ON FAST CHARGED PARTICLES SCATTERING ON A SYSTEM OF PERIODIC CRYSTALLINE PLANES

V.D. Omelchenko

National Science Center “Kharkiv Institute of Physics and Technology”

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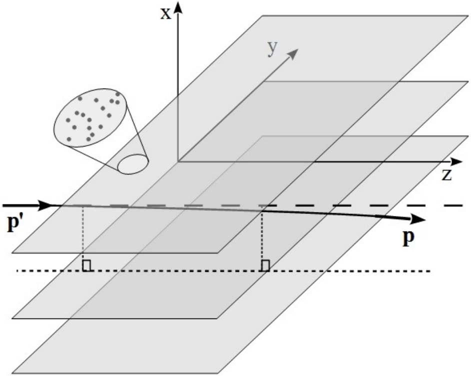
In memory of N.F. Shul'ga.
Lord rest his soul.

Scattering Problem

$$x_j = aj;$$

$$g(y_n) = \begin{cases} \frac{1}{L_y}, & -\frac{L_y}{2} \le y_n \le \frac{L_y}{2}; \\ 0, & |y_n| > \frac{L_y}{2}. \end{cases}$$



Differential Scattering Cross Section in Eikonal Approximation

$$\frac{d^2\sigma}{dq_x dq_y} = |a(\vec{q}_\perp)|^2,$$

$$a(\vec{q}_\perp) = \int_{-\infty}^{\infty} d^2\rho e^{\frac{i}{\hbar}\vec{q}\vec{\rho}} \left\{1 - exp\left[\frac{i}{\hbar}\chi_0^{(N)}(\vec{\rho})\right]\right\},$$

where $\vec{\rho} = (x, y)$, $\vec{q}_\perp = (q_x, q_y)$.

For $q_\perp \neq 0$: $\frac{d\sigma}{d^2 q_\perp} = \frac{1}{4\pi^2}\int_{\mathbb{R}^4} d^2\rho d^2\rho' e^{i\vec{q}_\perp\vec{\rho}} e^{i[\chi_0^{(N)}(\vec{\rho}) - \chi_0^{(N)}(\vec{\rho}')]}$,

$$\chi_0^{(N)} = -e\int_{-\infty}^{\infty} dz U^{(N)}(\vec{\rho}, z),$$

$$\chi_0^{(N)}(\vec{\rho}) = \sum_{n=1}^{N} \chi_0(\vec{\rho} - \vec{\rho}_n),$$

$$\chi_0(\vec{\rho} - \vec{\rho}_n) = -e\int_{-\infty}^{\infty} dz u(\vec{\rho} - \vec{\rho}_n, z).$$

Differential Scattering Cross Section in Eikonal Approximation

$$\langle e^{i[\chi_0^{(N)}(\vec{\rho})-\chi_0^{(N)}(\vec{\rho}')]}\rangle = \left(\prod_{n=1}^{N}\int_{-\infty}^{\infty} dy_n g(y_n)\right) \times$$

$$\times \prod_{j=1}^{N_x}\prod_{k=1}^{N_p} \exp\left(i[\chi_0(x-aj, y-y_n)] - \chi_0(x'-aj, y'-y_n)]\right).$$

$$\langle e^{i[\chi_0^{(N)}(\vec{\rho})-\chi_0^{(N)}(\vec{\rho}')]}\rangle = \exp\left\{N_p\left[i\sum_{j=1}^{N_x}\langle\chi_{(j)}-\chi'_{(j)}\rangle -\right.\right.$$

$$\left.\left.-\frac{1}{2}\sum_{j=1}^{N_x}\langle\left(\chi_{(j)}-\chi'_{(j)}\right)^2\rangle + \frac{1}{2}\sum_{j=1}^{N_x}\langle\chi_{(j)}-\chi'_{(j)}\rangle^2 + \ldots\right]\right\},$$

where $\chi_{(j)} = \chi_0(x-aj, y-y_0)$, $\chi'_{(j)} = \chi_0(x'-aj, y'-y_0)$, the brackets $\langle\ldots\rangle$ denote averaging.

