

Relativistic mean field within the chiral confining model

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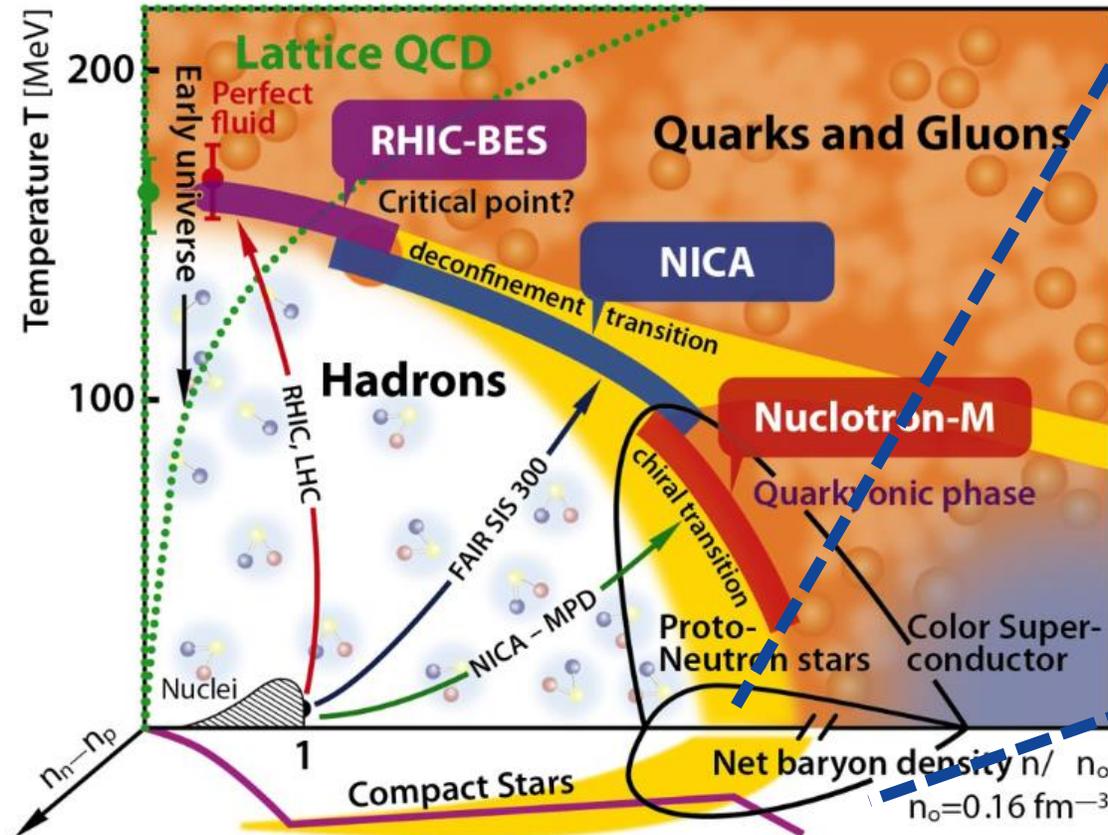
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Motivation for studying dense matter

The status of QCD dense matter

- The state of matter at high densities remains a mystery (quark-gluon plasma, hyperons, color superconductivity, ...)
- QCD is perturbative but above $\sim 40n_{\text{sat}}$!!
- No theory, only models apply in the regime of low- T and large densities.



A. Yu. Kotov, 2019

INSIDE A NEUTRON STAR

A NASA mission will use X-ray spectroscopy to gather clues about the interior of neutron stars — the Universe's densest forms of matter.

Outer crust

Atomic nuclei, free electrons

Inner crust

Heavier atomic nuclei, free neutrons and electrons

Outer core

Quantum liquid where neutrons, protons and electrons exist in a soup

Inner core

Unknown ultra-dense matter. Neutrons and protons may remain as particles, break down into their constituent quarks, or even become 'hyperons'.

Atmosphere

Hydrogen, helium, carbon

Beam of X-rays coming from the neutron star's poles, which sweeps around as the star rotates.

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- The remnant of massive dead stars
- Densest matter in the universe: 6-8 times saturation density !
- Excellent laboratory to study dense matter
- Their core remains a mystery

Why relativistic models ?

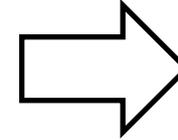
Many models for nuclear matter exist, with **chiral effective theory** being one of them: a perturbative expansion in (q/Λ) with a hierarchy of leading orders

Advantages

- A control of the uncertainty as a function of q/Λ , allowing to set the limitation of the EFT

Limitations

- Breaks down at $\sim 1-2n_{\text{sat}}$, whereas we need to describe nuclear matter at higher densities



Needs a complementary approach to describe higher densities (piecewise polytropes, sound-speed model, meta-model, etc)

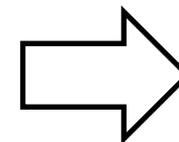
At high density, we need a **relativistic approach** since the sound speed in NS cores is expected to be larger than 10% of the light speed, as confirmed by analyses of recent radio as well as X-ray observations from NICER of massive NSs.

Advantages

- Built-in relativistic structure (spin, spin-orbit potential...) + can go beyond $2n_{\text{sat}}$

Limitations

- No simple way to decide where the model breaks down, or to quantify the uncertainties.



