



I. Axion Quality Problem [Cox,TG, Nguyen 1911.09385]

5D metric: $ds^2 = A^2(z)(dx^2 + dz^2) \equiv g_{MN}dx^M dx^N$ $A(z) = \frac{1}{kz}$
AdS curvature scale



[Randall, Sundrum '99]
 [Dienes, Dudas, TG '99]
 [Choi '03; Flacke, Gripaios, March-Russell, Maybury '06]

$$m_\Phi^2 = \Delta(\Delta - 4)k^2$$

“Bulk mass”

Explains
PQ scale

PQ symmetry
breaking



$$\eta(z) = k^{3/2}(\lambda(kz)^{4-\Delta} + \sigma(kz)^\Delta)$$

“Bulk VEV”

$$\lambda = \frac{\ell_{UV}}{\Delta - 4 + b_{UV}}(kz_{UV})^{\Delta-4},$$

“explicit” breaking

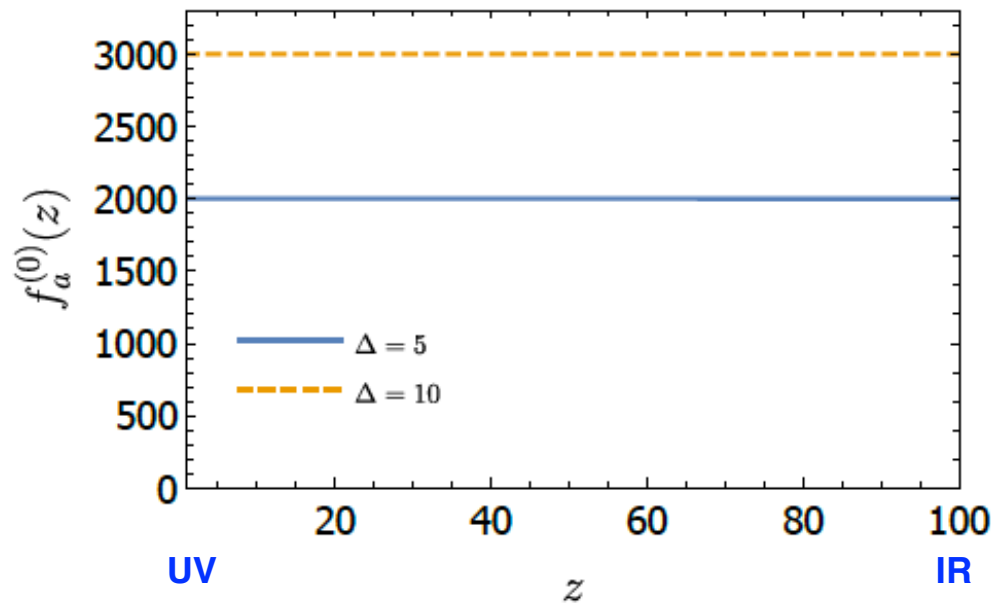
$$\sigma = \sqrt{v_{IR}^2 - \frac{\Delta}{2\lambda_{IR}}}(kz_{IR})^{-\Delta} \equiv \sigma_0(kz_{IR})^{-\Delta}$$

“spontaneous” breaking

No UV PQ-breaking ($\lambda = 0$)

→
($z_{IR} \gg z_{UV}$)

$$f_a^{(0)}(z) \simeq \frac{z_{IR}}{\sigma_0} \sqrt{\Delta - 1} \left(1 + \frac{g_5^2 k \sigma_0^2}{4\Delta(\Delta - 1)} \left(\frac{(\Delta - 1)^2}{2\Delta - 1} + \frac{z^2}{z_{IR}^2} \left(\left(\frac{z}{z_{IR}} \right)^{2(\Delta - 1)} - \Delta \right) \right) + \mathcal{O}(\sigma_0^4) \right)$$



Global $U(1)_{PQ}$ symmetry:

