Refining density functional approaches for nuclear reactions and neutron stars

Low-energy nuclear theory: From few nucleons to the stars

IJCLab, Orsay (France)

13^{th} - 14^{th} November 2025

MOdeling Nuclear STructure and REactions

Speaker: S. Burrello

INFN - Laboratori Nazionali del Sud, Catania

Outline of the presentation

1. **Theoretical approaches for nuclear many-body problem**
   - Ab-initio vs phenomenological models based on energy density functionals (EDF)
   - Effective interaction and nuclear matter (NM) Equation of State (EOS)

2. **Extended EDF-based models: recent developments and results**

   ✏ **Unified description of many-body correlations and clustering phenomena**
   - Phenomenological models with clusters as explicit degrees of freedom (DOF)
   - Dynamics of dilute nuclear matter with light-clusters and in-medium effects

   ✏ **Bridging phenomenological models with ab-initio approches**
   - Effective field theory (EFT)-inspired EDFs for nuclear matter and nuclei
   - Bayesian inference of neutron stars within ab-initio-benchmarked meta-model

3. **Further developments and outlooks**
   - Consistent description of fragmentation mechanisms in heavy-ion collisions (HICs)
   - Joint Bayesian analyses combining nuclear structure and HIC studies

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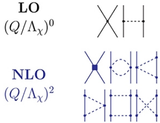
- **Ab-initio** approaches based on **many-body** expansion
  - Realistic or **effective field theory** (EFT) interactions

    $\Rightarrow$ Diagrammatic hierarchy (**power counting**)



- **Phenomenological** models with **effective** interaction
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- **Bridging** EDFs & ab-initio
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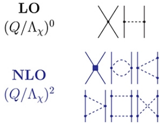

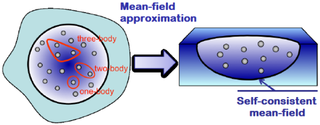

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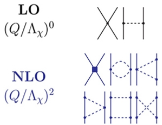

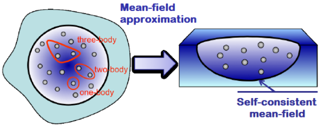

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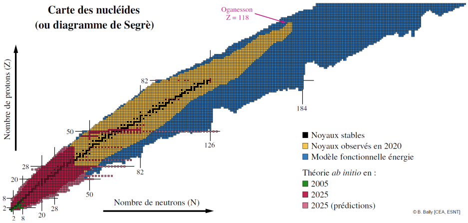

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Recent attempts to bridge EFT with EDF theories

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Contents lists available at ScienceDirect

Physics Letters B

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Towards a power counting in nuclear energy–density–functional theories through a perturbative analysis

Stefano Burrello [a,*], Marcella Grasso [a], Chieh-Jen Yang [b]

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PHYSICAL REVIEW C **106**, L011305 (2022)

Letter

Calculations for nuclear matter and finite nuclei within and beyond energy-density-functional theories through interactions guided by effective field theory

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[1] Department of Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden
[2] Nuclear Physics Institute of the Czech Academy of Sciences, 25069 Řež, Czech Republic
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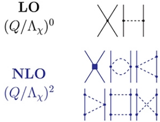

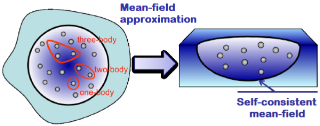

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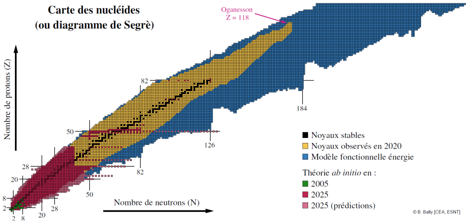

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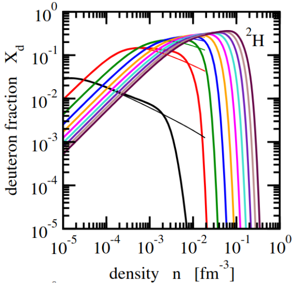
  [S. Typel et al., PRC 81, 015803 (2010)]



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  - (Effective) binding energy → $B^{\rm eff} = B - \Delta m$
- $\Delta m^{\rm (low)}$ from **in-medium** Schrödinger equation (**SE**)

  [G. Röpke, NPA 867 (2011) 66–80]

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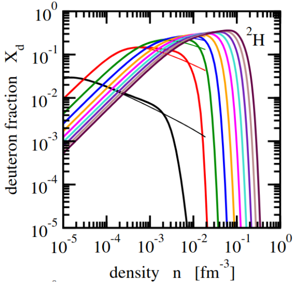

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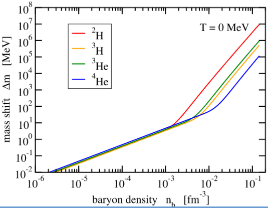