We thus employ Bayesian statistics to explore the relation between observables uncertainties and the one in the model predictions

Motivation for studying dense matter

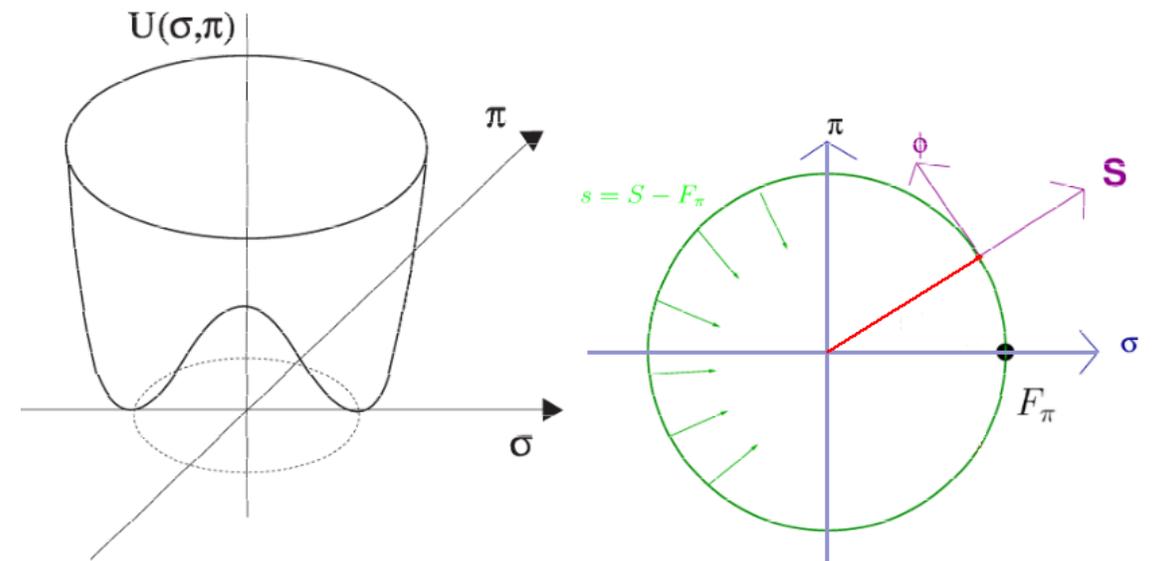
Aspects of the Strong Interactions

1) Chiral symmetry

- At the limit of zero quark masses (u,d), QCD has a chiral symmetry (non-interacting quarks with opposite parity are indistinguishable and do not couple to each other)
- Had the chiral symmetry been realised in nature, we would have observed for each meson, a partner meson with the SAME mass but opposite parity => the symmetry is broken

The radial component of the chiral field corresponds to the σ meson of Walecka, first identified by *Chanfray, Ericson, Guichon (PRC 63 (2001))*, and the phase component corresponds to the massless Goldstone boson, the pion

But since the quarks have a small mass, the symmetry is also explicitly broken and the pion acquires a small mass!



2) Confinement

- It is well established that in QCD, only colour neutral objects can be observed
- Nucleons, being made of quarks and gluons, are polarised by external fields
- In Guichon's work (*Guichon, Phys. Lett. B 200 (1988)*), the quarks wave functions get modified by the scalar field => the nucleon mass depends on the surrounding scalar field:
- We parametrize the nucleon mass as ^[1,2]:

$$M_N(s) = M_N + g_S s + \frac{1}{2} \kappa_{NS} \left(s^2 + \frac{s^3}{3 f_\pi} \right) \text{Nucleon polarisation}$$

The response parameters g_s and κ_{NS} might be given by an underlying confining model (for example NJL + confining potential)

[1] Chanfray and Ericson, *EPJA* (2005)

[2] Chanfray and Ericson, *PRC75* (2007)

Motivation for studying dense matter

Aspects of the Strong Interactions

The Chiral confining model

1. *Chamseddine, Margueron, Chanfray, et al. Relativistic Hartree–Fock chiral Lagrangians with confinement, nucleon finite size and short-range effects. EPJA59 (2024)*
2. *Chamseddine, Margueron, Hansen, Chanfray, Hartree-Fock Lagrangians with a Nambu-Jona–Lasino scalar potential. EPJA60 (2024)*
3. *Chanfray, EPJA (2024),*
4. *Chanfray, Ericson, Martini, Universe 9.7 (2023)*
5. *Massot, Chanfray, PRC78 (2008)*
6. *Chanfray and Ericson, PRC75 (2007)*
7. *Chanfray and Ericson, EPJA (2005)*

LQCD and confining models

κ_{NS} , or the dimensionless constant $C_{NS} = (\kappa_{NS} F_\pi^2 / 2M_N)$, is expected by realistic confining potentials to **always be smaller than one** : MIT bag model, Simple models^[1], QCD connected chiral confining model^[2]

- It is possible to link g_s and κ_{NS} to LQCD which expresses the nucleon mass as:

$$M_N(m_\pi^2) = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \dots + \Sigma_\pi(m_\pi^2, \Lambda)$$

- When using the Linear Sigma Model (LσM) for the chiral potential, we get^[3]:

$$a_2 = \frac{F_\pi g_s}{m_s^2} \quad \text{and} \quad a_4 = -\frac{F_\pi g_s}{2m_s^4} \left(3 - 2 \frac{M_N}{g_s F_\pi} C_{NS} \right)$$

[1] Chanfray and Ericson, PRC83 (2011)

[2] Chanfray, Ericson, Martini, Universe 9.7 (2023)

[3] Chanfray and Ericson, PRC75 (2007)