$$a^{(0)}(x^\mu) \rightarrow a^{(0)}(x^\mu) + \alpha_0$$

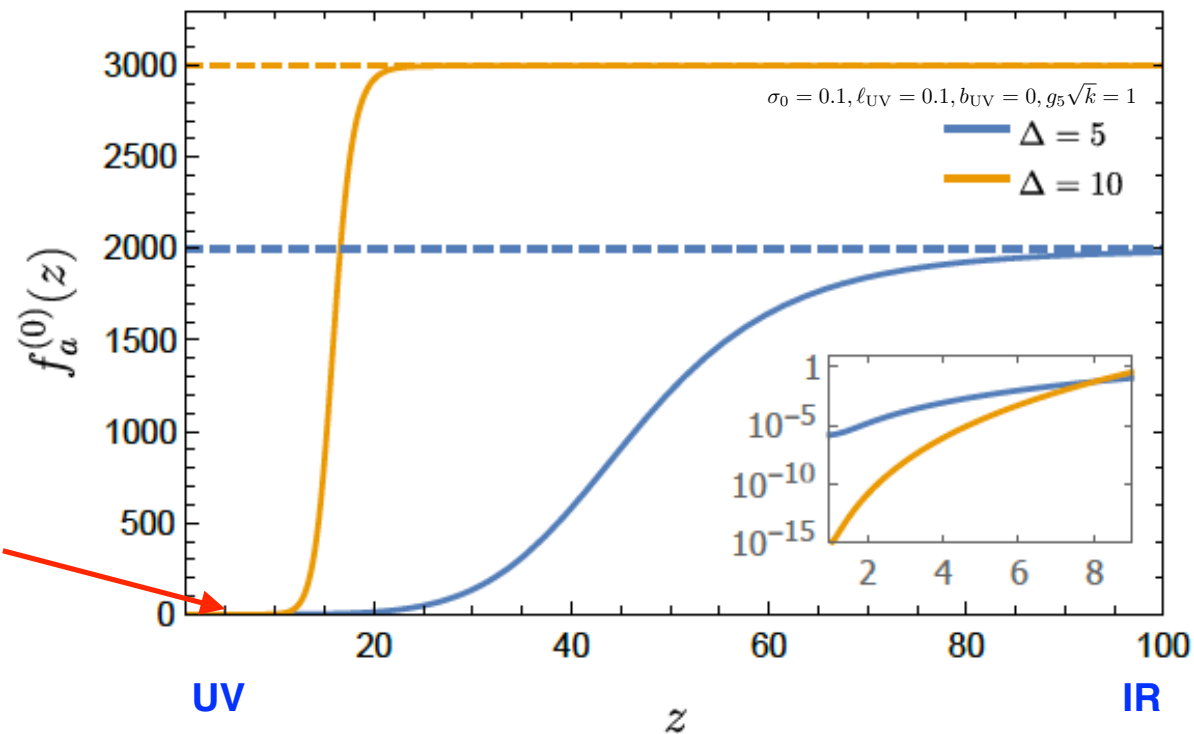
Massless axion

UV PQ-breaking ($\lambda \neq 0$)

$$U_{UV}(\Phi) \supset -2\ell_{UV}k^{5/2}\eta\cos(a) = -2\ell_{UV}k^{5/2}\eta\left(1 - \frac{1}{2}a^2 + \dots\right)$$

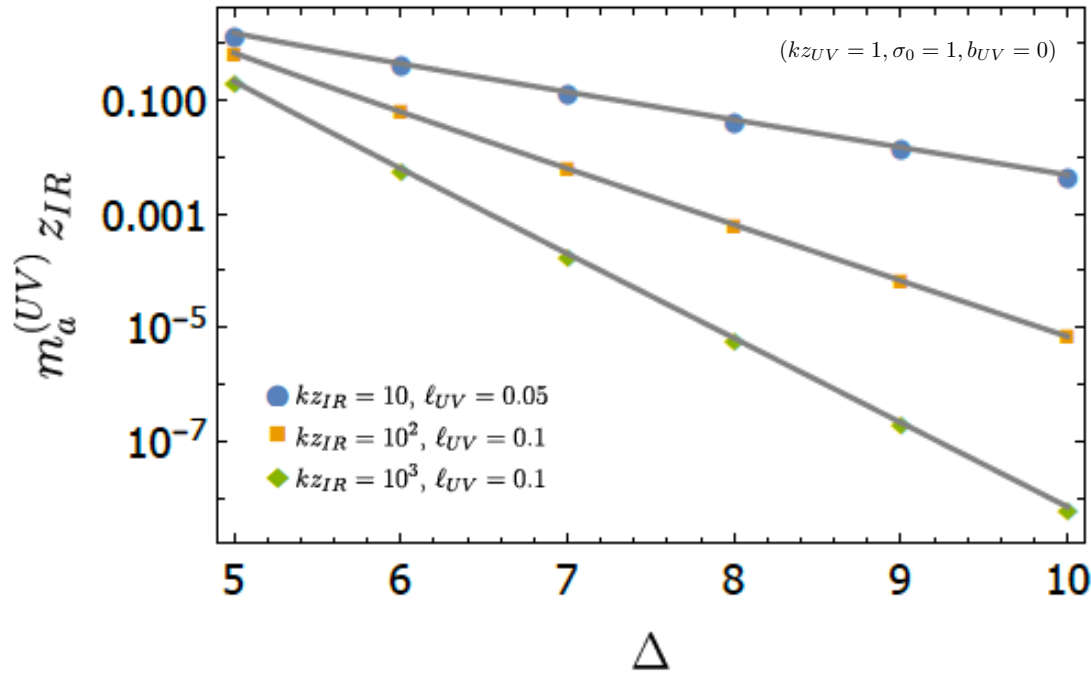
→
($z_{IR} \gg z_{UV}$)

$$f_a^{(0)}(z) \simeq z_{IR} \frac{k^{3/2}}{\eta(z)} \sqrt{\Delta - 1} \left(\frac{z}{z_{IR}}\right)^\Delta \left[1 + \frac{2\lambda(\Delta - 2)(kz_{UV})^\Delta (kz)^{2(2-\Delta)}}{\ell_{UV} + 2\sigma_0(\Delta - 2)(z_{UV}/z_{IR})^\Delta}\right]$$



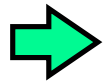
Axion profile
suppressed!

UV axion mass ($\lambda \neq 0$)



UV axion mass
suppressed for large Δ

Bulk Chern-Simons term: $-\frac{\kappa}{32\pi^2} \int_{z_{UV}}^{z_{IR}} d^5x \epsilon^{MNPQR} V_M G_{NP}^a G_{QR}^a$ ← generates axion-gluon coupling



$$(m_a^{(UV)})^2 = \frac{4\ell_{UV}\sigma_0(\Delta-2)}{\kappa^2(\Delta-4+b_{UV})} \left(\frac{\kappa\sqrt{\Delta-1}}{\sigma_0} \right)^\Delta \left(\frac{F_a}{\Lambda_{UV}} \right)^{\Delta-4} F_a^2$$

