



Charm and Beauty Physics complementarity: A theoretical overview

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Outline

- simplest way to relate charm to beauty:
Heavy Quark Flavour symmetry , the large mass limit and the Heavy Quark Expansion (HQE)
- selected application of HQE:
 - spectroscopy of open flavour hadrons
(D_α and B_α mesons, α light flavour index)
- cases with limited possibilities to apply HQE:
 - spectroscopy of hidden flavour hadrons
(charmonium, bottomonium)
- discussion on the determination of the angle γ from the decay chain $B \rightarrow KD$, $D \rightarrow X$
- where differences matter:
 - mixing and oscillations
- concluding remarks

Hadrons containing a single heavy quark Q

New symmetries arise in QCD in the limit $m_Q \rightarrow \infty$:
Heavy quark spin and flavour symmetries.

Static properties (masses, decay constants) and decays (form factors) of hadrons which differ only for the flavour of the heavy quark or its spin orientation can be related.

Two main realms:

Heavy Quark Effective Theory (**HQET**) (rigorous infinite mass limit)

Heavy Quark Expansion (**HQE**) (inclusion of subleading effects in the heavy quark mass)

Hadrons containing a single heavy quark Q

QCD Lagrangian for the Heavy Quark only:

$$L = \bar{Q} (i\mathcal{D} - m_Q) Q$$

Defining:

$$Q(x) = e^{-im_Q \cdot x} h_v(x)$$

Heavy quark field in HQET



$$L_{HQET} = \bar{h}_v i v \cdot D h_v$$

HQET Lagrangian

No dependence on heavy quark mass \rightarrow **flavour symmetry**
No gamma matrices \rightarrow **spin symmetry**

Corrections to the heavy quark limit can be included and lead to new lagrangian terms suppressed by inverse powers of m_Q

Can be treated as perturbations \rightarrow basis of the **Heavy Quark Expansion**

First corrections:

$$L_{1/m_Q} = \frac{1}{2m_Q} \left[\bar{h}_v (iD)^2 h_v + \frac{g}{2} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v \right]$$

Kinetic energy operator,
breaks **flavour symmetry**.

Its matrix element is denoted as μ_π^2 .

Chromomagnetic operator,
breaks **spin symmetry**

Its matrix element is μ_G^2 .

Hadrons containing a single heavy quark Q

Consequences of **spin symmetry**:

Spin of the heavy quark and of the light degrees of freedom decoupled in the $m_Q \rightarrow \infty$ limit

$$\vec{J}_M = \vec{s}_\ell + \vec{s}_Q \quad \text{spin}$$

$$\vec{s}_\ell = \vec{L} + \vec{s}_q$$

angular momentum
of the light degrees of freedom (conserved)

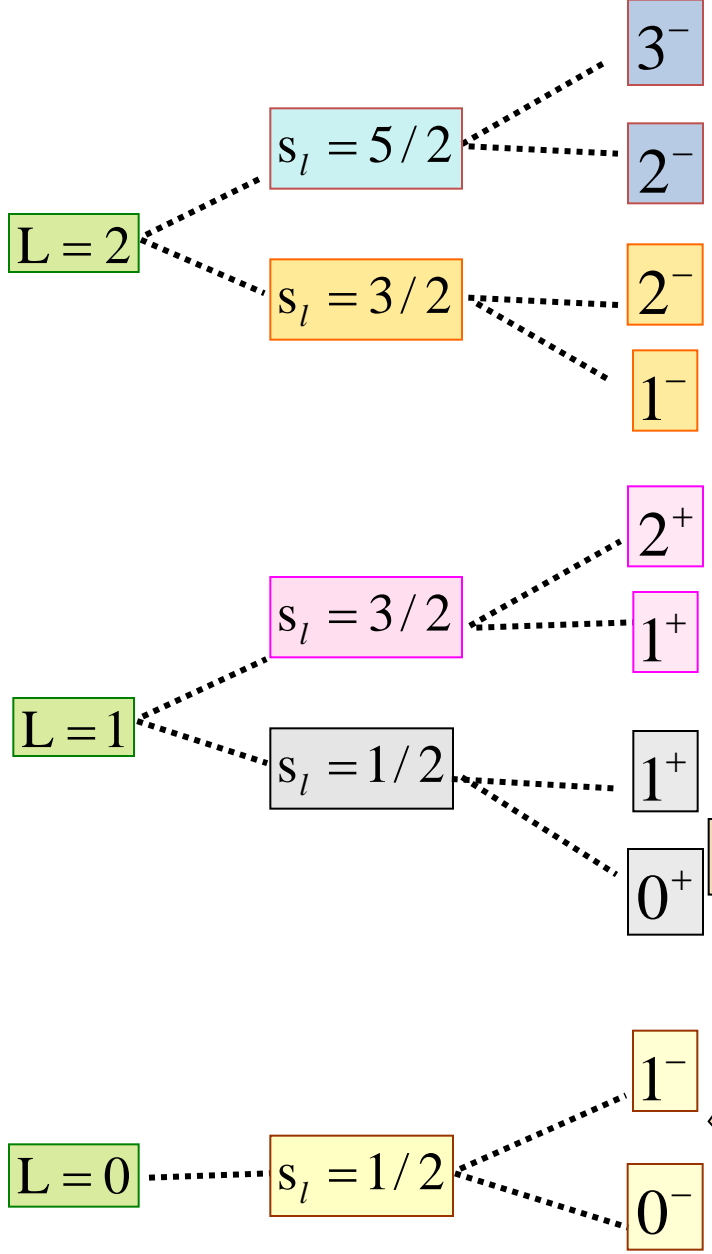
Mesons classified as doublets

- In the HQ limit:
 - states with the same S_ℓ^P degenerate
- finite m_Q corrections
 - remove degeneracy between the states of the same doublet
 - induce mixing between states with the same J^P

Flavour symmetry identifies the doublets which differ only for the flavour of Q
In practice: **charm and beauty mesons**

$Q\bar{q}$ multiplets

J^P



Strong transitions between multiplets

$\frac{3^+}{2} \rightarrow \frac{1^-}{2} + \text{pseudoscalar meson}$
d-wave transition
 $\frac{3^+}{2}$ mesons are expected to be narrow

$\frac{1^+}{2} \rightarrow \frac{1^-}{2} + \text{pseudoscalar meson}$
s-wave transition
 $\frac{1^+}{2}$ mesons are expected to be **broad**

Hadrons containing a single heavy quark Q

Examples of relations exploiting flavour and spin symmetries:

- decay constants:

$$f_{HQ} = \frac{\hat{F}}{\sqrt{m_Q}}$$

Independent on m_Q . Predicts that

$$\frac{f_B}{f_D} = \frac{\sqrt{m_c}}{\sqrt{m_b}}$$

- coupling constants:

$$g_{P^*P\pi} = \frac{2m_P}{f_\pi} g$$

Fundamental quantity: allows to describe the strong decays of all the members of the *fundamental* doublet ($D, D_s, D^*, D_s^*, B, B_s, B^*, B_s^*$) to a light pseudoscalar meson

Spectroscopy of open flavoured heavy hadrons

Mass formula

Including subleading terms:

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_Q} + \dots$$

with

$$\mu_G^2 = -2 \left[J(J+1) - \frac{3}{2} \right] \cdot \lambda_2$$

$$\bar{\Lambda}, \mu_\pi^2, \lambda_2$$



Independent of m_Q and of the light quark flavour
They are specific of a given doublet

The mass formula allows to relate the masses of hadrons in a given doublet if experimental data are available for any member of such a doublet.