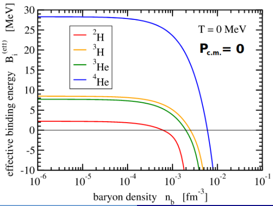

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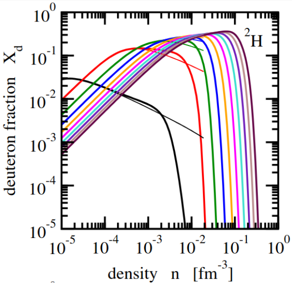

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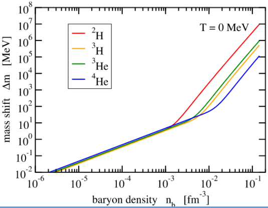

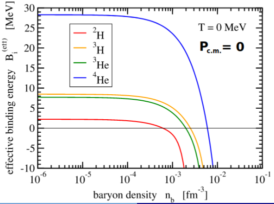

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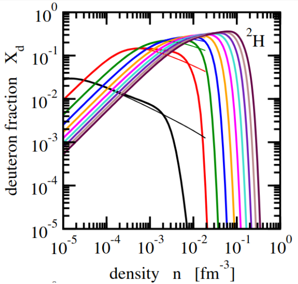

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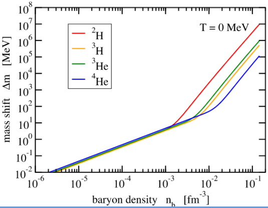
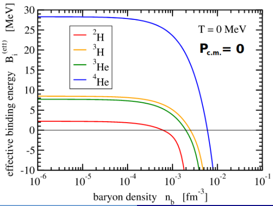
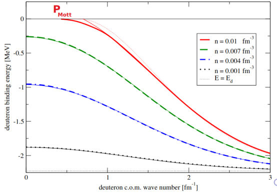


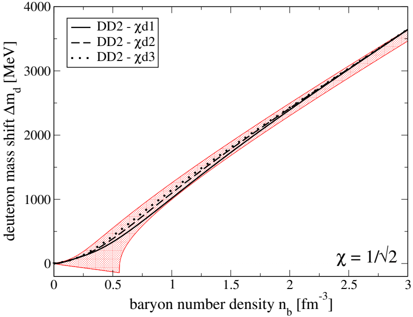


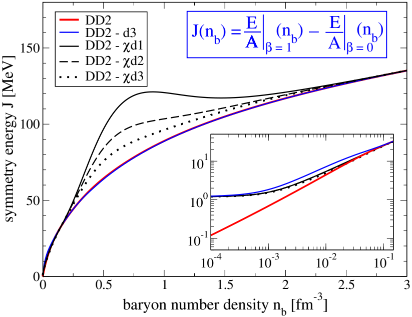


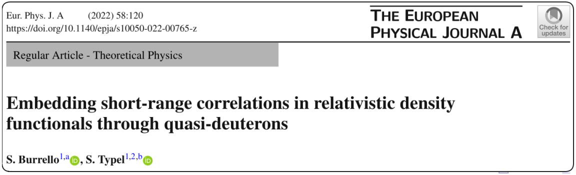
Clusters as surrogate for SRCs in extended EDFs

- **Unified** mass-shift **parameterization** for bound *d* / *np* SRCs ($\rho_B \lesssim \rho_{\rm Mott}$)

$$\Delta m_d(x) = \frac{ax}{1+bx} + cx^{\eta+1}\left[1-\tanh(x)\right] + fx\tanh(gx), \qquad x = \frac{\rho_b}{\rho_0}$$






Eur. Phys. J. A (2022) 58:120
https://doi.org/10.1140/epja/s10050-022-00765-z
THE EUROPEAN PHYSICAL JOURNAL A
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Regular Article - Theoretical Physics

Embedding short-range correlations in relativistic density functionals through quasi-deuterons

S. Burrello[1,a], S. Typel[1,2,b]

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Modeling HICs with EDF-based transport models

- Modeling **dynamics** of **HIC** at $E_{\mathrm{beam}} \approx (30 - 300)\, A\mathrm{MeV} \Rightarrow$ EOS
  - Boltzmann–Uehling–Uhlenbeck (**BUU**) equation for the **distribution function** f_τ

$$\left(\partial_t + \nabla_{\mathbf{p}}\varepsilon_\tau \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_\tau \cdot \nabla_{\mathbf{p}}\right) f_\tau = I_\tau^{\mathrm{coll}}[f_n, f_p, \ldots], \qquad \tau = n, p, d, t, h, \alpha$$

- **Extended** kinetic (**transport**) approaches $\Rightarrow$ Light **clusters** & **in-medium** effects
  - Solving **in-medium SE** $\forall$ $(\mathbf{r}, t)$ point (computationally very demanding)
  - Phase-space excluded-volume approach

- **Cut-off** momentum $P_j^{\mathrm{Mott}} \equiv \Lambda_j(\rho_b, \beta, T)$

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arXiv > nucl-th > arXiv:2506.16437

Nuclear Theory

[Submitted on 19 Jun 2025]

