Black Holes in Modified Gravity

David Langlois
(APC, Paris)

Works with Christos Charmousis, Simon Iteanu, Karim Noui & Hugo Roussille

Introduction

- So far, **GR** seems compatible with all observations.
- **Modified gravity to test GR** in future observations
- **DHOST theories:** include traditional scalar-tensor theories, Horndeski, Beyond Horndeski, EsGB ...
- Here:

$$S = \int \mathrm{d}^4x\sqrt{-g}\,\Big[P(X) + F(X)R + A_1(X)\left(\phi^{\mu\nu}\phi_{\mu\nu} - (\Box\phi)^2\right)$$
$$+A_3(X)\left(\phi^\alpha_\alpha\phi^\mu\phi_{\mu\nu}\phi^\nu - \phi^\lambda\phi_{\lambda\mu}\phi^{\mu\nu}\phi_\nu\right)\Big]$$

with $\quad X \equiv -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi \qquad \phi_\mu \equiv \nabla_\mu\phi \qquad \phi_{\mu\nu} \equiv \nabla_\nu\nabla_\mu\phi$

Non-rotating black holes

- **Static** solution with a **nontrivial** scalar field:
  - Metric: $$\mathrm{d}s^2 = -\mathcal{A}(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathcal{B}(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$
  - Scalar field: $$\phi(t,r) = q\,t + \psi(r)$$ [Babichev & Charmousis '13]

- **BHs with primary hair** [Bakopoulos et al. '23; Baake et al. 23']

$$P(X) = -\frac{2\alpha}{\lambda^2}X^p\,,\;\; F(X) = 1 - 2XA_1\,,\;\; A_1(X) = \frac{\alpha}{2}X^{p-1}\,,\;\; A_3(X) = \frac{\alpha}{2}(2p-1)X^{p-2}$$

$$\mathcal{A}(r) = \mathcal{B}(r) = 1 - \frac{2\mu}{r} - \frac{2\xi_p}{r}\int_0^r \frac{u^2\mathrm{d}u}{(1+u^2)^p} \qquad X = \frac{q^2}{2(1+r^2)}$$

$$\xi_p \equiv \alpha(2p-1)\left(\frac{q^2}{2}\right)^p$$

Black Holes with primary hair

- $p = 1/2$: « stealth » Schwarzschild (Horndeski theory)

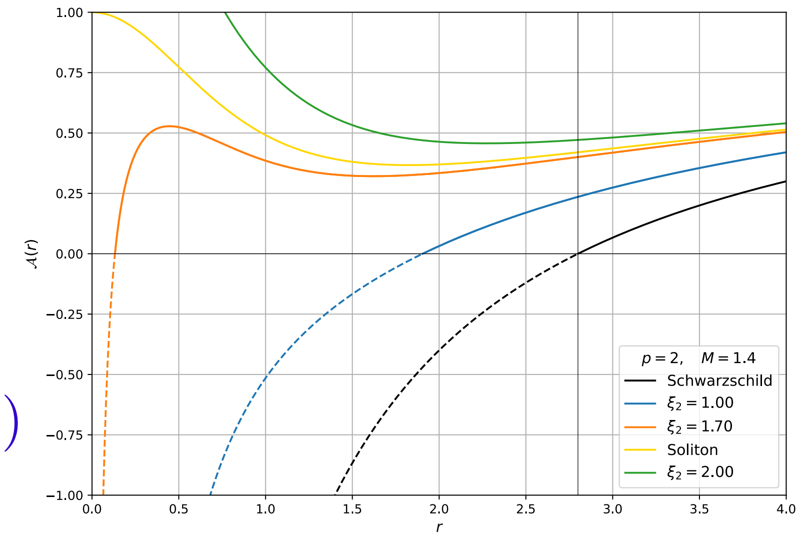
- $p = 2$: deformation of Schwarzschild

$$\mathcal{A}(r) = 1 - \frac{2M}{r} + \xi_2 \left(\frac{\pi/2 - \arctan r}{r} + \frac{1}{1+r^2} \right)$$