Subleading terms allow to estimate corrections to the asymptotic degeneracy condition

Spectroscopy of heavy-light mesons: an instructive example

HQ limit: the members of the doublets are described by effective fields:

$$L=0 \left\{ \begin{array}{l} \mathbf{S}_\ell^{\mathbf{P}} = \frac{1}{2}^- \end{array} \right.$$

$$H_a = \frac{1 + \not{v}}{2} [P_{a\mu}^* \gamma^\mu - P_a \gamma_5]$$

$$L=1 \left\{ \begin{array}{l} \mathbf{S}_\ell^{\mathbf{P}} = \frac{1}{2}^+ \end{array} \right.$$

$$S_a = \frac{1 + \not{v}}{2} [P_{1a}^{\prime\mu} \gamma_\mu \gamma_5 - P_{0a}^*]$$

$$\mathbf{S}_\ell^{\mathbf{P}} = \frac{3}{2}^+$$

$$T_a^\mu = \frac{1 + \not{v}}{2} \left\{ P_{2a}^{\mu\nu} \gamma_\nu - P_{1av} \sqrt{\frac{3}{2}} \gamma_5 \left[g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu - v^\mu) \right] \right\}$$

Spectroscopy of heavy-light mesons: an instructive example

Mass splittings between positive and negative parity doublets:

$$\Delta_S = \bar{M}_S - \bar{M}_H \quad \Delta_T = \bar{M}_T - \bar{M}_H$$

Spin-averaged masses:

$$\bar{M}_H = \frac{3M_{P^*} + M_P}{4} \quad \bar{M}_S = \frac{3M_{P'_1} + M_{P_0^*}}{4} \quad \bar{M}_T = \frac{5M_{P_2^*} + M_{P_1}}{8}$$

Mass splittings between the two members of a given doublet:

$$\lambda_H = \frac{1}{8}(M_{P^*}^2 - M_P^2) \quad \lambda_s = \frac{1}{8}(M_{P'_1}^2 - M_{P_0^*}^2) \quad \lambda_T = \frac{3}{8}(M_{P_2^*}^2 - M_{P_1}^2)$$

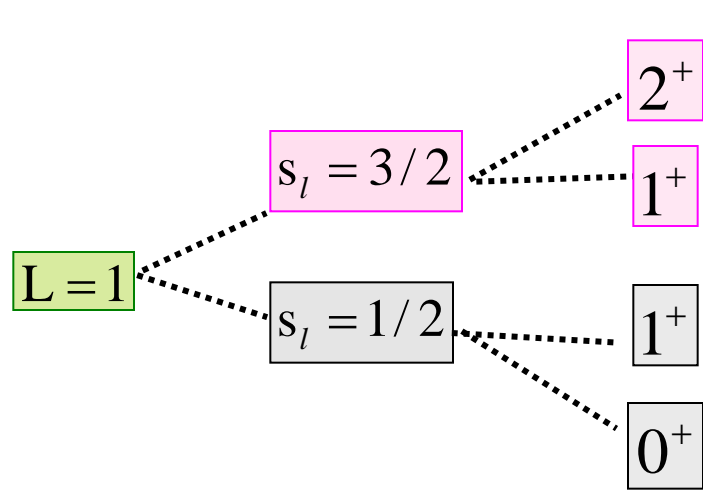
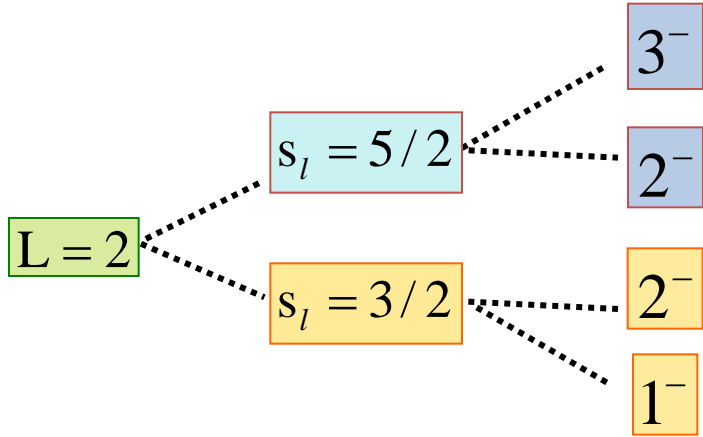


Many recent observations in the charm sector allow us to make predictions for beauty

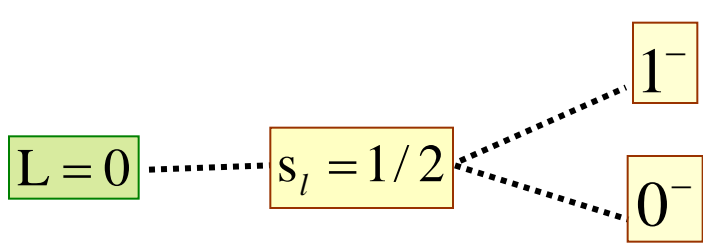
cq multiplets

J^P

Low lying



$D_2^{\pm,0} (2460)$	$\Gamma_{D_2} \approx 40 \text{ MeV}$
$D_1^{\pm} (2420)$	$\Gamma_{D_1} \approx 20 \text{ MeV}$
$D_1^{\prime 0} (2430)$	$\Gamma_{D_1} \approx 384 \text{ MeV}$
$D_0^{*0} (2308)$	$\Gamma_{D_0} \approx 260 \text{ MeV}$