Phase-space excluded-volume approach for light clusters in nuclear medium

Rui Wang, Zhen Zhang, Stefano Burrello, Maria Colonna, Edoardo G. Lanza

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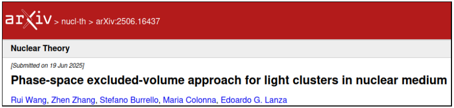
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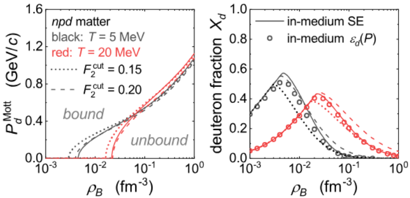
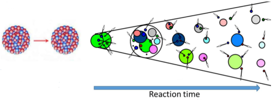
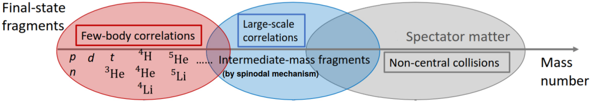


Fragment formation mechanisms in HICs



- **Kinetic** approaches for HIC at intermediate energies
  - No **consistent** light & heavier **fragments** production



- Spinodal instability ⇒ **Mean-field** (**Vlasov**) dynamics

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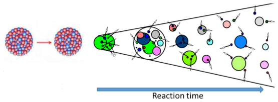

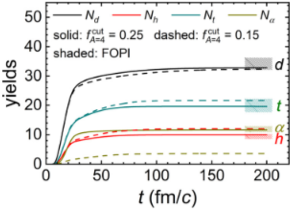

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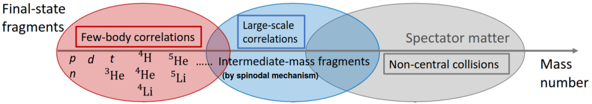

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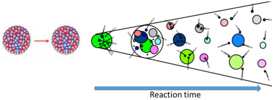

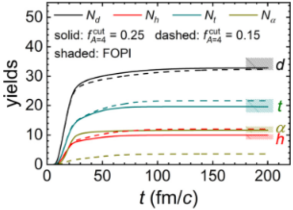

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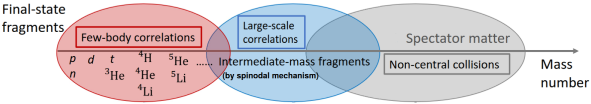

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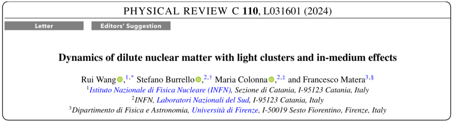

PHYSICAL REVIEW C **110**, L031601 (2024)

Letter | Editors' Suggestion

Dynamics of dilute nuclear matter with light clusters and in-medium effects

Rui Wang[1,*] Stefano Burrello[2,†] Maria Colonna[2,‡] and Francesco Matera[3,§]
[1]Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Catania, I-95123 Catania, Italy
[2]INFN, Laboratori Nazionali del Sud, I-95123 Catania, Italy
[3]Dipartimento di Fisica e Astronomia, Università di Firenze, I-50019 Sesto Fiorentino, Firenze, Italy

Question

Does the light **clusters** formation (during the **compression** phase) affect the **spinodal** instability (in the **expansion** stage)?

Linearized Vlasov equations for NM+deuterons

- **Linear** response to **collision-less** Boltzmann $\Rightarrow$ linearized **Vlasov** equations for NMd

$$\partial_t\,(\delta f_j)+\nabla_{\mathbf{r}}(\delta f_j)\cdot\nabla_{\mathbf{p}}\varepsilon_j-\nabla_{\mathbf{p}} f_j\cdot\nabla_{\mathbf{r}}(\delta\varepsilon_j)=0 \quad\Rightarrow\quad \boxed{\delta\rho_j=-\chi_j\sum_l\left(F_0^{jl}+\tilde{F}_\lambda^{jl}\right)\delta\rho_l-\delta_{jd}\sum_l\Phi_\lambda^{dl}\delta\rho_l}$$

- **Single-particle** energy $\varepsilon_j\equiv\dfrac{\delta\mathcal{E}}{\delta f_j(\mathbf{p})}$ (from **EDF** $\mathcal{E}=\mathcal{K}+\mathcal{U}$)

$$\varepsilon_j=\frac{p^2}{2m_j}+U_j+\tilde{\varepsilon}_j^\lambda \qquad \left(\tilde{\varepsilon}_j^\lambda\propto\Phi_\lambda^{dj}\sim\frac{\partial\Lambda_d}{\partial\rho_j}\right)$$

- Momentum-independent **Skyrme**-like interaction (= for bound and free nucleons)

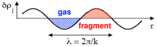
$$\mathcal{U}=\frac{A}{2}\frac{\rho_b^2}{\rho_0}+\frac{B}{\alpha+2}\frac{\rho_b^{\alpha+2}}{\rho_0^{\alpha+1}}+\frac{C(\rho)}{2}\frac{\rho_3^2}{\rho_0}+\frac{D}{2}(\nabla_{\mathbf{r}}\rho_b)^2-\frac{D_3}{2}(\nabla_{\mathbf{r}}\rho_3)^2$$

