Nuclei as an Assembly of Fermions and Bosons: RMF Description with Alpha Clusters

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Outline

Context and Motivations

What we observe here[a]:

- Non-vanishing alpha cluster density in neutron-skin
- Has been done with a Thomas-Fermi approximation from Symmetric Nuclear Matter

Clustering model we develop:

- Different from Quartetting
- Suppose that Nucleus is composed of nucleons and alpha clusters

[a] S. Typel, arXiv:1403.2851 (2018)

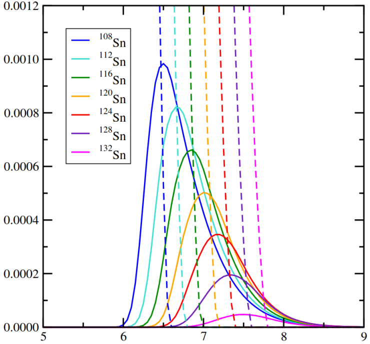

Figure: Alpha densities as function of the radius.

- DD ME2-like[a] (Meson Coupling Exchanges) model for nucleons **and** alpha clusters

$$\mathcal{L} = \overline{\psi}\left(\gamma^\mu \mathrm{i}\mathcal{D}_{N\mu} - M_N\right)\psi + \frac{1}{2}\left(\mathrm{i}\mathcal{D}_{\alpha\mu}\phi_\alpha\right)^\dagger\left(\mathrm{i}\mathcal{D}^\mu_\alpha\phi_\alpha\right) - \frac{1}{2}\phi^\dagger_\alpha M^2_\alpha\phi_\alpha$$
$$+ \frac{1}{2}\Bigg(\partial_\mu\sigma\partial^\mu\sigma - m^2_\sigma\sigma^2 - \frac{1}{2}\Omega^{\mu\nu}\Omega_{\mu\nu} + m^2_\omega\omega_\mu\omega^\mu - \frac{1}{2}\boldsymbol{R}^{\mu\nu}\cdot\boldsymbol{R}_{\mu\nu}$$
$$+ m^2_\rho\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \frac{1}{2}F^{\mu\nu}F_{\mu\nu}\Bigg)$$

- Two baryonic fields: nucleons (ψ) and alpha clusters (ϕ_α)

[a] S. Typel arXiv:0908.2344 (2010)

Alpha Clustering Model - How?

- **Our goal**: Simulating Nucleus as a mixing between nucleons and alpha clusters (for $T = 0$ K)
- Using an adapted Hartree-Fock method for nucleons and alpha
- Solving the problem self-consistensly

- We need two type of densities (both from Noether Theorem):
  1. Nucleonic density:
$$n_N = \overline{\psi}\gamma^0\psi \tag{1}$$
  2. Alpha density:
$$n_\alpha = \frac{1}{2}\left[\phi_\alpha^\dagger \left(\mathrm{i}\mathcal{D}_{\alpha,0}\phi_\alpha\right) + \text{h.c.}\right] \tag{2}$$

     n_α depends on the vector fields (see later).
- We define the total baryonic density:

$$n = n_p + n_n + 4n_\alpha \tag{3}$$

- Obtaining the Hamiltonian $\hat{\mathrm{H}}$ from a Legendre transform
- Choosing the Nucleus state as a product of a HFB state (nucleons) and a symmetric bosonic state (alpha):

$$|\text{Nucleus}\rangle = |\text{HFB}\rangle \otimes |\text{Bosons}\rangle \tag{4}$$

- Obtaining the Energy Density Functional (EDF):

$$\mathcal{E}[n_n, n_p, n_\alpha] = \langle\text{Nucleus}|\, \hat{\mathrm{H}}\, |\text{Nucleus}\rangle \tag{5}$$

- Stationary densities for nucleons and alpha. This leads us to:

$$n_\alpha(\boldsymbol{r}) = (\varepsilon_\alpha - 4g_\omega\omega_0(\boldsymbol{r}) - 2eA_0(\boldsymbol{r}))|\varphi_\alpha(\boldsymbol{r})|^2 \quad (6)$$

- From minimization of $\mathcal{E}$, we then obtain:
  1. For nucleons[a] (f_i are the nucleons wave functions):

$$\begin{pmatrix} h_d[n_n, n_p, n_\alpha] - \mu & \Delta \\ -\Delta^* & -h_d^*[n_n, n_p, n_\alpha] + \mu \end{pmatrix} f_i = \varepsilon_i f_i \quad (7)$$

     where h_d and Δ are the Dirac matrix and the pairing field
  2. For alpha (φ_α is the alpha wave function):

$$\left(-\nabla^2 + M_\alpha^2 - \Sigma_\alpha^0\right)\varphi_\alpha = \varepsilon_\alpha(\varepsilon_\alpha - 2\Sigma_\alpha^0)\varphi_\alpha \quad (8)$$

     where $M_\alpha = m_\alpha^0 - 4g_\sigma\sigma - B_\alpha(n)$ and $\Sigma_\alpha^0 = 4g_\omega\omega_0 + 2eA_0$.