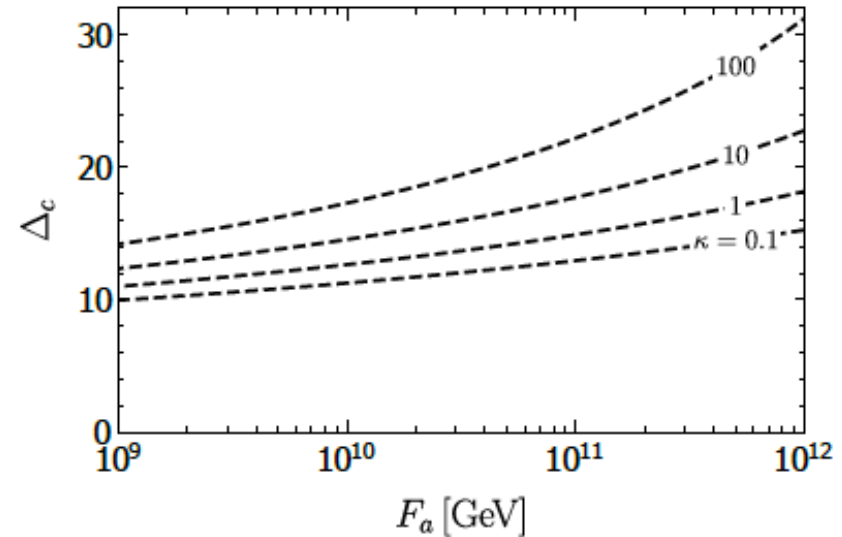
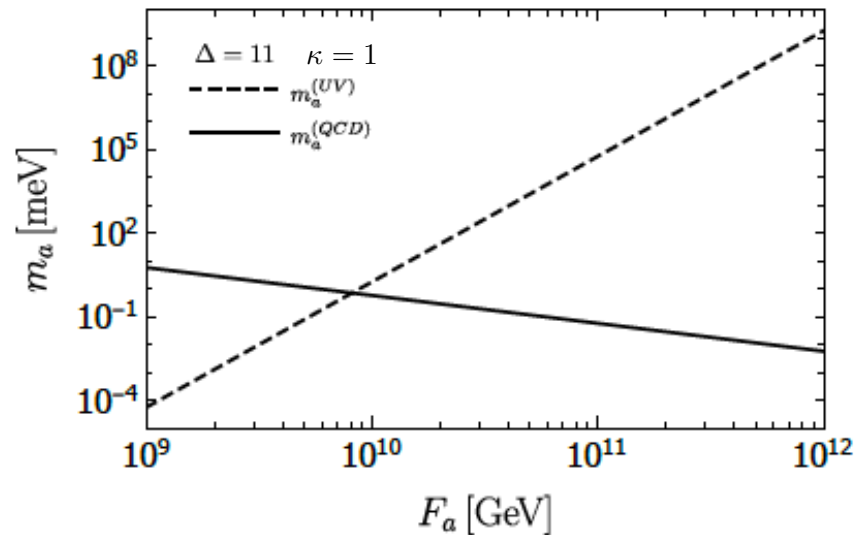
where

$$F_a \simeq \frac{1}{\kappa} \frac{\sigma_0}{\sqrt{\Delta-1}} z_{IR}^{-1}$$

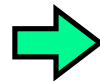
Axion potential:

$$V(a^{(0)}) \simeq -(m_a^{(QCD)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \bar{\theta}\right) - \underbrace{(m_a^{(UV)})^2}_{\text{UV explicit PQ violation}} F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \underbrace{\delta}_{\text{relative phase}}\right)$$

Require: $(m_a^{(UV)})^2 \lesssim 10^{-10} (m_a^{(QCD)})^2$



$$10^9 \text{ GeV} \lesssim F_a \lesssim 10^{12} \text{ GeV}$$



$$\Delta_c \gtrsim 10$$

Solves
axion-quality
problem!

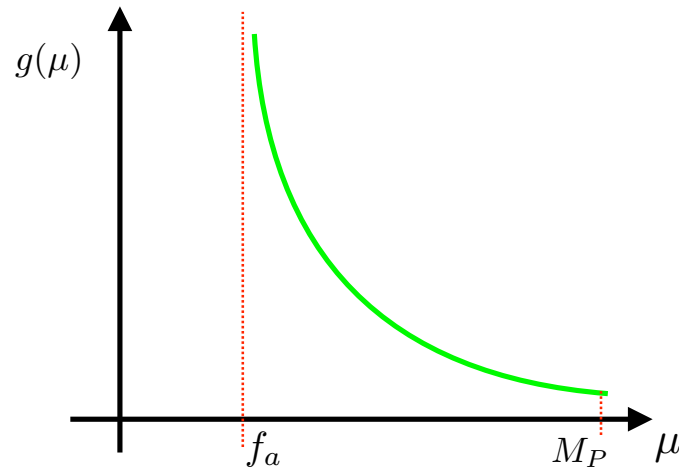
AdS/CFT dictionary

5D		4D
local $U(1)_{PQ}$ symmetry	$\leftrightarrow$	global $U(1)_{PQ}$ symmetry
$\Phi, m_\Phi^2 = \Delta(\Delta - 4)k^2$	$\leftrightarrow$	$\mathcal{O}$, dimension Δ
$\Lambda_{UV} e^{-\pi k R} = \Lambda_{IR}$	$\leftrightarrow$	$\Lambda_{IR} = \Lambda_{UV} e^{-\frac{8\pi^2}{b_{CFT} g^2}}$
σ	$\leftrightarrow$	$\langle \mathcal{O} \rangle$
λ	$\leftrightarrow$	$\lambda \phi_0 \mathcal{O}$

Holographic interpretation:

5D axion, local $U(1)$ PQ symmetry	$\leftrightarrow$	4D composite axion, accidental global $U(1)$ PQ symmetry
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Composite Axion: [Kim 1985; Choi, Kim 1985];



$$f_a \ll M_P$$

No need to tune mass parameters in KSVZ potential (or DFSZ) potential!