- **Density-dependent** (Mott) momentum **cut-off** $\Rightarrow$ extra-terms in both $\delta\rho_j$ and ε_j

$$\rho_j=g_j\int_{|\mathbf{p}|>\Lambda_j}\frac{d\mathbf{p}}{(2\pi\hbar)^3}f_j \quad j=n,p,d \quad\rightarrow\quad \delta\rho_j(\mathbf{r},t)=g_j\int_{|\mathbf{p}|>\Lambda_j}\frac{d\mathbf{p}}{(2\pi\hbar)^3}\delta f_j-\delta_{jd}\sum_{l=n,p,d}\Phi_\lambda^{dl}\delta\rho_l$$

- $\Phi_\lambda^{dl}\neq 0\Rightarrow$ adding **in-medium** effects for cluster appearance/dissolution in **dynamics**

- **Landau** procedure $\left(F_0^{jl}\sim\dfrac{\partial U_j}{\partial\rho_l},\tilde{F}_\lambda^{jl}\sim\dfrac{\partial\tilde{\varepsilon}_j^\lambda}{\partial\rho_l}\right)$ for $\delta f_j\sim\sum_{\mathbf{k}}\delta f_j^{\mathbf{k}}\,e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

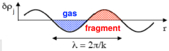


Spinodal with light-clusters & Mott effect

- **Solving** linearized Vlasov equations for symmetric NM (SNM) $\Rightarrow \omega = \omega(k)$

$$\delta\rho_j = -\chi_j \sum_l \left(F_0^{jl} + \tilde{F}_\lambda^{jl}\right) \delta\rho_l - \delta_{jd} \sum_l \Phi_\lambda^{dl} \delta\rho_l$$

- $\omega = \mathrm{Im}(\omega) \Leftrightarrow$ unstable mode (spinodal region)



[R. Wang, S. Burrello, M. Colonna, F. Matera, PRC 110, L031601 (2024)]

- $\omega = 0$ $(\chi_j = 1) \Rightarrow$ border

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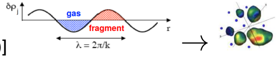

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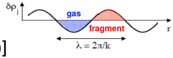

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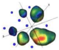

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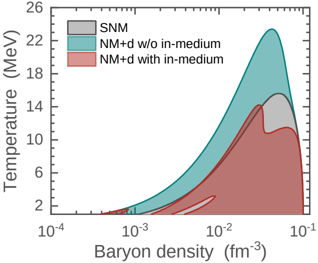

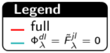

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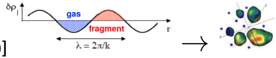

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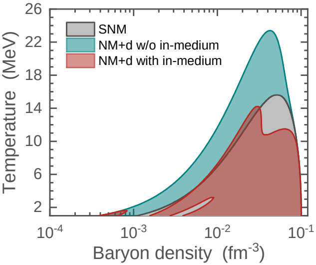

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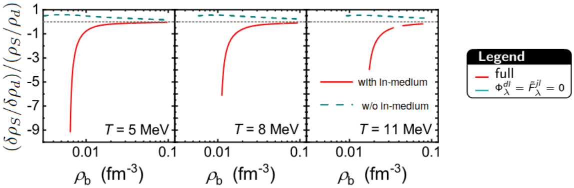

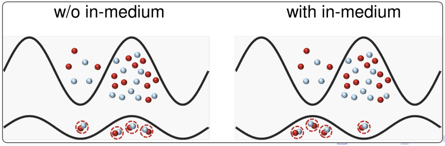

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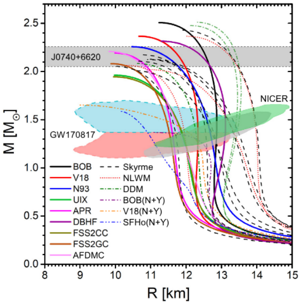

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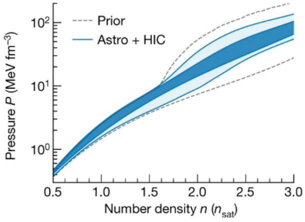

Huth et al. Nature 606, 276–280 (2022)

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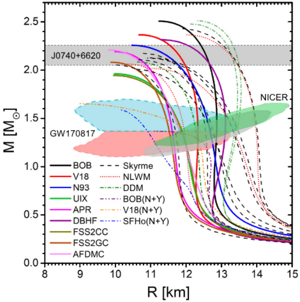

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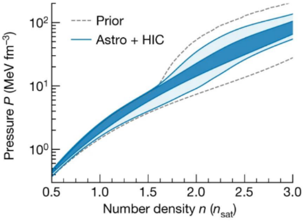

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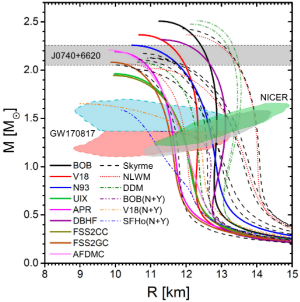