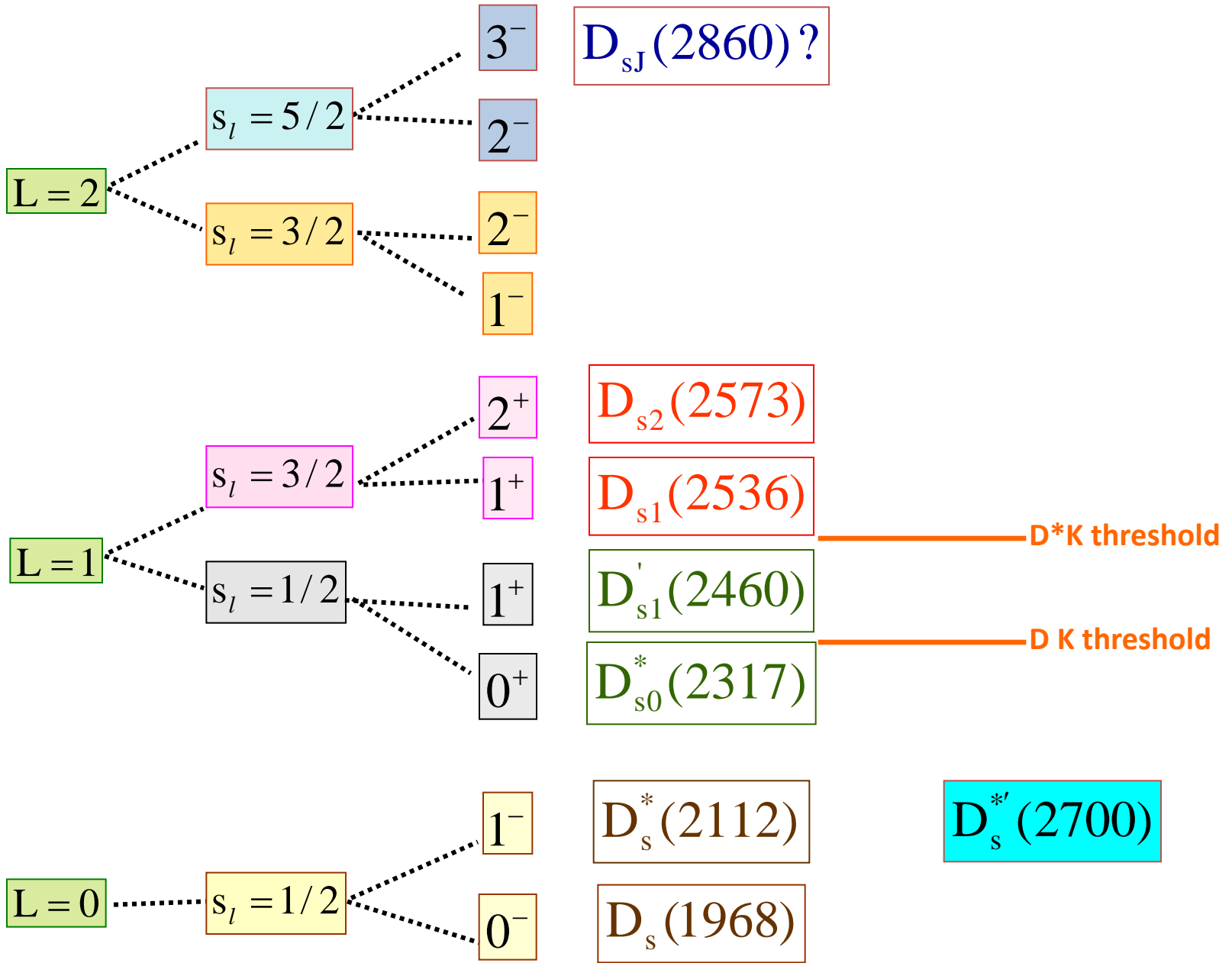
$D^{*\pm} (2010), D^{*0} (2007)$
$D^{\pm} (1869), D^0 (1865)$

cS multiplets

J^P

Low lying

Rad excitations



Spectroscopy of heavy-light mesons: an instructive example

P. Colangelo, R. Ferrandes, FDF
 MPLA 19 (2004) 2083;
 PLB 634 (2006) 235

	$c\bar{q}$	$c\bar{s}$	$b\bar{q}$	$b\bar{s}$
λ_H	$(261.1 \pm 0.7 \text{ MeV})^2$	$(270.8 \pm 0.8 \text{ MeV})^2$	$(247 \pm 2 \text{ MeV})^2$	$(252 \pm 10 \text{ MeV})^2$
λ_S	$(265 \pm 57 \text{ MeV})^2$	$(291 \pm 2 \text{ MeV})^2$		
λ_T	$(259 \pm 10 \text{ MeV})^2$	$(266 \pm 6 \text{ MeV})^2$		
\bar{M}_H	$1974.8 \pm 0.4 \text{ MeV}$	$2076.1 \pm 0.5 \text{ MeV}$	$5313.5 \pm 0.5 \text{ MeV}$	$5404 \pm 3 \text{ MeV}$
\bar{M}_S	$2397 \pm 28 \text{ MeV}$	$2424 \pm 1 \text{ MeV}$		
\bar{M}_T	$2445.1 \pm 1.4 \text{ MeV}$	$2558 \pm 1 \text{ MeV}$		
Δ_S	$422 \pm 28 \text{ MeV}$	$348 \pm 1 \text{ MeV}$		
Δ_T	$470.3 \pm 1.5 \text{ MeV}$	$482 \pm 1 \text{ MeV}$		

Extracting these quantities from data allows to make predictions:



	$B_{(s)0}^*(0^+)$	$B'_{(s)1}(1^+)$	$B_{(s)1}(1^+)$	$B_{(s)2}^*(2^+)$
$b\bar{q}$	$5.70 \pm 0.025 \text{ GeV}$	$5.75 \pm 0.03 \text{ GeV}$	$5.774 \pm 0.002 \text{ GeV}$	$5.790 \pm 0.002 \text{ GeV}$
$b\bar{s}$	$5.71 \pm 0.03 \text{ GeV}$	$5.77 \pm 0.03 \text{ GeV}$	$5.877 \pm 0.003 \text{ GeV}$	$5.893 \pm 0.003 \text{ GeV}$

In particular....