  - **regular solution** if $M = \frac{\pi}{4}\xi_2$

  - regular BH (otherwise soliton) if

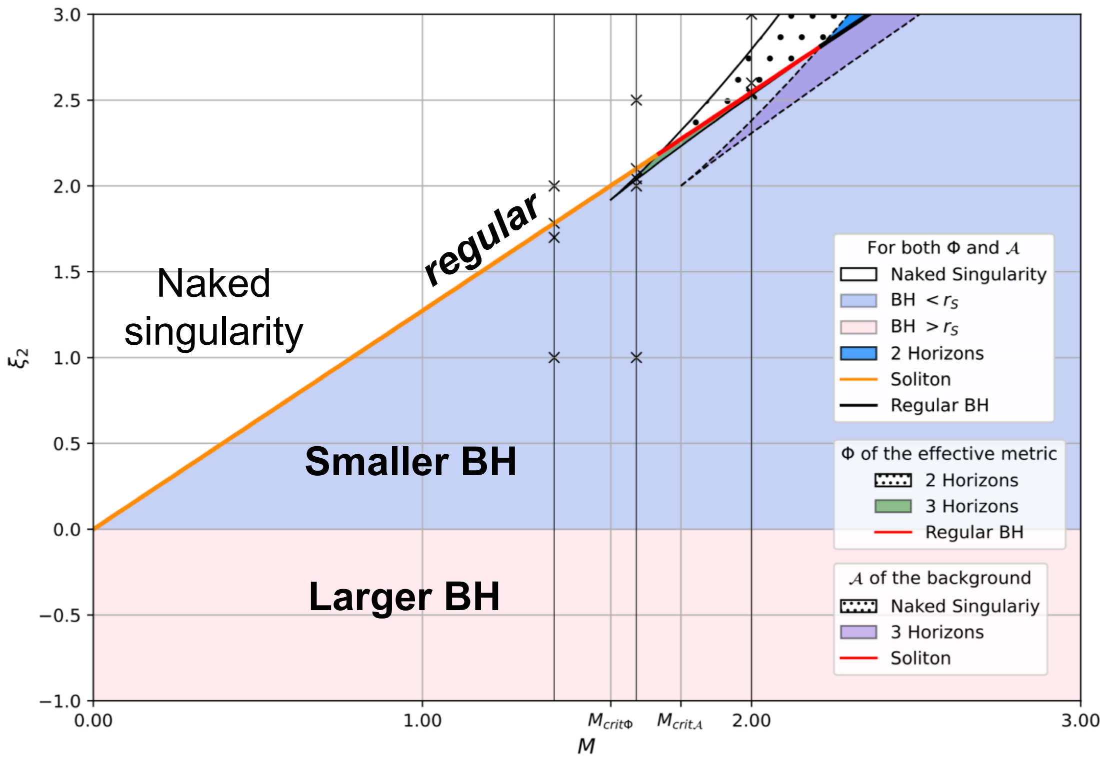
$$M > M_{\rm BH}^{\rm reg} \simeq 2.21 \quad (\text{i.e. } \xi_2 > \xi_{2,\rm BH}^{\rm reg} \simeq 2.82)$$

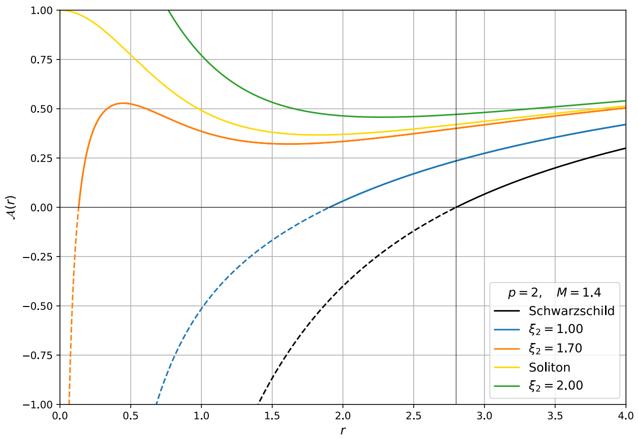


Black Holes with primary hair

Phase diagram

[Charmousis, Iteanu, DL & Noui 25']





New BHs via disformal transformations

- Transformation $g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X,\phi)\, g_{\mu\nu} + D(X,\phi)\, \partial_\mu\phi\, \partial_\nu\phi$