Axion quality:

$$\frac{|c_n|e^{i\delta_n}}{M_P^{n-4}}\Phi^n \quad (n \gtrsim 10) \quad \longleftrightarrow \quad \frac{c_\Delta}{M_P^{\Delta-4}}\mathcal{O} \sim \frac{c_\Delta}{M_P^{\Delta-4}}(\underbrace{\psi_i\psi_j\cdots\psi_k}_{\text{due to gauge + Lorentz symmetry!}}) \quad (\Delta \gtrsim 10)$$

Examples: [Randall 1992]; [Cheng, Kaplan 2001]; [Redi, Sato 2016]; [Lillard, Tait 2018]; [Contino, Podo, Revello: 2112.09635]

Build models which better incorporate SM features:

- Gives rise to QCD axion
- SM part of strong dynamics
- Connection to GUT physics?



Will give an example

I. High-quality composite Pati-Salam axion

[TG, Murayama, Quilez: 2505.08866]

strong dynamics: $SU(N_c) \quad 10 \times Q(\square) \quad 10 \times P(\bar{\square})$

flavor symmetry: $SU(10)_L \times SU(10)_R$

$\cup \quad \cup$

“weakly” gauged: $SO(6) \times SO(4) \quad Sp(10)$

	G_{local}				G_{global}	
	$SU(N_c)$	$Sp(10)$	$SO(6)$	$SO(4)$	$U(1)_B$	$U(1)_{PQ}$
Q_6	$\mathbf{N}_c$	1	6	1	+1	+2
Q_4	$\mathbf{N}_c$	1	1	4	+1	-3
P	$\bar{\mathbf{N}}_c$	10	1	1	-1	0

➡ P, Q fermions are naturally massless!

Symmetry breaking: $\langle P^r Q_s \rangle = \Lambda^3 \delta_s^r$ (r,s = flavor index)

$$SO(6) \times SO(4) \times Sp(10) \rightarrow SU(3) \times U(1)_{B-L} \times SU(2)_L \times U(1)_{I_{3R}}$$

Nambu-Goldstone bosons: $\dim[SU(10)_L \times SU(10)_R / SU(10)_V] = 99$ NGBs

$\dim[SO(6) \times SO(4) \times Sp(10) / U(3) \times U(2)] = 63$ NGBs eaten

36 NGBs remaining

Under $U(3) \times U(2)$

$$(\mathbf{8}_0, \mathbf{1}_0) \oplus (\mathbf{1}_0, \mathbf{3}_0) \oplus (\mathbf{3}_1 + \bar{\mathbf{3}}_{-1}, \mathbf{2}_1 + \mathbf{2}_{-1}) \oplus (\mathbf{1}_0, \mathbf{1}_0)$$

Obtain mass $\sim g^2 \Lambda^2$ QCD axion!

PQ-violating operators

Need $SO(6)$ ($SO(4)$), $Sp(10)$ gauge invariants:

$$\begin{aligned}
 e.g. \quad (Q_6^2)^{\overbrace{\{a,b\}}^{\text{SU(Nc) symmetric}}} &\equiv \underbrace{\delta^{ij}}_{SO(6)} (Q_{6i\alpha}^a \epsilon^{\alpha\beta} Q_{6j\beta}^b) & (Q_6^2)_{\mu\nu}^{[a,b]} &\equiv \delta^{ij} (Q_{6i\alpha}^a \sigma_{\mu\nu}^{\alpha\beta} Q_{6j\beta}^b) \\
 (P^2)^{\overbrace{[a,b]}^{\text{SU(Nc) antisymmetric}}} &\equiv \underbrace{J_{rs}}_{Sp(10)} (P_a^{r\alpha} \epsilon_{\alpha\beta} P_b^{s\beta}) & (P^2)^{\mu\nu}_{\{a,b\}} &\equiv J_{rs} (P_{\{a}^{r\alpha} \sigma_{\alpha\beta}^{\mu\nu} P_{b\}}^{s\beta})
 \end{aligned}$$

$SU(N_c)$ invariants $\Rightarrow$ no. P fermions = no. Q fermions
 $\Rightarrow$ mass terms and four-fermion operators (QQPP) are forbidden!
 $\Rightarrow$ obtain (QQPP)(QQPP) $\leftarrow$ eight-fermion operators!

$$e.g. \quad \mathcal{O}_{12} \equiv \frac{1}{M_P^8} (Q_{4,6}^2)^{\{a,b\}} (Q_{4,6}^2)_{\mu\nu}^{[c,d]} (P^2)_{[c,d]} (P^2)^{\mu\nu}_{\{a,b\}} \quad \leftarrow \text{dimension 12}$$

$$\frac{1}{M_P^8} \mathcal{O}_{12} \sim \frac{f_a^{12}}{M_P^8} e^{i \frac{8a}{f_a}} \quad \Rightarrow \quad V(a) \supset f_a^4 \left(\frac{f_a}{M_P} \right)^8 \cos \theta_{eff} \quad (\text{like warped model with } \Delta = 12 !)$$

Dark matter

(i) Pre-inflation

Dark matter via misalignment provided $10^{10}\text{GeV} \lesssim f_a \lesssim 10^{12}\text{GeV}$

$$\frac{1}{M_P^8} \mathcal{O}_{12} \quad \Rightarrow \quad \theta_{eff} \lesssim 10^{-10} \quad \text{for } f_a \lesssim 10^{11} \text{ GeV}$$

→ although $H_I \lesssim 10^{-5} f_a \sim 10^6 \text{ GeV}$ to suppress isocurvature perturbations