Spectroscopy of heavy-light mesons: an instructive example

Two narrow states observed in 2003 by BaBar, Belle, CLEO, Focus with charm and strangeness: $D_{sJ}(2317)$ and $D_{sJ}(2460)$

Peculiar features:

- Mass below the (DK) , (D^*K) thresholds, respectively
- Narrow states – width compatible with experimental resolution
- Isospin violating decays to $D_s\pi$ and $D_s^*\pi$, respectively
- Preferred spin-parity assignments: $J^P=0^+$ and $J^P=1^+$

Debated identification, by now mostly accepted as the two members of the S- doublet with

$$J_{s_\ell}^P = (0^+, 1^+)_{1/2}$$



Existence of beauty hadrons with similar features predicted :

- masses below the (BK) , (B^*K) thresholds
- decaying to $(B\pi)$, $(B^*\pi)$ (isospin violation) \rightarrow very narrow states

Their observation with such features required to definitively confirm the identification of $D_{sJ}(2317)$ and $D_{sJ}(2460)$


Some remarks

Manifold reasons to study excited charmed mesons (D^{**}) in B decays:

- can be used for tagging at B factories
- old and debated question of their role in semileptonic B decays:
D and D^* do not saturate the total $B \rightarrow X_c \ell \nu$ rate
- play a role as intermediate states in some three-body B decays useful for CP violation studies
-

Heavy quarkonium

- For a $Q\bar{Q}$ state only **spin symmetry holds**
- Divergences show up when studying these systems in the heavy quark mass limit cured taking the kinetic energy operator into account \rightarrow **breaking of flavour symmetry**



In the HQ limit one can relate again quarkonium states which differ only for the relative orientation of the quark spins such as $(\eta_c, J/\psi)$ or (η_b, Υ)
But cannot relate (at least in principle) η_c to η_b , J/ψ to Υ , etc.

Nevertheless complementarity between beauty and charm exists

1

Only production of charmonium with $J^P=1^-$ is possible in $e^+ - e^-$ annihilation
States with different quantum numbers can be accessed from B decays

Recent discoveries of $\eta_c(2S)$, $h_c(3525)$, $X(3872)$ (still to be understood), ...

\Rightarrow **B decays are a bridge to charmonium discoveries**

2

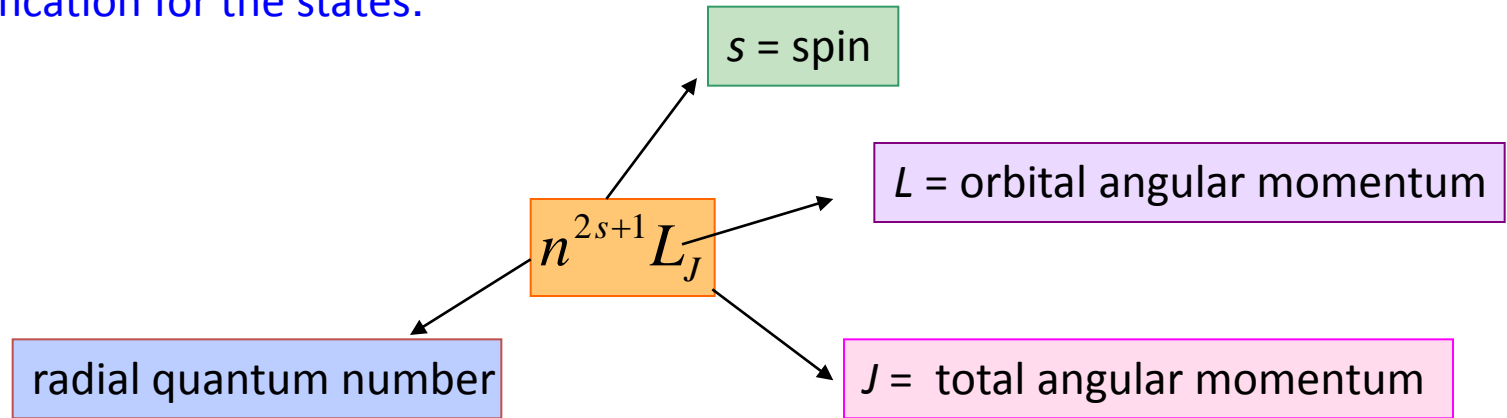
Can we say something about X(3872) interpretation starting from bottomonium?

Considered interpretations:

- hybrid state $c\bar{c}g$
- tetraquark state
- DD^* molecule
- ordinary charmonium with $J^{PC}=1^{++}$ (previously preferred assignment)
or $J^{PC}=2^{-+}$ (latest preferred assignment) → BaBar,
PRD 82 (2010) 011101

Heavy quark mass limit for heavy quarkonium states

Usual classification for the states:



with:

$$P = (-1)^{L+1}$$

$$C = (-1)^{L+s}$$

$L=0 \leftrightarrow$ S- wave states

$L=1 \leftrightarrow$ P- wave states

$L=2 \leftrightarrow$ D- wave states

.....

- HQ spin symmetry **YES**
- HQ flavour symmetry **NO**

Multiplets for heavy quarkonium states

- $L=0$ multiplet

$$J = \frac{1 + \not{v}}{2} \left[\underbrace{H_1^\mu \gamma_\mu}_{\text{spin 1 state}} - \underbrace{H_0 \gamma_5}_{\text{spin 0 state}} \right] \frac{1 - \not{v}}{2} \implies \begin{cases} n=1 & (\eta_c(1S), \psi(1S)) \\ n=2 & (\eta_c(2S), \psi(2S)) \end{cases}$$

- $L=1$ multiplet

triplet

$$J^\mu = \frac{1 + \not{v}}{2} \left\{ \underbrace{H_2^{\mu\alpha} \gamma_\alpha}_{\text{spin 2}} + \frac{1}{\sqrt{2}} \epsilon^{\mu\alpha\beta\gamma} v_\alpha \gamma_\beta \underbrace{H_{1\gamma}}_{\text{spin 1}} + \frac{1}{\sqrt{3}} (\gamma^\mu - v^\mu) \underbrace{H_0}_{\text{spin 0}} + \underbrace{K_1^\mu \gamma_5}_{\text{singlet spin 1 state}} \right\} \frac{1 - \not{v}}{2}$$

$n=1$ $\left[(\chi_{c0}(1P), \chi_{c1}(1P), \chi_{c2}(1P)), h_c(1P) \right]$

- $L=2$ multiplet

$$\left[(n^3 D_1, n^3 D_2, n^3 D_3), n^1 D_2 \right]$$

Triplet, spin 1,2,3

Singlet, spin 2

• $P \leftrightarrow S$

$$\mathcal{L}_{nP \leftrightarrow mS} = \delta_Q^{nPmS} \text{Tr} [\bar{J}(mS) J_\mu(nP)] v_\nu F^{\mu\nu} + \text{h.c.}$$

photon

a single constant describes all the transitions
among the various members of the P and S multiplets