- From an action $\tilde{S}\left[\phi, \tilde{g}_{\mu\nu}\right]$, one gets the new action

$$S[\phi, g_{\mu\nu}] \equiv \tilde{S}\left[\phi, \tilde{g}_{\mu\nu} = C\, g_{\mu\nu} + D\, \phi_\mu \phi_\nu\right]$$

- Disformally related theories are **physically inequivalent,** assuming matter (& light) **minimally coupled** to the metric.

$$S[g_{\mu\nu}, \phi] + S_m[\Psi_m, g_{\mu\nu}] \quad \neq \quad \tilde{S}[\tilde{g}_{\mu\nu}, \phi] + S_m[\Psi_m, \tilde{g}_{\mu\nu}]$$

- New BH solutions

Metric 1 in theory 1 ⟷ ($C = 1\,,\ D(X)$) ⟷ **Metric 2 in theory 2**

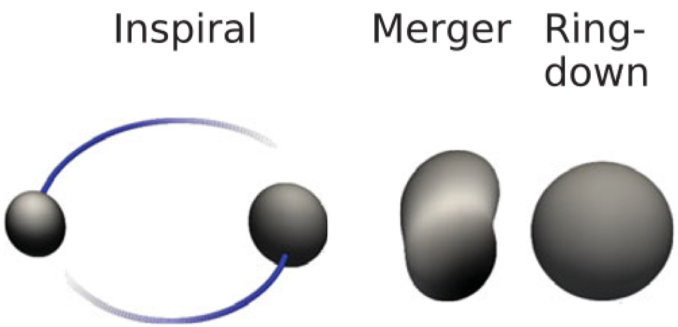
$$\mathrm{d}s^2 = -\mathcal{A}(r)\, \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathcal{A}(r)} + r^2\, \mathrm{d}\Omega^2$$

$$\mathrm{d}\tilde{s}^2 = -\tilde{\mathcal{A}}(r)\, \mathrm{d}\tilde{t}^{\,2} + \frac{\mathrm{d}r^2}{\tilde{\mathcal{B}}(r)} + r^2\, \mathrm{d}\Omega^2$$

Black hole perturbations

Black hole perturbations

- **GW astronomy** provides a new window to **test GR,** in particular in the strong field regime.



- **Ringdown phase** of a BH merger can be described by BH perturbations.

- **Deviations** in the context of **DHOST** theories ?

Black hole perturbations

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{bgd}} + h_{\mu\nu}$$

- In the frequency domain: $f(t,r) = f(r)\,e^{-i\omega t}$

- **Axial** (or odd) modes: $h_0(r), h_1(r)$ [Regge-Wheeler gauge]

$$h_{\mu\nu} = \sum_{\ell,m} \begin{pmatrix} 0 & 0 & \frac{1}{\sin\theta} h_0^{\ell m}\partial_\varphi & -\sin\theta h_0^{\ell m}\partial_\theta \\ 0 & 0 & \frac{1}{\sin\theta} h_1^{\ell m}\partial_\varphi & -\sin\theta h_1^{\ell m}\partial_\theta \\ \mathrm{sym} & \mathrm{sym} & 0 & 0 \\ \mathrm{sym} & \mathrm{sym} & 0 & 0 \end{pmatrix} Y_{\ell m}(\theta,\varphi)$$

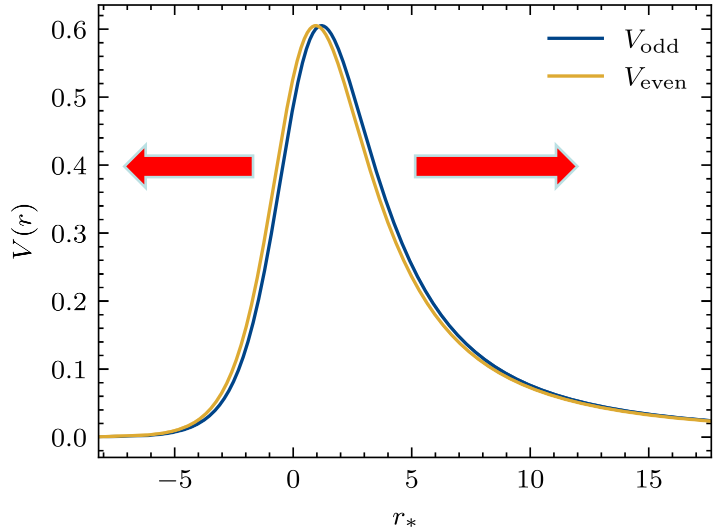
- **Polar** (or even) modes: H_0, H_1, H_2, K (and $\delta\phi$)