(ii) Post-inflation

Axion potential: $U(1)_{PQ} \rightarrow \mathbb{Z}_{4N_c}$

$\mathcal{O}_{12}$ explicitly breaks $\mathbb{Z}_{4N_c}$ discrete symmetry

→ provides bias potential $\frac{1}{M_{PQ}^8} \mathcal{O}_{12}$ ← causes domain wall/string decay

→ dark matter abundance provided $f_a \sim 10^9 \text{ GeV}, \delta = 10^{-2}, M_{PQ} = 10^{15-17} \text{ GeV}$ [Kawasaki, Saikawa, Sekiguchi: 1412.0789]

Axion-photon coupling

$$U(1)_{\text{EM}} \supset SO(6) \times SO(4) \times Sp(10)$$



P, Q electromagnetic charges fixed!

$$\frac{E}{N} = -\frac{7}{3}$$



$$g_{a\gamma\gamma} = \frac{\alpha_{EM}}{2\pi f_a} \left(-\frac{7}{3} - 1.92(4) \right)$$

Bonus feature

Grand Unification

$$SO(6) \times SO(4) \rightarrow SO(10)$$

$$SU(4) \times SU(2)_L \times SU(2)_R \quad \text{Pati-Salam!}$$

SM: $3 \times (4, 2, 1) \quad Q, L$

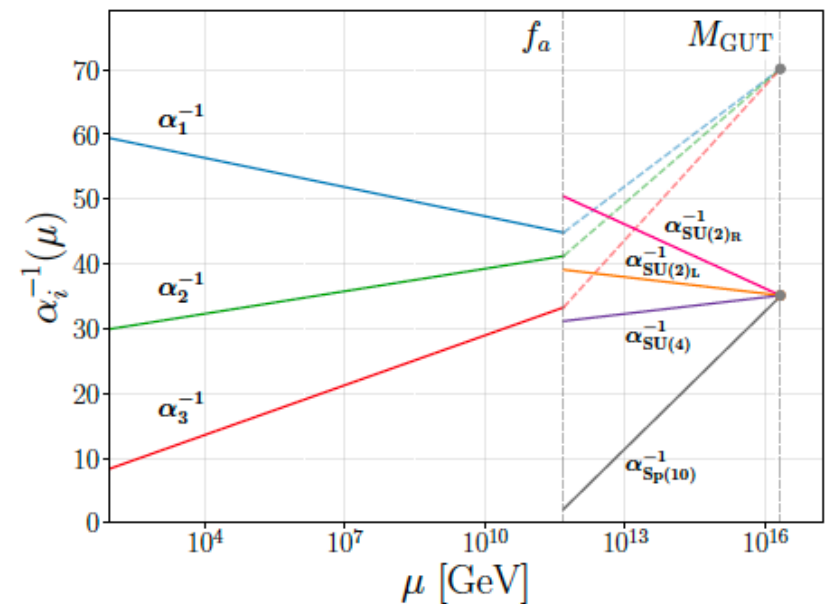
$3 \times (\bar{4}, 2, 1) \quad \bar{u}, \bar{d}, \bar{e}, N$

$(1, 2, 2) \subset \mathbf{10} \quad \text{Higgs}$

$\Phi = (10, 1, 3) \subset \mathbf{126} \quad \text{to break } U(1)_{B-L} \times U(1)_{I_3}$

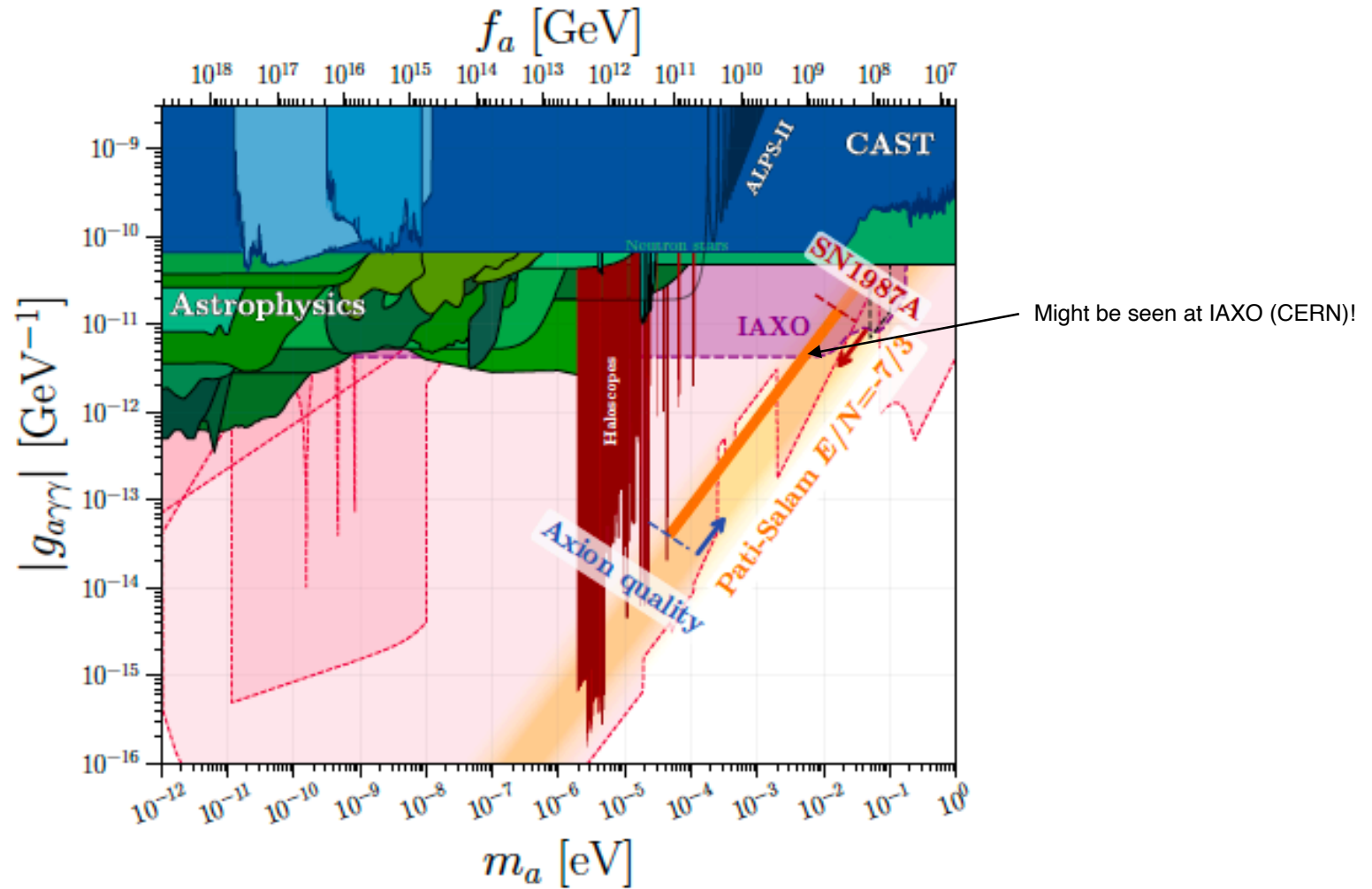
→ dimension 8 PQ-violating op. $\Phi^\dagger \Phi Q Q P P$