[a] Vreternar *et al* (2005)

- We already have a RHB code for solving HFB equations on a Cylindrical Harmonic Oscillator Basis (CHO)[a]
- We now need to solve the RMF - alpha equation, in the same basis:

$$\varphi_\alpha(r,z) = \frac{1}{\sqrt{2\pi}} \sum_{\substack{\beta=(n_r,n_z) \\ 2n_r+n_z \leq N_{0B}}} \varphi_\beta^{(\alpha)} \Phi_{n_r}^0(r) \Phi_{n_z}(z) \tag{9}$$

- This leads us to a projected alpha equation on CHO Basis:

$$\mathrm{A}\overrightarrow{\varphi^{(\alpha)}} = \varepsilon_\alpha(\varepsilon_\alpha - \mathrm{C})\overrightarrow{\varphi^{(\alpha)}} \tag{10}$$

$\rightarrow$ Generalized Quadratic Eigenvalues Problem.

[a] Ring *et al* arXiv:9710052 (1997)

We have many possibilities for solving this equation. We chose to double the dimension of the problem:

$$\begin{pmatrix} 0 & I \\ A & C \end{pmatrix} \begin{pmatrix} \overrightarrow{\varphi^{(\alpha)}} \\ \overrightarrow{\varepsilon_\alpha \varphi^{(\alpha)}} \end{pmatrix} = \varepsilon_\alpha \begin{pmatrix} \overrightarrow{\varphi^{(\alpha)}} \\ \overrightarrow{\varepsilon_\alpha \varphi^{(\alpha)}} \end{pmatrix} \Longrightarrow \mathsf{M}\overrightarrow{w} = \varepsilon_\alpha \overrightarrow{w} \tag{11}$$

Which is a standard Eigenvalues Problem.

Alpha Clustering Model - RMF

$$\mathrm{M}\overrightarrow{w} = \varepsilon_\alpha \overrightarrow{w} \tag{12}$$

Be careful:

1. We doubled the number of eigenvalues
2. Thanks to the alpha density, we have to take into account the positive eigenvalues of the matrix M
3. The condition $T = 0$ K implies all alpha (bosons) condensate in the groundstate: we are then interested by the **lowest positive** eigenvalue of M
4. The decomposition coefficient of φ_α are given by the first half components of $\overrightarrow{w}$
5. All energies different from the groundstate might be the excited states of the alpha clusters ?

Toy model (not realistic potentials):

$$M_\alpha = \mathrm{e}^{-z^2} \text{ and } \Sigma_\alpha^0 = \mathrm{e}^{-r^2/0.01} \tag{13}$$

The next step is to put alpha motion equation into the RHB code with the coupling between nucleons and alpha.

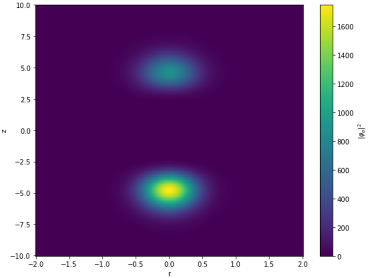

Figure: Alpha density in arbitrary units (r and z in fm)

Outlook

When the code will run:

- We'll be able to simulate clustering in light nuclei:
  1. Clustering is experimentally quite visible (*e.g.* ^{20}Ne like $\alpha +^{16}$ O)
  2. Verifying E/A is lower than the one without clustering
- Simulating clustering in heavy nuclei:
  1. We observe fraction of alpha clusters (*e.g.* Tin isotopes): simulating heavy nuclei like a superposition of wave functions with different alpha clusters numbers
  2. Verifying E/A is greater than the one without clustering (alpha radioactivity?)
- Using a nuclear reaction model with this model of structure.

Thank you!

Collaborators:

- Elias KHAN (IJCLab)
- Jean-Paul EBRAN (CEA)
- Stefan TYPEL (TU Darmstadt)