$$h_{\mu\nu} = \sum_{\ell,m} \begin{pmatrix} A(r)H_0^{\ell m}(r) & H_1^{\ell m}(r) & 0 & 0 \\ H_1^{\ell m}(r) & A^{-1}(r)H_2^{\ell m}(r) & 0 & 0 \\ 0 & 0 & K^{\ell m}(r)\,r^2 & 0 \\ 0 & 0 & 0 & K^{\ell m}(r)\,r^2\sin^2\theta \end{pmatrix} Y_{\ell m}(\theta,\varphi)$$

Axial modes in GR

- Linearised Einstein eqs yield 2 independent equations

$$\frac{dY}{dr} = M(r)\,Y(r)\,, \quad Y = \begin{pmatrix} h_0 \\ h_1/\omega \end{pmatrix}$$



or, in a **Schrödinger** form, [Regge & Wheeler '57]

$$\frac{d^2\hat{Y}}{dr_*^2} + \left(\omega^2 - V(r)\right)\hat{Y} = 0$$

[r_* tortoise coord: $\mathrm{d}s^2 = \mathcal{A}(r)\left(-\mathrm{d}t^2 + \mathrm{d}r_*^2\right)$]

- **Asymptotically** ($r_* \to -\infty, +\infty$)

$$e^{-i\omega t}\,\hat{Y}(r) \approx a_+\,e^{-i\omega(t-r_*)} + a_-\,e^{-i\omega(t+r_*)}$$

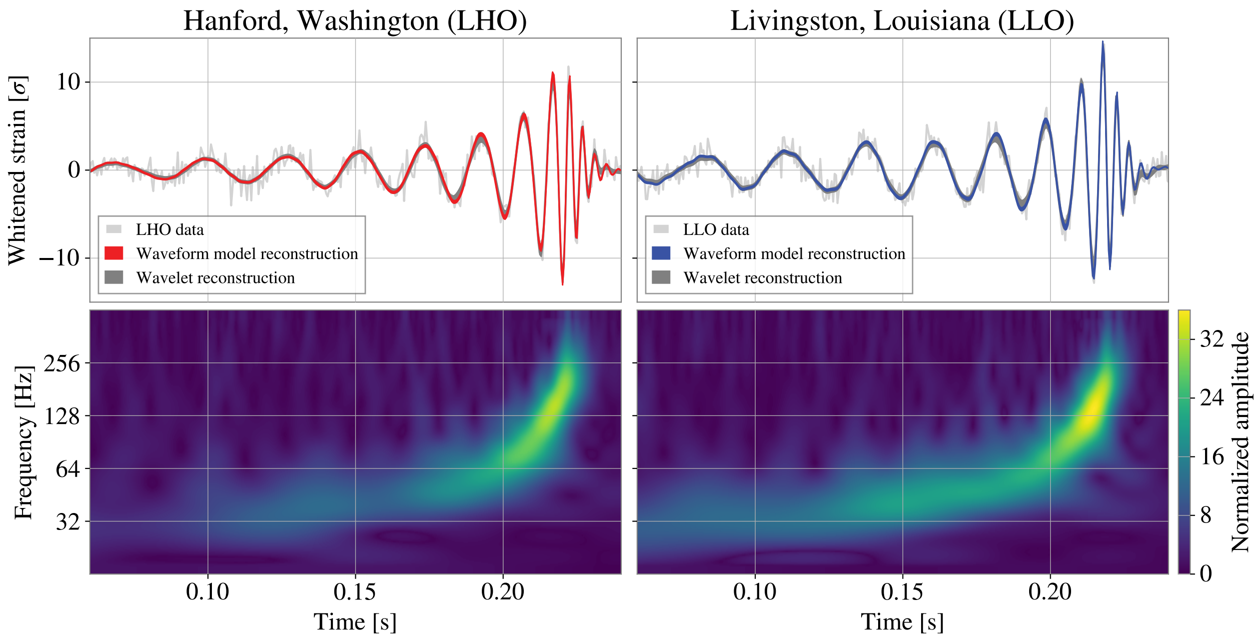
outgoing ingoing

- **Quasi-normal modes**: $a_+^{\rm hor} = 0 \quad \text{and} \quad a_-^{\infty} = 0$