→ forbidden by anomaly-free $\mathbb{Z}_8$ symmetry!



Axion-photon coupling limits on Pati-Salam axion

[TG, Murayama, Quilez: 2505.08866]



ALSO: High-quality axion from SUSY chiral dynamics [TG, Murayama, Noether, Quilez: 2508.21813] [see also Sato, Takeshita: 2508.21820]

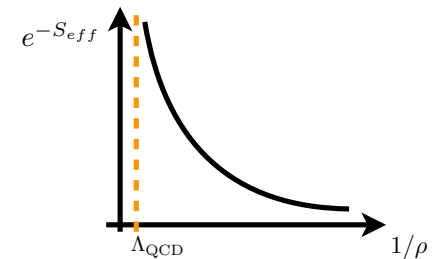
2. Axion mass from 5D small instantons

QCD axion mass:

$$m_a^2 = \frac{\mathcal{T}}{f_a^2} \quad \mathcal{T} \equiv -i \int d^4x \langle 0 | T \left[\frac{1}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}(x), \frac{1}{32\pi^2} G_{\rho\sigma}^b \tilde{G}^{b\rho\sigma}(0) \right] | 0 \rangle \quad \text{“topological susceptibility”}$$

Dilute instanton gas approximation:

$$\mathcal{T} \propto \int \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(1/\rho)} \right)^6 e^{-\frac{2\pi}{\alpha_s(1/\rho)}}$$



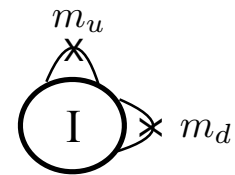
QCD asymptotically free



$$\mathcal{T} \propto \Lambda_{QCD}^4$$

Fermion zero modes:

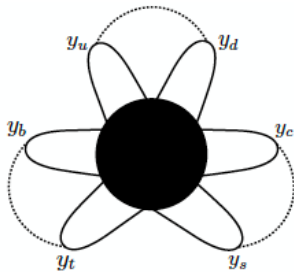
$$(\rho m_f)^{N_f} \longrightarrow \text{suppression} \frac{\prod_f m_f}{\Lambda_{QCD}^{N_f}}$$



$$m_{a,QCD}^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

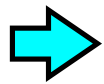
How to enhance QCD axion mass?

- Change QCD coupling in UV $\alpha_s(1/\rho) \sim 1$ “Small instantons” $\rho \sim 1/\Lambda_{UV}$
- Close fermion loops with Higgs boson



$$\kappa_f = \frac{y_u}{4\pi} \frac{y_d}{4\pi} \frac{y_c}{4\pi} \frac{y_s}{4\pi} \frac{y_t}{4\pi} \frac{y_b}{4\pi} \approx 10^{-23}$$

(otherwise $\frac{m_u m_d m_c m_s m_b m_t}{\Lambda_{UV}^6}$)