Parameterisation

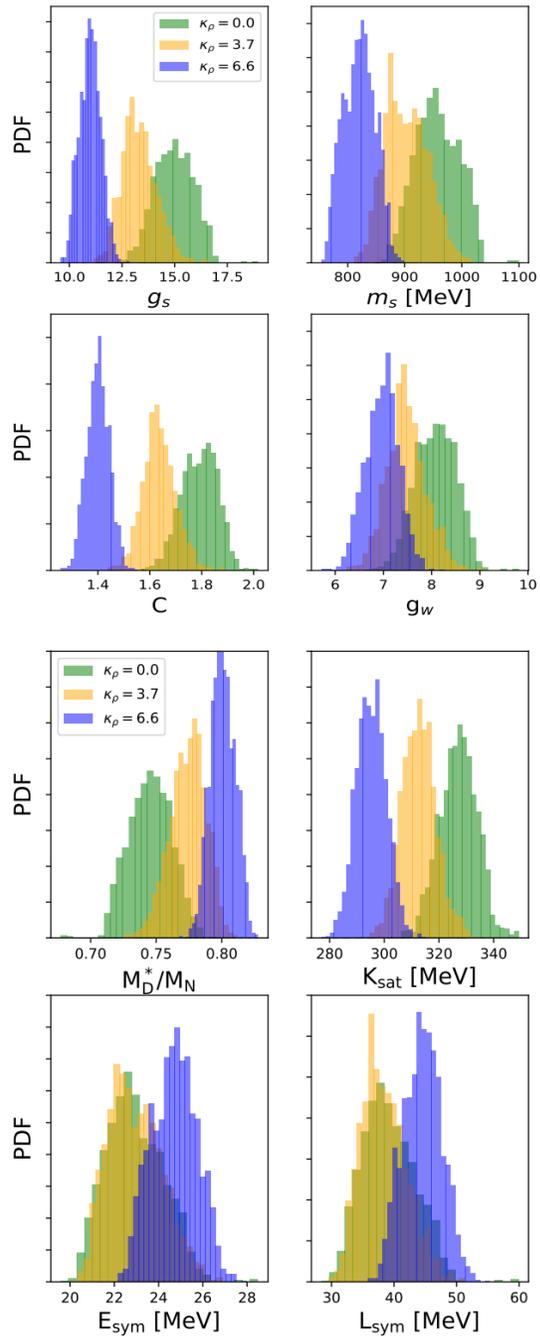
4 free parameters :

$$m_s, g_s, g_\omega, C$$

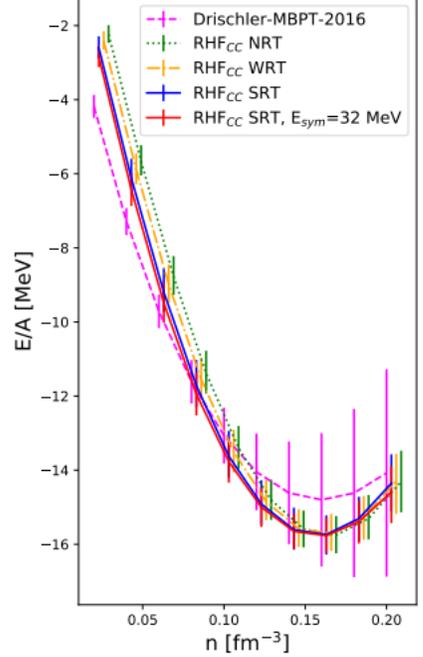
Tensor interaction coupling κ_ρ :

- $\kappa_\rho = 6.6$ suggested by scattering data
- $\kappa_\rho = 3.7$ suggested by the Vector Dominance Model (VDM)
- $\kappa_\rho = 0$ as a reference case

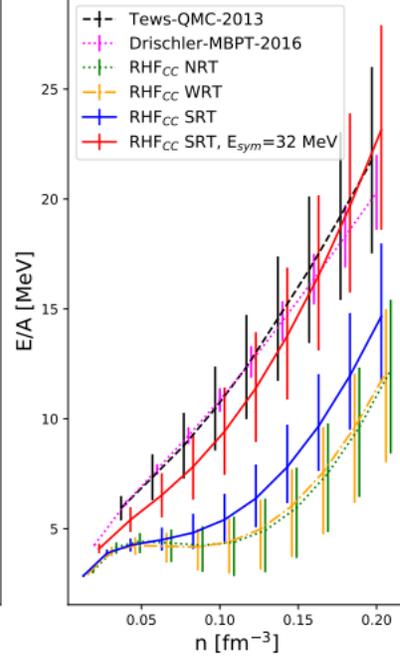
Constraints	centroid	std. dev.
a_2 (GeV^{-1})	1.553	0.136
a_4 (GeV^{-3})	-0.509	0.054
E_{sat} (MeV)	-15.8	0.3
n_{sat} (fm^{-3})	0.155	0.005



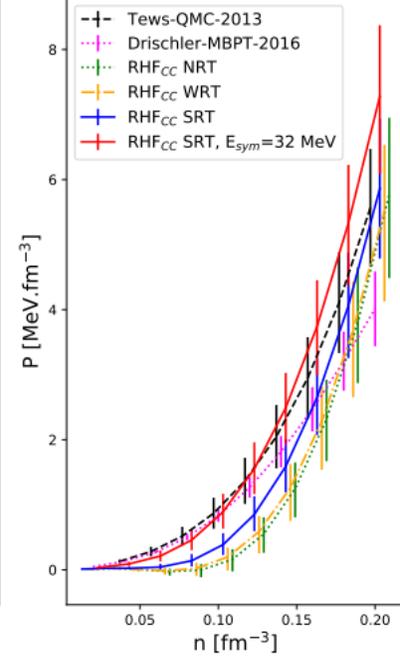
Symmetric matter binding energy



Neutron matter binding energy



Neutron matter pressure



Chamseddine, M. et al EPJA59 (2024)