GW250114

- **GW250114** observed by the two LIGO detectors (SNR=80)

[arXiv:2509.08054]

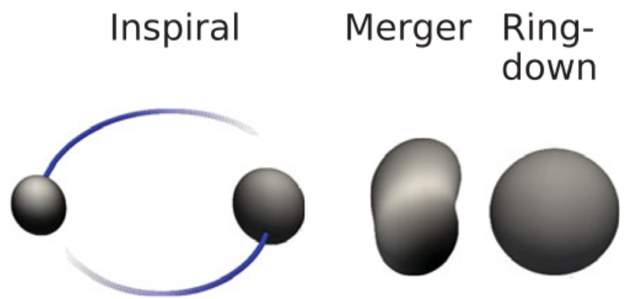


- Two initial black holes:

$$m_1 \simeq 34M_\odot\,, \quad m_2 \simeq 32M_\odot$$

- Final black hole:

$$M_f \simeq 63M_\odot\,, \quad \chi_f \simeq 0.68$$



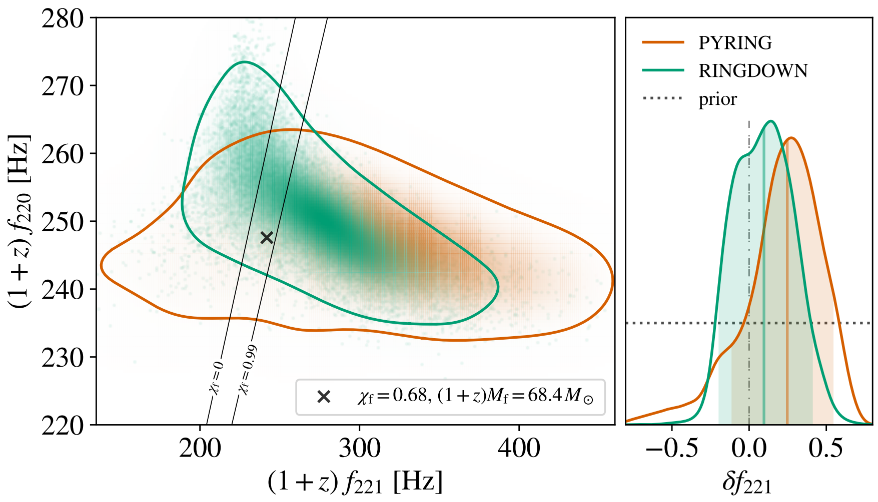
GW250114: Kerr BH

- **Kerr nature of the final black hole ?**
  GW signal: Kerr BH ringing

$$h \sim \sum_{\ell,m,n} A_{\ell mn} e^{-i\omega_{\ell mn} t}$$

$$\omega_{\ell mn}(M_f, \chi_f) = 2\pi f_{\ell mn}(M_f, \chi_f) - i\gamma_{\ell mn}(M_f, \chi_f)$$

- Analysis of the ringdown signal with modes (220) & (221)



arXiv:2509.08054

Axial modes in DHOST

[DL, Noui & Roussille '22]

- The equations have a similar structure

$$\frac{dY}{dr} = MY \,, \qquad M \equiv \begin{pmatrix} 2/r + i\omega\Psi & -i\omega^2 + 2i\lambda\Phi/r^2 \\ -i\Gamma & \Delta + i\omega\Psi \end{pmatrix}$$

$$\lambda \equiv \frac{\ell(\ell+1)}{2} - 1$$

where Ψ, Φ, Γ and Δ depend on the Lagrangian's functions and on the background.