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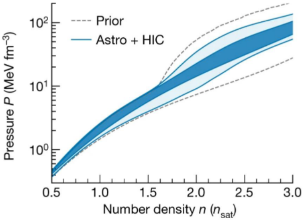

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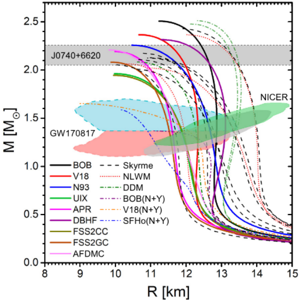

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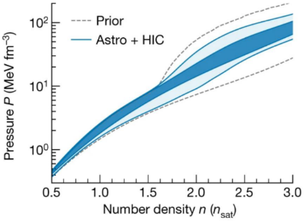

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Unified EOS: phenomenological meta-model

- **Unified crust-core** models based on **EDFs** ⇒ meta-modeling (**MM**) approach

[J. Margueron et al., PRC 97, 025805 (2018)]

$$e_{\mathrm{MM}}(n_{\mathrm{B}},\delta) = t^{*}_{\mathrm{FG}}(n_{\mathrm{B}},\delta) + v_{\mathrm{MM}}(n_{\mathrm{B}},\delta) \qquad \delta = (n_{\mathrm{n}} - n_{\mathrm{p}})/n_{\mathrm{B}}$$

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  - ✓ **Truncation** for $\mathcal{N} = 4$ ($E_{\mathrm{sat}}, K_{\mathrm{sat}}, Q_{\mathrm{sat}}, Z_{\mathrm{sat}}$ & $E_{\mathrm{sym}}, L_{\mathrm{sym}}, K_{\mathrm{sym}}, Q_{\mathrm{sym}}, Z_{\mathrm{sym}}$)

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- 👍 **Span** over EDFs existing in literature
- 👍 **Probe** of **novel** n_{B}-dependencies
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$$\frac{e_{\mathrm{B}}}{t_{\mathrm{FG}}} = 1 + \frac{10}{9\pi}(ak_F) + \frac{4}{21\pi^2}(11 - 2\ln 2)(ak_F)^2 + \ldots$$

⇒ **microscopic** ab-initio calculations

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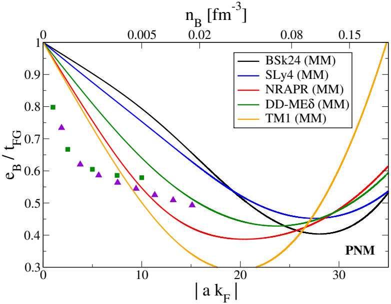

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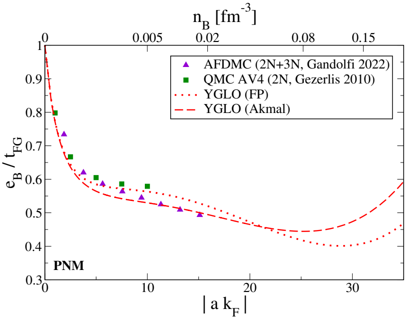
EFT-inspired EDFs for nuclear matter & nuclei

- **EFT**-inspired **EDFs**: YGLO [C.J. Yang, M. Grasso, D. Lacroix, PRC 94, 031301 (2016)]

$$e_Y(n_B,\delta) = t_{FG}(n_B,\delta) + v_Y(n_B,\delta) \qquad v_Y(n_B,\delta) = \frac{1}{n_B}\left[\mathcal{V}^Y_{SNM} + \left(\mathcal{V}^Y_{PNM} - \mathcal{V}^Y_{SNM}\right)\delta^2\right]$$



$$\mathcal{V}^Y_i = Y_i[n_B]n_B^2 + D_i n_B^{8/3} + F_i n_B^{\alpha+2}$$

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$(\nu_i = 2, 4$ for PNM, SNM$)$

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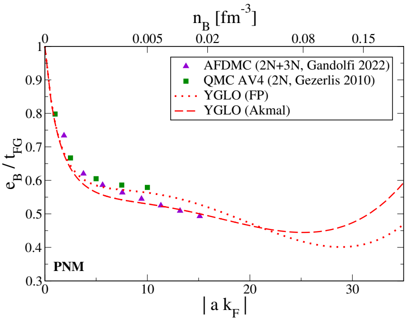

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PHYSICAL REVIEW C **103**, 064317 (2021)

Application of an *ab-initio*-inspired energy density functional to nuclei: Impact of the effective mass and the slope of the symmetry energy on bulk and surface properties

Stefano Burrello [1,*] Jérémy Bonnard [2,†] and Marcella Grasso [1,‡]

Eur. Phys. J. A (2022) 58:22
https://doi.org/10.1140/epja/s10050-022-00665-2
THE EUROPEAN PHYSICAL JOURNAL A
Regular Article - Theoretical Physics