Motivation for relativistic models

Aspects of the Strong Interactions

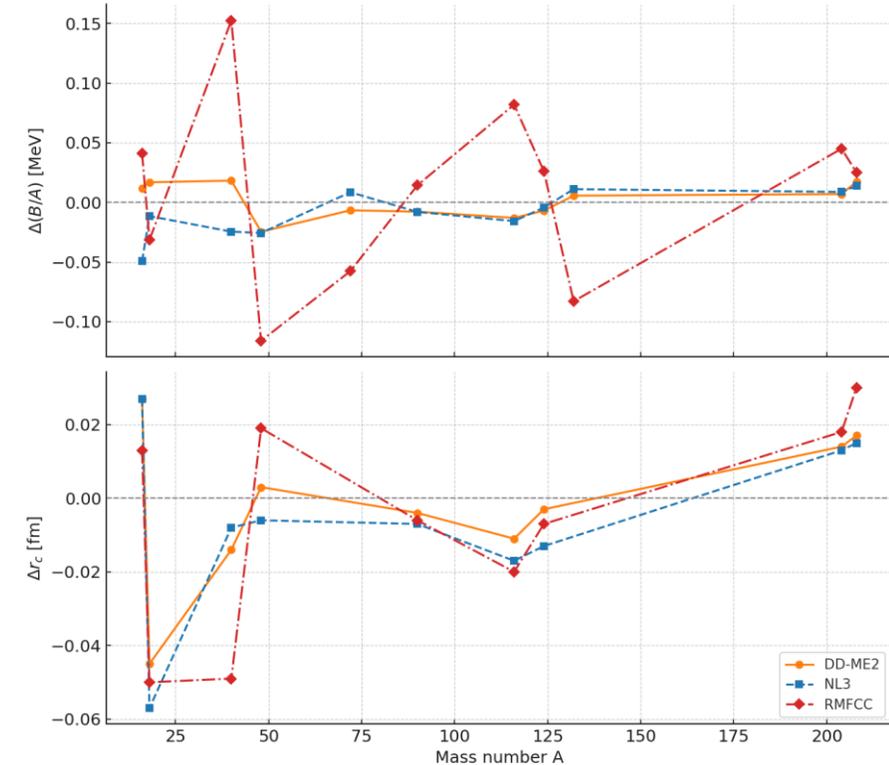
The Chiral Confining model

Finite nuclei

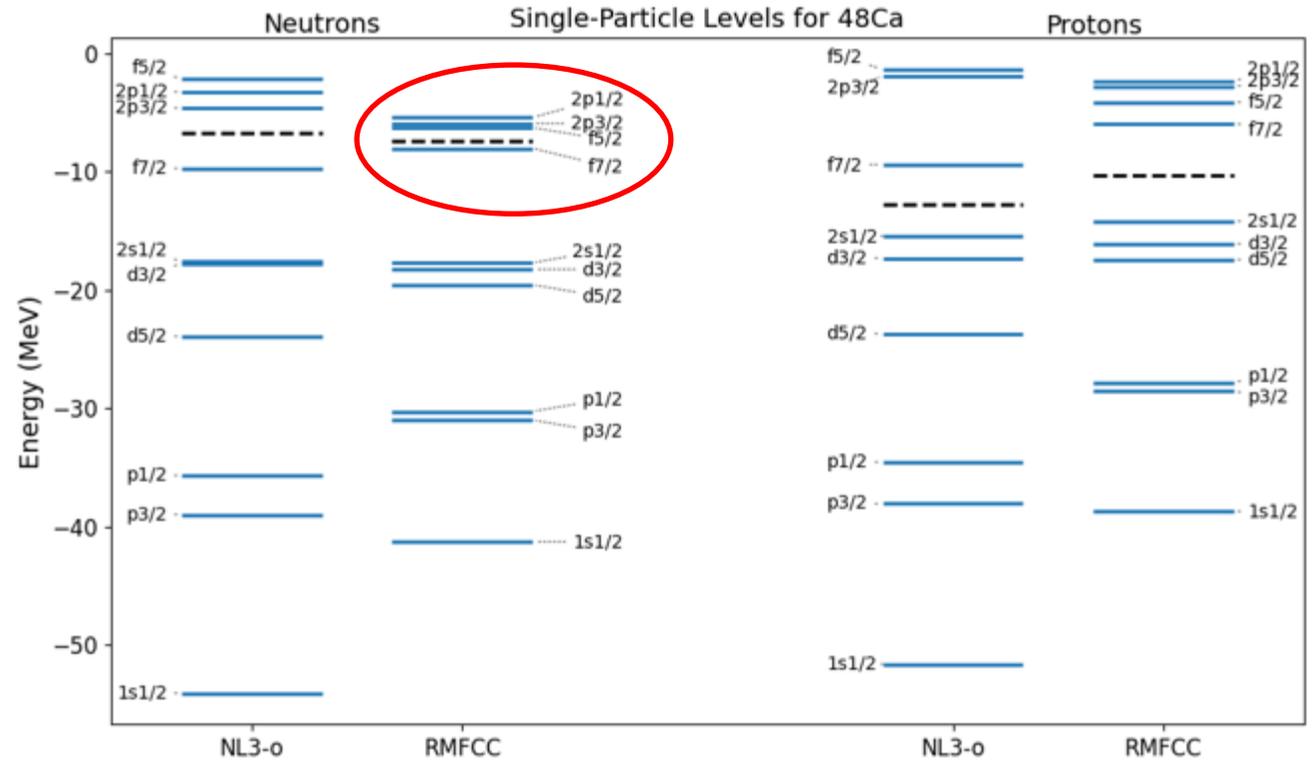
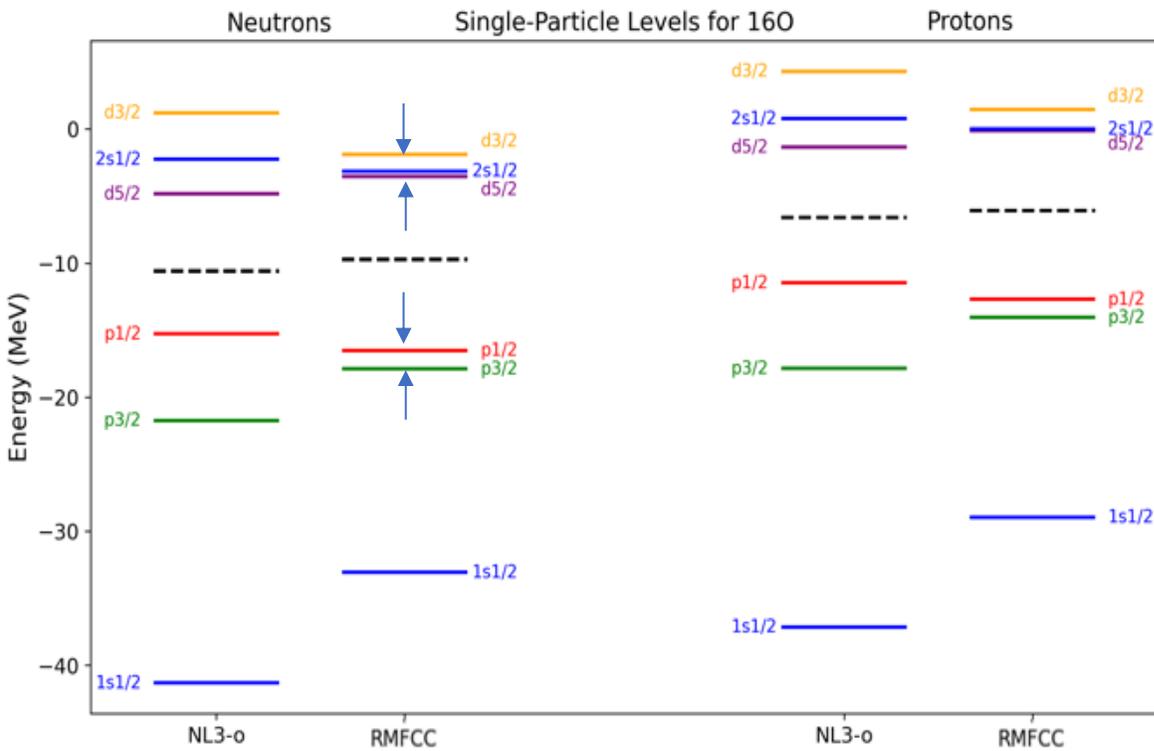
Spherical nuclei