$$m_a^2 f_a^2 \sim m_q \Lambda_{\text{QCD}}^3 + \Lambda_I^4$$

new contribution

where $\Lambda_I \gg \Lambda_{\text{QCD}}$

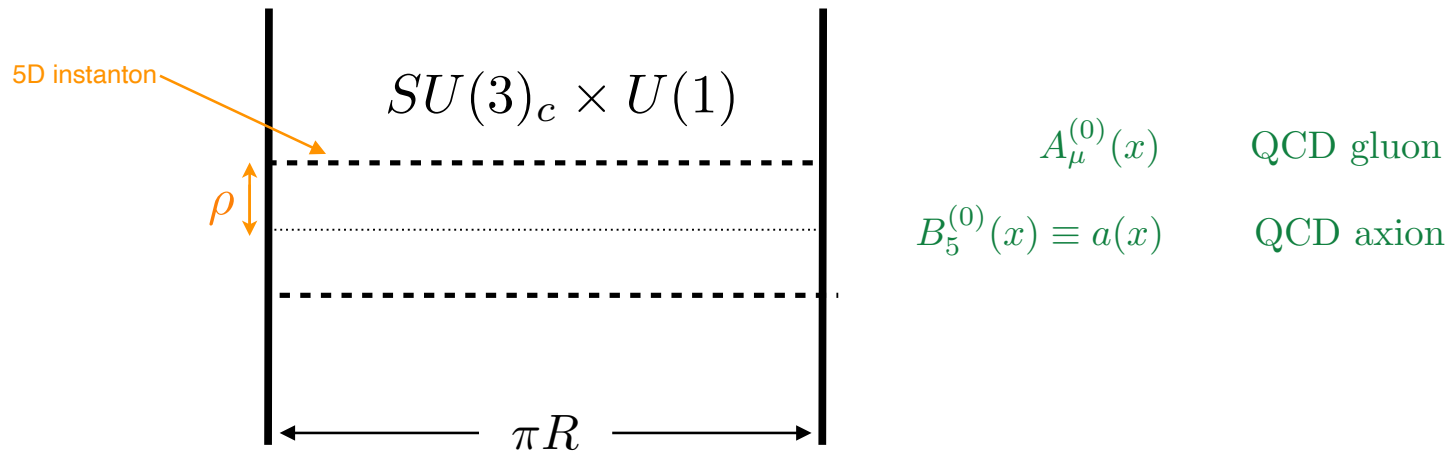
Use 5th dimension to make QCD axion heavy!

QCD in 5D

Flat space 5D metric:

$$ds^2 = dx^2 + dy^2$$

$$S_5 = - \int d^4x \int_0^L dy \left(\frac{1}{4g_5^2} \text{Tr}[G_{MN}^2] + \frac{b_{CS}}{32\pi^2} \epsilon^{MNRST} B_M \text{Tr}[G_{NR} G_{ST}] + \frac{1}{4g_5^2} F_{MN}^2 + \dots \right)$$



5D instanton:

$$A_\mu^a(x, y) = A_\mu^{(I)a}(x) = \frac{2\eta_{a\mu\nu}(x-x_0)_\nu}{(x-x_0)^2 + \rho^2}, \quad A_5^a(x, y) = 0$$



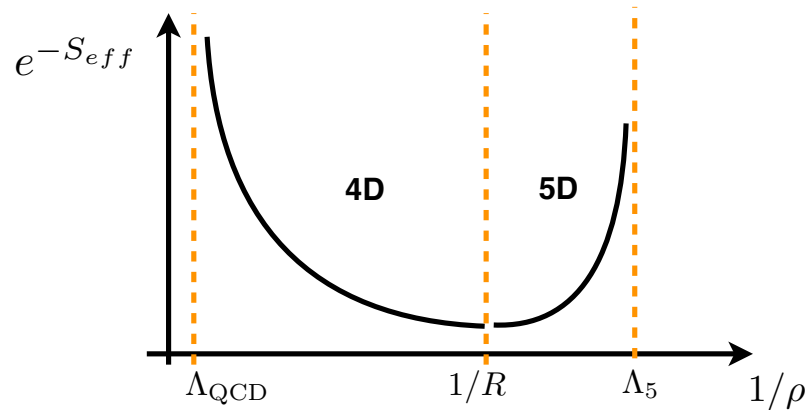
$$S_5^{(I)} = \frac{8\pi^3 R}{g_5^2} = \frac{2\pi}{\alpha_s}$$

Finite action

5D small instantons

[Poppitz, Shirman '02] [TG, Khoze, Pomarol, Shirman: 2001.05610]

Fluctuations + Kaluza-Klein contributions



small instantons!

$$\mathcal{T} \sim \int_{1/\Lambda_5}^R \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(1/R)} \right)^6 e^{-S_{eff}} \equiv \frac{K}{R^4} \propto m_a^2 f_a^2$$



$$S_{eff} = \frac{2\pi}{\alpha_s(1/R)} - 3\xi(R/\rho) \frac{R}{\rho} + b_0 \ln \frac{R}{\rho}$$

power law term!