Finite-temperature infinite matter with effective-field-theory-inspired energy-density functionals

Stefano Burrello[1,2,a], **Marcella Grasso**[2]
[1] Institut für Kernphysik, Technische Universität Darmstadt, Darmstadt, Germany
[2] IJCLab, Université Paris-Saclay, CNRS/IN2P3, 91405 Orsay, France

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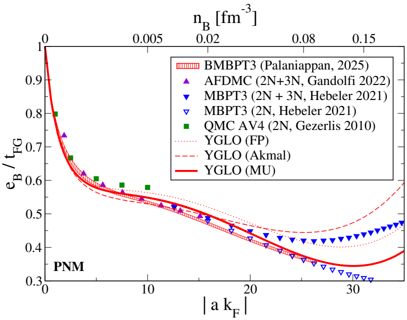

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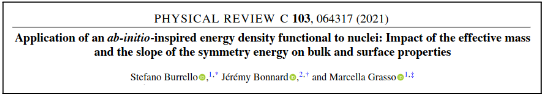

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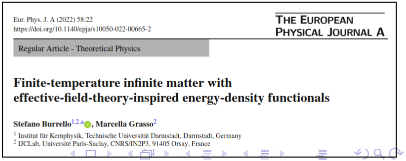

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THE EUROPEAN PHYSICAL JOURNAL A

Regular Article - Theoretical Physics

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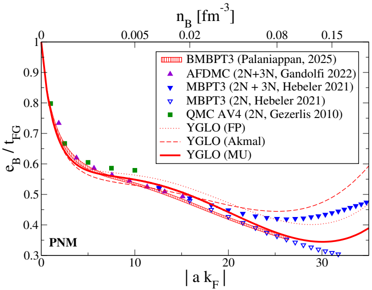
- **Newly** devised parameterization ⇒ **YGLO (MU)**

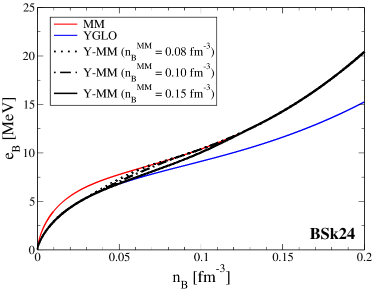
Ab-initio-benchmarked meta-model: Y-MM

- **Y-MM** $\Rightarrow$ smooth **interpolation** of MM with YGLO (MU) at low-density
  - Smooth-step transition **function** $\eta_\chi^{\mathrm{MM}} : \left[n_{\mathrm{B}}^{\chi}, n_{\mathrm{B}}^{\mathrm{MM}}\right] \to [0,1]$

$$e_{\mathrm{B}}(n_{\mathrm{B}},\delta) = e_{\mathrm{Y}}(n_{\mathrm{B}},\delta)\left(1-\eta_\chi^{\mathrm{MM}}\right) + e_{\mathrm{MM}}(n_{\mathrm{B}},\delta)\eta_\chi^{\mathrm{MM}}$$

[S. Burrello, F. Gulminelli, M. Antonelli, M. Colonna, A. Fantina, Phys. Rev. C 112, 035802 (2025)]





- $n_{\mathrm{B}} < n_{\mathrm{B}}^{\chi} = 0.02\ \mathrm{fm}^{-3}$ only **Monte-Carlo** (or Brueckner) calculations exist
- $n_{\mathrm{B}}^{\chi} \le n_{\mathrm{B}} < n_{\mathrm{B}}^{\mathrm{MM}}$ uncertainties on **3-body** forces of χEFT
- $n_{\mathrm{B}} \ge n_{\mathrm{B}}^{\mathrm{MM}}$ **empirical** MM, with $n_{\mathrm{B}}^{\mathrm{MM}} \le n_{\mathrm{sat}}$ variable **final endpoint**

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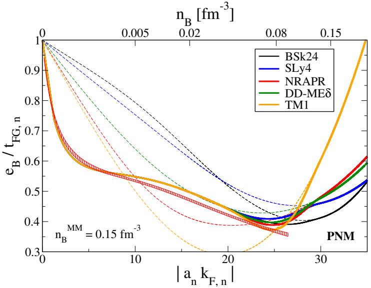

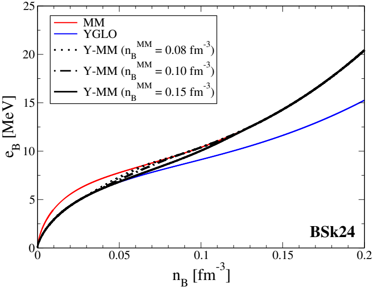

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Bayesian analysis: (informed) prior & posterior

- **Bayes'** principle: by filtering **prior** ⇒ **posterior** probability density functions (**PDF**)

$$p_{\text{post}}(\mathbf{X}) = \mathcal{C}\, w_{\text{EFT}}(\mathbf{X})\, w_{\text{IP}}(\mathbf{X})\, e^{-\chi^2(\mathbf{X})/2}\, p_{\text{prior}}(\mathbf{X})$$