- DIRHBS code (*Computer Physics Communications Volume 185, Issue 6, June 2014, Pages 1808-1821*) T. Nikšić et al
- Pairing is included (separable the Gogny force D1S)
- Hartree level only (direct terms)

Parameters	m_s (MeV)	C	g_s	g_ω	g_ρ
FN + NEP	455.76	0.593	5.970	5.614	4.309
FN + SPE	685.15	0.891	13.760	13.122	3.781
FN + NEP + NP	682.13	0.841	12.690	11.890	3.820



Single Particle Energies



Motivation for relativistic models

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The Chiral Confining model

Finite nuclei

Conclusions and outlooks

Conclusion

- We managed to propose a model with properties from QCD anchored at the fundamental level
- The model presents reasonable predictions for nuclear structure (binding energies, charge radii,...) at the RMF level, however the spin-orbit potential is too weak which leads to uncorrect predicitions for SPE
- At the Hartree level, the pion does not contribute, nor does the tensor interaction of the rho meson, important contributors to the spin-orbit

Outlooks

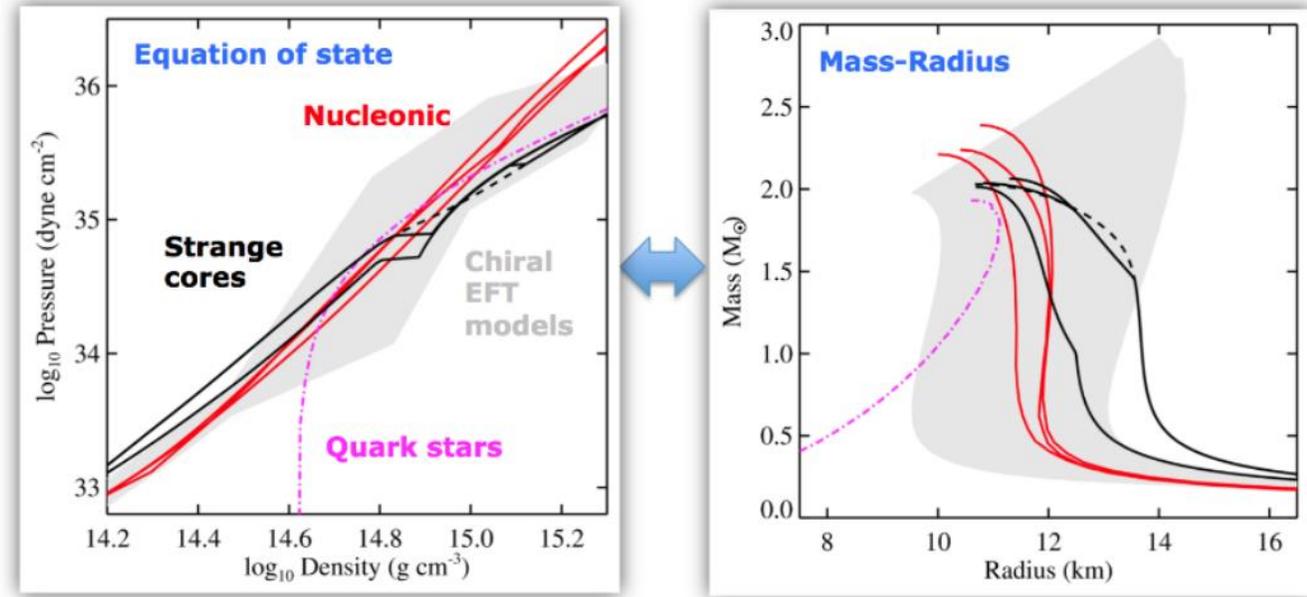
- Exploring high densities by connecting the model to a quark phase via some phase transitions
- Including Fock terms, and see whether it can solve the single particle energies problem by boosting the spin-orbit potential

THANK YOU !