- **Correspondence (in quadratic DHOST theories)**

DHOST axial modes in $g_{\mu\nu}$ ⟷ **GR** axial modes in $\hat{g}_{\mu\nu}$

with the **effective metric**

$$d\hat{s}^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = |\mathcal{F}|\sqrt{\frac{\Gamma\mathcal{B}}{\mathcal{A}}}\left(-\Phi\left(dt - \Psi dr\right)^2 + \Gamma\Phi dr^2 + r^2 d\Omega^2\right)$$

Effective metric for axial modes

[Charmousis, Iteanu, DL & Noui 25']

- For **BHs with primary hair**, the effective metric reduces to

$$d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = \sqrt{F}\left(-\Phi\, dt_*^2 + \frac{F}{\Phi} dr^2 + r^2 d\Omega^2\right)$$

with $\Phi = \mathcal{A} - q^2 A_1$

- In **quadratic** DHOST theories, the **effective metric** corresponds to a disformal transformation such that $\hat{F} = 1$ and $\hat{A}_1 = 0$.

- Photons & gravitons **«see» different geometries**: $g_{\mu\nu}$ and $\hat{g}_{\mu\nu}$, and thus different horizons (for BHs), determined by

$$\mathcal{A}(r_\ell) = 0\,, \qquad \Phi(r_g) = 0$$

$$\text{For } \xi_2 > 0: \quad r_\ell < r_g < r_S$$

Axial quasi-normal modes

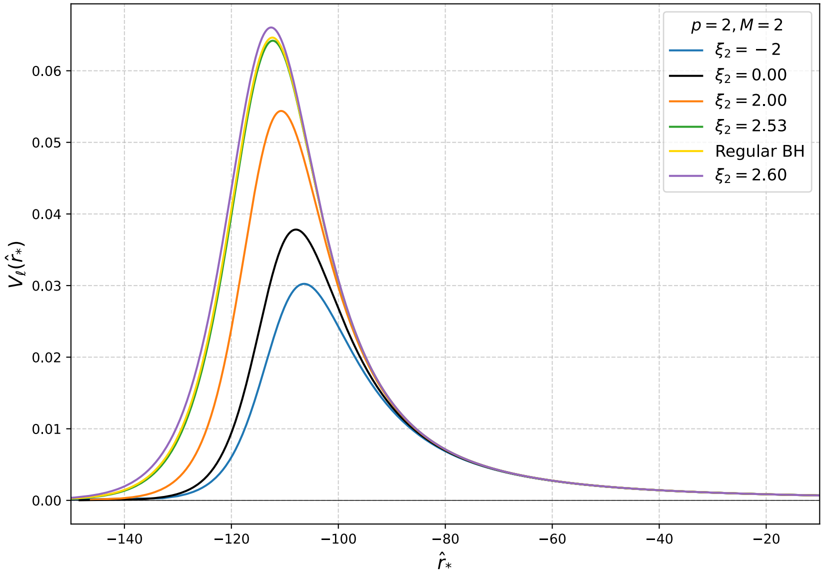
- As in GR, one can write a **Schrödinger-like** equation

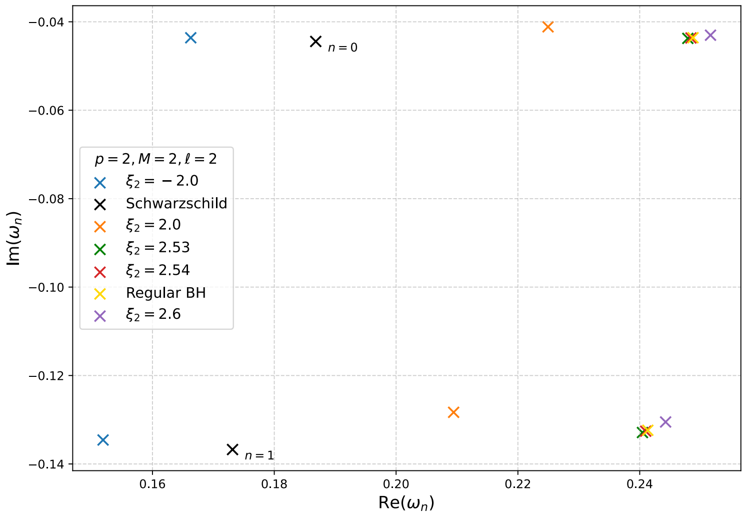
$$-\frac{d^2\mathcal{Y}}{d\hat{r}_*^2} + V_\ell\,\mathcal{Y} \;=\; \omega^2\,\mathcal{Y}$$

$$\mathcal{Y} = \frac{\Phi}{rF^{1/4}\omega}(h_1 + \Psi h_0)$$

where

$$V_\ell(r) = \Phi\left[\frac{\ell^2+\ell-2}{r^2} - \frac{1}{r}\kappa_1(F,F',r)\Phi' + \frac{2}{r^2}\kappa_2(F,F',F'',r)\Phi\right]$$