[S. Burrello, F. Gulminelli, M. Antonelli, M. Colonna, A. Fantina, Phys. Rev. C 112, 035802 (2025)]

- **Prior**: flat distributions $f(X_k)$ in empirical $[X_k^{\text{min}}, X_k^{\text{max}}]$

$$p_{\text{prior}}(\mathbf{X}) = \prod_{k=1}^{2(\mathcal{N}+2)} f(X_k^{\text{min}}, X_k^{\text{max}}; X_k)$$

X_k	X_k^{min}	X_k^{max}
n_{sat} [fm^{-3}]	0.15	0.17
E_{sat} [MeV]	-17	-15
K_{sat} [MeV]	190	270
Q_{sat} [MeV]	-1000	1000
Z_{sat} [MeV]	-3000	3000
E_{sym} [MeV]	26	38
L_{sym} [MeV]	10	80
K_{sym} [MeV]	-400	200
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m^*_{sat}/m	0.6	0.8
$\Delta m^*_{\text{sat}}/m$	0.0	0.2

- Filters always active ⇒ **"Informed" prior (IP)**
  - **Likelihood (exp) filter** ⇒ χ^2 fit of **nuclear masses**

- Toggled w_{EFT} **strict band filter**[a] in $[n_{\text{B}}^{\chi}, 0.20]$ fm^{-3}
  ⇒ Probe its **effectiveness** on (Y-)MM

[a] [S. Huth et al., Nature 606, 276 (2022)] with ±5% margin

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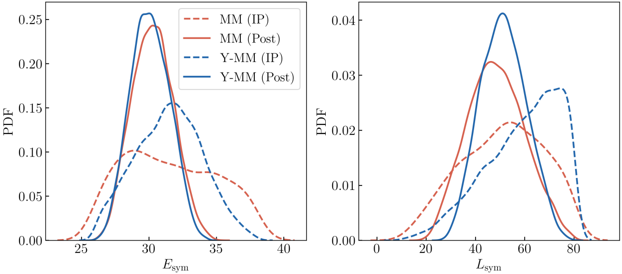
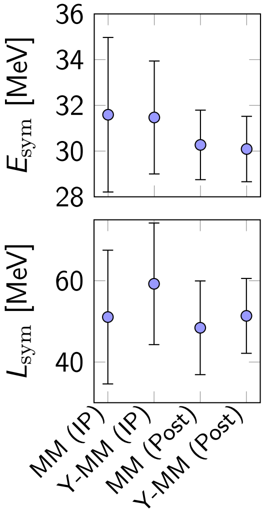
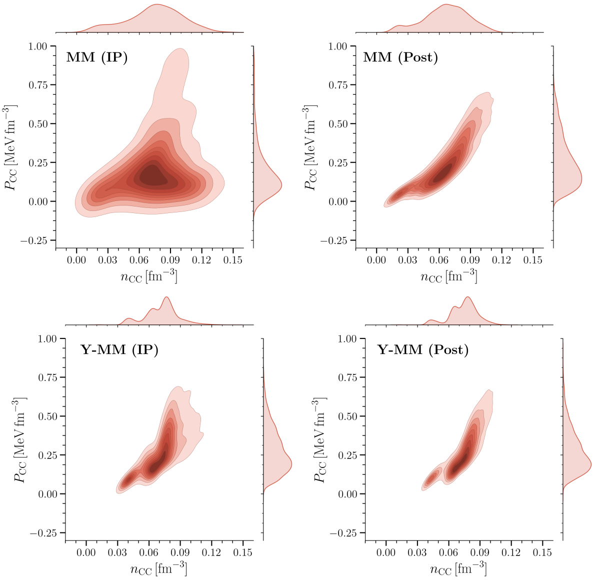
Isovector empirical parameters: $E_{\rm sym}$ and $L_{\rm sym}$

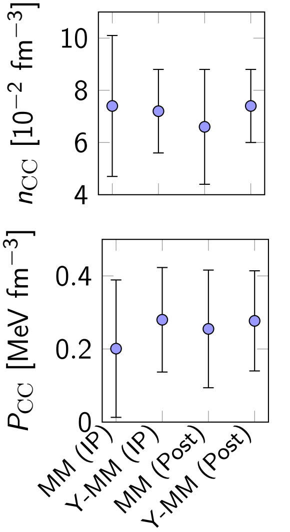




- IP filters **weakly** constraint $E_{\rm sym}$ & $L_{\rm sym}$
  - EFT filter crucial to constraint $E_{\rm sym} \approx 30$ MeV
- Y-MM reduces **dispersion** & shifts PDF to stiffer EOS ($L_{\rm sym} \approx 51$ MeV)