Differential Scattering Cross Section in Eikonal Approximation

$$\frac{d\sigma}{d^2 q_\perp} = \frac{1}{4\pi^2} |\tilde{a}(\vec{q}_\perp)|^2,$$

$$\tilde{a}(\vec{q}_\perp) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy e^{i\vec{q}_\perp \vec{\rho}} \left\{ 1 - \exp\left[i \sum_{j=1}^{N_x} \langle \chi_0(\vec{\rho} - \vec{\rho}_j) \rangle \right] \right\}.$$

Differential Cross Section of Scattering on Isolated "Substructures"

$$\tilde{a}(\vec{q}_\perp) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy e^{i\vec{q}_\perp \vec{\rho}} \left\{ 1 - \exp\left[i \sum_{k=1}^{M} \chi_0(\vec{\rho} - \vec{\rho}_k) \right] \right\} .$$

$$\tilde{a}(\vec{q}_\perp) \approx \int_{-\infty}^{l_1} dx \int_{-\infty}^{s_1} dy e^{i\vec{q}_\perp \vec{\rho}} \{1 - \exp[i\chi_0(\vec{\rho} - \vec{\rho}_1)]\} +$$

$$+ \int_{l_1}^{l_2} dx \int_{s_1}^{s_2} dy e^{i\vec{q}_\perp \vec{\rho}} \{1 - \exp[i\chi_0(\vec{\rho} - \vec{\rho}_2)]\} +$$

$$+ \ldots + \int_{l_{m-1}}^{l_m} dx \int_{s_{m-1}}^{s_m} dy e^{i\vec{q}_\perp \vec{\rho}} \{1 - \exp[i\chi_0(\vec{\rho} - \vec{\rho}_m)]\} + \ldots +$$

$$+ \int_{l_{M-1}}^{\infty} dx \int_{s_{M-1}}^{\infty} dy e^{i\vec{q}_\perp \vec{\rho}} \{1 - \exp[i\chi_0(\vec{\rho} - \vec{\rho}_M)]\} .$$

Differential Cross Section of Scattering on Isolated "Substructures"

$$\tilde{a}(\vec{q}_\perp) \approx \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy e^{i\vec{q}_\perp\vec{\rho}} \left\{1 - \exp\left[i\chi_0(\vec{\rho} - \vec{\rho}_1)\right]\right\} +$$

$$+ \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy e^{i\vec{q}_\perp\vec{\rho}} \left\{1 - \exp\left[i\chi_0(\vec{\rho} - \vec{\rho}_2)\right]\right\} +$$

$$+ ... + \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy e^{i\vec{q}_\perp\vec{\rho}} \left\{1 - \exp\left[i\chi_0(\vec{\rho} - \vec{\rho}_m)\right]\right\} + ... +$$

$$+ \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy e^{i\vec{q}_\perp\vec{\rho}} \left\{1 - \exp\left[i\chi_0(\vec{\rho} - \vec{\rho}_M)\right]\right\}.$$

$$\tilde{a}_m = \exp[i(q_x x_m + q_y y_m)] \times$$

$$\times \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\eta \ \exp[i(q_x\xi + q_y\eta)] \left\{1 - e^{[i\chi_0(\xi,\eta)]}\right\}$$

$$\tilde{a}_m = \exp[i(q_x x_m + q_y y_m)]\tilde{a}^{(1)}.$$

Differential Cross Section of Scattering on Isolated "Substructures"

$$\tilde{a} = \tilde{a}^{(1)} S_M,$$

where $S_M = \sum_{k=1}^{M} e^{i\vec{q}_\perp \vec{\rho}_k}$.

$$\frac{d\sigma_{eik}^{(M)}}{d^2 q_\perp} = \frac{d\sigma_{eik}^{(1)}}{d^2 q_\perp} D_M,$$

where $D_M = \left| \sum_{k=1}^{M} e^{i\vec{q}_\perp \vec{\rho}_k} \right|^2$.