- reduced theoretical uncertainty
- model independence

• $D \leftrightarrow P$

$$\mathcal{L}_{nD \leftrightarrow mP} = \delta_Q^{nDmP} \text{Tr} [\bar{J}_\alpha(mP) J_\mu^\alpha(nD)] v_\nu F^{\mu\nu} + \text{h.c.}$$

idem for transitions among members of D and P multiplets

1P → 1S transitions

Exploiting known data:

$$\left. \begin{aligned} \mathcal{B}(\chi_{c0}(1P) \rightarrow J/\psi \gamma) &= (1.28 \pm 0.11) \times 10^{-2} \\ \mathcal{B}(\chi_{c1}(1P) \rightarrow J/\psi \gamma) &= (36.0 \pm 1.9) \times 10^{-2} \\ \mathcal{B}(\chi_{c2}(1P) \rightarrow J/\psi \gamma) &= (20.0 \pm 1.0) \times 10^{-2} \\ &+ \text{total widths of } \chi_{cJ} \text{ states} \end{aligned} \right\} \Rightarrow \begin{aligned} \delta_c^{1P1S} &= 0.227 \pm 0.013 \text{ GeV}^{-1} \\ \delta_c^{1P1S} &= 0.241 \pm 0.009 \text{ GeV}^{-1} \\ \delta_c^{1P1S} &= 0.233 \pm 0.010 \text{ GeV}^{-1} \end{aligned}$$



spin symmetry turns out to be experimentally well satisfied

averaged result:

$$\delta_c^{1P1S} = 0.235 \pm 0.006 \text{ GeV}^{-1}$$

can be used to predict:

$$\Gamma(h_c(1P) \rightarrow \eta_c(1P) \gamma) = 634 \pm 32 \text{ KeV}$$

$2P \rightarrow 1S, 2S$ transitions

possibility to exploit data in the beauty sector:

$$\mathcal{B}(\chi_{b0}(2P) \rightarrow \Upsilon(1S) \gamma) = (9 \pm 6) \times 10^{-3}$$

$$\mathcal{B}(\chi_{b0}(2P) \rightarrow \Upsilon(2S) \gamma) = (4.6 \pm 2.1) \times 10^{-2}$$

$$\mathcal{B}(\chi_{b1}(2P) \rightarrow \Upsilon(1S) \gamma) = (8.5 \pm 1.3) \times 10^{-2}$$

$$\mathcal{B}(\chi_{b1}(2P) \rightarrow \Upsilon(2S) \gamma) = (21 \pm 4) \times 10^{-2}$$

$$\mathcal{B}(\chi_{b2}(2P) \rightarrow \Upsilon(1S) \gamma) = (7.1 \pm 1.0) \times 10^{-2}$$

$$\mathcal{B}(\chi_{b2}(2P) \rightarrow \Upsilon(2S) \gamma) = (16.2 \pm 2.4) \times 10^{-2}$$

χ'_{b0}

χ'_{b1}

χ'_{b2}

define width ratio

$$R_J^{(b)} = \frac{\Gamma(\chi_{bJ}(2P) \rightarrow \Upsilon(2S) \gamma)}{\Gamma(\chi_{bJ}(2P) \rightarrow \Upsilon(1S) \gamma)}$$

and coupling ratio

$$R_\delta^{(b)} = \frac{\delta_b^{2P1S}}{\delta_b^{2P2S}}$$



$$R_\delta^{(b)} = 8.8 \pm 0.7$$

even though the coupling might be different passing from beauty to charm, it is reasonable to assume that the ratios of the couplings stay stable



we can predict analogous charm ratios $R_J^{(c)}$

2P → 1S, 2S transitions

prediction for $J=1$:

$$R_1^{(c)} = \frac{\Gamma(\chi_{c1}(2P) \rightarrow \psi(2S) \gamma)}{\Gamma(\chi_{c1}(2P) \rightarrow \psi(1S) \gamma)} = 1.64 \pm 0.25$$

FDF,
PRD 79 (09) 054015

Identifying X(3872) with $\chi_{c1}(2P)$ and using the data:

$$\begin{aligned} \mathcal{B}(B^+ \rightarrow XK^+, X \rightarrow J/\psi \gamma) &= (2.8 \pm 0.8 \pm 0.2) \times 10^{-6} \\ \mathcal{B}(B^+ \rightarrow XK^+, X \rightarrow \psi(2S) \gamma) &= (9.9 \pm 2.9 \pm 0.6) \times 10^{-6} \end{aligned}$$



$$R_X = \frac{\Gamma(X(3872) \rightarrow \psi(2S) \gamma)}{\Gamma(X(3872) \rightarrow \psi(1S) \gamma)} = 3.5 \pm 1.4$$

BaBar,
PRL 102 (2009) 132001

$R_1^{(c)}$ and R_X are close enough to consider the assumption $X(3872) = \chi_{c1}(2P)$ plausible

to be contrasted with composite scenarios in which $X(3872) \rightarrow \psi(2S)\gamma$ is suppressed with respect to $X(3872) \rightarrow \psi(1S)\gamma$

A similar analysis cannot be repeated for $J^{PC}=2^{-+}$ due to lack of data in the beauty sector

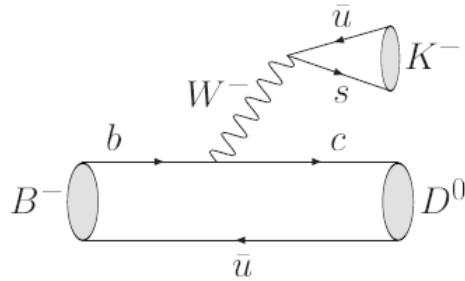
Discussion on the determination of γ from the decay chain $B \rightarrow KD, D \rightarrow X$

Discussion on the determination of γ from the decay chains

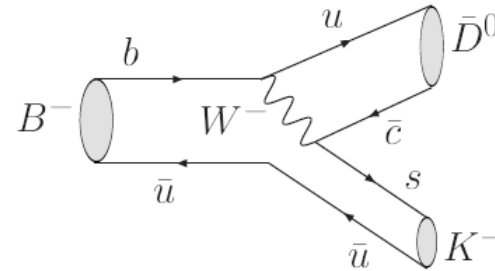
$$B^\pm \rightarrow DK^\pm$$

└─> final state common to D^0 and \bar{D}^0

indicates a generic D meson (D or D^*)