Radial perturbations

Radial perturbations

[Charmousis, Iteanu, DL & Noui, to appear]

- In **GR,** no radial perturbation for Schwarzschild (Birkhoff's theorem)

- In **DHOST** theories, radial perturbations no longer vanish

- **One dof**: mixture of scalar and metric perturbations

- By introducing **new time and radial** coordinates, one finds it obeys the wave equation $\left\{ \begin{array}{l} t_* = \phi/q \\ r_* = r_*(r) \end{array} \right.$

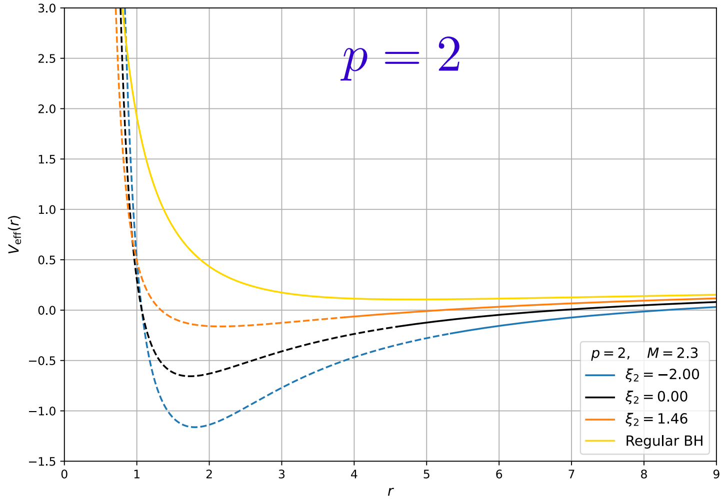
$$\frac{\partial^2 \chi}{\partial t_*^2} - \frac{\partial^2 \chi}{\partial r_*^2} + V\,\chi \ = 0$$

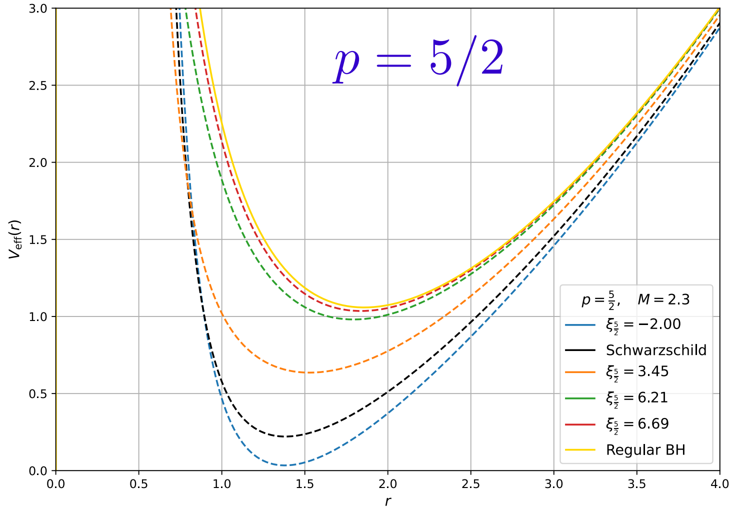
Radial perturbations

[Charmousis, Iteanu, DL & Noui, to appear]

- In the frequency domain, this yields a **Schrödinger-like** equation

$$-\frac{\mathrm{d}^2\chi}{\mathrm{d}r_*^2} + V\chi = \omega^2\chi$$





- **Stability** (positive self-adjoint operator)

Conclusion

- **DHOST theories:** most general framework for scalar-tensor theories propagating a single scalar dof.

- Interesting **subfamily** of DHOST theories allowing for **exact static BHs** solutions with **primary hair**:
  - Axial modes: effective metric & QNMs
  - Radial perturbations
  - Non-radial polar modes...

- More realistic BHs: rotating solutions...

- Neutron stars...