Extra slides

NS observables

- We solve the hydrostatic equations in GR for spherical and nonrotating stars (TOV equations).
- One-to-one correspondence between EoS and M-R curve
- We can extract tidal deformabilities from gravitational waves (LIGO/VIRGO) or compactness from X-ray measurements (e.g NICER)



Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4} \frac{(mc^2 + 4\pi pr^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$
$$\frac{dm}{dr} = 4\pi \frac{\epsilon}{c^2} r^2$$

Contact term problem and SRCs

The pion term and rho tensor have a derivative coupling which induces a UV divergence:

$$V(q) = \frac{q^2}{q^2 + m^2} = 1 - \underbrace{\frac{m^2}{q^2 + m^2}}$$

Contact term (hard core)

→ can be suppressed by SRC

Normal Yukawa
potential
(attractive)

Improvement of the chiral confining model

- The model being an effective one, doesn't have a good resolution at short ranges ($q \sim M_N$), where we expect it to start to break
- Short range effects should be treated by hand, but maintaining as much as possible a connection with underlying microscopic descriptions
- We use form factors (FF) for nucleon finite size, and the Jastrow function approach for SRC to simulate the G-matrix: the mesons' propagators are convoluted with a correlation function forbidding the presence of 2 nucleons at the same point

Improvement of the chiral confining model

Chamseddine, M. et al EPJA59 (2024)

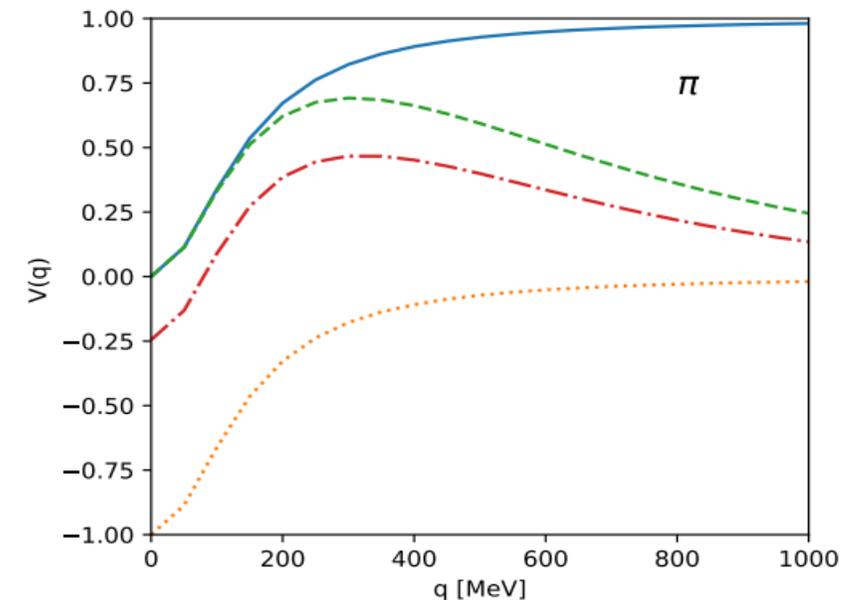
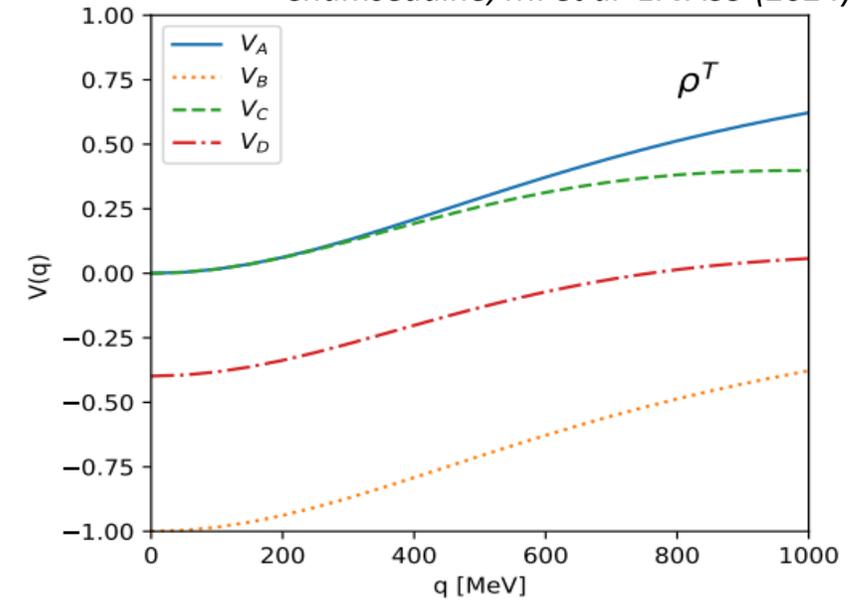
$$V_A = 1 - \frac{m^2}{q^2 + m^2}$$

can be treated in different ways:

- «Orsay» prescription:
$$V_B = -\frac{m^2}{q^2 + m^2}$$

In this case, the repulsive short range component of the pion disappears and is attractive, which is not really physical.