$$V_{cb} V_{us}^* \Rightarrow \text{real}$$



$$V_{ub} V_{us}^* \Rightarrow \text{weak phase } \gamma$$



Two interfering amplitudes leading to the same final state

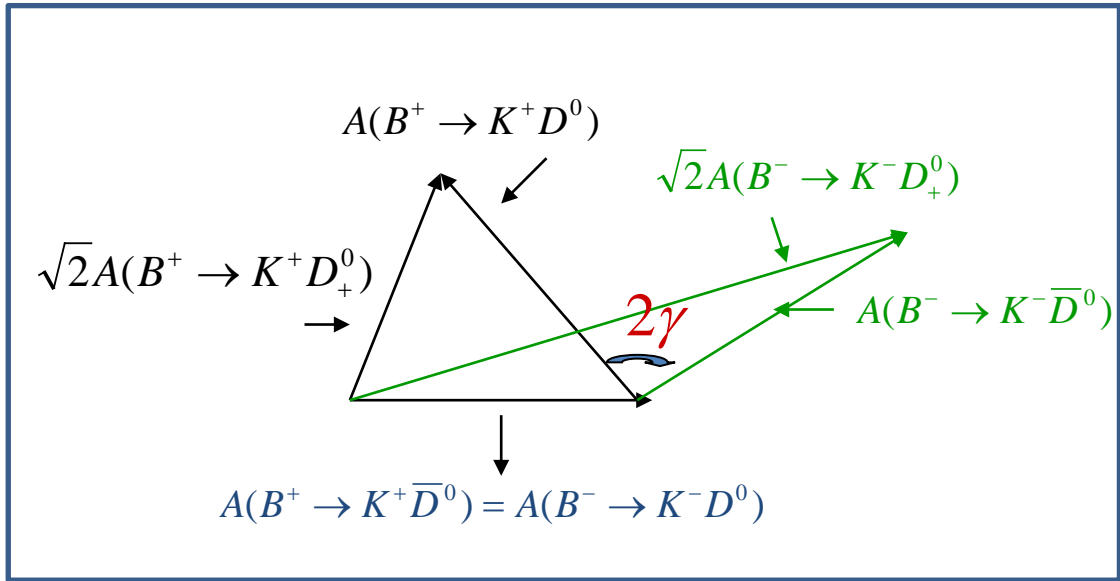
- possibility to determine γ from CP asymmetries
- requires difference in the strong phases
- comparable amplitudes necessary in practice

The original method made use of CP-eigenstates D_+ and D_- with final states being CP eigenstates

$$|D_{\pm}^0\rangle = \frac{|D^0\rangle \pm |\bar{D}^0\rangle}{\sqrt{2}}$$

$$\begin{aligned} \sqrt{2}A(B^+ \rightarrow K^+ D_+^0) &= A(B^+ \rightarrow K^+ D^0) + A(B^+ \rightarrow K^+ \bar{D}^0) \\ \sqrt{2}A(B^- \rightarrow K^- D_+^0) &= A(B^- \rightarrow K^- D^0) + A(B^- \rightarrow K^- \bar{D}^0) \end{aligned}$$

Using: $A(B^+ \rightarrow K^+ \bar{D}^0) = A(B^- \rightarrow K^- D^0)$
 $A(B^+ \rightarrow K^+ D^0) = e^{2i\gamma} A(B^- \rightarrow K^- \bar{D}^0)$

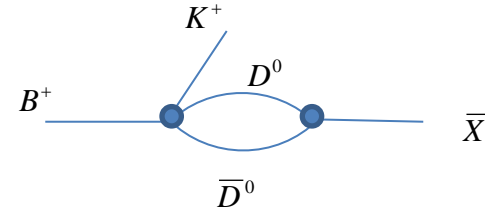
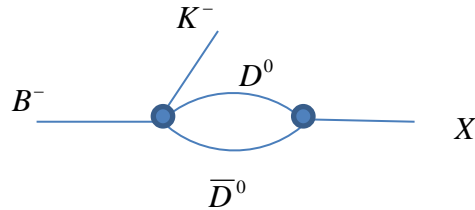


The decay to a CP eigenstate allows to tag the D_{\pm}
 The decay to D^0 can be measured through leptonic or hadronic D^0 decays

The decay $B^- \rightarrow K^- \bar{D}^0$ (colour suppressed) is the problematic one

Possible way out:

Decay chains in which the D decays to **CP non-eigenstate** final state X



Amplitudes: $d(K, X) \rightarrow \cos(\delta_{\text{strong}} + \gamma)$

$\bar{d}(K, \bar{X}) \rightarrow \cos(\delta_{\text{strong}} - \gamma)$

If the amplitudes of the subprocesses are known, these two quantities allow to access both δ_{strong} and γ

Otherwise

- more than one channel should be used
- a multibody D- decay can be considered together with a Dalitz plot analysis

each point to be considered as an individual channel if the final 3-body decay can be assumed reliably to proceed through intermediate resonances (quasi 2-body mode)

Main example: $D \rightarrow K \pi \pi$

$$M(D \rightarrow K \pi \pi) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r}$$

sum over intermediate resonances

Discussion on γ

Results already obtained
from $B \rightarrow KD$, $D \rightarrow K\pi\pi$
(also $D \rightarrow KKK$ considered)

BaBar, PRD 78 (2008) 03402
PRD 79 (2009) 072003;
1007.0504

Belle, PRD 81 (2010) 112002

However requires:

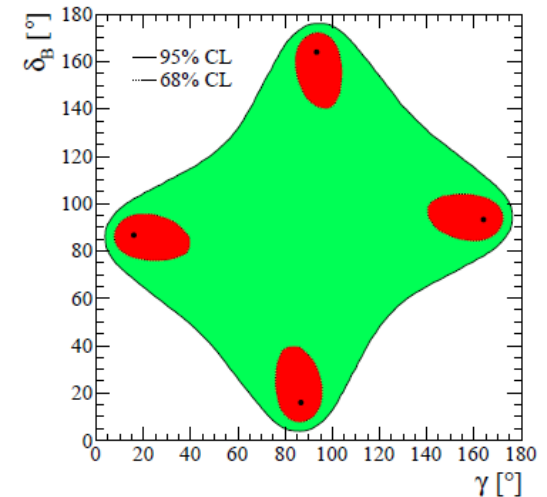
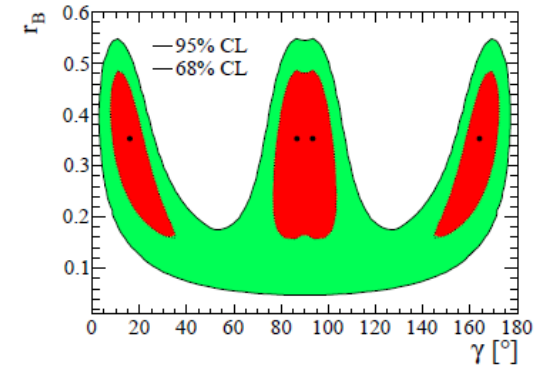
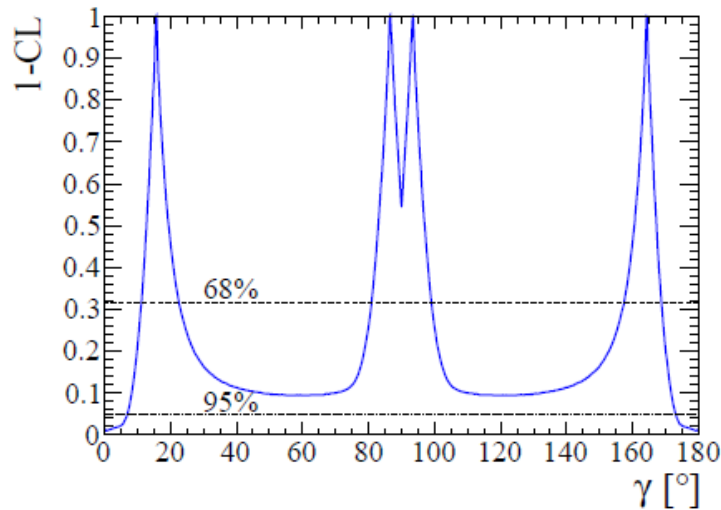
- Knowledge of the parameters of a (finite number) of intermediate resonances
- Model to take into account the contribution of the various resonances (particularly true for those decaying in S-wave to $\pi\pi$)
- Model for hadronic form factors in D decay