$$\xi(R/\rho) \sim 1/3$$

$$R/\rho \gtrsim 20$$

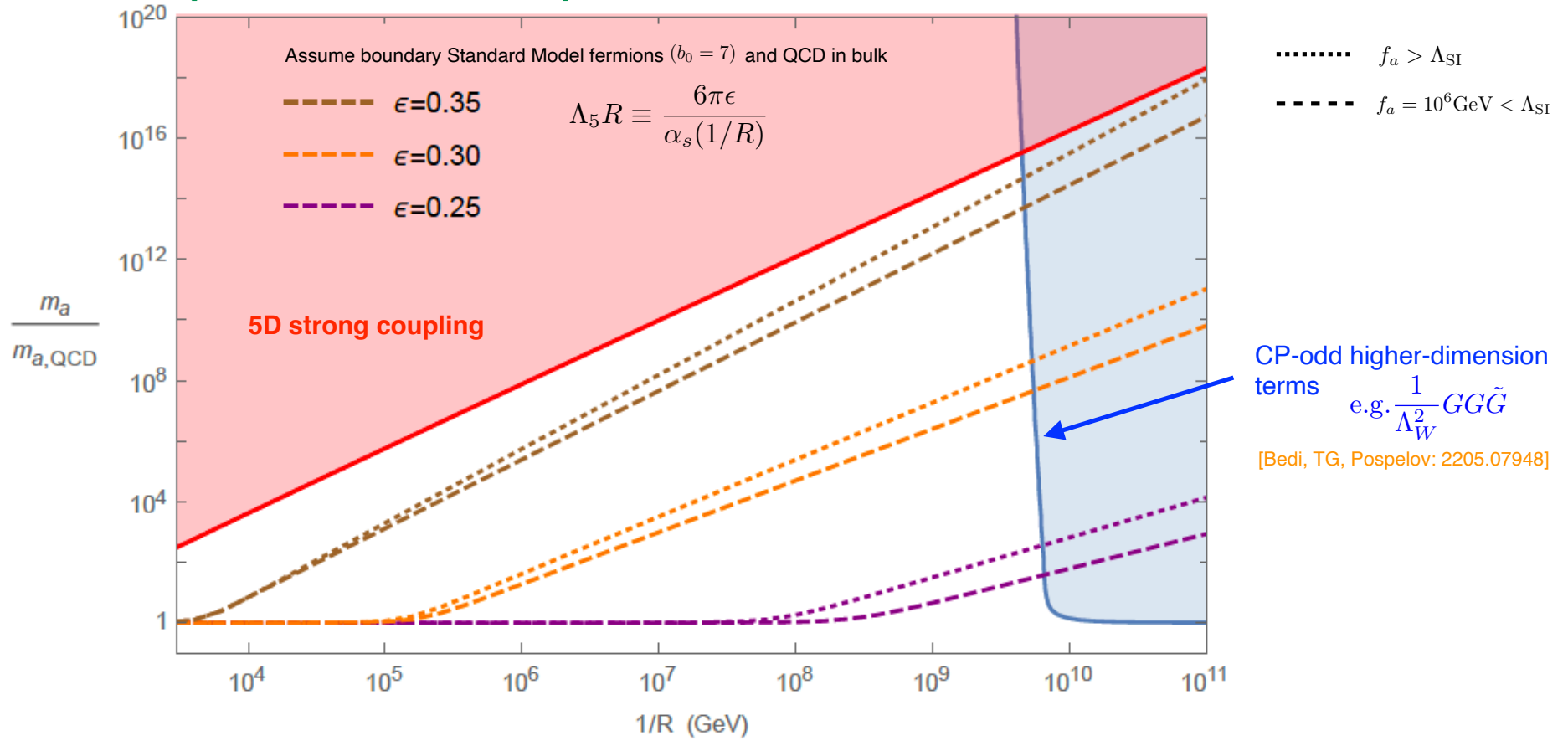


$$K \simeq C[3] \left(\frac{2\pi}{\alpha_s(1/R)} \right)^6 (\Lambda_5 R)^{3-b_0} e^{-\frac{2\pi}{\alpha_s(1/R)} + \Lambda_5 R}$$

power law contribution can overcome suppression

Valid up to $\frac{g_5^2 \Lambda_5}{24\pi^3} \sim 1$ or $\Lambda_5 R \lesssim \frac{6\pi}{\alpha_s}$

[TG, Khoze, Pomarol, Shirman: 2001.05610]



Small 5D instantons can dominate for $\frac{1}{R} \gtrsim 100 \text{ TeV}$

Enhanced EDMs

[Bedi, TG, Pospelov: 2205.07948]

Higher-dimension CP-odd sources are enhanced by small instantons

Weinberg operator: $\mathcal{L} \supset \frac{1}{\Lambda_W^2} GG\tilde{G}$ [Also fermion operator: $\mathcal{L} \supset \sum_{ijkl} \frac{\lambda_{ijkl}}{\Lambda_F^2} \bar{\psi}_i \psi_j \bar{\psi}_k \psi_l$] [see also
Bedi, TG, Grojean, Guedes,
Kley, Vuong: 2402.09361]

➡ $V(a) = \chi_W(0) \left(\frac{a}{f_a} \right) + \frac{1}{2} \chi(0) \left(\frac{a}{f_a} \right)^2$

where $\chi_W(0) = -i \lim_{k \rightarrow 0} \int d^4x e^{ikx} \left\langle 0 \left| T \left\{ \frac{1}{32\pi^2} G\tilde{G}(x), \frac{1}{\Lambda_W^2} GG\tilde{G}(0) \right\} \right| 0 \right\rangle$

➡ $\left\langle \frac{a}{f_a} \right\rangle \equiv \theta_{\text{ind}} = -\frac{\chi_W(0)}{\chi(0)} \propto \frac{\Lambda_{\text{SI}}^2}{\Lambda_W^2}$ ← Enhanced by $\frac{\Lambda_{\text{SI}}}{\Lambda_{\text{QCD}}}$

Neutron EDM:

$|\theta_{\text{ind}}| \lesssim 10^{-10}$



$\frac{\Lambda_{\text{SI}}}{\Lambda_W} \lesssim 10^{-6}$

Questions/Future Work

- Incorporate neutrino sector?

- e.g. right handed neutrinos at $f_a \sim 10^{12}$ GeV [Cox,TG, Paul: 2310.08557]

- Explore other possible strong dynamics

- completely break GUT group by strong dynamics?

- Study $SU(N)/Sp(2N)$ phase transition in early Universe

- 1st order phase transition for gravitational wave signal?

- Small instantons in weakly-gauged holographic models

- [TG, Pomarol: 2110.01762]
$$A_\mu^a(x, z) = 2\eta_{\mu\nu}^a \frac{x_\nu}{x^2} \frac{(x^2 + z^2)^2}{x^2 \rho^2 + (x^2 + z^2)^2}$$

- other solutions that give axion mass enhancement?

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Summary

QCD axion elegantly solves the strong CP problem, but

- Origin of $U(1)$ and axion decay constant?
 - *naturally explained by new strong dynamics*
 - *can be part of GUT or supersymmetry breaking*
- Axion quality problem?
 - *Avoided by gauge/discrete symmetries which are part of axion strong dynamics*
- Phenomenology
 - *gravitational wave signal from confining phase transition?*
 - *possible colored states near the TeV scale*
- 5D small instantons
 - *can enhance axion mass and not spoil strong CP solution*

Axion properties naturally emerge from strong dynamics!