- Form factors
$$V_C = V_A F^2(q) = V_A(q) \left(\frac{\Lambda^2}{\Lambda^2 + q^2} \right)^2$$
- FF + Jastrow ansatz
$$V_D = V_C(q) - V_C(q^2 \rightarrow q^2 + q_c^2)$$



Landau and Dirac effective masses

Chamseddine, M. et al EPJA59 (2024)

- From the Dirac equation we extract a Schrödinger equivalent potential V_{eq}
- The group velocity v_g is defined as the physical velocity of the wave packet

$$v_g^* = \frac{k}{M_L^*} \equiv \frac{d\epsilon}{dk}$$

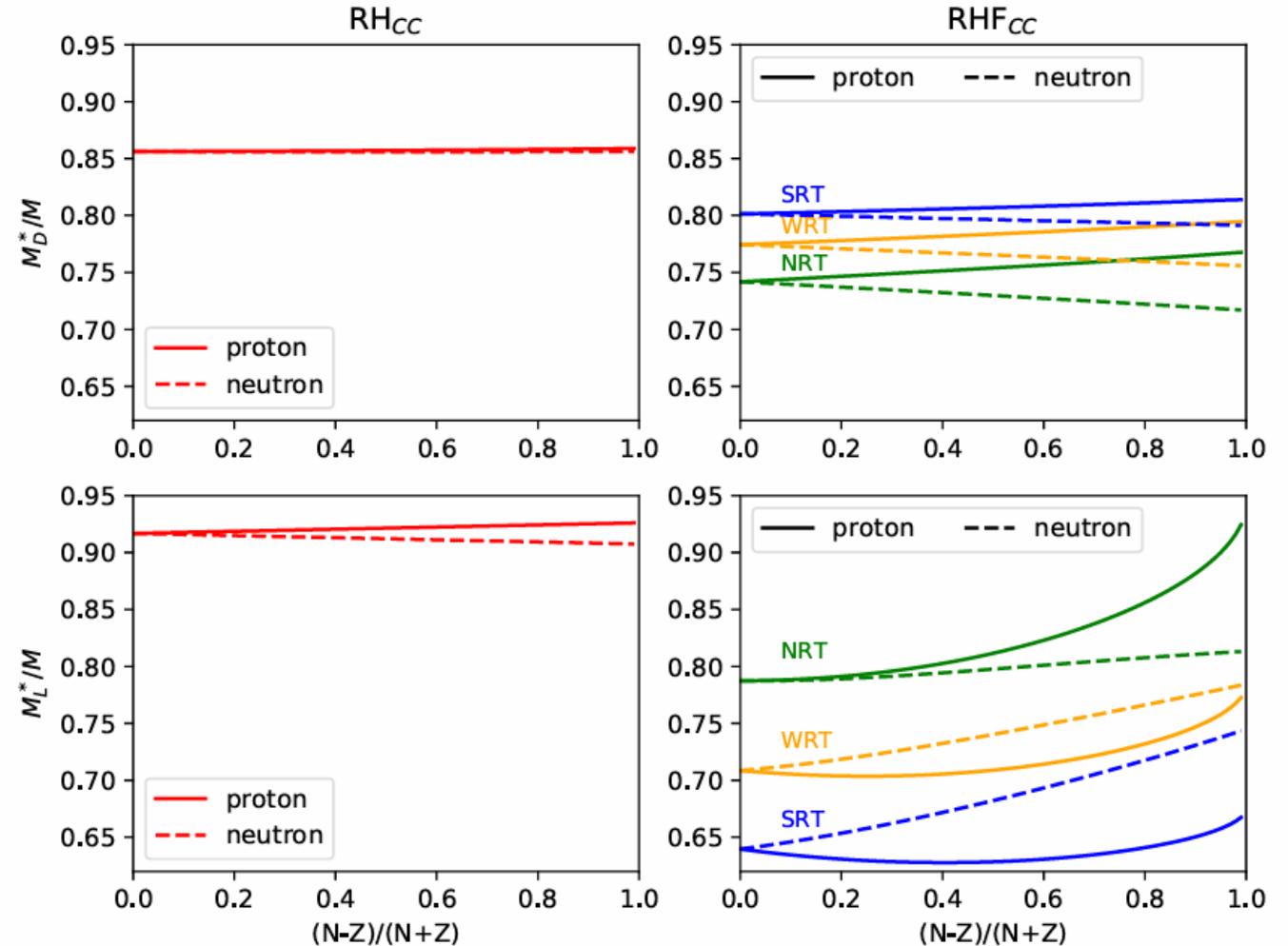
$$M_{D,N}^* = M_N + \Sigma_{S,N}^D$$

$$M_{L,N}^* = M_N - \Sigma_{0,N}^D$$

With

$$\Sigma_{S,N}^D = (M_N(\bar{s}) - M_N) - \frac{g_\delta^2}{m_\delta^2} (n_{sN} - n_{s\bar{N}})$$

$$\Sigma_{0,N}^D = \frac{g_\omega^2}{m_\omega^2} n + \frac{g_\rho^2}{m_\rho^2} (n_{sN} - n_{s\bar{N}})$$



Nucleus	Model	E_{tot} (MeV)	$E_{\text{pairing}}(\text{n/p})$ (MeV)	R_C (fm)
^{16}O	NL3	-126.83	0.0/0.0	2.726
	DD-ME2	-127.81	0.0/0.0	2.726
	PDM	-124.70	0.0/0.0	2.762
	PDM+SO	-131.52	0.0/0.0	2.637
	RMFCC1	-128.28	0.0/0.0	2.712
	RMFCC2	-134.77	0.0/0.0	2.806
	Exp	-128.00	0.0/0.0	2.699
^{18}O	NL3	-139.60	-5.37/0.0	2.718
	DD-ME2	-140.11	-5.12/0.0	2.730
	PDM	-136.70	-8.72/0.0	2.776
	PDM+SO	-142.13	-7.44/0.0	2.665
	RMFCC1	-139.24	-9.17/0.0	2.725
	RMFCC2	-147.78	-5.71/0.0	2.817
	Exp	-141.12	-6.0/0.0	2.773
^{40}Ca	NL3	-341.07	0.0/0.0	3.470
	DD-ME2	-342.78	0.0/0.0	3.464
	PDM	-332.54	0.0/0.0	3.469
	PDM+SO	-348.93	0.0/0.0	3.378
	RMFCC1	-348.15	0.0/0.0	3.429
	RMFCC2	-345.88	0.0/0.0	3.572