The experimental analysis measures: $x=r \cos(\delta_{\text{strong}}+\gamma)$ and $y=r \sin(\delta_{\text{strong}}-\gamma)$
then a frequentist approach is used to obtain γ



Very difficult to assess the attached theoretical uncertainty

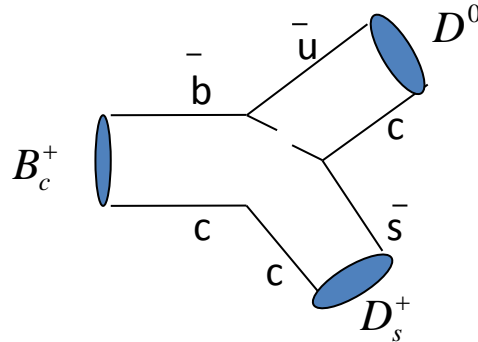
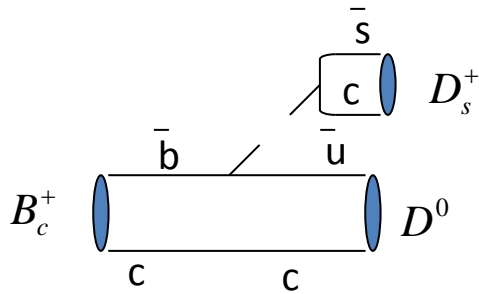
Plots from BaBar 1007.0504



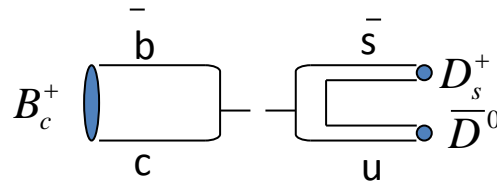
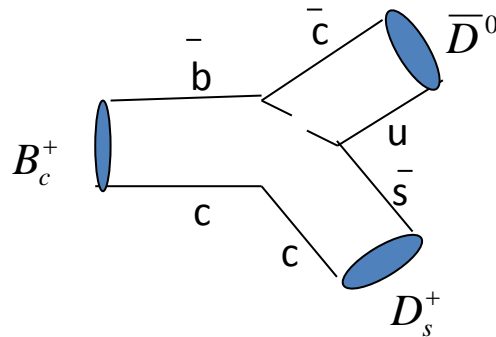
Another possibility for LHC: B_c decays

R. Fleischer, D. Wyler,
PRD62 (00) 057503

$$B_c^\pm \rightarrow D_s^\pm \{D^0, \bar{D}^0, D^+\} \Rightarrow \left. \begin{aligned} \sqrt{2}A(B_c^+ \rightarrow D_s^+ D^0) &= A(B_c^+ \rightarrow D_s^+ D^0) + A(B_c^+ \rightarrow D_s^+ \bar{D}^0) \\ \sqrt{2}A(B_c^- \rightarrow D_s^- D^0) &= A(B_c^- \rightarrow D_s^- D^0) + A(B_c^- \rightarrow D_s^- \bar{D}^0) \end{aligned} \right\} \text{Comparable amplitudes}$$



$B_c^+ \rightarrow D_s^+ \bar{D}^0$ colour-suppressed, prop to V_{cb}
 $B_c^+ \rightarrow D_s^+ D^0$ prop to V_{ub} , colour-allowed.



↪ Difference with respect $B^\pm \rightarrow K^\pm D$
 where V_{ub} enters in
 colour-suppressed diagrams



should be accessible:

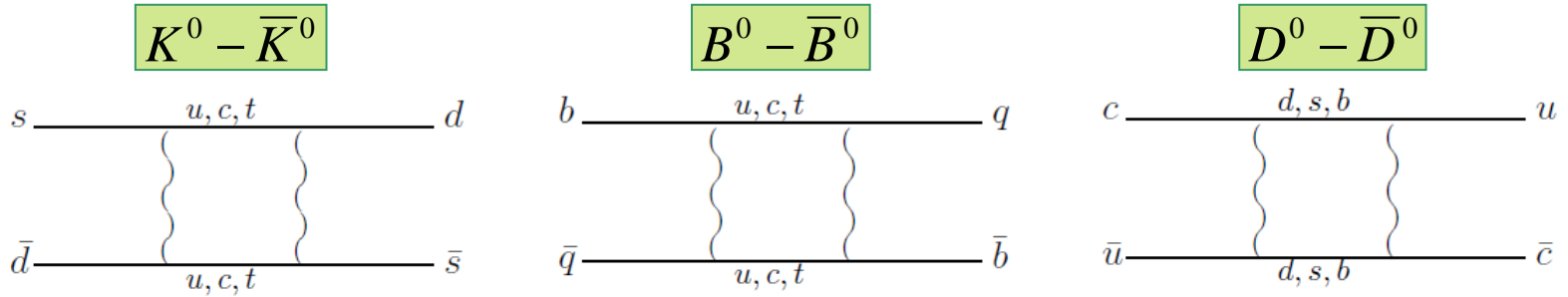
$$\begin{aligned} B(B_c^+ \rightarrow D_s^+ \bar{D}^0) &\approx O(10^{-6}) \\ B(B_c^+ \rightarrow D_s^{*+} \bar{D}^0) &\approx O(10^{-5}) \end{aligned}$$

P. Colangelo, FDF,
PRD 61 (00) 034012

$D^0 - \bar{D}^0$ mixing

$D^0 - \bar{D}^0$ mixing and oscillations: comparison with K and B cases

Typical quark-level box diagrams for mixing in the SM:



Intermediate quarks:

up-type

up-type

down-type

Customary quantities describing neutral mesons' mixing:

$$x = \frac{\Delta M}{\bar{\Gamma}}, \quad y = \frac{\Delta \Gamma}{\bar{\Gamma}}$$

$$\Delta M = M_2 - M_1 \quad \Delta \Gamma = \Gamma_2 - \Gamma_1$$

$$\bar{\Gamma} = \frac{\Gamma_1 + \Gamma_2}{2}$$

The Im part takes contribution from real intermediate states $\rightarrow \Delta \Gamma$
 The Real part from virtual ones $\rightarrow \Delta M$

Sensitive to NP!