Scattering on Periodic Planes of Atoms

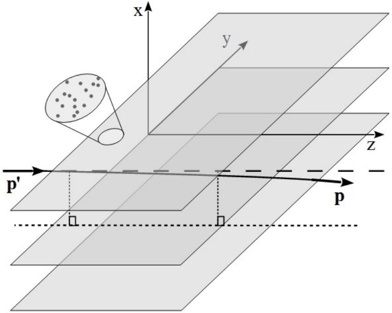
$$\{x_k\}_{k=1}^{M} = \begin{cases} \{ak\}_{k=-(M-1)/2}^{(M-1)/2}, & M = 2m+1; \\ \{a(k-1/2)\}_{k=-M/2+1}^{M/2}, & M = 2m. \end{cases}$$

$$S_M = \begin{cases} 1 + 2\sum_{k=1}^{(M-1)/2} cos[q_x ak], & M = 2m+1; \\ 2\sum_{k=1}^{M/2} cos[q_x a(k-1/2)], & M = 2m. \end{cases}$$

For screened Coulomb potential $u(\vec{r}) = \frac{Ze}{r}\exp(-\frac{r}{R})$:

$$\chi_0 = A\exp\left[-\frac{|x|}{R}\right],$$

where $A = 2Z\alpha N_p \frac{R}{L_y}$.



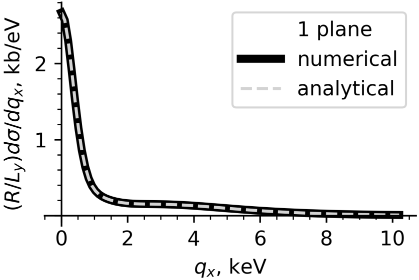
Scattering on a Single Plane of Atoms

$$\tilde{a} = -\frac{A}{q_x R} \left\{ (-iA)^{-1+iq_x R} \gamma(1 - iq_x R, -iA) - \right.$$

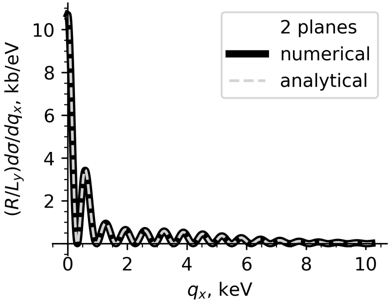
$$\left. -(-iA)^{-1-iq_x R} \gamma(1 + iq_x R, -iA) \right\},$$

where $\gamma(s, x)$ is the lower incomplete gamma function.

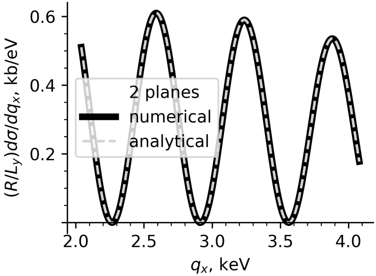
$$\tilde{a} = -2 \sum_{k=0}^{\infty} \frac{(iA)^{k+1}}{k! \left[(k+1)^2 + q_x^2 R^2 \right]}.$$