Nucleus	Model	E_{tot} (MeV)	$E_{\text{pairing}}(\text{n/p})$ (MeV)	R_C (fm)
^{48}Ca	Exp	-342.69	0.0/0.0	3.478
	NL3	-414.76	0.0/0.0	3.471
	DD-ME2	-414.81	0.0/0.0	3.480
	PDM	-397.72	-21.65/0.0	3.531
	PDM+SO	-413.00	-14.67/0.0	3.468
	RMFCC1	-410.42	-23.04/0.0	3.496
	RMFCC2	-420.31	0.0/0.0	3.614
	Exp	-414.33	0.0/0.0	3.477
^{72}Ni	NL3	-613.78	-8.43/0.0	3.895
	DD-ME2	-612.69	-7.15/0.0	3.914
	PDM	-614.98	-15.43/-20.45	3.945
	RMFCC1	-609.02	-18.24/-23.49	3.926
	RMFCC2	-611.93	-9.01/0.0	4.074
	Exp	-613.173	-9.0/0.0	3.914*
	^{90}Zr	NL3	-783.17	0.0/-8.40
DD-ME2		-783.19	0.0/-5.55	4.268
PDM		-778.30	-22.55/-5.35	4.281
RMFCC1		-785.18	-27.08/-8.61	4.266
RMFCC2		-776.03	0.0/-7.85	4.274
Exp		-783.89	0.0/-8.0	4.272
^{116}Sn		NL3	-986.86	-14.15/0.0
	DD-ME2	-987.17	-9.43/0.0	4.615
	PDM	-990.74	-14.09/-23.76	4.630
	RMFCC1	-998.18	-16.30/-28.11	4.606
	RMFCC2	-975.39	-14.27/0.0	4.798
	Exp	-988.681	-17./0.0	4.626
	^{124}Sn	NL3	-1049.42	-14.49/0.0
DD-ME2		-1049.10	-11.17/0.0	4.671
PDM		-1055.69	-19.31/-24.59	4.690
RMFCC1		-1053.19	-23.06/-29.04	4.667
RMFCC2		-1043.29	-15.79/0.0	4.854
Exp		-1049.96	-15./0.0	4.674
^{132}Sn		NL3	-1104.32	0.0/0.0
	DD-ME2	-1103.60	0.0/0.0	4.718
	PDM	-1107.44	-25.02/-25.22	4.764
	RMFCC1	-1091.92	-32.04/-29.67	4.742
	RMFCC2	-1105.70	0.0/0.0	4.907
	Exp	-1102.860	0.0/0.0	4.718*

Table 1: The values of the data to be reproduced by adjusting the parameters m_s , C , g_s , g_ω and g_ρ : the NEPs E_{sat} , n_{sat} , E_{sym} and K_{sat} .

Parameters	centroid	std. dev.
E_{sat} (MeV)	-16	5%
n_{sat} (fm^{-3})	0.16	10%
E_{sym} (MeV)	31	10%
K_{sat} (MeV)	250	10%

The following table shows the nuclei and their properties used for the fitting procedure:

TABLE II. The total binding energies BE , charge radii r_c , and the differences between the radii of neutron and proton density distributions $r_{np} = (r_n - r_p)$, used to adjust the interaction DD-ME2. The calculated values are compared with experimental data (values in parentheses). In the last three columns the corresponding deviations dE , dr_c , and dr_{np} (all in %) are included.

Nucleus	BE (MeV)	r_c (fm)	$r_n - r_p$ (fm)	dE	dr_c	dr_{np}
^{16}O	127.801 (127.619)	2.727 (2.730)	-0.03	0.1	-0.1	
^{40}Ca	342.741 (342.052)	3.464 (3.485)	-0.05	0.2	-0.6	
^{48}Ca	414.770 (415.991)	3.481 (3.484)	0.18	-0.3	-0.1	
^{72}Ni	612.655 (613.173)	3.914	0.28	-0.1		
^{90}Zr	783.155 (783.893)	4.275 (4.272)	0.07	-0.1	0.1	
^{116}Sn	986.928 (988.681)	4.615 (4.626)	0.12 (0.12)	-0.2	-0.2	3.8
^{124}Sn	1048.859 (1049.962)	4.671 (4.674)	0.21 (0.19)	-0.1	-0.1	10.7
^{132}Sn	1103.469 (1102.860)	4.718	0.26	0.1		
^{204}Pb	1608.506 (1607.520)	5.500 (5.486)	0.17	0.1	0.3	
^{208}Pb	1638.426 (1636.446)	5.518 (5.505)	0.19 (0.20)	0.1	0.2	-4.7
^{214}Pb	1661.182 (1663.298)	5.568 (5.562)	0.24	-0.1	0.1	
^{210}Po	1649.695 (1645.228)	5.552	0.17	0.3		

Pairing

- Since the calculations involving the finite-range Gogny force in the pairing channel require considerable computational effort, a separable form of the Gogny force has been introduced for RHB calculations in spherical and deformed nuclei
- The force is separable in momentum space, and is completely determined by two parameters that are adjusted to reproduce the pairing gap of the Gogny force in symmetric nuclear matter
- The two parameters G and a have been adjusted to reproduce the density dependence of the gap at the Fermi surface, calculated with a Gogny force (D1S parameterization)

$$V^{pp}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}'_1, \mathbf{r}'_2) = -G\delta(\mathbf{R} - \mathbf{R}') P(\mathbf{r})P(\mathbf{r}')$$

where $\mathbf{R} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 + \mathbf{r}_2)$ and $\mathbf{r} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2)$ denote the center-of-mass and the relative coordinates, respectively, and $P(\mathbf{r})$ is the Fourier transform of $p(k)$:

$$P(\mathbf{r}) = \frac{1}{(4\pi a^2)^{3/2}} e^{-\mathbf{r}^2/2a^2}$$