$D^0 - \bar{D}^0$ mixing and oscillations: comparison with K and B cases

Evidence of $D^0 - \bar{D}^0$ oscillations first reported in 2007 by Belle and BaBar

Later confirmed by CDF

Belle PRL 98 (07) 212803

BaBar PRL 98 (07) 212802

CDF PRL 100 (08) 121802

World average reported by HFAG :

<http://www.slac.stanford.edu/xorg/hfag/>

$$x_D = (0.59 \pm 0.20) \times 10^{-2}$$

$$y_D = (0.80 \pm 0.13) \times 10^{-2}$$

$D^0 - \bar{D}^0$ mixing and oscillations: comparison with K and B cases

- K mixing box diagram is *charm-dominated*; B mixing box diagram is *top-dominated*



internal fields are much heavier than external ones: can be integrated out;
 ΔM_K and ΔM_B are described by the expectation value of a local operator.
This procedure reproduces data fairly well

- x_D and y_D are expected small in the SM since:

- most of charm decays are Cabibbo allowed (CA), mixing is doubly Cabibbo suppressed (DCS) : the ratios $\Delta M_D/\Gamma$ and $\Delta\Gamma_D/\Gamma$ are small
- for K and B both mixing and decays show the same suppression: $x_K, x_B \sim O(1)$

In the limit of exact SU(3) GIM completely suppresses mixing:

- for kaons the breaking of SU(3) is due to $m_c^2 \neq m_u^2$ → GIM is ineffective
- for D is due to $m_s^2 \neq m_d^2$ → GIM is very effective

$D^0 - \bar{D}^0$ mixing and oscillations

Quark level contribution

Assuming $m_c \gg \Lambda$

→ the result is an expansion in powers of m_c^{-1} ;

including QCD corrections gives also an expansion in powers of α_s

Intermediate b quark contributes only to ΔM ($m_b > M_D$),

its contribution is CKM suppressed (not negligible: may introduce a sizable phase)

→ possible intermediate states are $s\bar{s}$, $d\bar{d}$, $s\bar{d} + d\bar{s}$

so the mixing functions depend on

$$z = \frac{m_s^2}{m_c^2} \approx 0.006$$

→ M. Bobrowski et al.,
JHEP 1003 (2010) 009



The result ($O(10^{-7} - 10^{-6})$) is an interplay of three expansion parameters:
What is the “true” leading term?

Hadron level contribution

Width difference obtained from an inclusive sum over intermediate hadronic states:

Requires the knowledge of decay amplitudes and strong phases

The dispersive part obtained through a dispersion relation in terms of the absorptive one

Several results in literature in the range $10^{-4} - 10^{-3}$

However: **uncertainties too large to be conclusive**

The discrepancy of the quark level result with experiment might be due to:

- HQE does not work @ m_c scale
- violation of quark-hadron duality: non perturbative long distance effects (in this respect $\Delta\Gamma$ probes the validity of duality in charm systems)
- severe GIM cancellations in the leading terms of the OPE may not occur for higher terms operators \rightarrow calculation of D=9, D=12 terms hopeful
- NP (may induce non-unitarity of CKM) \longrightarrow See discussion in:
M. Bobrowski et al.,
JHEP 1003 (2010) 009

Instead of conclusions I

Many other interesting topics have not been touched upon.

Just one of them is:

what can we learn about light mesons from D/B decays.

1st example

Decays $B_s \rightarrow (\eta, \eta') J/\psi$ may be considered as possible modes to determine the B_s mixing phase β_s

M. Carlucci, P. Colangelo,, FDF
PRD 80 (2009) 055023

⇒ knowledge of $B_s \rightarrow \eta(\eta')$ form factors (FF) required.

If η' has a **glue content** determinations of such FF would be modified with respect to calculations based on ordinary $q\bar{q}$ content and η - η' mixing

Establishing how large could be such a modification is hard task for theory. However, the existing measurement:

$$R_{D_s} = \frac{\text{BR}(D_s \rightarrow \eta' \ell^+ \nu_\ell)}{\text{BR}(D_s \rightarrow \eta \ell^+ \nu_\ell)} = 0.35 \pm 0.12$$

can be reproduced just using the η - η' mixing angle determined by KLOE from ϕ decays (41.5°) and neglecting any term stemming from the glue content of η'

⇒ the same should hold in B_s decays

Instead of conclusions II

2nd example

$$R_{f_0/\phi}^{B_s} = \frac{\mathcal{BR}(B_s \rightarrow J/\psi f_0)}{\mathcal{BR}(B_s \rightarrow J/\psi_L \phi_L)}$$

is interesting in two respects:

- to understand if $B_s \rightarrow J/\psi f_0$ is sizable as background to $B_s \rightarrow J/\psi \phi$
- to understand if it can be used to determine β_s as well

Proposal:

$$R_{f_0/\phi}^{B_s} \simeq R_{f_0/\phi}^{D_s} = \frac{\frac{d\Gamma}{dq^2}(D_s^+ \rightarrow f_0 e^+ \nu, f_0 \rightarrow \pi^+ \pi^-)|_{q^2=0}}{\frac{d\Gamma}{dq^2}(D_s^+ \rightarrow \phi e^+ \nu, \phi \rightarrow K^+ K^-)|_{q^2=0}}$$

Stone and Zhang, PRD 79 (2009) 074024
arXiv:0909.5442

For which CLEO gives:

$$R_{f_0/\phi}^{D_s} = (0.42 \pm 0.11)$$

CLEO, PRD 80 (2009) 052009

To be compared to the calculated ratio:
(depending on the FF)

$$R_{f_0/\phi}^{B_s} = \begin{cases} 0.13 \pm 0.06 \\ 0.22 \pm 0.10 \end{cases}$$

P. Colangelo, W. Wang, FDF
PRD 81 (2010) 074001