Differential Cross Section of Scattering on 2 Planes of Atoms

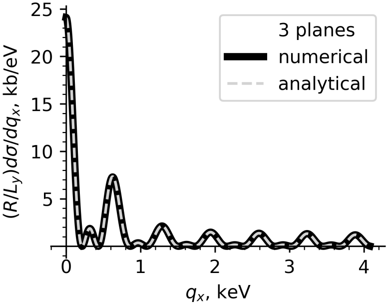


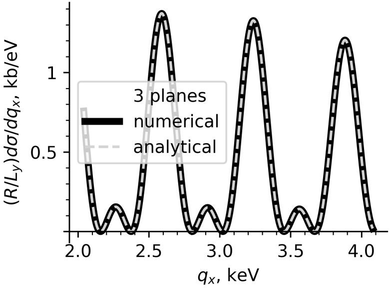
(a) $q_x \in [0, 10.2]$ keV

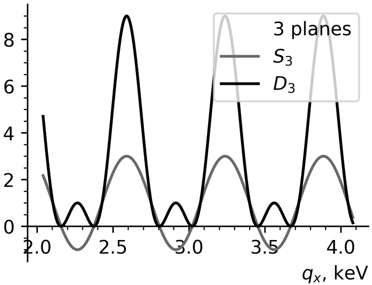


(b) $q_x \in [2.04, 4.08]$ keV

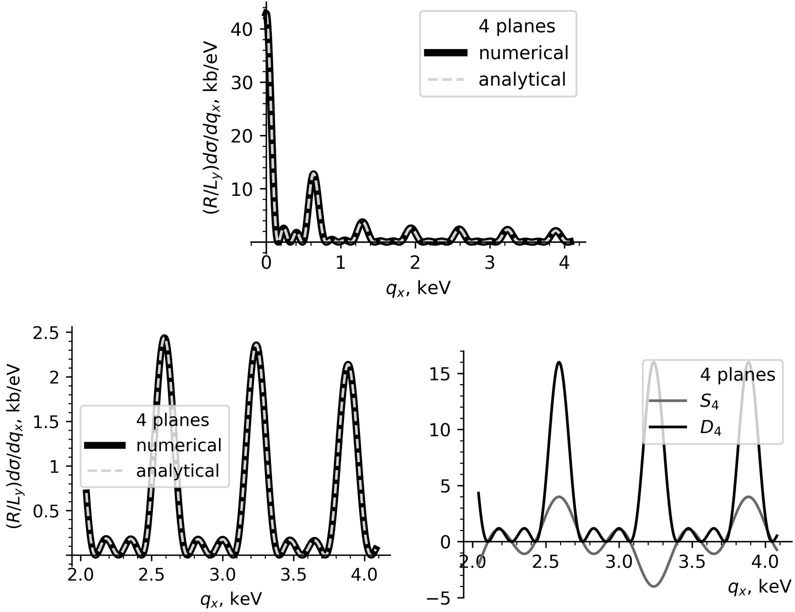
Differential Cross Section of Scattering on 3 Planes



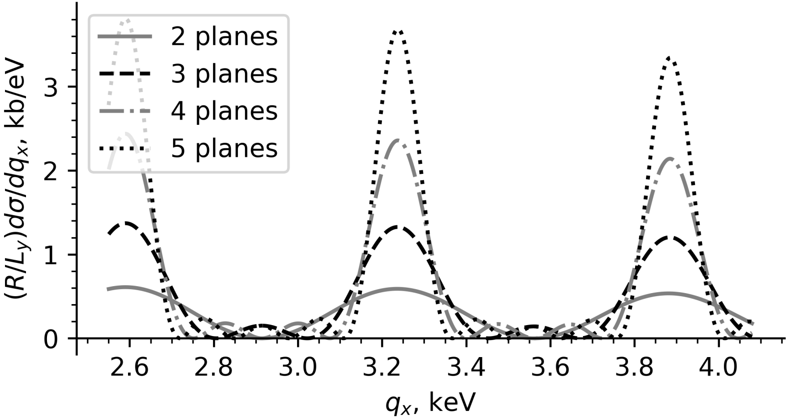




Differential Cross Section of Scattering on 4 Planes



Differential Cross Section of Scattering on 2-5 Planes of Atoms



Differential Cross Section of Scattering on Large Number of Planes of Atoms

The differential scattering cross section for $N_x \gg 1$ planes in the eikonal approximation:

$$\frac{d\sigma}{dq_x} = \frac{L_y N_x}{a} |\tilde{a}^{(1)}|^2 \sum_{j=-\infty}^{\infty} \delta\left(q_x - g(j)\right).$$

The differential scattering cross section for $N_x \gg 1$ planes in the Born approximation:

$$\langle \frac{d\sigma}{d^2 q_\perp} \rangle = N \left\{ 1 + n_\perp L_z \frac{(2\pi)^2}{a} \delta\left(\frac{q_y}{\hbar}\right) \sum_g \delta\left(\frac{q_x}{\hbar} - g\right) \right\} \langle \frac{d\sigma^{(1)}}{d^2 q_\perp} \rangle.$$

Conclusions

- We obtained the differential cross section in the Eikonal approximation for a fast charged particle scattering on sets of periodic planes of atoms numerically and analytically.
- Numerically and analytically obtained results agree well.
- Suggested analytical approach allows considering scattering on targets of complicated structure in a relatively easy way using less computing power and time comparing to the numerical approach.
- Obtained cross sections are sensitive to number of planes in the target.
- The case large number of planes in the target was also considered.

THANK YOU FOR ATTENTION

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