Crust-core transition density n_{CC} and pressure P_{CC}



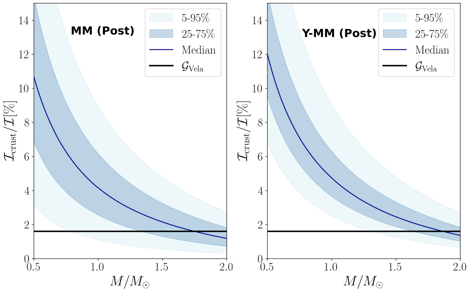


Crustal fraction of the moment of inertia $\mathcal{I}_{\rm crust}/\mathcal{I}$

- **Key** role for interpreting **glitch** activity
- **Slow-rotation** approximation $\left(\frac{\Omega^2 R^3}{GM} \ll 1\right)$
- $\frac{\mathcal{I}_{\rm crust}}{\mathcal{I}}$ **decreases** while increasing M

  $\Rightarrow$ **enhanced** role of crust in **lighter** NS
- Y-MM distributions shifted **upward**

  $\Rightarrow$ lower $\frac{\mathcal{I}_{\rm crust}}{\mathcal{I}}$ values are **ruled out**



- Negligible **crustal entrainment** $\Rightarrow \frac{\mathcal{I}_{\rm crust}}{\mathcal{I}} > \mathcal{G}_{\rm Vela} \approx 1.6\%$ (**wide** range of $M/M_\odot$)

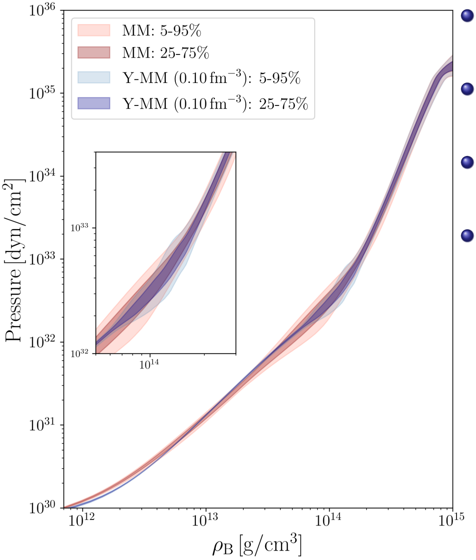
PHYSICAL REVIEW C **112**, 035802 (2025)

Bayesian inference of neutron star crust properties using an *ab-initio*–benchmarked metamodel

S. Burrello,[1,*] F. Gulminelli,[2] M. Antonelli,[2] M. Colonna,[1] and A. F. Fantina[3]
[1] *INFN, Laboratori Nazionali del Sud, I-95123 Catania, Italy*
[2] *Université de Caen Normandie, ENSICAEN, CNRS/IN2P3, LPC Caen UMR6534, 14000 Caen, France*
[3] *Grand Accélérateur National d'Ions Lourds (GANIL), CEA/DRF - CNRS/IN2P3, Boulevard Henri Becquerel, 14076 Caen, France*

(Received 6 June 2025; revised 23 July 2025; accepted 8 August 2025; published 5 September 2025)

Neutron star EOS: MM vs Y-MM



- Y-MM **reduces** MM **crust-uncertainties**
  ($\rho_B \lesssim 5 \cdot 10^{13}$ g/cm^3)
- **Distinct** behavior in outer layers
  ($\rho_B \lesssim 5 \cdot 10^{12}$ g/cm^3)
- **Overlapping** at **supra-saturation**
  ($\rho_B > 3 \cdot 10^{14}$ g/cm^3)
- **Widening** of blue bands at **saturation**
  ($\rho_B \simeq 2 \cdot 10^{14}$ g/cm^3)

Model	Post [%]
MM	0.31
Y-MM (n_B^{MM} =0.10 fm^{-3})	0.36
Y-MM (n_B^{MM} =0.12 fm^{-3})	0.74
Y-MM (n_B^{MM} =0.14 fm^{-3})	1.33
Y-MM ($n_B^{MM} = n_{sat}$)	2.38

Outline of the presentation

1. Theoretical approaches for nuclear many-body problem
   - Ab-initio vs phenomenological models based on energy density functionals (EDF)
   - Effective interaction and nuclear matter (NM) Equation of State (EOS)
2. Extended EDF-based models: recent developments and results
   - Unified description of many-body correlations and clustering phenomena
   - Bridging phenomenological models with ab-initio approaches
3. Further developments and outlooks
   - Consistent description of fragmentation mechanisms in heavy-ion collisions (HICs)
   - Joint Bayesian analyses combining nuclear structure and HIC studies
4. Summary

Final remarks and conclusions

Main topic

- **Extending** EDFs including **clusters** as explicit DOF and Mott effect
- **Bridging** **phenomenological** models with microscopic ab-initio approches

Main results

- Validation of **phase-space** **excluded-volume** approach against in-medium SE
- Role of clusters on SNM **spinodal** instability and fragmentation **dynamics**
- Low-density refinement through **benchmark** on ab-initio **PNM** calculations
- **Bayesian** inference of crustal observables with reduced **uncertainties**

Further developments and outlooks

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- **Joint** analyses with also nuclear structure & **HICs** for tighter **bounds** on EOS

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THANK YOU FOR YOUR ATTENTION!